AnRS

RET/12/Test B

904	Mathematics

020

Question Booklet No. ....

(*To be filled up by the candidate by* ***blue/black ball-point pen***)

Roll No. | | | | | | | | |

Roll No. (Write the digits in words) ....

Serial No. of OMR Answer Sheet ....

Day and Date ....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only ***blue/black ball-point pen*** in the space above and on both sides of the ***Answer Sheet***)

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope.*
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. *No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.*
7. *Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.*
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed.* If you *do not wish to attempt* a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit *both OMR Answer Sheet and Question Booklet* at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

[उपर्युक्त निर्देश हिन्दी में अन्तिम आवरण-पृष्ठ पर दिये गये हैं।]

Total No. of Printed Pages : 20

SEAL

6. Who was appointed the first Vice-Chancellor of Banaras Hindu University ?

(1) Madan Mohan Malviya (2) S. Radhakrishnan

(3) Shanti Swaroop Bhatnagar (4) Sir Sundar Lal

काशी हिन्दू विश्वविद्यालय का प्रथम कुलपति कौन नियुक्त हुआ था ?

(1) मदन मोहन मालवीय (2) एस० राधाकृष्णनन

(3) शांति स्वरूप भटनागर (4) सर सुन्दर लाल

7. Rabindra Nath Tagore was awarded 'Nobel Prize' in the year :

(1) 1911 (2) 1912 (3) 1913 (4) 1914

रबिन्द्र नाथ टैगोर को नोबेल पुरस्कार से किस वर्ष सम्मानित किया गया ?

(1) 1911 (2) 1912 (3) 1913 (4) 1914

8. Who is famous as 'Devbani' ?

(1) Sanskrit (2) Hindi (3) Bangla (4) Telgu

'देववाणी' के रूप में कौन चर्चित है ?

(1) संस्कृत (2) हिन्दी (3) बंगला (4) तेलगू

9. When ancient Olympic games were started ?

(1) 394 B.C. (2) 493 B.C. (3) 676 B.C. (4) 776 B.C.

प्राचीन ओलम्पिक खेल कब प्रारंभ हुए ?

(1) 394 ई० पू० (2) 493 ई० पू० (3) 676 ई० पू० (4) 776 ई० पू०

10. 'Indian Institute of Mass Communication' is situated in :

(1) New Delhi (2) Bangalore (3) Ahmedabad (4) Chennai

'इंडियन इंस्टीट्यूट ऑफ मास कम्युनिकेशन' कहाँ स्थित है ?

(1) नई दिल्ली में (2) बैंगलोर में (3) अहमदाबाद में (4) चेन्नई में

11. The radius of convergence of the power series $\sum_{n=0}^{\infty} n! Z^{n^2}$ is :

(1) e (2) $\frac{1}{e}$ (3) e^2 (4) 0

12. A sequence $\{f_n\}_{n=1}^{\infty}$ of functions is defined by $f_n(x) = \frac{n^2 x}{1+n^4 x^2} \forall x \in \mathbb{R}$. Then i

is :

(1) neither point - wise nor uniformly convergent.

(2) point - wise convergent but not uniformly convergent.

(3) point - wise as well as uniformly convergent.

(4) not point - wise convergent but uniformly convergent.

13. Let $C_{\mathbb{R}}[a,b]$ be the vector space of all continuous real - valued on the close interval $[a, b]$ in $\mathbb{R}$ under usual operations. For $f \in C_{\mathbb{R}}[a,b]$, define :

$$\|f\|_1 = \int_a^b |f(x)| dx, \quad \|f\|_{\infty} = \sup |f(x)| \,.\, x \in [a,b] \text{ then}$$

(1) $(C_{\mathbb{R}}[a,b], \| \ \|_1)$ is a normed linear space but not a Banach space.

(2) $(C_{\mathbb{R}}[a,b], \| \ \|_{\infty})$ is a normed linear space but not a Banach space.

(3) $(C_{\mathbb{R}}[a,b], \| \ \|_1)$ is a Banach space but not a Hilbert space.

(4) $(C_{\mathbb{R}}[a,b], \| \ \|_{\infty})$ is an inner product space but not a Hilbert space.

14. A sequence $\{f_n\}_{n=1}^{\infty}$ of functions is defined by $f_n(x) = \begin{cases} 2n & \text{if } x \in \left[\frac{1}{2n}, \frac{1}{n}\right] \\ 0 & \text{if } x \in [0,1] - \left[\frac{1}{2n}, \frac{1}{n}\right] \end{cases}$

Then for this sequence of functions :

(1) Fatou's lemma does not apply

(2) Lebesgue dominated convergence theorem applies.

(3) Fatou's lemma applies but Lebesgue dominated convergence theorem do not apply.

(4) Fatou's lemma as well as Lebesgue dominated convergence theorem apply

15. The function $f(z) = \sin\frac{1}{z}, z \neq 0$ has :

(1) an isolated essential singularity at $z = 0$.

(2) non - isolated essential singularity at $z = 0$.

(3) a pole of order 2 at $z = 0$.

(4) a simple pole at $z = 0$.

16. Let X and Y be normed linear spaces over a field $K(=\mathbb{R}$ or $\mathbb{C})$ and $B(X, Y)$ be the normed linear space of all bounded linear transformation on X into Y. then :

(1) B (X, Y) is a Banach space $\Rightarrow$ X is a Banach space.

(2) B (X, Y) is a Banach space $\Rightarrow$ Y is a Banach space.

(3) X is a Banach space $\Rightarrow$ B (X, Y) is a Banach space.

(4) Y is a Banach space $\Rightarrow$ B (X, Y) is a Banach space.

17. Let X = $\{a, b, c\}$

and T= $\{\phi, X, \{a\}, \{b\}, \{a,b\}\}$

Then T is :

(1) the indiscrete topology on X.

(2) the discrete topology on X.

(3) a topology on X which can not be induce by a metric on X.

(4) a Hausdorff topology on X.

18. If G is a non - abelian group of order 12 having a normal sylow 3 - subgroup then G has an element of order :

(1) 2 (2) 3 (3) 4 (4) 6

19. The total number of composition series of the group $\mathbb{Z}_{30}$ is :

(1) 3 (2) 4 (3) 5 (4) 6

20. If T is a nilpotent operator with index of nilpotency k then T^n, $n > 1$, is also a nilpotent operator with index of nilpotency :

(1) greator than k. (2) equal to k.

(3) less than or equal to k. (4) less than k.

21. The splitting field of the polynomial $x^5 - 1$ over the field Q of rationals, and its degree, are given by :

(1) $Q\left(e^{\pi i/5}\right)$, 5 (2) $Q\left(e^{3\pi i/5}\right)$, 4 (3) $Q\left(e^{2\pi i/5}\right)$, 4 (4) $Q\left(e^{2\pi i/5}\right)$, 5

22. If P_1 and P_2 are the prime subfields of the two fields F_1 and F_2 respectively, where $F_1 \subset F_2$, then :

(1) $P_1 \supset P_2$ (2) $P_2 \supset P_1$ (3) $P_1 = P_2$ (4) $P_1 \neq P_2$

23. The real projective space $\mathbb{R}P^n$ is :

(1) orientable if n is even. (2) always orientable.

(3) not orientable if n is even. (4) always non - orientable.

24. Let X be a non - empty set an R a relation on X. Then R is an equivalence relation in X if :

(1) R is symmetric and circular in X (2) R is reflexive and triangular in X

(3) R is reflexive and transitive in X (4) R is symmetric and transitive in X

25. Which one of the following statements is not correct for a compact metric space (X, d) :

(1) (X, d) satisfies Bolzano - Weierstrass property.

(2) (X, d) is sequentially compact.

(3) (X, d) is totally bounded and complete.

(4) (X, d) is totally bounded and no open cover of (X, d) has a Lebesgue number.

SEAL

26. Which one of the following statements is not correct :

(1) Every $C^r n$ - manifold (M, $\mathfrak{C}$) satisfies the first axion of countability.

(2) Every $C^r n$ - manifold (M, $\mathfrak{C}$) is totally connected.

(3) If a $C^r n$ - manifold (M, $\mathfrak{C}$) is Hausdorff then it is locally compact.

(4) Every $C^r n$ - manifold (M, $\mathfrak{C}$) is paracompact.

27. Poincare conjecture was proved by :

(1) R. S. Kulkarni (2) R. S. Hamilton. (3) R. S. Mishra (4) G. Perelman

28. Which one of the following statements is not correct :

(1) The odd dimensional spheres S^{2n+1} always admit a sasakian structure.

(2) The odd dimensional euclidean spaces R^{2n+1} always admit a Sasakian structure.

(3) The even dimensional spheres S^{2n} always admit a Kaehler structure.

(4) Every three dimensional compact Riemannian manifold admits a contact structure.

29. If bisection method is started with interval [50, 55] then the number of steps n needed to compute root of an equation with relative accuracy of 10^{-2} are :

(1) $n \geq 2$ (2) $n \geq 3$ (3) $n \geq 4$ (4) $n \geq 5$

30. If f is a continuous function of (t, s) where $0 \leq t \leq 1$ and $-\infty < s < \infty$ and assume that on this domain :

$$|f(t,s_1) - f(t,s_2)| \leq k\,|s_1 - s_2|,$$

then the two point boundary value problem

$$x'' = f(t,x), \quad x(0) = 0, \quad x(1) = 0,$$

has a unique solution if

(1) $k = 2$ (2) $k = 3$ (3) $k = 4$ (4) $k = 8$

31. The differential equation describing the characteristic curve of the partial differential equation :

$y\,U_{xx} + (x+y^2)\,U_{xy} + xy\,U_{yy} = 0$ is

(1) $y\left(\frac{dy}{dx}\right)^2 - \left(x+y^2\right)\frac{dy}{dx} + xy = 0$

(2) $y\left(\frac{d^2y}{dx^2}\right) - \left(x+y^2\right)\frac{dy}{dx} + xy = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + \left(x+y^2\right)\frac{dy}{dx} + xy = 0$

(4) $y\left(\frac{d^2y}{dx^2}\right) + \left(x+y^2\right)\frac{dy}{dx} + xy = 0$

32. System Ax = b is called ill conditioned if condition number K(A) is large and is computed as :

(1) $K(A) = \| A \| \| A^T \|$

(2) $K(A) = \| A \| \| A^{-1} \|$

(3) $K(A) = | A | | A^{-1} |$

(4) $K(A) = | A | | A^T |$

33. Which one of the following is in general not a convex set :

(1) Subspace of $\mathbb{R}^n$

(2) Ball B $(\bar{x}^0, r) = \left\{ \bar{x} \in \mathbb{R}^n : \left| \bar{x} - \bar{x}^0 \right| < r \right\}$

(3) Intersection of all Convex sets

(4) Union of all convex sets

34. Which one of the following is not a method of solving a non - linear programming problem :

(1) Frank and Wolfe's method.

(2) Penalti function method.

(3) Rosen gradient projection method

(4) Karmarkar's algorithm.

35. The degree of freedom of a dynamical system of n particles with k holonomic constraints is :

(1) 3n - k (2) k - 3n. (3) 3k - n. (4) n - 3k.

36. The equations of motion of a dynamical system in the Lagrangian formulation are in terms of :

(1) two sets of first order differential equations.

(2) two sets of second order differential equations.

(3) one set of second order differential equations.

(4) one set of first order differential equations.

37. q_i and $\dot{q}_i$ respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system then Hamilton's equations of motion are :

(1) $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = \frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$

(3) $\dot{q}_i = -\frac{\partial H}{\partial p_i}$, $\dot{p}_i = \frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i = -\frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$

38. The path which allows a particle to move under the influence of gravity only, in the least possible time, starting at rest from one fixed point to some other lower fixed point is a :

(1) Catenary (2) parabola (3) hyperbola (4) Cycloid

39. If a function $F(q_i, p_i, t)$ is a constant of motion of a mechanical system and H is the Hamiltonian of the system, then

(1) $\frac{\partial H}{\partial t} + [F, H] = 0$ (2) $\frac{\partial H}{\partial t} + [H, F] = 0$

(3) $\frac{\partial F}{\partial t} + [F, H] = 0$ (4) $\frac{\partial F}{\partial t} + [H, F] = 0$

40. If the motion of a fluid is rotational then the Vorticity vector $\vec{\Omega}$ and the angular velocity vector $\vec{w}$ are related by :

(1) $\vec{\Omega}=\frac{1}{2}\vec{w}$

(2) $\vec{\Omega}=2\vec{w}$

(3) $\left|\vec{\Omega}\right|^2=2\left|\vec{w}\right|^2$

(4) $\left|\vec{\Omega}\right|^2=\frac{1}{2}\left|\vec{w}\right|^2$

Short Answer Questions. Attempt any five :

1. Let $(B, \|\ \|)$ be a real Banach space in which norm satisfies the parallelogram law i.e.

$$\| x+y \|^2 + \| x-y \|^2 = 2\left[\| x \|^2 + \| y \|^2\right] \forall x, y \in B$$

If the function < > is defined by

$$< x, y > = \frac{1}{4}\left[\| x+y \|^2 - \| x-y \|^2\right] \forall x, y \in B$$

then show that B is a Hilbert space with < > as an inner product.

2. If a real valued function f is Riemann - Stieltjes integrable with respect to a monotonically increasing function α on [a, b] in $\mathbb{R}$ i. e. $f \in R(\alpha)$

then show that

$$|f| \in R(\alpha)]$$

$$\text{and } \left|\int_a^b f\, d\alpha\right| \leq \int_a^b |f|\, d\alpha .$$

3. Show that a group of prime power order is solvable.

4. Let M be an R - module. If every sub module of M is finitely generated then show that M satisfies ascending chain condition for its sub modules.

5. Prove that a C^rn-atlas on a set M induces a natural topology on M.

6. Let (M^n, g) be an n - dimensional Riemannian manifold. If all membres of the matrix (g_P) of g_P be strictly positive real numbers at a point $p \in M$ then show that

$$Z_n \leq n^2 - 2n$$

Where Z_n is the number of zero elements in the inverse matrix (g_p^{-1}) of (g_P).

7. Find the solution satisfying kuhn - Tucker conditions of the problem,

$$\text{min. } x_1^2 + 2x_2^2,$$

$$\text{subject to } x_1^2 + 2x_2^2 \leq 5,$$

$$2x_1 - 2x_2 = 1.$$

8. Briefly describe finite element methods for solution of differential equations.

9. Verify whether the transformation

$$Q = \log\left(\frac{\sin p}{q}\right),$$

$$p = q \cot p$$

is a contact transformation.

10. For a steady motion of an inviseid incompressible fluid of uniform density under conservative forces, show that the Vorticity $\vec{w}$ and velocity $\vec{q}$ satisfy the equation

$$(\vec{q}.\vec{\nabla})\vec{w} = (\vec{w}.\vec{\nabla})\vec{q}.$$