Mathematics - I

Algebra

Based on Maharashtra State Board Syllabus

Std. X

Target Publications Pvt. Ltd.

Written as per the revised syllabus prescribed by the Maharashtra State Board
of Secondary and Higher Secondary Education, Pune.

STD. X

Mathematics I

Algebra

Fifth Edition: March 2015

Salient Features

- Written as per the new textbook.
- Exhaustive coverage of entire syllabus.
- Topic–wise distribution of all textual questions and practice problems at the beginning of every chapter.
- Precise theory for every topic.
- Covers answers to all textual exercises and problem set.
- Includes additional problems for practice.
- Indicative marks for all problems.
- Comprehensive solution to Question Bank.
- Attractive layout of the content.
- Self evaluative in nature.
- Includes Board Question Papers of 2013, 2014 and 2015

Printed at: **India Printing Works,** Mumbai

TEID : 869

Preface

Algebra is the branch of mathematics which deals with the study of rules of operations and relations and the concepts arising from them. It has wide applications in different fields of science and technology. It deals with concepts like linear equations, quadratic equations, Arithmetic progressions etc. Its application in statistics deals with measures of central tendency, representation of statistical data, etc.

The study of Algebra requires a deep and intrinsic understanding of concepts, terms and formulae. Hence, to ease this task we bring to you **"Std. X: Algebra",** a complete and thorough guide extensively drafted to boost the students confidence. The question answer format of this book helps the student to understand and grasp each and every concept thoroughly. The book is based on the new text book and covers the entire syllabus. At the beginning of every chapter, topic–wise distribution of all textual questions and practice problems has been provided for simpler understanding of different types of questions. The book contains answers to textual exercises, problems sets and Question bank. It also includes additional questions for practice. Graphs are drawn with proper scale and pie diagrams are with correct measures. Another feature of the book is its layout which is attractive and inspires the student to read.

Marks are provided for each and every problem. However, marks mentioned are indicative and are subject to change as per Maharashtra State Board's discretion.

There is always room for improvement and hence we welcome all suggestions and regret any errors that may have occurred in the making of this book.

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

Your's faithfully,

Publisher

MARKING SCHEME

Marking Scheme (for March 2014 exam and onwards)

Written Exam

Algebra	40 Marks	Time: 2 hrs.
Geometry	40 Marks	Time: 2 hrs.
* Internal Assessment	20 Marks	
Total	**100 Marks**	

* Internal Assessment		
Home Assignment:	10 Marks	5-5 Home assignment for Algebra and Geometry of 10 marks each would be given. Marks obtained out of 100 would be converted to marks out of 10.
Test of multiple choice question:	10 Marks	Depending upon the entire syllabus, internal test for Algebra and Geometry with 20 marks each would be taken at the end of second semester. Marks obtained out of 40 would be converted to marks out of 10.
Total	**20 marks**	

ALGEBRA AND GEOMETRY

Mark Wise Distribution of Questions

	Marks	Marks with Option
6 sub questions of 1 mark each: Attempt any 5	05	06
6 sub questions of 2 marks each: Attempt any 4	08	12
5 sub questions of 3 marks each: Attempt any 3	09	15
3 sub questions of 4 marks each: Attempt any 2	08	12
3 sub questions of 5 marks each: Attempt any 2	10	15
Total:	**40**	**60**

Weightage to Types of Questions

Sr. No.	Type of Questions	Marks	Percentage of Marks
1.	Very short answer	06	10
2.	Short answer	27	45
3 .	Long answer	27	45
	Total:	**60**	**100**

Weightage to Objectives

Sr. No	Objectives	Algebra Percentage marks	Geometry Percentage marks
1.	Knowledge	15	15
2.	Understanding	15	15
3.	Application	60	50
4.	Skill	10	20
	Total:	100	100

Unit wise Distribution: Algebra

Sr. No.	Unit	Marks with option
1.	Arithmetic Progression	12
2.	Quadratic equations	12
3.	Linear equation in two variables	12
4.	Probability	10
5.	Statistics – I	06
6.	Statistics – II	08
	Total:	**60**

Unit wise Distribution: Geometry

Sr. No.	Unit	Marks with option
1.	Similarity	12
2.	Circle	10
3.	Geometric Constructions	10
4.	Trigonometry	10
5.	Co-ordinate Geometry	08
6.	Mensuration	10
	Total:	**60**

Contents

In this book, we have deliberately included the Board Question Papers for 2013 although it follows the old paper pattern.

***** marked questions in the above board papers are deleted from the new syllabus as compared to the earlier syllabus.

01 Arithmetic Progression

Type of Problems	Exercise	Q. Nos.
Find the next two/three/four terms for the given sequence.	1.1	Q.1
	Practice Problems (Based on Exercise 1.1)	Q.1
For the given n^{th} term find first two/three/four/five terms of the sequence.	1.1	Q.2
	Practice Problems (Based on Exercise 1.1)	Q.2 (i), 3
Find the first two/three terms of the sequence for which S_n is given.	1.1	Q. 3
	Practice Problems (Based on Exercise 1.1)	Q.2 (ii)
Identify Arithmetic Progressions for the given list of numbers.	1.2	Q.1
	Practice Problems (Based on Exercise 1.2)	Q.4 (check for A.P.), 5
	1.3	Q.6
Find the terms of A.P. by using the given values of 'a' and 'd'.	1.2	Q.2
	Practice Problems (Based on Exercise 1.2)	Q.6, 7
Find the n terms using $t_n = a + (n - 1)d$ /first term/common difference, Find the number of terms.	Practice Problems (Based on Exercise 1.2)	Q.4(d, t_n)
	1.3	Q.1, 2, 3, 4, 5, 7, 8
	Practice Problems (Based on Exercise 1.3)	Q.8, 9, 10, 11, 12
	Problem set-1	Q.1, 2
Find the sum of n terms using $S_n = \frac{n}{2}[2a + (n - 1)d]$, Find the number of terms.	1.4	Q.1, 2, 3, 4, 5, 6, 7, 8
	Practice Problems (Based on Exercise 1.4)	Q.13, 14, 15, 16, 17
	Problem set-1	Q.3, 4, 5, 6, 7, 8
Consecutive Terms of an A.P.	1.5	Q.1, 2, 3, 4
	Practice Problems (Based on Exercise 1.5)	Q.18, 19
Applications of A.P.	1.6	Q.1, 2, 3, 4, 5, 6, 7, 8, 9, 10
	Practice Problems (Based on Exercise 1.6)	Q.20, 21, 22, 23

Introduction

We have observed different relations or specific patterns in some numbers while studying the operations on numbers like addition, subtraction, multiplication and division.

Examples:

i. 1, 2, 3, …
This is a collection of all the positive integers in which the difference between two consecutive numbers is 1.

ii. 1, 3, 5, 7, 9, …
This is a collection of all the odd natural numbers in which the difference between two consecutive numbers is 2.

Such patterns are also observed in our day-to-day life.

1.1 Sequence

a. Definition of Sequence:
A sequence is a collection of numbers arranged in a definite order according to some definite rule.

Examples:

i. 1, 4, 9, 16, … (Collection of perfect squares of natural numbers)
ii. 2, 4, 6, 8, 10, … (Collection of positive even integers)
iii. 1, 3, 5, 7, … (Collection of positive odd integers)
iv. –2, –4, –6, … (Collection of negative even integers)
v. 5, 10, 15, 20, … 100 (Collection of first 20 integral multiples of 5)

b. Term:
Each number in the sequence is called a term of the sequence.
The number in the first position is called the first term and is denoted by t_1.
The number in the second position is called the second term and is denoted by t_2.
Similarly, the number in the 'n^{th}' position of the sequence is called the n^{th} term and is denoted by t_n.
If t_n is given, then a sequence can be formed.
Example: If $t_n = 2n + 1$, then
by putting n = 1, 2, 3, …, we get
$t_1 = 2 \times 1 + 1 = 3$,
$t_2 = 2 \times 2 + 1 = 5$,
$t_3 = 2 \times 3 + 1 = 7$ and so on

$\therefore$ The sequence can be written as 3, 5, 7,...

c. Sum of the first n terms of a sequence:
If a sequence consists of n terms, then its sum can be represented as
$S_n = t_1 + t_2 + t_3 + \ldots + t_n$
Putting n = 1, 2, 3, … , we get
$S_1 = t_1$
$S_2 = t_1 + t_2$
$S_3 = t_1 + t_2 + t_3$
⋮
$S_n = t_1 + t_2 + t_3 + \ldots + t_n$

d. n^{th} term from S_n:
If S_n is given, then t_n can also be found out.
Since, $S_n = t_1 + t_2 + t_3 + \ldots + t_n$
$S_1 = t_1$
$S_2 = t_1 + t_2$
$S_3 = t_1 + t_2 + t_3$

$\therefore$ $S_2 - S_1 = (t_1 + t_2) - t_1 = t_2$
$S_3 - S_2 = (t_1 + t_2 + t_3) - (t_1 + t_2) = t_3$
Similarly,
$S_n - S_{n-1}$
$= (t_1 + t_2 + t_3 + .. + t_n) - (t_1 + t_2 + t_3 + .. + t_{n-1})$
$= t_n$

$\therefore$ $t_n = S_n - S_{n-1}$, for $n > 1$

1.2 Types of Sequences

There are two types of sequences:

a. Finite Sequence: If the number of terms in a sequence is finite (countable) i.e. if there is an end of terms in the sequence then it is called a Finite Sequence.

Examples:
i. 1, 2, 3, … 20.
ii. 4, 6, 8, … 50.
iii. 1, 4, 9, 16, … 100.

b. Infinite Sequence: If the number of terms in a sequence is infinite (uncountable) i.e. there is no end of terms in the sequence then it is called an Infinite Sequence.

Examples:
i. 1, 3, 5, 7, …
ii. 5, 10, 15, …
iii. 2, 4, 6, 8, …

<u>Differences between a Sequence and Set:</u>

	Sequence	*Set*
1.	The elements of a sequence are in a specific order, so they cannot be rearranged.	Elements are at random, so they can be rearranged.
2.	The same value can appear many times.	Any value can appear only once.

Exercise 1.1

1. For each of the following sequences, find the next four terms: [1 mark each]

i. 1, 2, 4, 7, 11, … [Oct 13]

ii. 3, 9, 27, 81, …

iii. 1, 3, 7, 15, 31, … [Mar 13]

iv. 192, –96, 48, – 24, …

v. 2, 6, 12, 20, 30, …

vi. 0.1, 0.01, 0.001, 0.0001, …

vii. 2, 5, 8, 11, …

viii. –25, –23, –21, –19, …

ix. 2, 4, 8, 16, … [Oct 12]

x. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots$

Solution:

i. The given sequence is 1, 2, 4, 7, 11, ...

Here, $t_1 = 1, t_2 = 2, t_3 = 4, t_4 = 7, t_5 = 11$

The differences between two consecutive terms are 1, 2, 3, 4, ...

∴ $t_6 = 11 + 5 = 16$

$t_7 = 16 + 6 = 22$

$t_8 = 22 + 7 = 29$

$t_9 = 29 + 8 = 37$

∴ **The next four terms are 16, 22, 29 and 37.**

---

ii. The given sequence is 3, 9, 27, 81, …

Here, $t_1 = 3, t_2 = 9, t_3 = 27, t_4 = 81$

This sequence is in the form $3^1, 3^2, 3^3, 3^4$

∴ $t_5 = 3^5 = 243$

$t_6 = 3^6 = 729$

$t_7 = 3^7 = 2187$

$t_8 = 3^8 = 6561$

∴ **The next four terms are 243, 729, 2187 and 6561.**

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iii. The given sequence is 1, 3, 7, 15, 31, …

Here, $t_1 = 1, t_2 = 3, t_3 = 7, t_4 = 15, t_5 = 31$

The differences between two consecutive terms are 2, 4, 8, 16, …

i.e. $2^1, 2^2, 2^3, 2^4, \ldots$

∴ $t_6 = 31 + 2^5 = 31 + 32 = 63$

$t_7 = 63 + 2^6 = 63 + 64 = 127$

$t_8 = 127 + 2^7 = 127 + 128 = 255$

$t_9 = 255 + 2^8 = 255 + 256 = 511$

∴ **The next four terms are 63, 127, 255 and 511.**

iv. The given sequence is 192, –96, 48, – 24, …

Here, $t_1 = 192, t_2 = -96, t_3 = 48, t_4 = -24$

The common ratio of two consecutive terms is $-\frac{1}{2}$

∴ $t_5 = -24 \times -\frac{1}{2} = 12$

$t_6 = 12 \times -\frac{1}{2} = -6$

$t_7 = -6 \times -\frac{1}{2} = 3$

$t_8 = 3 \times -\frac{1}{2} = -\frac{3}{2}$

∴ **The next four terms are 12, –6, 3 and** $-\frac{3}{2}$**.**

---

v. The given sequence is 2, 6, 12, 20, 30, …

Here, $t_1 = 2, t_2 = 6, t_3 = 12, t_4 = 20, t_5 = 30$

The differences between two consecutive terms are 4, 6, 8, 10...

∴ $t_6 = 30 + 12 = 42$

$t_7 = 42 + 14 = 56$

$t_8 = 56 + 16 = 72$

$t_9 = 72 + 18 = 90$

∴ **The next four terms are 42, 56, 72 and 90.**

---

vi. The given sequence is 0.1, 0.01, 0.001, 0.0001, …

Here, $t_1 = 0.1, t_2 = 0.01, t_3 = 0.001, t_4 = 0.0001$.

The common ratio of two consecutive terms is 0.1

∴ $t_5 = 0.0001 \times 0.1 = 0.00001$

$t_6 = 0.00001 \times 0.1 = 0.000001$

$t_7 = 0.000001 \times 0.1 = 0.0000001$

$t_8 = 0.0000001 \times 0.1 = 0.00000001$

∴ **The next four terms are 0.00001, 0.000001, 0.0000001 and 0.00000001.**

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vii. The given sequence is 2, 5, 8, 11, …

Here, $t_1 = 2, t_2 = 5, t_3 = 8, t_4 = 11$

The common difference between two consecutive terms is 3

∴ $t_5 = 11 + 3 = 14$

$t_6 = 14 + 3 = 17$

$t_7 = 17 + 3 = 20$

$t_8 = 20 + 3 = 23$

∴ **The next four terms are 14, 17, 20 and 23.**

viii. The given sequence is –25, –23, –21, –19, …
Here, $t_1 = -25$, $t_2 = -23$, $t_3 = -21$, $t_4 = -19$
The common difference between two consecutive terms is 2

$\therefore$ $t_5 = -19 + 2 = -17$
$t_6 = -17 + 2 = -15$
$t_7 = -15 + 2 = -13$
$t_8 = -13 + 2 = -11$

$\therefore$ **The next four terms are –17, –15, –13 and –11.**

---

ix. The given sequence is 2, 4, 8, 16, …
Here, $t_1 = 2$, $t_2 = 4$, $t_3 = 8$, $t_4 = 16$
The common ratio of two consecutive terms is 2

$\therefore$ $t_5 = 16 \times 2 = 32$
$t_6 = 32 \times 2 = 64$
$t_7 = 64 \times 2 = 128$
$t_8 = 128 \times 2 = 256$

$\therefore$ **The next four terms are 32, 64, 128 and 256.**

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x. The given sequence is $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots$
Here, $t_1 = \frac{1}{2}$, $t_2 = \frac{1}{6}$, $t_3 = \frac{1}{18}$, $t_4 = \frac{1}{54}$
The common ratio of two consecutive terms is $\frac{1}{3}$

$\therefore$ $t_5 = \frac{1}{54} \times \frac{1}{3} = \frac{1}{162}$
$t_6 = \frac{1}{162} \times \frac{1}{3} = \frac{1}{486}$
$t_7 = \frac{1}{486} \times \frac{1}{3} = \frac{1}{1458}$
$t_8 = \frac{1}{1458} \times \frac{1}{3} = \frac{1}{4374}$

$\therefore$ **The next four terms are $\frac{1}{162}, \frac{1}{486}, \frac{1}{1458}$ and $\frac{1}{4374}$.**

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2. Find the first five terms of the following sequences, whose 'n^th^' terms are given:
[2 marks each]

i. $t_n = 4n - 3$ **[Mar 13]**
ii. $t_n = 2n - 5$ **[Mar 13]**
iii. $t_n = n + 2$ **[Mar 13]**
iv. $t_n = n^2 - 2n$ **[Mar 13]**
v. $t_n = n^3$
vi. $t_n = \frac{1}{n+1}$

Solution:

i. Given, $t_n = 4n - 3$
For n = 1, $t_1 = 4(1) - 3 = 4 - 3 = 1$
For n = 2, $t_2 = 4(2) - 3 = 8 - 3 = 5$
For n = 3, $t_3 = 4(3) - 3 = 12 - 3 = 9$
For n = 4, $t_4 = 4(4) - 3 = 16 - 3 = 13$
For n = 5, $t_5 = 4(5) - 3 = 20 - 3 = 17$

$\therefore$ **The first five terms are 1, 5, 9, 13 and 17.**

---

ii. Given, $t_n = 2n - 5$
For n = 1, $t_1 = 2(1) - 5 = -3$
For n = 2, $t_2 = 2(2) - 5 = -1$
For n = 3, $t_3 = 2(3) - 5 = 1$
For n = 4, $t_4 = 2(4) - 5 = 3$
For n = 5, $t_5 = 2(5) - 5 = 5$

$\therefore$ **The first five terms are –3, –1, 1, 3 and 5.**

---

iii. Given, $t_n = n + 2$
For n = 1, $t_1 = 1 + 2 = 3$
For n = 2, $t_2 = 2 + 2 = 4$
For n = 3, $t_3 = 3 + 2 = 5$
For n = 4, $t_4 = 4 + 2 = 6$
For n = 5, $t_5 = 5 + 2 = 7$

$\therefore$ **The first five terms are 3, 4, 5, 6 and 7.**

---

iv. Given, $t_n = n^2 - 2n$
For n = 1, $t_1 = (1)^2 - 2(1) = 1 - 2 = -1$
For n = 2, $t_2 = (2)^2 - 2(2) = 4 - 4 = 0$
For n = 3, $t_3 = (3)^2 - 2(3) = 9 - 6 = 3$
For n = 4, $t_4 = (4)^2 - 2(4) = 16 - 8 = 8$
For n = 5, $t_5 = (5)^2 - 2(5) = 25 - 10 = 15$

$\therefore$ **The first five terms are –1, 0, 3, 8 and 15.**

---

v. Given, $t_n = n^3$
For n = 1, $t_1 = (1)^3 = 1$
For n = 2, $t_2 = (2)^3 = 8$
For n = 3, $t_3 = (3)^3 = 27$
For n = 4, $t_4 = (4)^3 = 64$
For n = 5, $t_5 = (5)^3 = 125$

$\therefore$ **The first five terms are 1, 8, 27, 64 and 125.**

---

vi. Given, $t_n = \frac{1}{n+1}$
For n = 1, $t_1 = \frac{1}{1+1} = \frac{1}{2}$
For n = 2, $t_2 = \frac{1}{2+1} = \frac{1}{3}$

For n = 3, $t_3 = \frac{1}{3+1} = \frac{1}{4}$

For n = 4, $t_4 = \frac{1}{4+1} = \frac{1}{5}$

For n = 5, $t_5 = \frac{1}{5+1} = \frac{1}{6}$

$\therefore$ **The first five terms are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$.**

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3. Find the first three terms of the sequences for which S_n is given below: **[2 marks each]**

i. $S_n = n^2(n+1)$

ii. $S_n = \frac{n^2(n+1)^2}{4}$

iii. $S_n = \frac{n(n+1)(2n+1)}{6}$

Solution:

i. Given, $S_n = n^2(n+1)$

For n = 1, $S_1 = (1)^2(1+1) = 1 \times 2 = 2$

For n = 2, $S_2 = (2)^2(2+1) = 4 \times 3 = 12$

For n = 3, $S_3 = (3)^2(3+1) = 9 \times 4 = 36$

Now, $t_1 = S_1$ and $t_n = S_n - S_{n-1}$, for n > 1

$\therefore$ $t_1 = 2$

$t_2 = S_2 - S_1 = 12 - 2 = 10$

$t_3 = S_3 - S_2 = 36 - 12 = 24$

$\therefore$ **The first three terms are 2, 10 and 24.**

---

ii. Given, $S_n = \frac{n^2(n+1)^2}{4}$

For n = 1, $S_1 = \frac{(1)^2(1+1)^2}{4} = \frac{1\times 2^2}{4} = \frac{1\times 4}{4} = 1$

For n = 2, $S_2 = \frac{(2)^2(2+1)^2}{4} = \frac{4\times 3^2}{4} = \frac{4\times 9}{4} = 9$

For n = 3, $S_3 = \frac{(3)^2(3+1)^2}{4} = \frac{9\times 4^2}{4} = \frac{9\times 16}{4} = 36$

Now, $t_1 = S_1$ and $t_n = S_n - S_{n-1}$, for n > 1

$\therefore$ $t_1 = 1$

$t_2 = S_2 - S_1 = 9 - 1 = 8$

$t_3 = S_3 - S_2 = 36 - 9 = 27$

$\therefore$ **The first three terms are 1, 8 and 27.**

---

iii. Given, $S_n = \frac{n(n+1)(2n+1)}{6}$

For n = 1, $S_1 = \frac{1(1+1)(2\times 1+1)}{6} = \frac{1\times 2\times 3}{6} = 1$

For n = 2, $S_2 = \frac{2(2+1)(2\times 2+1)}{6} = \frac{2\times 3\times 5}{6} = 5$

For n = 3, $S_3 = \frac{3(3+1)(2\times 3+1)}{6} = \frac{3\times 4\times 7}{6} = 14$

Now, $t_1 = S_1$ and $t_n = S_n - S_{n-1}$, for n > 1

$\therefore$ $t_1 = 1$

$t_2 = S_2 - S_1 = 5 - 1 = 4$

$t_3 = S_3 - S_2 = 14 - 5 = 9$

$\therefore$ **The first three terms are 1, 4 and 9.**

1.3 Progressions

a. Definition:

A progression is a special type of sequence in which the relationship between any two consecutive terms is the same.

Examples:

i. 3, 6, 9, 12, … 27

Here, $t_2 - t_1 = t_3 - t_2 = \ldots = 3$ = constant

ii. 2, 4, 8, 16, …

Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \ldots = 2$ = constant

iii. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

Here, $\frac{1}{t_2} - \frac{1}{t_1} = \frac{1}{t_3} - \frac{1}{t_2} = \ldots = 2$ = constant

Hence, each example represents a progression.

Think it over

The following are not progressions. Explain why?

i. 1, 4, 9, 16, …

ii. 3, 5, 8, 13, …

Solution:

i. The given sequence is 1, 4, 9, 16, …

Here, 4 – 1 = 3

9 – 4 = 5

16 – 9 = 7

Since, there is no fixed (same) relationship between any two consecutive terms, the given sequence is not a progression.

ii. The given sequence is 3, 5, 8, 13, …

Here, 5 – 3 = 2

8 – 5 = 3

13 – 8 = 5

Since, there is no fixed (same) relationship between any two consecutive terms, the given sequence is not a progression.

b. Types of Progressions:

There are three types of progressions:

i. Arithmetic progression (A.P.)

ii. Geometric progression (G.P.)

iii. Harmonic progression (H.P.)

We shall study only A.P. here.

1.4 Arithmetic Progression (A.P.)

Definition: An Arithmetic Progression is a sequence in which the difference between any two consecutive terms is constant. This constant is called the common difference of that A.P.

Examples:

i. 10, 20, 30, 40, ...

Here, $t_2 - t_1 = t_3 - t_2 = \dots = 10$ = constant

ii. 18, 16, 14, ...

Here, $t_2 - t_1 = t_3 - t_2 = \dots = -2$ = constant

iii. $\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$

Here, $t_2 - t_1 = t_3 - t_2 = \dots = \frac{1}{5}$ = constant

Note:

i. If $t_{n+1} - t_n$ is constant, for all $n \in N$, then the sequence is an A.P.

ii. In an A.P., the first term is denoted by 'a' and the common difference is denoted by 'd'.

iii. The value of d may be positive, negative or zero.

General representation of an A.P.

If $t_1, t_2, t_3, t_4 \dots$ are terms of an A.P., then

Now, $t_1 = a$

$t_2 = t_1 + d = a + d = a + (2 - 1)d$

$t_3 = t_2 + d = a + d + d = a + 2d = a + (3 - 1)d$

.

.

.

$\therefore$ $t_n = t_{n-1} + d = a + (n - 1)d$

Think it over

The triplets 1, 25, 49 form an A.P

Can you find some more such triplets?

Solution:

Triplets like 2, 4, 6 and 17, 14, 11 form an A.P.

Exercise 1.2

1. Which of the following lists of numbers are Arithmetic Progressions? Justify.

[2 marks each]

i. 1, 3, 6, 10, … [Mar 14]

ii. 3, 5, 7, 9, 11, …

iii. 1, 4, 7, 10, … [Mar 14, 15]

iv. 3, 6, 12, 24, …

v. 22, 26, 28, 31, …

vi. 0.5, 2, 3.5, 5, …

vii. 4, 3, 2, 1, …

viii. −10, −13, −16, −19, …

Solution:

i. The given list of numbers is 1, 3, 6, 10, …

Here, $t_1 = 1, t_2 = 3, t_3 = 6, t_4 = 10$

$\therefore$ $t_2 - t_1 = 3 - 1 = 2$

$t_3 - t_2 = 6 - 3 = 3$

$t_4 - t_3 = 10 - 6 = 4$

$\therefore$ $t_2 - t_1 \neq t_3 - t_2 \neq t_4 - t_3$

The difference between two consecutive terms is not constant.

$\therefore$ **The given list of numbers is not an A.P.**

---

ii. The given list of numbers is 3, 5, 7, 9, 11, ...

Here, $t_1 = 3, t_2 = 5, t_3 = 7, t_4 = 9, t_5 = 11$

$\therefore$ $t_2 - t_1 = 5 - 3 = 2$

$t_3 - t_2 = 7 - 5 = 2$

$t_4 - t_3 = 9 - 7 = 2$

$t_5 - t_4 = 11 - 9 = 2$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = \dots = 2$ = constant

The difference between two consecutive terms is constant.

$\therefore$ **The given list of numbers is an A.P.**

---

iii. The given list of numbers is 1, 4, 7, 10, …

Here, $t_1 = 1, t_2 = 4, t_3 = 7, t_4 = 10$

$\therefore$ $t_2 - t_1 = 4 - 1 = 3$

$t_3 - t_2 = 7 - 4 = 3$

$t_4 - t_3 = 10 - 7 = 3$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = \dots = 3$ = constant

The difference between two consecutive terms is constant.

$\therefore$ **The given list of numbers is an A.P.**

iv. The given list of numbers is 3, 6, 12, 24, …
Here, $t_1 = 3$, $t_2 = 6$, $t_3 = 12$, $t_4 = 24$

$\therefore$ $t_2 - t_1 = 6 - 3 = 3$

$t_3 - t_2 = 12 - 6 = 6$

$t_4 - t_3 = 24 - 12 = 12$

$\therefore$ $t_2 - t_1 \neq t_3 - t_2 \neq t_4 - t_3$

The difference between two consecutive terms is not constant.

$\therefore$ **The given list of numbers is not an A.P.**

---

v. The given list of numbers is 22, 26, 28, 31, …
Here, $t_1 = 22$, $t_2 = 26$, $t_3 = 28$, $t_4 = 31$

$\therefore$ $t_2 - t_1 = 26 - 22 = 4$

$t_3 - t_2 = 28 - 26 = 2$

$t_4 - t_3 = 31 - 28 = 3$

$\therefore$ $t_2 - t_1 \neq t_3 - t_2 \neq t_4 - t_3$

The difference between two consecutive terms is not constant.

$\therefore$ **The given list of numbers is not an A.P.**

---

vi. The given list of numbers is 0.5, 2, 3.5, 5, …
Here, $t_1 = 0.5$, $t_2 = 2$, $t_3 = 3.5$, $t_4 = 5$

$\therefore$ $t_2 - t_1 = 2 - 0.5 = 1.5$

$t_3 - t_2 = 3.5 - 2 = 1.5$

$t_4 - t_3 = 5 - 3.5 = 1.5$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = 1.5 =$ constant

The difference between two consecutive terms is constant.

$\therefore$ **The given list of numbers is an A.P.**

---

vii. The given list of numbers is 4, 3, 2, 1, …
Here, $t_1 = 4$, $t_2 = 3$, $t_3 = 2$, $t_4 = 1$

$\therefore$ $t_2 - t_1 = 3 - 4 = -1$

$t_3 - t_2 = 2 - 3 = -1$

$t_4 - t_3 = 1 - 2 = -1$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = -1 =$ constant

The difference between two consecutive terms is constant.

$\therefore$ **The given list of numbers is an A.P.**

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viii. The given list of numbers is
−10, −13, −16, −19, …
Here, $t_1 = -10$, $t_2 = -13$, $t_3 = -16$, $t_4 = -19$

$\therefore$ $t_2 - t_1 = -13 - (-10) = -3$

$t_3 - t_2 = -16 - (-13) = -3$

$t_4 - t_3 = -19 - (-16) = -3$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = -3 =$ constant

The difference between two consecutive terms is constant.

$\therefore$ **The given list of numbers is an A.P.**

2. Write the first five terms of the following Arithmetic Progressions where, the common difference 'd' and the first term 'a' are given: [2 marks each]

i. a = 2, d = 2.5 ii. a = 10, d = – 3
iii. a = 4, d = 0 iv. a = 5, d = 2
v. a = 3, d = 4 [Mar 12]
vi. a = 6, d = 6

Solution :

i. Given, a = 2, d = 2.5

$\therefore$ $t_1 = a = 2$

$t_2 = t_1 + d = 2 + 2.5 = 4.5$

$t_3 = t_2 + d = 4.5 + 2.5 = 7$

$t_4 = t_3 + d = 7 + 2.5 = 9.5$

$t_5 = t_4 + d = 9.5 + 2.5 = 12$

$\therefore$ **The first five terms of the A.P. are 2, 4.5, 7, 9.5 and 12.**

---

ii. Given, a = 10, d = – 3

$\therefore$ $t_1 = a = 10$

$t_2 = t_1 + d = 10 + (-3) = 10 - 3 = 7$

$t_3 = t_2 + d = 7 + (-3) = 7 - 3 = 4$

$t_4 = t_3 + d = 4 + (-3) = 4 - 3 = 1$

$t_5 = t_4 + d = 1 + (-3) = 1 - 3 = -2$

$\therefore$ **The first five terms of the A.P. are 10, 7, 4, 1 and –2.**

---

iii. Given, a = 4, d = 0

$\therefore$ $t_1 = a = 4$

$t_2 = t_1 + d = 4 + 0 = 4$

$t_3 = t_2 + d = 4 + 0 = 4$

$t_4 = t_3 + d = 4 + 0 = 4$

$t_5 = t_4 + d = 4 + 0 = 4$

$\therefore$ **The first five terms of the A.P. are 4, 4, 4, 4 and 4.**

---

iv. Given, a = 5, d = 2

$\therefore$ $t_1 = a = 5$

$t_2 = t_1 + d = 5 + 2 = 7$

$t_3 = t_2 + d = 7 + 2 = 9$

$t_4 = t_3 + d = 9 + 2 = 11$

$t_5 = t_4 + d = 11 + 2 = 13$

$\therefore$ **The first five terms of the A.P. are 5, 7, 9, 11 and 13.**

---

v. Given, a = 3, d = 4

$\therefore$ $t_1 = a = 3$

$t_2 = t_1 + d = 3 + 4 = 7$

$t_3 = t_2 + d = 7 + 4 = 11$

$t_4 = t_3 + d = 11 + 4 = 15$

$t_5 = t_4 + d = 15 + 4 = 19$

$\therefore$ **The first five terms of the A.P. are 3, 7, 11, 15 and 19.**

vi. Given, a = 6, d = 6

$\therefore$ $t_1 = a = 6$

$t_2 = t_1 + d = 6 + 6 = 12$

$t_3 = t_2 + d = 12 + 6 = 18$

$t_4 = t_3 + d = 18 + 6 = 24$

$t_5 = t_4 + d = 24 + 6 = 30$

$\therefore$ **The first five terms of the A.P. are 6, 12, 18, 24 and 30.**

The General (n^{th}) term of an A.P.

Consider the A.P. a, a + d, a + 2d, a + 3d, ...

Here, $t_1 = a$ … (i)

$t_2 - t_1 = d$ … (ii)

$t_3 - t_2 = d$ … (iii)

$t_4 - t_3 = d$ … (iv)

.

.

.

$t_{n-1} - t_{n-2} = d$ … (n – 1)

$t_n - t_{n-1} = d$ … (n)

Adding all the above equations, we get

$t_1 + (t_2 - t_1) + (t_3 - t_2) + \ldots + (t_n - t_{n-1})$

$= a + d + d + \ldots + d$ [d is added (n – 1) times]

$\therefore$ $t_n = a + (n - 1)\,d$.

This is the General (n^{th}) term of an A.P. with first term ‘a’ and common difference ‘d’.

Note: For an A.P., if d = 0 then the sequence is a constant sequence.

Exercise 1.3

1. Find the twenty fifth term of the A.P.: 12, 16, 20, 24, … [2 marks]

Solution:

The given A.P. is 12, 16, 20, 24, ...

Here, a = 12, d = 16 – 12 = 4

Since, $t_n = a + (n - 1)d$

$\therefore$ $t_{25} = 12 + (25 - 1)4$

$= 12 + 24 \times 4$

$= 12 + 96$

$\therefore$ $t_{25} = 108$

$\therefore$ **The twenty fifth term of the given A.P. is 108.**

2. Find the eighteenth term of the A.P.: 1, 7, 13, 19, … [2 marks]

Solution:

The given A.P. is 1, 7, 13, 19, ...

Here, a = 1, d = 7 – 1 = 6

Since, $t_n = a + (n - 1)d$

$\therefore$ $t_{18} = 1 + (18 - 1)6$

$= 1 + 17 \times 6$

$= 1 + 102$

$\therefore$ $t_{18} = 103$

$\therefore$ **The eighteenth term of the given A.P. is 103.**

3. Find t_n for an Arithmetic Progression where $t_3 = 22$, $t_{17} = -20$. [4 marks]

Solution:

Given, $t_3 = 22$, $t_{17} = -20$

Since, $t_n = a + (n - 1)d$

$\therefore$ $t_3 = a + (3 - 1)d$

$\therefore$ $22 = a + 2d$

$\therefore$ $a + 2d = 22$ ...(i)

Also, $t_{17} = a + (17 - 1)d$

$\therefore$ $-20 = a + 16d$

$\therefore$ $a + 16d = -20$ ...(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{rcrcr} a & + & 16d & = & -20 \\ a & + & 2d & = & 22 \\ (-) & & (-) & & (-) \\ \hline & & 14d & = & -42 \end{array}$$

$\therefore$ $d = \dfrac{-42}{14}$

$\therefore$ $d = -3$

Substituting d = –3 in equation (i), we get

$a + 2(-3) = 22$

$\therefore$ $a - 6 = 22$

$\therefore$ $a = 22 + 6$

$\therefore$ $a = 28$

$\therefore$ $t_n = a + (n - 1)d$

$\therefore$ $t_n = 28 + (n - 1)(-3)$

$= 28 - 3n + 3$

$\therefore$ **$t_n = -3n + 31$**

4. For an A.P., if $t_4 = 12$ and $d = -10$, then find its general term. [3 marks]

Solution:

Given, $t_4 = 12$, $d = -10$

Since, $t_n = a + (n - 1)d$

$\therefore$ $t_4 = a + (4 - 1)(-10)$

$\therefore$ $12 = a + 3 \times (-10)$

$\therefore$ $12 = a - 30$

$\therefore$ $12 + 30 = a$

$\therefore$ $a = 42$

Now, $t_n = a + (n - 1)d$

$\therefore$ $t_n = 42 + (n - 1)(-10)$

$\therefore$ $t_n = 42 - 10n + 10$

$\therefore$ $t_n = -10n + 52$

$\therefore$ **The general term t_n is $-10n + 52$.**

5. Given the following sequence, determine whether it is arithmetic progression or not. If it is an Arithmetic Progression, find its general term:
$-5, 2, 9, 16, 23, 30, \ldots$ [3 marks]

Solution:

The given sequence is $-5, 2, 9, 16, 23, 30, \ldots$
Here, $t_1 = -5$, $t_2 = 2$, $t_3 = 9$, $t_4 = 16$, $t_5 = 23$, $t_6 = 30$

$\therefore$ $t_2 - t_1 = 2 - (-5) = 2 + 5 = 7$
$t_3 - t_2 = 9 - 2 = 7$
$t_4 - t_3 = 16 - 9 = 7$
$t_5 - t_4 = 23 - 16 = 7$
$t_6 - t_5 = 30 - 23 = 7$

Since, the common difference i.e.,7 is a constant, the given sequence is an A.P.
Here, $a = -5$, $d = 7$
Since, $t_n = a + (n - 1)d$
$\therefore$ $t_n = -5 + (n - 1)\,7$
$= -5 + 7n - 7$
$\therefore$ $t_n = 7n - 12$
$\therefore$ **The given sequence is an A.P. and its general term is $7n - 12$.**

---

6. Given the following sequence, determine whether it is an arithmetic progression or not. If it is an Arithmetic Progression, find its general term.
$5, 2, -2, -6, -11, \ldots$ [2 marks]

Solution:

The given sequence is $5, 2, -2, -6, -11, \ldots$
Here, $t_1 = 5$, $t_2 = 2$, $t_3 = -2$, $t_4 = -6$, $t_5 = -11$
$\therefore$ $t_2 - t_1 = 2 - 5 = -3$
$t_3 - t_2 = -2 - 2 = -4$
$\therefore$ $t_2 - t_1 \neq t_3 - t_2$
The difference between two consecutive terms is not constant.
$\therefore$ **The given sequence is not an A.P.**

---

7. How many three digit natural numbers are divisible by 4? [4 marks]

Solution:

Let n be the number of 3 digit natural numbers divisible by 4.
The three digit natural numbers which are divisible by 4 are 100, 104, 108, ..., 996.
This sequence is an A.P. with $a = 100$, $d = 4$, $t_n = 996$
But, $t_n = a + (n - 1)d$
$\therefore$ $996 = 100 + (n - 1)4$
$\therefore$ $996 - 100 = (n - 1)4$
$\therefore$ $896 = (n - 1)4$
$\therefore$ $\frac{896}{4} = n - 1$
$\therefore$ $224 = n - 1$
$\therefore$ $n = 224 + 1 = 225$
$\therefore$ **There are 225 three digit natural numbers divisible by 4.**

---

8. The 11^{th} term and the 21^{st} term of an A.P. are 16 and 29 respectively. Find
i. the 1^{st} term and the common difference
ii. the 34^{th} term
iii. 'n' such that $t_n = 55$. [5 marks]

Solution:

Given, $t_{11} = 16$, $t_{21} = 29$

i. Since, $t_n = a + (n - 1)d$
$\therefore$ $t_{11} = a + (11 - 1)d$
$\therefore$ $16 = a + 10d$
$\therefore$ $a + 10d = 16$ ... (i)
Also, $t_{21} = a + (21 - 1)d$
$\therefore$ $29 = a + 20d$
$\therefore$ $a + 20d = 29$ ... (ii)
Subtracting (i) from (ii), we get

$$\begin{array}{rcl} a + 20d &=& 29 \\ a + 10d &=& 16 \\ (-)\ \ (-) && (-) \\ \hline 10d &=& 13 \end{array}$$

$\therefore$ $d = \frac{13}{10}$

Substituting $d = \frac{13}{10}$ in (i), we get

$a + 10 \times \frac{13}{10} = 16$
$\therefore$ $a + 13 = 16$
$\therefore$ $a = 16 - 13$
$\therefore$ $a = 3$
$\therefore$ $a = 3$ and $d = \frac{13}{10} = 1.3$
$\therefore$ **The 1^{st} term is 3 and the common difference is 1.3**

---

ii. Now, $t_n = a + (n - 1)d$
$\therefore$ $t_{34} = 3 + (34 - 1)1.3$
$= 3 + 33 \times 1.3$
$= 3 + 42.9$
$\therefore$ $t_{34} = 45.9$
$\therefore$ **The 34^{th} term is 45.9**

iii. Given, $t_n = 55$

Since, $t_n = a + (n - 1)d$

$\therefore$ $55 = 3 + (n - 1)1.3$

$55 - 3 = (n - 1)1.3$

$\therefore$ $52 = (n - 1)1.3$

$\therefore$ $\frac{52}{1.3} = n - 1$

$\therefore$ $\frac{520}{13} = n - 1$

$\therefore$ $40 = n - 1$

$\therefore$ $n = 40 + 1 = 41$

$\therefore$ **$t_n = 55$ for $n = 41$**

Sum of the first n terms of an A.P.

If a, a + d, a + 2d, … a + (n − 1)d is an A.P. with first term 'a' and common difference 'd', then the sum of first n terms of the A.P. is

$S_n = [a] + [a + d] + \ldots + [a + (n - 2)d] + [a + (n - 1)d]$ ...(i)

Reversing the terms and rewriting (i), we get

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + [a + d] + [a]$ ...(ii)

Now, adding equations (i) and (ii), we get

$2S_n = [a + a + (n - 1)d] + [a + d + a + (n - 2)d] + \ldots + [a + (n - 2)d + a + d] + [a + (n - 1)d + a]$

$\therefore$ $2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + \ldots$ (n times)

$\therefore$ $2S_n = n\,[2a + (n - 1)d]$

$\therefore$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

Thus the sum of the first n terms of an A.P. is

$S_n = \frac{n}{2}[2a + (n - 1)d]$.

Think it over

1. Derive the formula for n^{th} term of the sequence of odd natural numbers and even natural numbers.
2. Find the sum of first n odd natural numbers and first n even natural numbers.

Solution:

1. Sequence of odd natural numbers is

1, 3, 5, 7, …

This sequence is an A.P. with

$a = 1, d = 3 - 1 = 2$

Now, $t_n = a + (n - 1)d$

$= 1 + (n - 1)2$

$= 1 + 2n - 2$

$= 2n - 1$

$\therefore$ **The n^{th} term of the sequence of odd natural numbers is $2n - 1$.**

Sequence of even natural numbers is

2, 4, 6, 8, …

This sequence is an A.P. with

$a = 2, d = 4 - 2 = 2$

$t_n = a + (n - 1)d$

$= 2 + (n - 1)2$

$= 2 + 2n - 2$

$= 2n$

$\therefore$ **The n^{th} term of the sequence of even natural numbers is 2n.**

2. Sequence of odd natural numbers is

1, 3, 5, 7, …

This sequence is an A.P. with

$a = 1, d = 3 - 1 = 2$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[2 \times 1 + (n - 1)2]$

$= \frac{n}{2}[2 + 2n - 2] = \frac{n}{2}[2n] = n^2$

$\therefore$ **The sum of first n odd natural numbers is n^2.**

Sequence of even natural numbers is

2, 4, 6, 8, …

This sequence is an A.P. with

$a = 2, d = 4 - 2 = 2$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[2 \times 2 + (n - 1)2]$

$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2n + 2]$

$= \frac{2n(n+1)}{2}$

$= n(n + 1)$

$\therefore$ **The sum of first n even natural numbers is $n(n + 1)$.**

Exercise 1.4

1. Find the sum of the first 'n' natural numbers and hence find the sum of the first 20 natural numbers. [4 marks]

Solution:

The first 'n' natural numbers are 1, 2, 3, ... , n.

This sequence is an A.P. with a = 1, d = 2 – 1 = 1

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)1]$

$= \frac{n}{2}[2 + n - 1] = \frac{n}{2} \times (n+1)$

$= \frac{n(n+1)}{2}$

$\therefore$ $S_{20} = \frac{20(20+1)}{2} = \frac{20 \times 21}{2} = 210$

$\therefore$ **The sum of first 'n' natural numbers is $\frac{n(n+1)}{2}$ and the sum of first 20 natural numbers is 210.**

---

2. Find the sum of all odd natural numbers from 1 to 150. [4 marks]

Solution:

The odd natural numbers from 1 to 150 are

1, 3, 5, ... , 149

This sequence is an A.P. with

a = 1, d = 3 – 1 = 2, t_n = 149

But, $t_n = a + (n-1)d$

$\therefore$ $149 = 1 + (n-1)2$ $\therefore$ $149 - 1 = (n-1)2$

$\therefore$ $148 = (n-1)2$ $\therefore$ $\frac{148}{2} = n - 1$

$\therefore$ $n - 1 = 74$ $\therefore$ $n = 75$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{75} = \frac{75}{2}[2 \times 1 + (75-1)2] = \frac{75}{2}[2 + (74)2]$

$= \frac{75}{2}[2 + 148] = \frac{75}{2} \times 150$

$\therefore$ $S_{75} = 75 \times 75 = 5625$

$\therefore$ **The sum of all the odd natural numbers from 1 to 150 is 5625.**

3. For an A.P., find S_{10}, if a = 6 and d = 3. [Mar 13][2 marks]

Solution:

Given, a = 6, d = 3

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{10} = \frac{10}{2}[2 \times 6 + (10-1)3] = 5[12 + (9)3]$

$= 5[12 + 27] = 5 \times 39$

$\therefore$ $\mathbf{S_{10} = 195}$

---

4. Find the sum of all numbers from 1 to 140 which are divisible by 4. [4 marks]

Solution:

The numbers from 1 to 140 which are divisible by 4 are 4, 8, 12, ... 140

This sequence is an A.P. with a = 4, d = 8 – 4 = 4, t_n = 140

But, $t_n = a + (n-1)d$

$\therefore$ $140 = 4 + (n-1)4$

$\therefore$ $140 - 4 = (n-1)4$

$\therefore$ $136 = (n-1)4$

$\therefore$ $\frac{136}{4} = n - 1$ $\therefore$ $34 + 1 = n$

$\therefore$ $n = 35$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{35} = \frac{35}{2}[2 \times 4 + (35-1)4]$

$= \frac{35}{2}[8 + (34)4]$

$= \frac{35}{2}[8 + 136] = \frac{35}{2} \times 144$

$= 35 \times 72$

$\therefore$ $S_{35} = 2520$

$\therefore$ **The sum of all numbers from 1 to 140 which are divisible by 4 is 2520.**

---

5. Find the sum of the first 'n' odd natural numbers. Hence, find 1 + 3 + 5 + … + 101. [5 marks]

Solution:

The sequence of odd natural numbers is 1, 3, 5, …

This sequence is an A.P. with a = 1, d = 3 – 1 = 2

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}[2 + 2n - 2] = \frac{n}{2}[2n]$

$\therefore \quad S_n = n^2$ …(i)

For 1, 3, 5, ..., 101, $t_n = 101$

But, $t_n = a + (n-1)d$

$\therefore \quad 101 = 1 + (n-1)2$

$\therefore \quad 101 - 1 = (n-1)2$

$\therefore \quad 100 = (n-1)2$

$\therefore \quad \frac{100}{2} = n - 1$

$\therefore \quad 50 = n - 1$

$\therefore \quad n = 50 + 1 = 51$

$\therefore \quad 1 + 3 + 5 + \ldots + 101 = S_{51}$

$\therefore \quad 1 + 3 + 5 + \ldots + 101 = (51)^2$ ...[From (i)]

$\therefore \quad 1 + 3 + 5 + \ldots + 101 = 2601$

$\therefore$ **The sum of the first 'n' odd natural numbers is n^2 and 1 + 3 + 5 + ... + 101 is 2601.**

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6. Obtain the sum of the 56 terms of an A.P., whose 19th and 38th terms are 52 and 148 respectively. [4 marks]

Solution:

Given, $t_{19} = 52$ and $t_{38} = 148$

Now, $t_n = a + (n-1)d$

$\therefore \quad t_{19} = a + (19-1)d$

$\therefore \quad 52 = a + 18d$

$\therefore \quad a + 18d = 52$ ... (i)

Also, $t_{38} = a + (38-1)d$

$\therefore \quad 148 = a + 37d$

$\therefore \quad a + 37d = 148$ ... (ii)

Adding (i) and (ii), we get

$$\begin{array}{r} a + 18d = 52 \\ a + 37d = 148 \\ \hline 2a + 55d = 200 \end{array}$$ … (iii)

Also, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{56} = \frac{56}{2}[2a + (56-1)d]$

$= 28[2a + 55d]$

$= 28(200)$ …[From (iii)]

$\therefore \quad S_{56} = 5600$

$\therefore$ **Sum of the 56 terms of an A.P. is 5600.**

7. The sum of the first 55 terms of an A.P. is 3300. Find the 28th term. [3 marks]

Solution:

Given, $S_{55} = 3300$

But, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore \quad S_{55} = \frac{55}{2}\left[2a + (55-1)d\right]$

$\therefore \quad 3300 = \frac{55}{2}[2a + 54d]$

$\therefore \quad 3300 = \frac{55}{2} \times 2(a + 27d)$

$\therefore \quad 3300 = 55(a + 27d)$

$\therefore \quad \frac{3300}{55} = a + 27d$

$\therefore \quad 60 = a + 27d$

$\therefore \quad a + 27d = 60$ ... (i)

Also, $t_n = a + (n-1)d$

$\therefore \quad t_{28} = a + (28-1)d$

$\therefore \quad t_{28} = a + 27d$

$t_{28} = 60$ …[From (i)]

$\therefore$ **28th term of an A.P. is 60.**

---

8. Find the sum of the 'n' even natural numbers. Hence find the sum of the first 20 even natural numbers. [3 marks]

Solution:

The sequence of even natural numbers is 2, 4, 6, ...

This sequence is an A.P. with a = 2, d = 4 – 2 = 2

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_n = \frac{n}{2}[2 \times 2 + (n-1)2]$

$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2n + 2]$

$= \frac{n}{2} \times 2(n+1) = n(n+1)$

$\therefore \quad S_n = n(n+1)$

$\therefore \quad S_{20} = 20(20+1)$

$= 20 \times 21 = 420$

$\therefore$ **The sum of the 'n' even natural numbers is n (n + 1) and the sum of the first 20 even natural numbers is 420.**

1.5 Properties of an A.P.

Property I:
For an A.P. with the first term 'a' and the common difference 'd', if any real number 'k' is added to each term, then the new sequence is also an A.P. with the first term 'a + k' and the same common difference 'd'.

Property II:
For an A.P. with the first term 'a' and the common difference 'd', if each term of an A.P. is multiplied by any real number k, then the new sequence is also an A.P. with the first term 'ak' and the common difference 'dk'.

Note:
1. If each term of an A.P. is multiplied by 0 then the new sequence will be 0, 0, 0, …
2. If each term of an A.P. is added, subtracted multiplied or divided by a certain constant then the new sequence is also an A.P.

1.6 Particular terms in an A.P.

To solve problems, we can consider three, four or five consecutive terms of an A.P. in the following way.

i. Three consecutive terms as $a - d, a, a + d$.
ii. Four consecutive terms as $a - 3d, a - d, a + d, a + 3d$. (common difference being 2d)
iii. Five consecutive terms as $a - 2d, a - d, a, a + d, a + 2d$.

Exercise 1.5

1. Find four consecutive terms in an A.P. whose sum is 12 and the sum of the 3rd and the 4th terms is 14. [4 marks]

Solution:
Let the four consecutive terms be $a - 3d$, $a - d$, $a + d$ and $a + 3d$.
According to the first condition,
$a - 3d + a - d + a + d + a + 3d = 12$
$\therefore$ $4a = 12$ $\quad\therefore$ $a = \frac{12}{4}$
$\therefore$ $a = 3$ ... (i)
According to the second condition,
$a + d + a + 3d = 14$
$\therefore$ $2a + 4d = 14$
$\therefore$ $2 \times 3 + 4d = 14$ ... [From (i)]
$\therefore$ $4d = 14 - 6$
$\therefore$ $4d = 8$ $\quad\therefore$ $d = 2$

Thus, $a - 3d = 3 - 3 \times 2 = -3$
$a - d = 3 - 2 = 1$
$a + d = 3 + 2 = 5$
$a + 3d = 3 + 3 \times 2 = 9$
$\therefore$ **The four consecutive terms of an A.P. are –3, 1, 5 and 9.**

2. Find four consecutive terms in an A.P. whose sum is – 54 and the sum of the 1st and the 3rd terms is –30. [4 marks]

Solution:
Let the four consecutive terms be $a - 3d$, $a - d$, $a + d$ and $a + 3d$.
According to the first condition,
$a - 3d + a - d + a + d + a + 3d = -54$
$\therefore$ $4a = -54$
$\therefore$ $a = \frac{-54}{4} = \frac{-27}{2} = -13.5$ ... (i)
According to the second condition,
$a - 3d + a + d = -30$
$\therefore$ $2a - 2d = -30$
$\therefore$ $a - d = -15$
$\therefore$ $-13.5 - d = -15$ ... [From (i)]
$\therefore$ $d = -13.5 + 15$
$\therefore$ $d = 1.5$
Thus, $a - 3d = -13.5 - 3 \times 1.5$
$= -13.5 - 4.5 = -18$
$a - d = -13.5 - 1.5 = -15$
$a + d = -13.5 + 1.5 = -12$
$a + 3d = -13.5 + 3 \times 1.5$
$= -13.5 + 4.5 = -9$
$\therefore$ **The four consecutive of an A.P. terms are –18, –15, –12, –9.**

3. Find three consecutive terms in an A.P. whose sum is –3 and the product of their cubes is 512. [4 marks]

Solution:
Let the three consecutive terms in an A.P be $a - d$, a and $a + d$
According to the first condition,
$a - d + a + a + d = -3$
$\therefore$ $3a = -3$
$\therefore$ $a = -1$ ... (i)
According to the second condition,
$(a - d)^3 (a)^3 (a + d)^3 = 512$
Taking cube root on both sides, we get
$(a - d)(a)(a + d) = 8$
$\therefore$ $a(a^2 - d^2) = 8$
$\therefore$ $-1[(-1)^2 - d^2] = 8$ … [From (i)]

$\therefore$ $-1\,(1 - d^2) = 8$

$\therefore$ $1 - d^2 = -8$

$\therefore$ $d^2 = 9$

$\therefore$ $d = \sqrt{9} = \pm 3$

For $d = 3$ and $a = -1$

$a - d = -1 - 3 = -4$

$a = -1$

$a + d = -1 + 3 = 2$

For $d = -3$ and $a = -1$

$a - d = -1 - (-3) = -1 + 3 = 2$

$a = -1$

$a + d = -1 + (-3) = -1 - 3 = -4$

$\therefore$ **The three consecutive terms of an A.P. are –4, –1 and 2 or 2, –1 and –4.**

---

4. In winter, the temperature at a hill station from Monday to Friday is in A.P. The sum of the temperatures of Monday, Tuesday and Wednesday is zero and the sum of the temperatures of Thursday and Friday is 15. Find the temperature of each of the five days. [4 marks]

Solution:

Let the temperatures from Monday to Friday in A.P be $a - 2d$, $a - d$, a, $a + d$, $a + 2d$.

According to the first condition,

$a - 2d + a - d + a = 0$

$\therefore$ $3a - 3d = 0$ $\therefore$ $a - d = 0$ $\therefore$ $a = d$

According to the second condition,

$a + d + a + 2d = 15$

$\therefore$ $2a + 3d = 15$

$\therefore$ $2a + 3a = 15$ ...[$\because$ d = a]

$\therefore$ $5a = 15$

$\therefore$ $a = 3$

$\therefore$ $d = 3$ ...[$\because$ d = a]

Thus, $a - 2d = 3 - 2 \times 3 = -3$

$a - d = 3 - 3 = 0$

$a = 3$

$a + d = 3 + 3 = 6$

$a + 2d = 3 + 2 \times 3 = 9$

$\therefore$ **The temperature of each of the five days is –3, 0, 3, 6 and 9 respectively.**

1.7 Applications of A.P.

In this section, we will study the application of theory and formulae of A.P. to solve various word problems.

Exercise 1.6

1. Mary got a job with a starting salary of ₹ 15,000/- per month. She will get an increment of ₹ 100/- per month. What will be her salary after 20 months? [3 marks]

Solution:

Mary's salaries are in A.P. with the first term 15,000 and common difference 100.

$\therefore$ $a = 15000$, $d = 100$, $n = 20$

Now, $t_n = a + (n - 1)\,d$

$\therefore$ $t_{20} = 15000 + (20 - 1)\,100$

$= 15000 + 19 \times 100$

$= 15000 + 1900$

$= 16900$

$\therefore$ **Mary's salary after 20 months will be ₹ 16,900.**

---

2. The taxi fare is ₹ 14 for the first kilometre and ₹ 2 for each additional kilometre. What will be the fare for 10 kilometres? [3 marks]

Solution:

The taxi fares are in A.P. with the first term 14 and common difference 2.

$\therefore$ $a = 14$, $d = 2$, $n = 10$

Now, $t_n = a + (n - 1)\,d$

$\therefore$ $t_{10} = 14 + (10 - 1) \times 2$

$= 14 + 9 \times 2$

$= 32$

$\therefore$ **The taxi fare for 10 kilometres will be ₹ 32.**

---

3. Mangala started doing physical exercise 10 minutes for the first day. She will increase the time of exercise by 5 minutes per day, till she reaches 45 minutes per day. How many days are required to reach 45 minutes? [3 marks]

Solution:

The daily time of exercise is an A.P. with the first term 10 and common difference 5.

$\therefore$ $a = 10$, $d = 5$, $t_n = 45$

Now, $t_n = a + (n - 1)\,d$

$\therefore$ $45 = 10 + (n - 1)\,5$

$\therefore$ $35 = (n - 1)\,5$

$\therefore$ $\frac{35}{5} = n - 1$

$\therefore$ $7 = n - 1$

$\therefore$ $n = 7 + 1 = 8$

$\therefore$ **The number of days required to reach 45 minutes will be 8.**

4. There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. Find the number of seats in the twenty fifth row. [3 marks]

Solution:

The number of seats arranged row wise are as follows :
20, 22, 24, ...
This sequence is an A.P. with
a = 20, d = 22 – 20 = 2, n = 25

Now, $t_n = a + (n - 1)d$

$\therefore \quad t_{25} = 20 + (25 - 1)2$
$= 20 + 24 \times 2$
$= 68$

$\therefore$ **The number of seats in the twenty fifth row is 68.**

---

5. A village has 4000 literate people in the year 2010 and this number increases by 400 per year. How many literate people will be there till the year 2020? Find a formula to know the number of literate people after n years. [4 marks]

Solution:

The number of literate people in the village is in A.P. with the first term 4000 and common difference 400.

$\therefore \quad a = 4000, d = 400, n = 10$

Now, $t_n = a + (n - 1)d$

$\therefore \quad t_{10} = 4000 + (10 - 1)400$
$= 4000 + 9 \times 400$
$= 4000 + 3600 = 7600$

Since, $t_n = a + (n - 1)d$
$= 4000 + (n - 1)400$
$= 4000 + 400n - 400$
$= 400n + 3600$

$\therefore$ **The number of literate people till the year 2020 will be 7600 and the formula to know the number of literate people after 'n' years is (400 n + 3600).**

---

6. Neela saves in a 'Mahila Bachat Gat' ₹ 2 on the first day of February, ₹ 4 on the second day, ₹ 6 on the third day and so on. What will be her savings in the month of February 2010? [4 marks]

Solution:

Neela's daily savings of February 2010 are as follows :
2, 4, 6, ...
This sequence is an A.P. with
a = 2, d = 4 – 2 = 2, n = 28
($\because$ Feb 2010 had 28 days as 2010 was not a leap year)

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{28} = \frac{28}{2}[2 \times 2 + (28 - 1)2]$
$= 14 [4 + 27 \times 2]$
$= 14 \times 58$
$= 812$

$\therefore$ **Neela's saving in the month of February 2010 will be ₹ 812.**

---

7. Babubhai borrows ₹ 4000 and agrees to repay with a total interest of ₹ 500 in 10 instalments, each instalment being less than the preceding instalment by ₹ 10. What should be the first and the last instalment? [Mar 14, 15; Oct 14][4 marks]

Solution:

The instalments are in A.P.
Here, $S_{10} = 4000 + 500 = 4500$
Also, n = 10, d = –10

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{10} = \frac{10}{2}[2a + (10 - 1)(-10)]$

$\therefore \quad 4500 = 5 [2a + 9 \times (-10)]$

$\therefore \quad \frac{4500}{5} = 2a - 90$

$\therefore \quad 900 + 90 = 2a$

$\therefore \quad 990 = 2a$

$\therefore \quad a = \frac{990}{2}$

$\therefore \quad a = 495$

Also, $t_n = a + (n - 1)d$

$\therefore \quad t_{10} = 495 + (10 - 1)(-10)$
$= 495 + 9 \times (-10)$
$= 495 - 90$
$= 405$

$\therefore$ **The first instalment is ₹ 495 and the last instalment is ₹ 405.**

---

8. A meeting hall has 20 seats in the first row, 24 seats in the second row, 28 seats in the third row, and so on and has in all 30 rows. How many seats are there in the meeting hall? [4 marks]

Solution:

The number of seats arranged row-wise are as follows:
20, 24, 28, …
This sequence is an A.P. with a = 20, d = 4, n = 30.

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{30} = \frac{30}{2}[2 \times 20 + (30-1)4]$

$= 15[40 + 29 \times 4]$

$= 15[40 + 116]$

$= 15 \times 156 = 2340$

$\therefore$ **The number of seats in the meeting hall is 2340.**

---

9. Vijay invests some amount in the National Saving Certificates. For the 1st year, he invests ₹ 500, for the 2nd year he invests ₹ 700, for the 3rd year he invests ₹ 900, and so on. How much amount he has invested in 12 years? [4 marks]

Solution:

Amount of investments year-wise are as follows:

500, 700, 900, ...

This sequence is an A.P. with

a = 500, d = 200, n = 12

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{12} = \frac{12}{2}[2 \times 500 + (12-1)200]$

$= 6[1000 + 11 \times 200]$

$= 6[1000 + 2200] = 6 \times 3200$

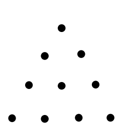
$= 19200$

$\therefore$ **Total amount invested in 12 years is ₹ 19200.**

---

10. In a school, a plantation programme was arranged on the occasion of 'World Environment Day', on a ground of triangular shape. The saplings are to be planted as shown in the figure.

One plant in the first row, two in the second row, three in the third row and so on. If there are 25 rows, then find the total number of plants to be planted. [4 marks]



Solution:

Number of saplings planted row-wise are as follows:

1, 2, 3, …

This sequence is an A.P. with

a = 1, d = 1, n = 25

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{25} = \frac{25}{2}[2 \times 1 + (25-1)1]$

$= \frac{25}{2}[2 + 24] = \frac{25}{2} \times 26$

$= 325$

$\therefore$ **The total number of plants to be planted is 325.**

Problem Set - 1

1. Find t_{11} from the following A.P. 4, 9, 14, … [2 marks]

Solution:

The given A.P. is 4, 9, 14, ...

Here, a = 4, d = 9 – 4 = 5

Now, $t_n = a + (n-1)d$

$\therefore \quad t_{11} = 4 + (11-1)5$

$= 4 + 10 \times 5$

$\therefore$ **$t_{11} = 54$**

---

2. For the following A.P., find the first n for which t_n is negative.

122, 116, 110, ...

(Note: Find smallest n, such that $t_n < 0$)

[4 marks]

Solution:

The given A.P. is 122, 116, 110, ...

Here, a = 122, d = 116 – 122 = –6

Now, $t_n = a + (n-1)d$

$= 122 + (n-1)(-6)$

$= 122 - 6n + 6$

$\therefore \quad t_n = 128 - 6n$

Since, $t_n < 0$

$\therefore \quad 128 - 6n < 0$

$\therefore \quad 128 < 6n$

$\therefore \quad \frac{128}{6} < n$

$\therefore \quad 21.33 < n$

$\therefore \quad n > 21.33$

$\therefore$ n should be 22.

$\therefore$ t_{22} is the first negative number.

$\therefore$ **The first negative term of the given A.P. is 22nd term.**

3. Find the sum of the first 11 positive numbers which are multiples of 6. [3 marks]

Solution:

The positive multiples of 6 are 6, 12, 18, …

This sequence is an A.P. with

$a = 6$, $d = 12 - 6 = 6$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{11} = \frac{11}{2}[2 \times 6 + (11 - 1)6]$

$= \frac{11}{2}[12 + 10 \times 6]$

$= \frac{11}{2}[12 + 60]$

$= \frac{11}{2} \times 72$

$= 11 \times 36$

$\therefore \quad S_{11} = 396$

$\therefore$ **The sum of the first 11 positive multiples of 6 is 396.**

---

4. In the A.P. 7, 14, 21, … how many terms are to be considered for getting the sum 5740? [4 marks]

Solution:

The given A. P. is 7, 14, 21, …

Also, $S_n = 5740$

Here, $a = 7$, $d = 14 - 7 = 7$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad 5740 = \frac{n}{2}[2 \times 7 + (n - 1)7]$

$\therefore \quad 5740 \times 2 = n(14 + 7n - 7)$

$\therefore \quad 11480 = n(7 + 7n)$

$\therefore \quad 11480 = 7n + 7n^2$

$\therefore \quad 7n^2 + 7n - 11480 = 0$

$\therefore \quad n^2 + n - 1640 = 0$ ….[Dividing both sides by 7]

$\therefore \quad n^2 + 41n - 40n - 1640 = 0$

$\therefore \quad n(n + 41) - 40(n + 41) = 0$

$\therefore \quad (n + 41)(n - 40) = 0$

$\therefore \quad n + 41 = 0$ or $n - 40 = 0$

$\therefore \quad n = -41$ or $n = 40$

But, n cannot be negative

$\therefore \quad n = 40$

$\therefore$ **The number of terms to be considered is 40.**

5. From an A.P., the first and the last terms are 13 and 216 respectively. Common difference is 7. How many terms are there in that A.P.? Find the sum of all the terms. [4 marks]

Solution:

Let there be n number of terms.

Given, $a = 13$, $t_n = 216$, $d = 7$

Now, $t_n = a + (n - 1)d$

$\therefore \quad 216 = 13 + (n - 1)7$

$\therefore \quad 216 - 13 = (n - 1)7$

$\therefore \quad 203 = (n - 1)7$

$\therefore \quad (n - 1) = \frac{203}{7}$ $\qquad \therefore \quad n - 1 = 29$

$\therefore \quad n = 29 + 1 = 30$ $\qquad \therefore \quad n = 30$

Also, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{30} = \frac{30}{2}[2 \times 13 + (30 - 1)7]$

$= 15[26 + 29 \times 7]$

$= 15[26 + 203]$

$\therefore \quad S_{30} = 15 \times 229 = 3435$

$\therefore$ **The number of terms in the A.P. is 30 and the sum of all 30 terms is 3435.**

---

6. The second and the fourth term of an A.P. is 12 and 20 respectively. Find the sum of the first 25 terms of that A.P. [4 marks]

Solution:

Given, $t_2 = 12$, $t_4 = 20$

Now, $t_n = a + (n - 1)d$

$\therefore \quad t_2 = a + (2 - 1)d$

$\therefore \quad 12 = a + d$

$\therefore \quad a + d = 12$ …(i)

Also, $t_4 = a + (4 - 1)d$

$\therefore \quad 20 = a + 3d$

$\therefore \quad a + 3d = 20$ …(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{rcl} a + 3d &=& 20 \\ a + d &=& 12 \\ (-) \quad (-) & & (-) \\ \hline 2d &=& 8 \end{array}$$

$\therefore \quad d = \frac{8}{2} = 4$

Substituting $d = 4$ in (i), we get

$a + 4 = 12$

$\therefore \quad a = 12 - 4 = 8$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{25} = \frac{25}{2}[2\times 8 + (25-1)4]$

$= \frac{25}{2}[16 + 24 \times 4]$

$= \frac{25}{2}[16 + 96] = \frac{25}{2} \times 112$

$= 25 \times 56$

$\therefore \quad S_{25} = 1400$

$\therefore$ **The sum of first 25 terms is 1400.**

---

7. The sum of the first n terms of an A.P. is $3n + n^2$.

i. Find the first term and the sum of the first two terms [2 marks]

ii. Find the second, third and the 15th term. [3 marks]

Solution :

Given, $S_n = 3n + n^2$

i. For n = 1, $S_1 = 3(1) + (1)^2$

$= 3 + 1 = 4$

For n = 2, $S_2 = 3(2) + (2)^2$

$= 6 + 4 = 10$

$\therefore \quad t_1 = S_1 = 4$ and $S_2 = 10$

$\therefore$ **The first term is 4 and the sum of the first two terms is 10.**

---

ii. Since, $t_n = S_n - S_{n-1}$, for $n > 1$

$\therefore \quad t_2 = S_2 - S_1$

$= 10 - 4 = 6$

$\therefore \quad a = 4, d = 6 - 4 = 2$

Now, $t_n = a + (n-1)d$

$\therefore \quad t_3 = 4 + (3-1)2$

$= 4 + 2 \times 2$

$= 4 + 4 = 8$

$t_{15} = 4 + (15-1)2$

$= 4 + 14 \times 2$

$= 4 + 28 = 32$

$\therefore$ **The second, third and the 15th terms are 6, 8 and 32 respectively.**

8. For an A.P. given below, find t_{20} and S_{10}.

$\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots$ **[4 marks]**

Solution:

The given A.P. is $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots$

Here, $a = \frac{1}{6}$, $d = \frac{1}{4} - \frac{1}{6} = \frac{2}{24} = \frac{1}{12}$

Now, $t_n = a + (n-1)d$

$\therefore \quad t_{20} = \frac{1}{6} + (20-1)\frac{1}{12}$

$= \frac{1}{6} + 19 \times \frac{1}{12} = \frac{1}{6} + \frac{19}{12}$

$= \frac{2+19}{12} = \frac{21}{12} = \frac{7}{4}$

Also, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{10} = \frac{10}{2}\left[2\times\frac{1}{6} + (10-1)\frac{1}{12}\right]$

$= 5\left[\frac{1}{3} + 9 \times \frac{1}{12}\right] = 5\left[\frac{1}{3} + \frac{3}{4}\right]$

$= 5\left[\frac{4+9}{12}\right] = 5 \times \frac{13}{12}$

$= \frac{65}{12}$

$\therefore$ $\mathbf{t_{20} = \frac{7}{4}}$ **and** $\mathbf{S_{10} = \frac{65}{12}}$

One-Mark Questions

1. Write first three terms of the A.P. when the first term is 10 and common difference is zero.

Solution:

The terms are 10, 10, 10.

---

2. For the A.P. $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \ldots$ write the common difference.

Solution:

The common difference is -1.

3. Find the first three terms of the sequence whose n^{th} term is given by $\frac{1}{n^2} + 1$.

Solution:

$t_n = \frac{1}{n^2} + 1$

$\therefore \quad t_1 = \frac{1}{1^2} + 1 = 2$

$\therefore \quad t_2 = \frac{1}{2^2} + 1 = \frac{5}{4}$

$\therefore \quad t_3 = \frac{1}{3^2} + 1 = \frac{10}{9}$

$\therefore$ **First three terms are 2, $\frac{5}{4}$, $\frac{10}{9}$.**

---

4. Write the next two terms of the following sequence: 1, –1, –3, –5, …

Solution:

The next two terms are –7 and –9.

---

5. Frame the A.P. for the following situation. The taxi fare after each km, when the fare is ₹ 17 for first km and ₹ 9 for each additional km.

Solution:

The A.P. is 17, 26, 35, …

---

6. In the given A.P., find the missing term: 2, __ , 26.

Solution:

The missing term is 14.

---

7. For a given A.P. if a = 6 and d = 3, find S_4. [Mar 13]

Solution:

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_4 = \frac{4}{2}[2(6) + (4-1)3]$

$= 2(12 + 9)$

$= 2(21)$

$\therefore \quad S_4 = 42$

---

8. For an A.P. $t_3 = 8$ and $t_4 = 12$, find the common difference d. [Mar 14]

Solution:

Given, $t_3 = 8$, $t_4 = 12$

$\therefore \quad d = t_4 - t_3 = 12 - 8 = 4$

9. Find S_2 for the following given A.P. 3, 5, 7, 9, ……. . [Oct 14]

Solution:

The given A.P. is 3, 5, 7, 9, ...

Here, $t_1 = 3$, $t_2 = 5$

Now, $S_2 = t_1 + t_2$

$\therefore \quad S_2 = 3 + 5 = 8$

Additional Problems for Practice

Based on Exercise 1.1

1. For each sequence, find the next four terms: **[1 mark each]**
   i. 2, 4, 6, 8, …
   ii. 0.2, 0.02, 0.002, 0.0002, …
2. Find the first five terms of the following: **[2 marks each]**
   i. $t_n = 1 + \frac{1}{n}$
   ii. $S_n = \frac{n(n+1)}{2}$
3. Find the first three terms of the following sequence, whose n^{th} term is $t_n = 2n + 2$. **[Oct 14] [3 marks]**

Based on Exercise 1.2

4. Check whether the sequence 7, 12, 17, 22, ….. is an A.P. If it is an A.P., find d and t_n. **[3 marks]**
5. Which of the following sequences are arithmetic progressions? Justify **[2 marks each]**
   i. 2, 6, 10, 14, …
   ii. 24, 21, 18, 15, …
   iii. 4, 12, 36, 108, …
   iv. $1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$
   v. – 50, – 75, – 100, …
   vi. 12, 2, – 8, – 18, …
6. Write the first four terms of the following Arithmetic Progression where the common difference 'd' and the first term 'a' are given. **[2 marks each]**
   i. a = 5, d = 7
   ii. a = 8, d = 0
7. Find the first four terms in an A.P. when a = 3 and d = 4. **[Oct 14][2 marks]**

Based on Exercise 1.3

8. For an A.P. if $t_4 = 20$ and $t_7 = 32$, find a, d and t_n. **[3 marks]**

9. Find the
   i. 10^{th} term of the A.P. 3, 1, −1, −3, … **[2 marks]**
   ii. 7^{th} term of the A.P. 6, 10, 14, … **[2 marks]**

10. How many terms are there in the A.P. 201, 208, 215, … 369? **[3 marks]**

11. If the 5^{th} and 12^{th} terms of an A.P. are 14 and 35 respectively, find the first term and the common difference. **[2 marks]**

12. Find t_n for an A.P. 1, 7, 13, 19, …… . **[Oct 14][2 marks]**

Based on Exercise 1.4

13. Find the sum of the first n terms of an A.P. 1, 4, 7, 10, … Also find S_{40}. **[4 marks]**

14. If for an A.P.
   i. a = 6, d = 3, find S_8 **[3 marks]**
   ii. a = 6, d = 3, find S_6 **[3 marks] [Mar 13]**

15. If for an A.P. $t_8 = 36$, find S_{15}. **[3 marks]**

16. If for an A.P. $S_{31} = 186$, find t_{16}. **[3 marks]**

17. Find the sum of all natural numbers from 50 to 250, which are exactly divisible by 4. **[4 marks]**

Based on Exercise 1.5

18. Find three consecutive terms in as A.P. whose sum is 21 and their product is 315. **[4 marks]**

19. Find four consecutive terms in an A.P. such that their sum is 26 and the product of the first and the fourth term is 40. **[4 marks]**

Based on Exercise 1.6

20. A man borrows ₹ 1,000 and agrees to repay without interest in 10 instalments, each instalment being less than the preceding instalment by ₹ 8. Find his first instalment. **[4 marks]**

21. A man saves ₹ 16,500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year? **[4 marks]**

22. A man borrows ₹2,000 and agrees to repay with a total interest of ₹ 340 in 12 monthly instalments, each instalment being less than the preceding one by ₹ 10. Find the amount of the first and the last instalment. **[4 marks]**

23. A sum of ₹ 6,240 is paid off in 30 instalments, such that each instalment is 10 more than the preceding instalment. Calculate the value of the first instalment. **[3 marks]**

Answers to additional problems for practice

1. i. 10, 12, 14, 16
   ii. 0.00002, 0.000002, 0.0000002, 0.00000002
2. i. $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}$
   ii. 1, 3, 6, 10, 15
3. 4, 6 and 8
4. d = 5 and $t_n = 5n + 2$
5. iii. is not an A.P.
   i, ii, iv, v, vi are A.P.
6. i. 5, 12, 19, 26, …
   ii. 8, 8, 8, 8, …
7. 3, 7, 11 and 15
8. a = 8, d = 4, $t_n = 4n + 4$
9. i. 10^{th} term is −15
   ii. 7^{th} term is 30
10. There are 25 terms in the given A.P.
11. The first term is 2 and common difference is 3.
12. $t_n = 6n - 5$
13. $S_n = \frac{n}{2}(3n-1)$, $S_{40} = 2380$
14. i. $S_8 = 132$ ii. $S_6 = 81$
15. $S_{15} = 540$
16. $t_{16} = 6$
17. The sum of all natural numbers from 50 to 250, that are divisible by 4 is 7500.
18. 5, 7 and 9 or 9, 7 and 5
19. 5, 6, 7 and 8 or 8, 7, 6 and 5
20. The first instalment is of ₹ 136.
21. The man saved ₹ 1200 in the first year.
22. The first instalment is ₹ 250 and the last instalment is ₹ 140.
23. The first instalment is ₹ 63.