INVENTORY SYSTEM REDESIGN

(Prerequisite: Independent demand inventory models)

- Before any changes are considered
  - Define the objectives of the inventory system and their relative importance
- Redesign is based on the resultant benefit

  Benefit – Incremental cost savings

  - Improved customer service
- Final decision should not be based solely on a cost analysis
  - Since other intangible or unquantifiable factors also must be considered.
- A simulation study is an excellent analysis tool for determining the impact of different system designs without actually implementing them.
- A pilot study of the inventory system can reveal the potential benefit of redesign

Pilot study procedure

1. A representative sample of inventory items is selected and analysed
2. Using appropriate inventory models, theoretical and actual performance are compared
3. The potential cost savings and resultant benefit are determined
4. A decision is made to continue the study, stop the study, revise the inventory system, or make no changes in the inventory system.

An example

An organization with an inventory of five items is considering a change from a periodic to a perpetual inventory system.

Currently each item is ordered at the end of the month.

The ordering cost per item is Rs 10 per order, and the holding cost is 12 % per year.

The relevant item data are listed in table below.

Should the organization adopt a perpetual inventory system?

Item, i	Annual Demand, R_i	Unit Cost, P_i
1	600	3
2	900	10
3	2,400	5
4	12,000	5
5	18,000	1

Solution

Periodic System

Item, i	Annual Demand, R_i	Unit Cost, P_i	Orders per year	Average Inventory, $P_iR_i/24$
1	600	3	12	75
2	900	10	12	375
3	2,400	5	12	500
4	12,000	5	12	2500
5	18,000	1	12	750
		60	Rs 4200	

Annual cost = 60(10) + 0.12(4200) = Rs 1104

Perpetual System

Item, i	Optimal order size in Rs, P_iQ_{oi}	Orders per year P_iR_i/P_iQ_{oi}
1	548	3.28
2	1225	7.35
3	1414	8.49
4	3162	18.98
5	1732	10.39
	Rs 8081	48.49

Annual cost = 48.49(10) + 0.12(8081/2) = Rs 970

INVENTORY SYSTEM CONSTRAINTS

- EOQ apply to single inventory items
- Appropriate to treat each item in inventory on an individual basis when no limitations are placed on aggregate inventory
- Limitations exist, and they arise because scarce resource exist such as capital, storage space and equipment capacities
- Stringent budget requirement may put restriction in inventory investment
- Reduction of investment in inventory means reducing order quantity
- This necessitates an increase in the holding cost fraction

Calculation of increase in holding cost fraction

I_c – Current investment in inventory

I_n – New investment in inventory

F_c – Current holding cost fraction

F_n – New holding cost fraction

P_i – Price of item i

${}_cQ_{oi}$ - Current optimum order quantity for item i

${}_nQ_{oi}$ - New optimum order quantity for item i

N – Number of items

- Same C and F for all the items included in the analysis
- If there is restriction in inventory investment and order cost is not changed, the new holding cost fraction due to the restriction in inventory investment can be found in the following way

$$\frac{\sum_{i=1}^{N} \frac{P_i \, {}_cQ_{oi}}{2}}{\sum_{i=1}^{N} \frac{P_i \, {}_nQ_{oi}}{2}} = \frac{I_c}{I_n}$$

$$\sqrt{\frac{F_n}{F_c}} = \frac{I_c}{I_n}$$

The new order quantity or the new review period can be calculated using F_n

Example

Consider a shop producing three items which are produced in lots with the following characteristics:

Item	1	2	3
Holding cost (Rs./unit/year)	0.20	0.20	0.20
Set-up cost (Rs./set-up)	50	40	60
Cost per units (Rs.)	6	7	5
Annual Demand (Units)	10,000	12,000	7,500

The replenishment rate of the above items is infinite. All the items have two week lead time. Currently, the organisation was following Q-system of inventory control.

(a) Determine the parameters of the Q-system.

(b) What is the average inventory level of the system?

(c) Now, the organisation desires to have an average inventory level of Rs 5000/-. What is the new holding cost fraction?

Answer:

(a)

Item	1	2	3
Q_{oi}	913	828	949
B_i	385	462	288

(b) Average inventory in Rs. = 8008

(c) New holding cost fraction = 0.5131

Exchange Curve

- Order cost and holding cost fraction are not explicitly available
- Some times undesirable to minimize cost on individual basis
- Management want to control aggregate inventory levels without being burdened with individual item decisions
- Under the above circumstances management attempt to use the total average inventory investment and total number of annual replenishments as decision variables
- Tradeoff is possible between these variable
- The tradeoff between management policies can be shown by a method called **exchange curve**
- An exchange curve shows the optimal tradeoffs that can be achieved between two or more aggregate measures of performance

Construction of Exchange Curve

Assumption

- Order cost and carrying cost fraction are same for all inventory items

Construction

If EOQ is used for each item in the system, the total average inventory investment (*AI*) for *N* items is

$$AI = \sum_{i=1}^{N} \frac{P_i Q_{oi}}{2} = \sum_{i=1}^{N} \sqrt{\frac{CP_i R_i}{2F}} = \sqrt{\frac{C}{F}} \sum_{i=1}^{N} \sqrt{\frac{P_i R_i}{2}}$$

The total number of annual replenishments (*AR*) is

$$AR = \sum_{i=1}^{N} \frac{R_i}{Q_{oi}} = \sum_{i=1}^{N} \sqrt{\frac{P_i R_i F}{2C}} = \sqrt{\frac{F}{C}} \sum_{i=1}^{N} \sqrt{\frac{P_i R_i}{2}}$$

$$(AI)(AR) = \left(\sum_{i=1}^{N} \sqrt{\frac{P_i R_i}{2}} \right)^2$$ - Hyperbolic curve

$$\frac{AI}{AR} = \frac{C}{F}$$

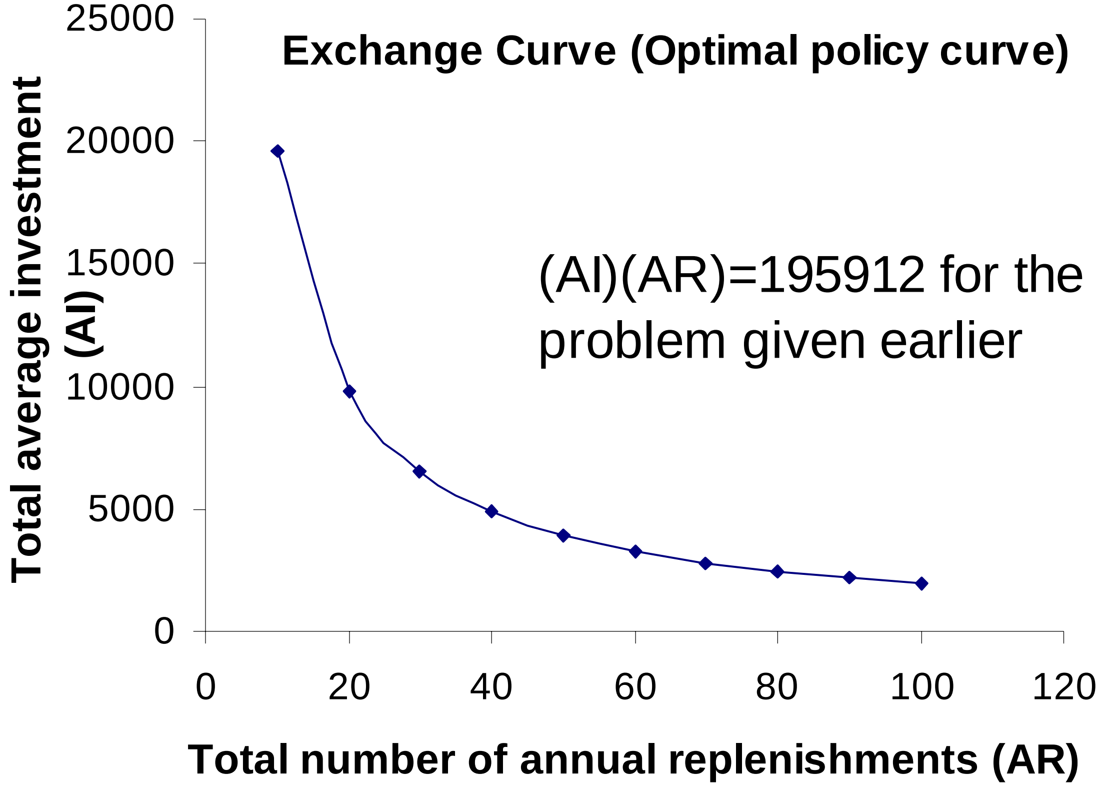

Fig. 1 Exchange curve or Optimal Policy Curve

- Management may select an acceptable operating point on an aggregate exchange curve
- At the selected point, value of *C* and *F* are implied
- Order quantities for individual items can be calculated based on the ratio, *C*/*F*
- These quantities will lie within the chosen limitations and conform to aggregate policies.
- Based on the inventory control system the parameters will be determined

<u>For Q-System</u>

$$Q_{0i} = \sqrt{\frac{2CR_i}{P_i F}}$$

$$Q_{0i} = \sqrt{\frac{C}{F}}\sqrt{\frac{2R_i}{P_i}} = \sqrt{\frac{AI}{AR}}\sqrt{\frac{2R_i}{P_i}}$$

For P-System

$$T_{0i} = \sqrt{\frac{2C}{P_i F R_i}}$$

$$T_{0i} = \sqrt{\frac{C}{F}}\sqrt{\frac{2}{P_i R_i}} = \sqrt{\frac{AI}{AR}}\sqrt{\frac{2}{P_i R_i}}$$

- The general effect of the ratio *C*/*F* apply to all items considered in the analysis
- As *C*/*F* increases, each order quantity increases, which in turn increases the total average inventory investment and decreases the total number of annual replenishment
- By varying *C*/*F*, the aggregate exchange curve can be traced

Example

An organisation uses periodic system of inventory control for the following items.

Item	1	2	3
Cost per units (Rs.)	6	7	5
Annual Demand (Units)	10,000	12,000	7,500
Lead Time in Weeks	2	3	2

The organisation desires to have an average inventory level of Rs 10,000/- for all items together. Determine the parameters of the periodic system for each item. Assume 52 weeks in a year.

Solution

$$(AI)(AR) = \left(\sum_{i=1}^{N}\sqrt{\frac{P_i R_i}{2}}\right)^2$$

$$\frac{AI}{AR} = \frac{C}{F}$$

$$T_{0i} = \sqrt{\frac{C}{F}}\sqrt{\frac{2}{P_i R_i}} = \sqrt{\frac{AI}{AR}}\sqrt{\frac{2}{P_i R_i}}$$

$$E_i = \frac{R(T_{0i} + L_i)}{52}$$

Item	1	2	3
Cost per units (Rs.)	6	7	5
Annual Demand (Units)	10,000	12,000	7,500
Lead Time in Weeks	2	3	2
$\frac{p_i R_i}{2}$	30,000	42,000	18,750

$\sqrt{\frac{p_i R_i}{2}}$	173.21	204.93	136.93
$\sqrt{\frac{2}{p_i R_i}}$	0.00577	0.004879	0.007303
T_o in years	0.1119	0.0947	0.1417
T_o in weeks	5.81 ≈ 6	4.92 ≈ 5	7.36 ≈ 7
E_i	1538	1846	1298

Optimising Constrained Functions

Lagrange Method

- Lagrange method specifically applies to equality constraints
- Problems with inequality constraints can be solved by combining Lagrange method with Kuhn-Tucker conditions

One Equality Constraint

Min. $f(X_1, X_2, ...X_n)$

Subjected to

$g(X_1, X_2, ...X_n) = a$

is equivalent to minimise the unconstrained function

$$h(X_1, X_2, ...X_n) = f(X_1, X_2, ...X_n) + \lambda(g(X_1, X_2, ...X_n) - a)$$

where λ is a Lagrange multiplier unrestricted in sign.

- To minimise this function

  Take partial derivative of *h* with respect to X_j and λ and set them equal to zero.

$$\frac{\partial h}{\partial X_j} = \frac{\partial f}{\partial X_j} + \lambda \frac{\partial g}{\partial X_j} = 0, \quad j = 1, ..., n$$

$$\frac{\partial h}{\partial \lambda} = g - a = 0$$

- By simultaneously solving the above equations for X_j and λ, the minimum point is obtained.
- If the multiplier is positive, it indicates the rate at which the objective function will decrease per unit increase in the parameter *a*.

More Than One Equality Constraint

$$h(X_1, X_2,..., X_n) = f(X_1, X_2,..., X_n) + \lambda_1[g_1(X_1, X_2,..., X_n) - a_1)] + \lambda_2[g_2(X_1, X_2,..., X_n) - a_2)]$$

$$\Downarrow$$

$$\frac{\partial h}{\partial X_j} = \frac{\partial f}{\partial X_j} + \lambda_1 \frac{\partial g_1}{\partial X_j} + \lambda_2 \frac{\partial g_2}{\partial X_j} = 0, \quad j = 1,..., n$$

$$\frac{\partial h}{\partial \lambda_1} = g_1 - a_1 = 0$$

$$\frac{\partial h}{\partial \lambda_2} = g_2 - a_2 = 0$$

One Inequality Constraint

$$g(X_1, X_2,...X_n) \le a$$

Lagrange method combined with Kuhn-Tucker Conditions

$$\downarrow$$

$$h(X_1, X_2,...X_n) = f(X_1, X_2,...X_n) + \lambda[g(X_1, X_2,...X_n) - a]$$

$$\downarrow$$

$$\frac{\partial h}{\partial X_j} = \frac{\partial f}{\partial X_j} + \lambda \frac{\partial g}{\partial X_j} = 0, \quad j = 1,..., n$$

$$\lambda(g - a) = 0$$

More Than One Inequality Constraint

$$\frac{\partial h}{\partial X_j} = \frac{\partial f}{\partial X_j} + \lambda_1 \frac{\partial g_1}{\partial X_j} + \lambda_2 \frac{\partial g_2}{\partial X_j} = 0, \quad j = 1,..., n$$

$$\lambda_1(g_1 - a_1) = 0$$

$$\lambda_2(g_2 - a_2) = 0$$

WORKING CAPITAL RESTRICTION

Lagrange-multiplier Method

Minimise the inventory cost subjected to average inventory investment *J* rupees

Min: Annual Variable Inventory Cost = Order cost + Holding cost

$$= \sum_{i=1}^{N} \left(\frac{R_i C}{Q_i} + \frac{Q_i P_i F}{2} \right)$$

Subjected to $g = \sum_{i=1}^{N} \frac{P_i Q_i}{2} = J$

The Lagrange expression is

$$h = C \sum_{i=1}^{N} \frac{R_i}{Q_i} + F \sum_{i=1}^{N} \frac{P_i Q_i}{2} + \lambda \left(\sum_{i=1}^{N} \frac{P_i Q_i}{2} - J \right)$$

To minimise the objective function subjected to the restriction requires the minimisation of h with respect to Q_i and λ

Take partial derivative and equate them to zero

$$\frac{\partial h}{\partial Q_i} = -\frac{C R_i}{Q_i^2} + \frac{F P_i}{2} + \frac{\lambda P_i}{2} = 0$$

$$\frac{\partial h}{\partial \lambda} = \sum_{i=1}^{n} \frac{P_i Q_i}{2} - J = 0$$

Q_i and λ can be found out by solving the above simultaneous equations

$$Q_{0i} = \sqrt{\frac{2 C R_i}{(F + \lambda) P_i}} \quad \text{and} \quad \lambda = \frac{C \left(\sum_{i=1}^{N} \sqrt{P_i R_i} \right)^2}{2 J^2} - F$$

Examples

1. Consider a shop with three items which are produced in lots with the following characteristics:

Item	1	2	3
Holding cost (Rs./unit/year)	20	20	20
Set-up cost (Rs./set-up)	50	40	60
Cost per units (Rs.)	6	7	5
Annual Demand (Units)	10,000	12,000	7,500

The replenishment rate is infinite. The organisation desires to have an average inventory level of Rs 1000/-. a) Show the problem formulation which minimise the total relevant cost b) Write the Lagrangian of the optimisation problem and derive the equation for order quantity. c) Determine the batch size for each item.

(Note: If all the items have same holding cost fraction and order cost (set-up cost), exchange curve method is a suitable procedure for batch size determination. Hence, the batch size is to be determined using Lagrange method)

Answer

Minimise

Total relevant cost $TC_v = \sum_{i=1}^{N}\left(\frac{R_i C_i}{Q_i} + \frac{Q_i}{2}H\right)$

Subjected to

$$\sum_{i=1}^{N}\frac{P_i Q_i}{2} = 2000$$

Langrangian expression

$$h = \sum_{i=1}^{N}\frac{R_i C_i}{Q_i} + \sum_{i=1}^{N}\frac{Q_i H_i}{2} + \lambda\left(\sum_{i=1}^{N}\frac{P_i Q_i}{2} - 2000\right)$$

$$\frac{\partial h}{\partial Q_i} = \frac{-R_i C}{Q_i^2} + \frac{H_i}{2} + \frac{\lambda P_i}{2} \qquad \text{i} \quad (1)$$

$$\frac{\partial h}{\partial \lambda} = \sum_{i=1}^{N}\frac{P_i Q_i}{2} - 2000 \qquad \text{i} \quad (2)$$

Equate equation (1) to zero and simplify

$$\frac{R_i C_i}{Q_i^2} = \frac{H_i}{2} + \lambda\frac{P_i}{2}$$

$$Q_i = \sqrt{\frac{2R_i C_i}{H_i + \lambda P_i}}$$

λ	Q_1	Q_2	Q_3	$\sum P_i Q_i$
10	118.8	103.28	113.39	1960.7
0.5	208.5	202.1	200	3665.7
0.1	220.3	215.3	209.5	3876.5
-1	267.26	271.75	244.95	4448.8
0	223.6	219.1	212.1	$3935.8 \approx 4000$

Therefore $\lambda = 0$, $Q_1 = 224$, $Q_2 = 219$ and $Q_3 = 212$

2. A firm with an inventory of three different items has the following information on the inventoried items.

Item	Annual Demand (Units)	Unit Cost (Rs.)
A	300	80
B	800	105
C	1500	40

The current policy is to order each item once in each month. What is the optimal ordering policy, which gives the same total number of orders? Assume same holding cost fraction and order cost for all items.

(Note: Exchange curve principle can be used)

3. A company producing three items has limited storage space of average 750 items of all types. Determine the optimal production quantities for each item separately, when the following information is given:

Product	1	2	3
Holding cost (Rs)	0.05	0.02	0.04
Set-up cost (Rs)	50	40	60
Demand rate	100	120	75

(Answer: $\lambda = 0.00235$, $Q_1 = 428$ units, $Q_2 = 623$ units and $Q_3 = 449$ units)