A FILL IN THE BLANKS

1. The larger of $\cos(\ln\theta)$ and $\ln(\cos\theta)$ if $e^{-\pi/2} < \theta < \frac{\pi}{2}$ is ... (IIT 1983; 1M)

2. The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real, is... (IIT 1986; 2M)

3. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \ge 0$, is... (IIT 1987; 2M)

4. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is...... (IIT 1996; 1M)

B TRUE / FALSE

1. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. (IIT 1984; 1M)

C OBJECTIVE QUESTION

➨ ***Only one option is correct :***

1. The equation
$$2\cos^2\left(\frac{1}{2}x\right)\sin^2 x = x^2 + x^{-2},\ x \le \frac{\pi}{9}$$ has : (IIT 1980)
(a) no real solution
(b) one real solution
(c) more than one real solution

2. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by : (IIT 1981; 2M)
(a) $x = 2n\pi;\ n = 0, \pm 1, \pm 2, \ldots\ldots$
(b) $x = 2n\pi + \pi/2;\ n = 0, \pm 1, \pm 2 \ldots.$
(c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4};\ n = 0, \pm 1, \pm 2 \ldots.$
(d) none of these

3. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0$ for all x is : (IIT 1987; 2M)
(a) zero (b) one
(c) three (d) infinite
(e) none of these .

4. The smallest positive root of the equation, $\tan x - x = 0$ lies in : (IIT 1987; 2M)
(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \pi\right)$
(c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(\frac{3\pi}{2}, 2\pi\right)$
(e) none of these

5. The general solution of
$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x - \cos 3x$ is : (IIT 1989; 2M)
(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$
(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\frac{3}{2}$

6. The equation
$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$$
in the variable x, has real roots. Then p can take any value in the interval : (IIT 1990; 2M)
(a) $(0, 2\pi)$ (b) $(-\pi, 0)$
(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

7. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation
$$3\sin x - 4\sin^3 x - k = 0,\ 0 < k < 1,$$
then the measure of angle C is : (IIT 1990)
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

8. The number of solutions of the equation
$$\sin(e)^x = 5^x + 5^{-x}$$ is : (IIT 1991)
(a) 0 (b) 1
(c) 2 (d) infinitely many

9. Number of solutions of the equation
$$\tan x + \sec x = 2\cos x$$
lying in the interval $[0, 2\pi]$ is : (IIT 1993; 1M)
(a) 0 (b) 1
(c) 2 (d) 3

10. The general value of θ satisfying the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$, is : (IIT 1995)

(a) $n\pi + (-1)^n \frac{\pi}{6}$ (b) $n\pi + (-1)^n \frac{\pi}{2}$

(c) $n\pi - (-1)^n \frac{5\pi}{6}$ (d) $n\pi + (-1)^n \frac{7\pi}{6}$

11. The graph of the function $\cos x \cos(x+2) - \cos^2(x+1)$ is : (IIT 1997)

(a) a straight line passing through $(0, -\sin^2 1)$ with slope 2

(b) a straight line passing through (0, 0)

(c) a parabola with vertex $(1, -\sin^2 1)$

(d) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the x-axis.

12. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is : (IIT 1998; 2M)

(a) 0 (b) 5 (c) 6 (d) 10

13. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then : (IIT 1998; 2M)

(a) $b_0 = 1, b_1 = 3$ (b) $b_0 = 0, b_1 = n$

(c) $b_0 = -1, b_1 = n$ (d) $b_0 = 0, b_1 = n^2 - 3n + 3$

14. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k - 1$ has a solution, is : (IIT 2002)

(a) 4 (b) 8

(c) 10 (d) 12

15. Let $0 < \alpha < \frac{\pi}{2}$ be a fixed angle. If $P = (\cos\theta, \sin\theta)$ and $Q = (\cos(\alpha - \theta), \sin(\alpha - \theta))$, then Q is obtained from P by (IIT 2002)

(a) clockwise rotation around origin through an angle α

(b) anticlockwise rotation around origin through an angle α

(c) reflection in the line through origin with slope $\tan\alpha$

(d) reflection in the line through origin with slope $\tan\frac{\alpha}{2}$

16. The set of values of θ satisfying the inequation $2\sin^2\theta - 5\sin\theta + 2 > 0$, where $0 < \theta < 2\pi$ is : (IIT 2006)

(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$

(c) $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, 2\pi\right]$ (d) none of these

17. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is : (IIT 2007)

(a) zero (b) one

(c) two (d) four

D OBJECTIVE QUESTIONS

➡ ***More than one options are correct :***

1. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0, \text{ are :}$$ (IIT 1988)

(a) $7\pi/24$ (b) $5\pi/24$

(c) $11\pi/24$ (d) $\pi/24$

2. For $0 < \phi < \pi/2$, if

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi,\ y = \sum_{n=0}^{\infty} \sin^{2n}\phi,\ z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi,$$ then : (IIT 1993; 2M)

(a) $xyz = xz + y$ (b) $xyz = xy + z$

(c) $xyz = x + y + z$ (d) $xyz = yz + x$

3. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if : (IIT 1996; 1M)

(a) $x + y \neq 0$ (b) $x = y, x \neq 0$

(c) $x = y$ (d) $x \neq 0, y \neq 0$

E SUBJECTIVE QUESTIONS

1. For all θ in $[0, \pi/2]$, show that $\cos(\sin\theta) \geq \sin(\cos\theta)$. (IIT 1981; 4M)

2. Find the coordinates of the points of intersection of the curves $y = \cos x$, $y = \sin 3x$, if $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. (IIT 1982; 3M)

3. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution. (IIT 1982; 2M)

4. Find all the solutions of : (IIT 1983; 2M)

$$4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$$

5. Find the values of $x(-\pi, +\pi)$ which satisfy the equation $2^{1+|\cos x|+|\cos^2 x|+\ldots\ldots} = 4$ (IIT 1984; 2M)

6. Consider the system of linear equations in x, y, z :

$$(\sin 3\theta)x - y + z = 0$$
$$(\cos 2\theta)x + 4y + 3z = 0$$
$$2x + 7y + 7z = 0$$

Find the values of θ for which this system has non-trivial solutions. (IIT 1986)

7. If $\exp\{(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty)\ln 2\}$ satisfies the equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$. (IIT 1991; 4M)

8. Show that the value of $\tan x / \tan 3x$, wherever defined never lies between 1 / 3 and 3. **(IIT 1992; 4M)**

9. Determine the smallest positive value of x (in degrees) for which

$$\tan(x+100°) = \tan(x+50°)\tan(x)\tan(x-50°)$$

(IIT 1993; 5M)

10. Find the smallest positive number p for which the equation

$$\cos(p\sin x) = \sin(p\cos x)$$ has a solution $x \in [0, 2\pi]$

(IIT 1995; 5M)

11. Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation

$$(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$$

(IIT 1996; 2M)

ANSWERS

A Fill in the Blanks

1. $\cos(\log\theta) > \log(\cos\theta)$ 2. no solution 3. $x \in \left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, 2\pi\right] \cup \left\{\frac{\pi}{2}\right\}$ 4. $\theta = m\pi, m\pi \pm \frac{\pi}{3}$

B True / False

1. True

C Objective Questions (Only one option)

1. (a)	2. (c)	3. (d)	4. (c)	5. (b)	6. (d)	7. (c)
8. (a)	9. (c)	10. (d)	11. (d)	12. (c)	13. (b)	14. (b)
15. (d)	16. (a)	17. (c)				

D Objective Questions (More than one option)

1. (a, c) 2. (b, c) 3. (a, b)

E Subjective Questions

2. $\left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)\left(\frac{\pi}{4}, \cos\frac{\pi}{4}\right)\left(-\frac{3\pi}{8}, \cos\frac{3\pi}{8}\right)$ 4. $\{x : x = n\pi\} \cup \left\{x : x = n\pi + (-1)^n\frac{\pi}{10}\right\} \cup \left\{x : x = n\pi + (-1)^n\left(-\frac{3\pi}{10}\right)\right\}$

5. $\left\{\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}\right\}$ 6. $\theta = n\pi$ or $n\pi + (-1)^n\left(\frac{\pi}{6}\right)$ 7. $\frac{\sqrt{3}-1}{2}$ 9. 30°

10. $\frac{\pi}{2\sqrt{2}}$ 11. $\theta = \pm\frac{\pi}{3}$

SOLUTIONS

A FILL IN THE BLANKS

1. As, $\cos\theta \le 1 \Rightarrow \log(\cos\theta) < 0$

and $\cos(\log\theta) > 0$

$\therefore \cos(\log\theta) > \log(\cos\theta)$

2. $$x + y = \frac{2\pi}{3}$$

and $$\cos x + \cos y = \frac{3}{2}$$

$$\Rightarrow \cos x + \cos\left(\frac{2\pi}{3} - x\right) = \frac{3}{2}$$

$$\Rightarrow \cos x + \left(-\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right) = \frac{3}{2}$$

$$\Rightarrow \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{3}{2}$$

$$\Rightarrow \sin\left(\frac{\pi}{6} + x\right) = \frac{3}{2}$$, which is never possible

Thus, no solution

3. $$2\sin^2 x - 3\sin x + 1 \ge 0$$

$$\Rightarrow 2\sin^2 x - 2\sin x - \sin x + 1 \ge 0$$

$$\Rightarrow (2\sin x - 1)(\sin x - 1) \ge 0$$

$$\Rightarrow 2\sin x - 1 \le 0$$

$$\Rightarrow \sin x \le \frac{1}{2} \quad \text{or} \quad \sin x = 1$$

$$\Rightarrow x \in \left[0, \frac{\pi}{3}\right] \cup \left[2\frac{\pi}{3}, \pi\right] \cup [\pi, 2\pi] \cup \left\{\frac{\pi}{2}\right\}$$

or $$x \in \left[0, \frac{\pi}{3}\right] \cup \left[2\frac{\pi}{3}, 2\pi\right] \cup \left\{\frac{\pi}{2}\right\}$$

4. $$\tan^2\theta + \sec 2\theta = 1 \quad \text{(given)}$$

$$\Rightarrow \tan^2\theta + \frac{1}{\cos 2\theta} = 1$$

$$\Rightarrow \quad \tan^2\theta + \frac{1+\tan^2\theta}{1-\tan^2\theta} = 1$$

$$\Rightarrow \quad \tan^2\theta(1-\tan^2\theta) + (1+\tan^2\theta) = 1-\tan^2\theta$$

$$\Rightarrow \quad 3\tan^2\theta - \tan^4\theta = 0$$

$$\Rightarrow \quad \tan^2\theta\,(3-\tan^2\theta) = 0$$

B TRUE / FALSE

1. Here, $\sin^4\theta - 2\sin^2\theta + 1 = 2$

or $(\sin^2\theta - 1)^2 = 2$

or $\sin^2\theta = \pm\sqrt{2} + 1$

C OBJECTIVE (ONLY ONE OPTION)

1. As $\text{L.H.S} = 2\cos^2\left(\frac{x}{2}\right)\sin^2 x < 2$

and $\text{R.H.S} = x^2 + \frac{1}{x^2} \geq 2$

$\therefore$ The equation has no solution.

2. $$\sin x - \cos x = 1$$

Dividing and multiplying by $\sqrt{2}$,

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\left(x + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$$

or $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$, $n \in$ integer

3. We have

$$a_1 + a_2\cos 2x + a_3\sin^2 x = 0, \text{ for all } x$$

$$\Rightarrow a_1 + a_2\cos 2x + a_3\left(\frac{1-\cos 2x}{2}\right) = 0 \text{ for all } x$$

$$\Rightarrow \left(a_1 + \frac{a_3}{2}\right) + \left(a_2 - \frac{a_3}{2}\right)\cos 2x = 0, \text{ for all } x$$

$$\Rightarrow \quad a_1 + \frac{1}{2}a_3 = 0 \text{ and } a_2 - \frac{a_3}{2} = 0$$

$$\Rightarrow \quad a_1 = -\frac{k}{2},\ a_2 = \frac{k}{2},\ a_3 = k, \text{ where } k \in R$$

Hence the solutions are $\left(-\frac{k}{2}, \frac{k}{2}, k\right)$, where k is any real number.

Thus the number of triplets is infinite.

4. Let $f(x) = \tan x - x$

We know for $0 < x < \frac{\pi}{2}$

$$\Rightarrow \quad \tan x > x$$

$$\Rightarrow \quad \tan\theta = 0 \quad \text{or} \quad \tan\theta = \pm\sqrt{3}$$

Now, $\tan\theta = 0$, $\theta = m\pi$ where m is an integer.

and $\tan\theta = \pm\sqrt{3} = \tan(\pm\pi/3)$

$$\theta = n\pi \pm \frac{\pi}{3}$$

$\Rightarrow \theta = m\pi,\ n\pi \pm \pi/3$ where m and n are integers.

where, $\sin^2\theta = 1 - \sqrt{2}$, neglecting $\sqrt{2} - 1$

$\therefore \quad \sin\theta = \pm\sqrt{1-\sqrt{2}}$ are possible solutions.

Hence, (True).

$\therefore$ $f(x) = \tan x - x$ has no root in $(0, \pi/2)$ for $\pi/2 < x < \pi$, $\tan x$ is negative.

$$\therefore \quad f(x) = \tan x - x < 0$$

so $f(x) = 0$ has no root in $\left(\frac{\pi}{2}, \pi\right)$

For $\frac{3\pi}{2} < x < 2\pi$, $\tan x$ is negative

$$\therefore \quad f(x) = \tan x - x < 0$$

So, $f(x) = 0$ has no root in $\left(\frac{3\pi}{2}, 2\pi\right)$

We have $f(\pi) = 0 - \pi < 0$

and $f\left(\frac{3\pi}{2}\right) = \tan\frac{3\pi}{2} - \frac{3\pi}{2} > 0$

$\therefore$ $f(x) = 0$ has at least one root between π and $\frac{3\pi}{2}$

5. $\sin 3x + \sin x - 3\sin 2x = \cos 3x + \cos x - 3\cos 2x$

$\Rightarrow 2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

$\Rightarrow \quad \sin 2x\,(2\cos x - 3) = \cos 2x\,(2\cos x - 3)$ $\quad \{\because 2\cos x - 3 \neq 0\}$

$$\Rightarrow \quad \sin 2x = \cos 2x$$

$$\Rightarrow \quad \tan 2x = 1 \quad \Rightarrow \quad 2x = n\pi + \frac{\pi}{4}$$

$$\Rightarrow \quad x = \frac{n\pi}{2} + \frac{\pi}{8}$$

6. For the quadratic equation to have real roots we must have

$$\cos^2 p - 4\sin p\,(\cos p - 1) \geq 0$$

$$\Rightarrow \quad (\cos p - 2\sin p)^2 - 4\sin^2 p + 4\sin p \geq 0$$

$$\Rightarrow \quad (\cos p - 2\sin p)^2 + 4\sin p\,(1 - \sin p) \geq 0$$

$\because$ $4\sin p\,(1 - \sin p) > 0$ for $0 < p < \pi$

and $(\cos p - 2\sin p)^2 \geq 0$

Thus, $(\cos p - 2\sin p)^2 + 4\sin p\,(1 - \sin p) \geq 0$ for $0 < p < \pi$

Hence, the equation has real roots for $0 < p < \pi$

7. We have $3\sin x - 4\sin^3 x = k$, $0 < k < 1$ which can also be written as $\sin 3x = k$.

It is given that A and B are solutions of this equation. Therefore

$\sin 3A = k$ and $\sin 3B = k$, where $0 < k < 1$

$\Rightarrow \quad 0 < 3A < \pi$ and $0 < 3B < \pi$

Now, $\sin 3A = k$ and $\sin 3B = k$

$\Rightarrow \quad \sin 3A - \sin 3B = 0$

$\Rightarrow \quad 2\cos\frac{3}{2}(A+B)\sin\frac{3}{2}(A-B) = 0$

$\Rightarrow \quad \cos 3\left(\frac{A+B}{2}\right) = 0,\ \sin\frac{3}{2}(A-B) = 0$

But it is given that $A > B$ and $0 < 3A < \pi$, $0 < 3B < \pi$

Therefore $\sin\frac{3}{2}(A-B) \neq 0$.

Hence, $\cos 3\left(\frac{A+B}{2}\right) = 0 \Rightarrow \frac{3}{2}(A+B) = \frac{\pi}{2}$

$\Rightarrow \quad A + B = \frac{\pi}{3}$

$\Rightarrow \quad C = \pi - (A+B) = \frac{2\pi}{3}$

8. Here, $L.H.S = \sin(e^x) < 1$ for all $x \in R$.

and $\quad R.H.S = 5^x + 5^{-x} \geq 2$

$\therefore \quad \sin(e^x) = 5^x + 5^{-x}$ has no solution

9. $\tan x - \sec x = 2\cos x$, $x \notin (2n+1)\frac{\pi}{2}$

$\Rightarrow \quad \sin x - 1 = 2\cos^2 x$

$\Rightarrow \quad \sin x + 1 = 2(1 - \sin^2 x)$

$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow \quad (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \quad \sin x = \frac{1}{2}, \sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $x = \frac{3\pi}{2}$ but $x \notin (2n-1)\frac{\pi}{2}$

$\therefore \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, number of solutions are two.

10. $2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow \quad (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \quad \sin\theta = -1/2$

{neglecting $\sin\theta = 2$, as $|\sin\theta| \leq 1$}

$\therefore \quad \theta = n\pi + (-1)^n(-\pi/6)$

$\Rightarrow \quad \theta = n\pi + (-1)^n(7\pi/6)$

11. Let $y = \cos x\cos(x+2) - \cos^2(x+1)$

$= \cos(x+1-1)\cos(x+1+1) - \cos^2(x+1)$

$= \cos^2(x+1) - \sin^2 1 - \cos^2(x+1)$

$y = -\sin^2 1$. This is a straight line which is parallel to x-axis.

It passes through $(\pi/2, -\sin^2 1)$.

Therefore, (d) is the answer.

12. $3\sin^2 x - 7\sin x + 2 = 0$ (given)

$\Rightarrow \quad 3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow \quad 3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$

$\Rightarrow \quad (3\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \quad \sin x = 1/3$ or $\sin x = 2$ ($\sin x = 2$ is rejected)

$\Rightarrow \quad x = n\pi + (-1)^n \sin^{-1}\frac{1}{3}, n \in I$

For $\quad 0 \leq n \leq 5, x \in [0, 5\pi]$

$\therefore$ There are six values of $x \in [0, 5\pi]$ which satisfy the equation

$3\sin^2 x - 7\sin x + 2 = 0$

Therefore, (c) is the answer.

13. $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r\theta$ (given)

Now, put $\theta = 0$, we get $0 = b_0$

$\therefore \quad \sin n\theta = \sum_{r=1}^{n} b_r \sin^r\theta$ is true

$\Rightarrow \quad \frac{\sin n\theta}{\sin\theta} = \sum_{r=1}^{n} b_r (\sin\theta)^{r-1}$

Taking limit as $\theta \to 0$

$\Rightarrow \quad \lim_{\theta\to 0}\frac{\sin n\theta}{\sin\theta} = \lim_{\theta\to 0}\sum_{r=1}^{n} b_r(\sin\theta)^{r-1}$

$\Rightarrow \quad \lim_{\theta\to 0}\frac{n\theta.\frac{\sin n\theta}{n\theta}}{\theta.\frac{\sin\theta}{\theta}} = b_1 + 0 + 0 + 0 \ldots$

Other values becomes zero for higher powers of $\sin\theta$

$\Rightarrow \quad \frac{n.1}{1} = b_1$

$\Rightarrow \quad b_1 = n$. Therefore, (b) is the answer.

14. We know,

$-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$

$\therefore \quad -\sqrt{74} \leq 7\cos x + 5\sin x \leq \sqrt{74}$

i.e., $-\sqrt{74} \leq 2k + 1 \leq \sqrt{74}$

Since k is integer.

$-9 < 2k + 1 < 9$

$-10 < 2k < 8$

$-5 < k < 4$

$\Rightarrow$ Number of possible integer values of $k = 8$.

15. In the argand plane, P is represented by $e^{i\theta}$ and Q is represented by $e^{i(\alpha-\theta)}$.

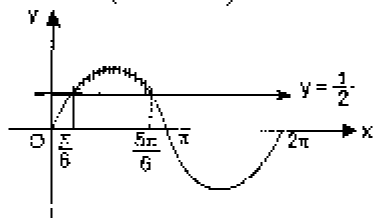
Now, rotation about a line with angle α is given by $e^{i\theta} \to e^{i(2\alpha-\theta)}$. Therefore Q is obtained from P by reflection in the line making an angle $\alpha/2$.

16. As, $2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow \quad (2\sin\theta - 1)(\sin\theta - 2) > 0$

{where, $(\sin\theta - 2) < 0$ for all $\theta \in R$}

$\therefore \quad (2\sin\theta - 1) < 0$



$\Rightarrow \quad \sin\theta < \frac{1}{2}$, shown as

$\therefore \quad \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

Hence (a) is the correct answer.

17. $2\sin^2\theta - \cos 2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{4}$

Also $2\cos^2\theta = 3\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow$ Two solutions in $[0, 2\pi]$.

D OBJECTIVE (MORE THAN ONE OPTION)

1. Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_1$, we get

$$\Rightarrow \begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

applying $C_1 \to C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 2 + 4\sin 4\theta = 0$

$\Rightarrow \quad \sin 4\theta = \frac{-1}{2}$

$\Rightarrow \quad 4\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\Rightarrow \quad \theta = \frac{n\pi}{4} + (-1)^{n+1}\left(\frac{\pi}{24}\right)$

Clearly, $\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$ are two values of θ lying between 0 and $\frac{\pi}{2}$.

2. For $0 < \phi < \pi/2$ we have

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi = 1 + \cos^2\phi + \cos^4\phi + \cos^6\phi + \ldots$$

it is clearly a G.P. with common ratio of $\cos^2\phi$ which is ≤ 1.

Hence, $x = \frac{1}{1-\cos^2\phi} = \frac{1}{\sin^2\phi}$ $\left[\because S_\infty = \frac{a}{1-r}, -1 < r < 1\right]$

Similarly, $\quad y = \frac{1}{\cos^2\phi}$

and $\quad z = \frac{1}{1 - \sin^2\phi\cos^2\phi}$

Now, $\quad x + y = \frac{1}{\sin^2\phi} + \frac{1}{\cos^2\phi}$

$= \frac{\cos^2\phi + \sin^2\phi}{\cos^2\phi\sin^2\phi} = \frac{1}{\cos^2\phi\sin^2\phi}$

again $\quad \frac{1}{z} = 1 - \sin^2\phi\cos^2\phi = 1 - \frac{1}{xy}$

$\Rightarrow \quad \frac{1}{z} = \frac{xy-1}{xy}$

$\Rightarrow \quad xy = xyz - z$

$\Rightarrow \quad xy + z = xyz.$

Therefore, (b) is the answer from (1) (putting the value of xy)

$\Rightarrow \quad xyz = x + y + z$

Therefore, (c) is the answer.

3. We know that $\sec^2\theta \ge 1$

$\Rightarrow \quad \frac{4xy}{(x+y)^2} \ge 1$

$\Rightarrow \quad 4xy \ge (x+y)^2$

$\Rightarrow \quad (x+y)^2 - 4xy \le 0$

$\Rightarrow \quad (x-y)^2 \le 0$

$\Rightarrow \quad x - y = 0$

$\Rightarrow \quad x = y$

Therefore, $x + y = 2x$ (add x both sides)

but $x + y \ne 0$ since it lies in the denominator.

$\Rightarrow \quad 2x \ne 0$

$\Rightarrow x \ne 0$ Hence $x = y, x \ne 0$ is the answer.

E SUBJECTIVE QUESTIONS

1. We have,

$$\cos\theta + \sin\theta = \sqrt{2}\left\{\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right\}$$

$$= \sqrt{2}\left(\sin\frac{\pi}{4}\cdot\cos\theta + \cos\frac{\pi}{4}\cdot\sin\theta\right)$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right)$$

$$\Rightarrow \quad \cos\theta + \sin\theta \le \sqrt{2} < \frac{\pi}{2}$$

$$\left\{\text{as } \sqrt{2} = 1.4141,\ \frac{\pi}{2} = 1.57\,(\text{approx})\right\}$$

$$\Rightarrow \quad \cos\theta + \sin\theta < \frac{\pi}{2}$$

$$\Rightarrow \quad \cos\theta < \frac{\pi}{2} - \sin\theta$$

Taking sin on both sides,

$$\sin(\cos\theta) < \sin\left(\frac{\pi}{2} - \sin\theta\right)$$

$$\sin(\cos\theta) < \cos(\sin\theta)$$

$$\Rightarrow \quad \cos(\sin\theta) > \sin(\cos\theta)$$

2. The point of intersection is given by

$$\sin 3x = \cos x = \sin\left(\frac{\pi}{2} - x\right) \Rightarrow 3x = n\pi + (-1)^n\left(\frac{\pi}{2} - x\right)$$

(i) Let n be even i.e., $n = 2m$

$$\Rightarrow \quad 3x = 2m\pi + \frac{\pi}{2} - x$$

$$\Rightarrow \quad x = \frac{m\pi}{2} + \frac{\pi}{8} \quad \ldots (1)$$

(ii) Let n be odd i.e., $n = (2m+1)$

$$\therefore \quad 3x = (2m+1)\pi - \left(\frac{\pi}{2} - x\right)$$

$$\Rightarrow \quad 3x = 2m\pi + \frac{\pi}{2} + x$$

$$\Rightarrow \quad x = m\pi + \frac{\pi}{4} \quad \ldots (2)$$

Now $\quad -\dfrac{\pi}{2} < x \le \dfrac{\pi}{2}$

$$\Rightarrow \quad x = \frac{\pi}{8}, \frac{\pi}{4}, -\frac{3\pi}{8} \quad \{\text{from (1) and (2)}\}$$

Thus point of intersection are,

$$\left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)\left(\frac{\pi}{4}, \cos\frac{\pi}{4}\right)\left(-3\frac{\pi}{8}, \cos 3\frac{\pi}{8}\right)$$

3. Here, $\quad e^{\sin x} - \dfrac{1}{e^{\sin x}} = 4$

$$\Rightarrow \quad (e^{\sin x})^2 - 4(e^{\sin x}) - 1 = 0$$

$$\Rightarrow \quad e^{\sin x} = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$$

But since $e \sim 2.72$ and we know $0 < e^{\sin x} < e$

$\therefore \quad e^{\sin x} = 2 \pm \sqrt{5}$ is not possible

So there does not exist any solution.

4. $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$

$$\Rightarrow \quad 4(1 - \sin^2 x)\sin x - 2\sin^2 x - 3\sin x = 0$$

$$\Rightarrow \quad 4\sin x - 4\sin^3 x - 2\sin^2 x - 3\sin x = 0$$

$$\Rightarrow \quad -4\sin^3 x - 2\sin^2 x + \sin x = 0$$

$$\Rightarrow \quad -\sin x(4\sin^2 x + 2\sin x - 1) = 0$$

$$\Rightarrow \quad \sin x = 0 \quad \text{or} \quad 4\sin^2 x + 2\sin x - 1 = 0$$

$$\Rightarrow \quad \sin x = \sin 0 \quad \text{or} \quad \sin x = \frac{-2 \pm \sqrt{4+16}}{2(4)}$$

$$\Rightarrow \quad x = n\pi \quad \text{or} \quad \sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$x = n\pi \text{ or } \sin x = \sin\frac{\pi}{10} \quad \text{or} \quad \sin x = \sin\left(-\frac{3\pi}{10}\right)$$

$$\Rightarrow \quad x = n\pi + (-1)^n\frac{\pi}{10},\ n\pi + (-1)^n\left(\frac{-3\pi}{10}\right)$$

$\therefore$ General solution set

$$\Rightarrow \quad \{x : x = n\pi\} \cup \left\{x : x = n\pi + (-1)^n\frac{\pi}{10}\right\}$$

$$\cup \left\{x : x = n\pi + (-1)^n\left(\frac{-3\pi}{10}\right)\right\}$$

5. Here, $2^{1+|\cos x|+|\cos^2 x|+|\cos^3 x|+\ldots} = 2^2$

$$\Rightarrow \quad 2^{\frac{1}{1-|\cos x|}} = 2^2 \Rightarrow \frac{1}{1-|\cos x|} = 2$$

$$\Rightarrow \quad |\cos x| = \frac{1}{2}$$

$$\Rightarrow \quad \cos x = \pm\frac{1}{2}$$

$$\therefore \quad x = \frac{\pi}{3}, \frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$$

Thus the solution set

$$\left\{\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}\right\}$$

6. The system has non-trivial solution, if

$$\begin{vmatrix} \sin 3\theta & 1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

on expanding, we get

$$\Rightarrow \quad 7\sin 3\theta + 14\cos 2\theta - 14 = 0$$

$$\Rightarrow \quad \sin 3\theta + 2\cos 2\theta - 2 = 0$$

⇒ $3\sin\theta - 4\sin^3\theta + 2(1 - 2\sin^2\theta) - 2 = 0$

⇒ $\sin\theta\{4\sin^2\theta + 4\sin\theta - 3\} = 0$

⇒ $\sin\theta = 0 \Rightarrow \theta = n\pi$ ....(1)

or $4\sin^2\theta + 4\sin\theta - 3 = 0$

⇒ $(2\sin\theta - 1)(2\sin\theta + 3) = 0$

⇒ $\sin\theta = \frac{1}{2}$ or $\sin\theta = -\frac{3}{2}$ (not possible)

∴ $\theta = n\pi + (-1)^n\left(\frac{\pi}{6}\right)$ ...(2)

∴ from (1) and (2), we get

$$\theta = n\pi \text{ or } n\pi + (-1)^n\left(\frac{\pi}{6}\right)$$

7. $\text{Exp}\{(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty)\log_e 2\}$

⇒ $e^{\frac{\sin^2 x}{1-\sin^2 x}\cdot\log_e 2}$

⇒ $e^{\log_e 2^{\frac{\sin^2 x}{\cos^2 x}}}$

⇒ $2^{\tan^2 x}$ satisfy $x^2 - 9x + 8 = 0$

⇒ $x = 1, 8$

∴ $2^{\tan^2 x} = 1$ and $2^{\tan^2 x} = 8$

⇒ $\tan^2 x = 0$ and $\tan^2 x = 3$

⇒ $x = n\pi$ and $\tan^2 x = \left(\tan\frac{\pi}{3}\right)^2$

and $x = n\pi \pm \frac{\pi}{3}$

Neglecting $x = n\pi$ as $0 < x < \frac{\pi}{2}$

⇒ $x = \frac{\pi}{3} \in \left(0, \frac{\pi}{2}\right)$

∴ $\dfrac{\cos x}{\cos x + \sin x} = \dfrac{\frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \dfrac{1}{1+\sqrt{3}} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{\sqrt{3}-1}{2}$

∴ $\dfrac{\cos x}{\cos x + \sin x} = \dfrac{\sqrt{3}-1}{2}$

8. $y = \dfrac{\tan x}{\tan 3x} = \dfrac{\tan x}{\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}}$

$= \dfrac{\tan x(1 - 3\tan^2 x)}{3\tan x - \tan^3 x}$

$= \dfrac{1 - 3\tan^2 x}{3 - \tan^2 x}$ [∵ $\tan 3x \neq 0 \Rightarrow 3x \neq 0$]

⇒ $x \neq 0 \Rightarrow \tan x \neq 0$

Let $\tan x = t$

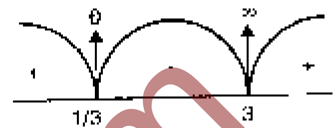
⇒ $y = \dfrac{1 - 3t^2}{3 - t^2}$

⇒ $3y - t^2 y = 1 - 3t^2$

⇒ $3y - 1 = t^2 y - 3t^2$

⇒ $3y - 1 = t^2(y - 3)$

⇒ $\dfrac{3y-1}{y-3} = t^2$



⇒ $\dfrac{3y-1}{y-3} \geq 0,\ t^2 \geq 0\ \forall\ t \in R$

⇒ $y \in (-\infty, 1/3) \cup (3, \infty)$

Therefore, y is not defined in between $(1/3, 3)$.

9. $\tan(x + 100°) = \tan(x + 50°)\tan x \tan(x - 50°)$

⇒ $\dfrac{\tan(x+100°)}{\tan x} = \tan(x + 50°)\tan(x - 50°)$.

⇒ $\dfrac{\sin(x+100°)}{\cos(x+100°)}\cdot\dfrac{\cos x}{\sin x} = \dfrac{\sin(x+50°)\sin(x-50°)}{\cos(x+50°)\cos(x-50°)}$

⇒ $\dfrac{\sin(2x+100°) + \sin 100°}{\sin(2x+100°) - \sin 100°} = \dfrac{\cos 100° - \cos 2x}{\cos 100° + \cos 2x}$

⇒ $[\sin(2x+100°) + \sin 100°][\cos 100° + \cos 2x]$

$= [\cos 100° - \cos 2x] \times [\sin(2x+100°) - \sin 100°]$

⇒ $\sin(2x+100°)\cdot\cos 100° + \sin(2x+100°)\cos 2x$

$+ \sin 100° \cos 100° + \sin 100° \cos 2x$

$= \cos 100° \sin(2x+100°) - \cos 100° \sin 100°$

$- \cos 2x \sin(2x+100°) + \cos 2x \sin 100°$

⇒ $2\sin(2x+100°)\cos 2x + 2\sin 100° \cos 100° = 0$

⇒ $\sin(4x+100°) + \sin 100° + \sin 200° = 0$

⇒ $\sin(4x+100°) + 2\sin 150° \cos 50° = 0$

⇒ $\sin(4x+100°) + 2\cdot\frac{1}{2}\sin(90° - 50°) = 0$

⇒ $\sin(4x+100°) + \sin 40° = 0$

⇒ $\sin(4x+100°) = -\sin 40°$

⇒ $\sin(4x+100°) = \sin(-40°)$

⇒ $4x + 100° = n\pi + (-1)^n(-40°)$

⇒ $4x = n(180°) + (-1)^n(-40°) - 100°$

⇒ $x = \frac{1}{4}[n(180°) + (-1)^n(-40°) - 100°]$

The smallest positive value of x is obtained when $n = 1$

Therefore, $x = \frac{1}{4}[180° + 40° - 100°]$

or $x = \frac{1}{4}[120°] = 30°$

10. $\cos(p\sin x) = \sin(p\cos x)$ (given) $\forall x \in [0, 2\pi]$

$\Rightarrow \quad \cos(p\sin x) = \cos\left(\frac{\pi}{2} - p\cos x\right)$

$\Rightarrow \quad p\sin x = 2n\pi \pm \left(\frac{\pi}{2} - p\cos x\right), n \in I.$

$[\because \cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I]$

$\Rightarrow \quad p\sin x + p\cos x = 2n\pi + \pi/2$

or $\quad p\sin x - p\cos x = 2n\pi - \pi/2, n \in I$

$\Rightarrow \quad p(\sin x + \cos x) = 2n\pi + \pi/2$

or $\quad p(\sin x - \cos x) = 2n\pi - \pi/2, n \in I$

$\Rightarrow \quad p.\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) = 2n\pi + \pi/2$

or $\quad p\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right) = 2n\pi - \pi/2, n \in I$

$\Rightarrow \quad p\sqrt{2}\left(\cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x\right) = 2n\pi + \frac{\pi}{2}$

or $\quad p\sqrt{2}\left(\cos\frac{\pi}{4}\sin x - \sin\frac{\pi}{4}\cos x\right) = 2n\pi - \pi/2, n \in I$

$\Rightarrow \quad p\sqrt{2}\,[\sin(x + \pi/4)] = \frac{(4n+1)\pi}{2}$

or $\quad p\sqrt{2}\,[\sin(x - \pi/4)] = (4n-1)\frac{\pi}{2}, n \in I$

Now, $\quad -1 \le \sin(x \pm \pi/4) \le 1$

$\Rightarrow \quad -p\sqrt{2} \le p\sqrt{2}\sin(x \pm \pi/4) \le p\sqrt{2}$

$\Rightarrow \quad -p\sqrt{2} \le \frac{(4n+1).\pi}{2} \le p\sqrt{2}, n \in I$

or $\quad -p\sqrt{2} \le \frac{(4n-1)\pi}{2} \le p\sqrt{2}, n \in I$

Second inequality is always a subset of first, therefore, we have to consider only first.

It is sufficient to consider $n \ge 0$, because for $n > 0$, the solution will be same for $n \ge 0$.

If $\quad n \ge 0, \therefore -\sqrt{2}p \le (4n+1)\pi/2$

$\Rightarrow \quad (4n+1)\pi/2 \le \sqrt{2}p$

For p to be least, n should be least

$\Rightarrow \quad n = 0$

$\Rightarrow \quad \sqrt{2}p \ge \pi/2$

$\Rightarrow \quad p \ge \frac{\pi}{2\sqrt{2}}$

Therefore least value of $p = \frac{\pi}{2\sqrt{2}}$.

11. $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$ (given)

$\Rightarrow \quad (1 - \tan^2\theta)\cdot(1 + \tan^2\theta) + 2^{\tan^2\theta} = 0$

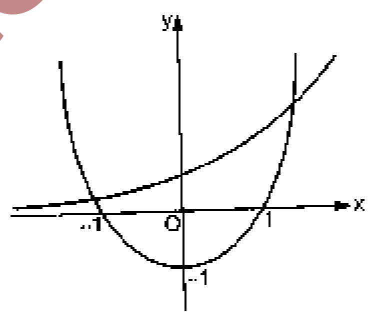
$\Rightarrow \quad 1 - \tan^4\theta + 2^{\tan^2\theta} = 0$

put $\quad \tan^2\theta = x$

$\Rightarrow \quad 1 - x^2 + 2^x = 0$

$\Rightarrow \quad x^2 - 1 = 2^x$

Imp. note : 2^x and $x^2 - 1$ are uncompatible functions, therefore, we have to consider range of both functions.



curves $\quad y = x^2 - 1$ and $y = 2^x$

intersect at one point (negative value will not consider)

$x = 3, \; y = 8$

Therefore, $\tan^2\theta = 3 \Rightarrow \tan\theta = \pm\sqrt{3} \Rightarrow \theta = \pm\pi/3$