F.E. (Rev) (CBGS) Sem I . M-2015
App. Maths - I .. 12/5/15 .

QP Code : **1002**

(REVISED COURSE)

(3 Hours) [Total Marks : 80

N. B. : (1) Question No. **1** is **compulsory**.
(2) Answer any **three** questions from remaining.
(3) Assume suitable data if necessary.

1. (a) If $\tan \frac{x}{2} = \tanh \frac{u}{2}$ then S.T. 3

$$u = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

(b) If $u = x^y$ find $\frac{\partial^3 u}{\partial x \partial y \partial x}$ 3

(c) If $ux = yz$, $vy = zx$, $wz = xy$ 3

find $J\left(\frac{u, v, w}{x, y, z} \right)$

(d) If $y = (x - 1)^n$ then P.T. $y + \frac{y_1}{1!} + \frac{y_2}{2!} + \frac{y_3}{3!} + \dots \frac{y_n}{n!} = x^n$ 3

(e) P.T. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} +$ 4

(f) Express the matrix A as sum of Hermition and skew Hermition matrix where 4

$$\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$

2. (a) Solve $x^7 + x^4 + i(x^3 + 1) = 0$ 6

(b) Reduce the matrix A to normal form and hence find its rank where 6

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 4 & 3 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

[TURN OVER

(c) State and prove Euler's theorem for three variables and hence find **8**

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial x} \text{ where}$$

$$u=\frac{x^3y^3z^3}{x^3+y^3+z^3}$$

3. (a) Solve the following system of equations **6**

$2x - 2y - 5z = 0$

$4x - y + z = 0$

$3z - 2y + 3z = 0$

$x - 3y + 7z = 0$

(b) Find the maximum and minimum values **6**

of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

(c) Separate into real and imaginary parts of $\tanh^{-1}(x + iy)$ **8**

4. (a) If $u = 2xy$, $v = x^2 - y^2$ and **6**

$x = r\cos\theta$, $y = r\sin\theta$ then find

$$\frac{\partial(u_1 v)}{\partial(\partial_1 \theta)}$$

(b) If $i^{i^{i^{\cdot^{\cdot^{\infty}}}}} = A + iB$, prove that **6**

$$\left(\frac{\pi A}{2}\right)=\frac{B}{A} \quad \text{and } A^2 + B^2 = e^{-\pi B}$$

(c) Solve by crouts method the system of equations **8**

$3x + 2y + 7z = 4$

$2x + 3y + z = 5$

$3x + 4y + z = 7$

5. (a) By using De Moiverse thm **6**

Express $\frac{\sin 7\theta}{\sin\theta}$ in powers of $\sin\theta$ only.

(b) By using Taylor's series expand $\tan^{-1} x$ in positive powers of $(x-1)$ upto first four non-zero terms. **6**

(c) If $y = \sin[\log(x^2 + 2x + 1)]$ prove that **8**

$(x + 1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + (n^2 + 4)y_n = 0$

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6. (a) Determine linear dependance or independance of vectors 6

$x_1 = [1, 3, 4, 2]$ $x_2 = [3, -5, 2, 6]$

$x_3 = [2, -1, 3, 4]$ and if dependent find the relation between them.

(b) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$ prove that 6

$$\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=4\sqrt{u^2+v^2}\left[\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2\right]$$

(c) (i) Evaluate $\lim_{x\to 0}\frac{\sin x.\sin^{-1}x - x^2}{x^6}$ 4

(ii) Fit straight line to the following data 4

$(x, y) = (-1, -5), (1, 1), (2, 4), (3, 7), (4, 10)$

Estimate y when x =7
