HINTS & SOLUTIONS

Physics

1. Because of large mass and large velocity, α-particles have large ionising power. Each α-particle produces thousands of ions before being absorbed. The β-particles ionise the gas through which they pass, but their ionising power is only $\frac{1}{100}$th that of α-particles. γ-rays have got small ionising power.

Because of large mass, the penetrating power of α-particles is very small, it being 1/100 times that to β-rays and 1/10000 times that of γ-rays. α-particles can be easily stopped by an Aluminium sheet, only 0.02 mm thick. β-particles have very small mass, so their penetrating power is large. γ-rays have very large penetrating power.

2. In time $t = T, N = \frac{N_0}{2}$

In another half-life, (*i.e.*, after 2 half-lives)

$$N = \frac{1}{2}\frac{N_0}{2} = \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^2$$

After yet another half-life (*i.e.*, after 3 half-lives) $N = \frac{1}{2}\left(\frac{N_0}{4}\right) = \frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^3$ and so on. Hence, after n half-lives

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$= N_0\left(\frac{1}{2}\right)^{t/T}$$

where $t = n \times T$ = total time of n half-lives.

Here, $n = \frac{t}{T} = \frac{19}{3.8}$

$= 5$

$\therefore$ The fraction left

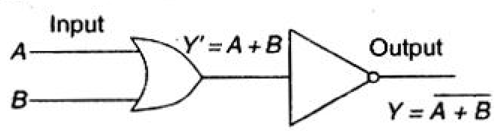
$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

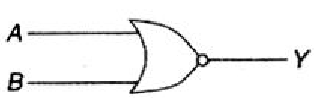
$= 0.031$

3. In circuit 2, each p-n junction is reverse biased, some current will flow giving equal potential difference across each p-n junction diode. In circuit 3, each p-n junction is forward biased, hence same current flows, giving same potential difference across p-n junctions.

4. Zener breakdown occurs in heavily doped p-n junction. The temperature coefficient of the zener mechanism is negative or the breakdown voltage for a particular diode decreases with increasing temperature.

5. NOR gate is a combination of OR gate and NOT gate. In other words, output of OR gate is connected to the input of a NOT gate as shown in figure. Note that output of OR gate is inverted to form NOR gate. This is illustrated in the truth table for NOR gate. It is clear that truth table for NOR gate is developed by inverting the output of the OR gate.





Input		Output	
A	**B**	**OR(Y′)**	**NOR(Y)**
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0

The Boolean expression for NOR function is

$$Y = \overline{A + B}$$

6. According to De-Morgan's theorem

$$\bar{A} \cdot \bar{B} = \overline{(A + B)}$$

$$\therefore \quad \overline{(\bar{A} \cdot \bar{B})} = \overline{\overline{(A + B)}}$$

$$= (A + B) \qquad (\because \bar{\bar{A}} = A)$$

$$\therefore \qquad \overline{(\bar{A} \cdot \bar{B})} = (A + B)$$

7. Here, $v_s = 0$ and v_L is negative where v_s is velocity of source and v_L is velocity of listener (aeroplane)

If apparent frequency is ν' and ν is actual frequency, then

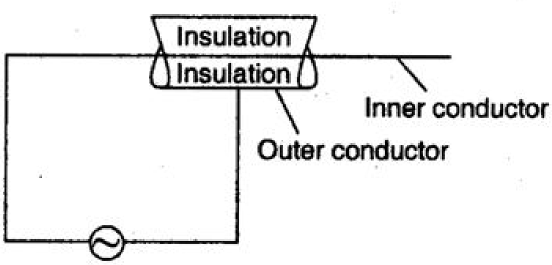
$$\nu' = \frac{v - (-v_L)}{v} \nu$$

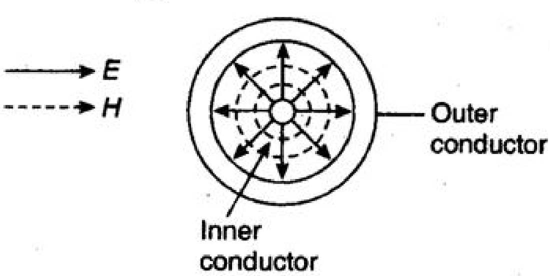
$$= \frac{v + v_L}{v} \nu$$

i.e., $\qquad \nu' > \nu$

So, apparent frequency will increase, it means apparent wavelength will decrease.

8. Basically, a coaxial cable consists of a hollow (outer) cylindrical conductor surrounding a single (inner) conductor along its axis. The two conductors are well insulated from each other. The electric field ($\vec{\mathbf{E}}$) and magnetic field ($\vec{\mathbf{H}}$) at the cross-sections are shown by solid lines and dotted lines, respectively. The outer conductor acts as the shield and minimises interference.





Different kinds of dielectric materials, such as teflon and polythene are covered over copper wire, it acts as a spacer.

In the transmission of power through coaxial cable, the dielectric medium separating the inner conductor from outer one plays a vital role. These dielectric materials are good insulators only at low frequencies. As the frequency increases, the energy loss becomes significant. That is why a coaxial cable can be used effectively for transmission upto a frequency of 20 MHz.

A steady signal flowing in a wire, uniformly distributes itself throughout the cross-section of the wire. A high frequency signal, on the other hand distributes itself uniformly, there being a concentration of current on the outer surface of the conductor. If the frequency of the current is very high, the current is almost wholly confined to the surface layers. This is called 'Skin effect'.

9. Because of their inherent distortion class *C* amplifiers are never used as audio amplifiers, but they are primarily employed in the final stage of TV transmitters. Also, owing to low efficiency of class *A* operations, these amplifiers are not employed where large RF (radio frequency) power is involved *e.g.,* to excite transmitting antenna. In such cases class *C* amplifiers are used, since a class *C* amplifier has a very high efficiency, it can deliver more load power. In out put stage of a TV transmitter grid modulated class *C* amplifier is used.

10. The power dissipated in any circuit is a function of the square of voltage across the circuit and the effective resistance of the circuit. Equation of AM wave reveals that it has three components of amplitude E_c, $mE_c/2$ and $mE_c/2$. Clearly, power output must be distributed among these components.

$$\text{Carrier Power,} \quad P_C = \frac{(E_c/\sqrt{2})^2}{R} = \frac{E_c^2}{2R}$$

Total power of side bands,

$$P_S = \frac{(mE_c/2\sqrt{2})^2}{R} + \frac{(mE_c/2\sqrt{2})^2}{R}$$

$$= \frac{m^2 E_c^2}{4R}$$

$$\therefore \qquad \frac{P_S}{P_C} = \frac{1}{2} m^2$$

and $\quad P_T = P_C + P_S = P_C \left(1 + \frac{m^2}{2}\right)$

$$\therefore \quad \frac{P_T}{P_C} = 1 + \frac{m^2}{2}$$

or $$\left(\frac{I_T}{I_C}\right)^2 = 1 + \frac{m^2}{2}$$

Given that, $I_T = 8.93$ A, $I_C = 8$ A, $m = ?$

$$\therefore \quad \left(\frac{8.93}{8}\right)^2 = 1 + \frac{m^2}{2}$$

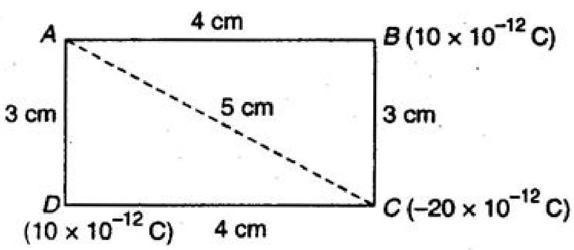
or $$1.246 = 1 + \frac{m^2}{2}$$

or $$\frac{m^2}{2} = 0.246$$

or $$m = \sqrt{2 \times 0.246} = 0.701$$

$$= 70.1\%$$

12.



The situation is summarised in figure. $BC = AD = 3$ cm, $AB = DC = 4$ cm, so $AC = 5$ cm.

Now, potential at A

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{AB} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C}{AC} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_D}{AD}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{10 \times 10^{-12}}{4 \times 10^{-2}} - \frac{20 \times 10^{-12}}{5 \times 10^{-2}} + \frac{10 \times 10^{-12}}{3 \times 10^{-2}}\right]$$

$$= 9 \times 10^9 \times 10^{-10}\left[\frac{10}{4} - \frac{20}{5} + \frac{10}{3}\right]$$

$$= \frac{9 \times 10^{-1} \times 11}{6}$$

$$= 16.5 \times 10^{-1} = 1.65 \text{ V}$$

13. The arrangement of n metal plates separated by dielectric acts as parallel combination of $(n-1)$ capacitors.

Therefore, $$C = \frac{(n-1)K\varepsilon_0 A}{d}$$

Here, $$C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F}$$

$$K = 4,\ \varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2$$

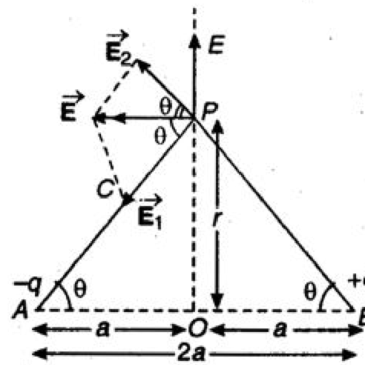
$$A = \pi r^2 = 3.14 \times (1 \times 10^{-2})^2$$

$$d = 1 \text{ mm} = 1 \times 10^{-3}$$

$$\therefore \quad 100 \times 10^{-12} = \frac{(n-1) \times 4 \times 8.85 \times 10^{-12} \times 3.14 \times (1 \times 10^{-2})^2}{1 \times 10^{-3}}$$

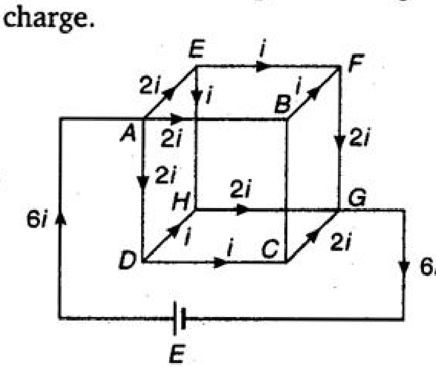
or $$n = \frac{1111.156}{111.156} = 9.99 \approx 10$$

14. Consider an electric dipole consisting of two point charges $-q$ and $+q$ separated by a small distance $AB = 2a$ with centre at O,



As shown in figure, on equatorial line, the resultant electric field $\vec{\mathbf{E}}$ of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ is parallel to the axis of the dipole but opposite to the direction of the dipole moment $\vec{\mathbf{p}}$ as it is directed from negative charge to positive charge.

15.



Let $ABCDEFGH$ be the skeleton cube formed by joining twelve equal wires each of resistance r. Let the current enters the cube at corner A and after passing through all twelve wires, let the current leaves at G, a corner diagonally opposite to corner A.

For the sake of convenience, let us suppose that the total current is $6i$. At A, this current is divided into three equal parts each $(2i)$ along AE, AB and AD as the resistance along these paths are equal and their end points are equidistant from exit point G. At the points E, B and D, each part is further divided into two equal parts each part equal to i. The distribution of current in the various arms of skeleton cube is shown according to Kirchhoff's first law. The current leaving the cube at G is again $6i$.

Applying Kirchhoff's second law to the closed circuit $ADCGA$, we get

$$2ir + ir + 2ir = E$$

or $$5ir = E \quad \text{...(i)}$$

where E is the emf of the cell of neglegible internal resistance.

If R is the resistance of the cube between the diagonally opposite corners A and G, then according to Ohm's law, we have

$$6i \times R = E \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$6iR = 5ir$$

or $$R = \frac{5}{6} r$$

Here, $$r = 6\,\Omega$$

$\therefore$ $$R = \frac{5}{6} \times 6$$

or $$R = 5\,\Omega$$

6. If the voltage of the DC source is increased then both conductor and semiconductor registers same current *i.e.*, semiconductor is in forward biased condition and it conducts. So, ammeters connected to both semiconductor and conductor will register the same current.

7. Consider a conductor of length l and of uniform area of cross-section A.

$\therefore$ Volume of the conductor = Al

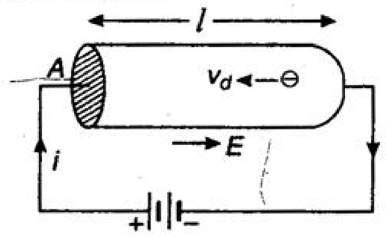
If n is the number of free electrons per unit volume of the conductor, then total number of free electrons in the conductor = Aln. If e is the charge on each electron, then total charge on all the free electrons in the conductor, $q = Alne$.

Let a constant potential difference V is applied across the ends of the conductor with the help of a battery.

The electric field set up across the conductor is given by

$$E = V/l$$

Due to this field, the free electrons present in the conductor will begin to move with a drift velocity v_d towards the left hand side as shown in figure.



Therefore, time taken by the free electrons to cross the conductor,

$$t = \frac{l}{v_d}$$

Hence, current $$i = \frac{q}{t} = \frac{Alne}{l/v_d}$$

or $$i = Anev_d$$

Here, $i = 1$ A, $n = 8 \times 10^{28}$ electron/m^3

$$A = 5 \times 10^{-7}\ \text{m}^2$$

$$\Rightarrow \quad 1 = 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-7} \times v_d$$

or $$v_d = \frac{1}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-7}}$$

Now, $$t = \frac{l}{v_d}$$

$$= 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-7}$$

$$= 64 \times 10^2$$

$$= 6.4 \times 10^3\ \text{s}$$

18. The resistance R_t of a metal conductor at temperature $t\,°$C is given by

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where α and β are temperature coefficients of resistance. R_0 is the resistance of conductor at 0°C. Their values vary from metal to metal. If the temperature $t\,°$C is not sufficiently large which is so in the most practical cases, the above relation may be expressed as

$$R_t = R_0(1 + \alpha t) \quad ...(i)$$

Given, $\alpha = 0.00125/K$

$R_{300} = 1\Omega$

From Eq. (i), we have

$$1 = R_0(1 + 0.00125 \times 300) \quad ...(ii)$$

and, $2 = R_0(1 + 0.00125 \times T) \quad ...(iii)$

$$\therefore \quad \frac{2}{1} = \frac{1 + 0.00125 \times T}{1 + 0.00125 \times 300}$$

or $2.75 = 1 + 0.00125 \times T$

or $T = \dfrac{1.75}{0.00125}$

$= 1400$ K

19. As the coil is rotated, angle θ (angle which normal to the coil makes with $\vec{\mathbf{B}}$ at any instant t) changes, therefore magnetic flux ϕ linked with the coil changes and hence an emf is induced in the coil. At this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NAB \cos \omega t)$$

where N is number of turns in the coil.

or $e = -NAB \dfrac{d}{dt}(\cos \omega t)$

$= -NAB(-\sin \omega t)\,\omega$

or $e = NAB\,\omega \sin \omega t \quad ...(i)$

The induced emf will be maximum

When $\sin \omega t$ = maximum = 1

$\therefore \quad e_{max} = e_0 = NAB\,\omega \times 1$

or $e = e_0 \sin \omega t$

Therefore, e would be maximum, hence current is maximum (as $i_0 = e_0/R$), when $\theta = 90°$, *i.e.*, normal to plane of coil is perpendicular to the field or plane of coil is parallel to magnetic field.

20. $$E = at + \frac{1}{2}bt^2 \quad ...(i)$$

Differentiating Eq. (i), w.r.t., t we have

$$\frac{dE}{dt} = a + bt$$

When $t = t_n$, *i.e.*, neutral temperature, then

$$\frac{dE}{dt} = 0$$

$\therefore \quad 0 = a + bt_n$

or $t_n = -\dfrac{a}{b}$

The temperature of inversion

$$t_i = 2t_n - t_0$$
$$= 2t_n - 0$$
$$= -\frac{2a}{b}$$

Thermoelectric power

$$P = \frac{dE}{dt} = a + bt$$

21. Cyclotron frequency is given by

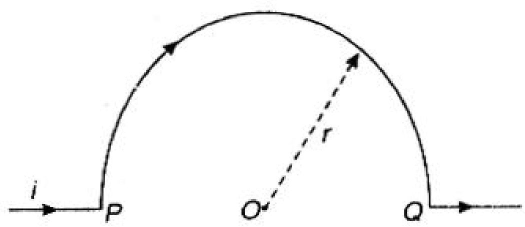
$$\nu = \frac{qB}{2\pi m}$$

$$\therefore \quad \nu = \frac{1.6 \times 10^{-19} \times 6.28 \times 10^{-4}}{2 \times 3.14 \times 1.7 \times 10^{-27}}$$

$= 0.94 \times 10^4$

$\approx 10^4$ Hz

22.



Magnetic field induction at $P = 0$, magnetic field induction at $O = B_0$. Average magnetic field induction from P to $O = \dfrac{0 + B_0}{2} = \dfrac{B_0}{2}$

Force experienced by a charge while moving along the axis of wire, *i.e.*, from P to O will be

$$= \frac{B_0}{2} \times q \times v$$
$$= \frac{1}{2} B_0 qv$$

23. When two solenoids of inductance L_0 are connected in series at large distance and current i is passed through them, the total flux linkage ϕ_{total} is the sum of the flux linkages $L_0 i$ and $L_0 i$. *i.e.*,

$$\phi_{total} = L_0 i + L_0 i$$

If L be the equivalent inductance of the system, then

$$\phi_{total} = Li$$

$$\therefore \quad Li = L_0 i + L_0 i$$

or $L = 2L_0$

When solenoids are connected in series with one inside the other and senses of the turns coinciding, then there will be a mutual inductance L between them. In this case the resultant induced emf in the coils is the sum of the emf's e_1 and e_2 in the respective coils, *i.e.*,

$$e = e_1 + e_2$$

$$= \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt}\right) + \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt}\right)$$

where (+) sign is for positive coupling and (–) sign for negative coupling.

But, $e = -L \cdot \frac{di}{dt}$

$$\therefore \quad -L\frac{di}{dt} = -L_0 \frac{di}{dt} - L_0 \frac{di}{dt} \pm 2L_0 \frac{di}{dt}$$

i.e., $L = L_0 + L_0 + 2L_0$

$= 4L_0$ (for positive coupling)

When solenoids are connected in series with one inside the other with senses of the turns opposite, then their is negative coupling.

So, $L = L_0 + L_0 - 2L_0$

$= 0$

24. The impedance (Z) of an R-L-C series circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

As frequency of alternating emf applied to the circuit is increased, X_L goes on increasing and X_C goes on decreasing. For a particular value of ω $(= \omega_r$ say)

$$X_L = X_C$$

i.e., $\omega_r L = \frac{1}{\omega_r C}$

or $\omega_r = \frac{1}{\sqrt{LC}}$

or $2\pi\nu_r = \frac{1}{\sqrt{LC}}$

or $\nu_r = \frac{1}{2\pi\sqrt{LC}}$

$$\therefore \quad \nu = \frac{1}{2\times 3.14\times\sqrt{5\times 80\times 10^{-6}}}$$

$$= \frac{1}{2\times 3.14\sqrt{(400\times 10^{-6})}}$$

$$= \frac{1}{2\times 3.14\times 2\times 10^{-2}}$$

$$= \frac{100}{3.14\times 4}$$

$$= \frac{25}{3.14} = \frac{25}{\pi}\text{ Hz}$$

25. The induced emf e in the secondary is given by

$$e = -\frac{d\phi}{dt} = -M\frac{dI}{dt}$$

or $|e| = M\frac{dI}{dt}$

$$\therefore \quad |e| = 5\times\frac{10}{5\times 10^{-4}}$$

$$= 1\times 10^5\text{ V}$$

26. A series resonance circuit admits maximum current, as

$$P = i^2 R$$

So, power dissipated is maximum at resonance.

So, frequency of the source at which maximum power is dissipated in the circuit is

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\times 3.14\sqrt{25\times 10^{-3}\times 400\times 10^{-6}}}$$

$$= \frac{1}{2\times 3.14\sqrt{10^{-5}}}$$

$$= 50.3\text{ Hz}$$

27. To see interference, we need two sources with the same frequency and with a constant phase difference. In the given waves,

$$X_1 = a_1 \sin \omega t$$

and $X_4 = a_1 \sin(\omega t + \delta)$

have a constant phase difference δ, so interference is possible between them.

For $X_1 = a_1 \sin \omega t,$

and $\quad X_2 = a_2 \sin 2\omega t,$

frequency is not equal and there is no constant phase difference.

For $\quad X_1 = a_1 \sin \omega t,$

and $\quad X_3 = a_1 \sin \omega_1 t,$

frequency is different and there is no constant phase difference.

28. The equation of nth principal maxima for wavelength λ is given by

$$(a + b) \sin \theta = n\lambda$$

where a is the width of transparent portion and b is that of opaque portion. The width $(a + b)$ is called the grating element.

The spectral lines will overlap, *i.e.*, they will have the same angle of diffraction if

$$\lambda_1 = \lambda_2$$

When a line of wavelength λ_1 in order n_1 coincides with a line of unknown wavelength λ_2 in order n_2, then

$$n_2\lambda_2 = n_1\lambda_1$$

or $$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

29. Given, $$\frac{I_{max}}{I_{min}} = 9 = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$\therefore$ $$\frac{a_1 + a_2}{a_1 - a_2} = 3$$

or $$2a_1 = 4a_2$$

or $$a_1 = 2a_2$$

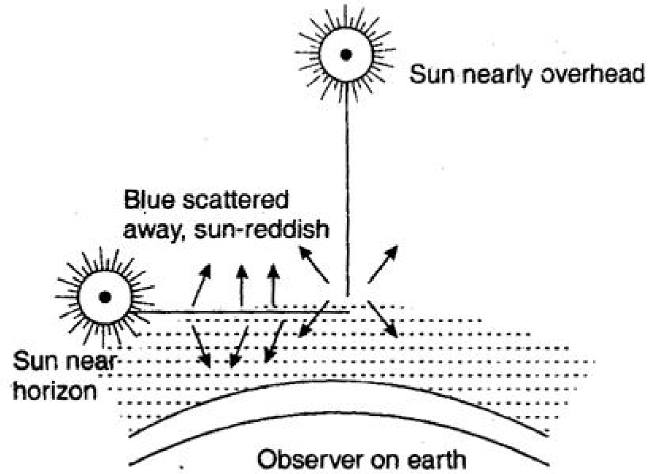
$\Rightarrow$ $$\frac{a_1}{a_2} = 2$$

Again, intensity ratio at the screen due to two slits

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{4}{1}$$

Hence, amplitude ratio is 2 and intensities at the screen due to two slits are 4 units and 1 units, respectively.

30. At the time of sunrise and sunset, the sun is near the horizon. The rays from the sun have to travel a larger part of the atmosphere, As $\lambda_b < \lambda_r$, and intensity of scattered light $\propto \frac{1}{\lambda^4}$, therefore, most of the blue light is scattered away, only red colour, which is least scattered enters our eyes and appears to come from the sun. Hence, the sun looks red both at the time of sunrise and sunset.



31. The magnetic moment of the ground state of an atom is

$$\mu = \sqrt{n(n + 2)}\, \mu_B$$

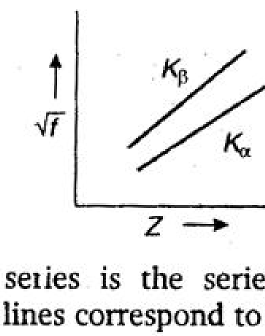
where μ_B is gyromagnetic moment. Here, open sub-shell is half-filled with 5 electrons. *i.e.*, $n = 5$

$\therefore$ $$\mu = \sqrt{5(5 + 2)} \cdot \mu_B$$
$$= \mu_B \sqrt{35}$$

33. Moseley studied the X-ray spectra of various elements. The spectral line observed were of (i) short wavelength – K series and (ii) long wavelength – L series. K_α line is most intense in the K-series. Moreover, he observed that the wavelength of the K_α line decreases with increase in the atomic number of the element as the target. If a graph is plotted between the square root of the frequency and the atomic number of the element emitting the line, it is a straight line

Thus, $$\sqrt{f} \propto Z$$

Where $\sqrt{f}$ is the frequency of the radiation and Z is the atomic number of the element.



34. Balmer series is the series in which the spectral lines correspond to the transition of electron from some higher energy state to the lower energy state corresponding to $n_f = 2$.

Therefore, for Balmer series, $n_f = 2$ and $n_i = 3, 4, 5, \ldots$

Frequency, of 1st spectral line of Balmer series

$$f = RZ^2 c\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

or $$f = RZ^2 c \times \frac{5}{36} \quad \ldots(i)$$

Frequency of 2nd spectral line of Balmer series

$$f' = RZ^2 c\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

or $$f' = RZ^2 c \times \frac{3}{16} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{f}{f'} = \frac{20}{27}$$

$$\therefore \quad f' = \frac{27}{20} f = 1.35 f$$

35. The relativistic kinetic energy of a particle of rest mass m_0 is given by

$$K = (m - m_0) c^2$$

$m = \dfrac{m_0}{\sqrt{1 - (v^2/c^2)}}$, where m is the mass of the particle moving with velocity v.

$$\therefore \quad K = \left[\frac{m_0}{\sqrt{1 - (v^2/c^2)}} - m_0\right] c^2 z$$

According to problem,

kinetic energy = rest energy

$$\therefore \left[\frac{m_0}{\sqrt{1 - (v^2/c^2)}} - m_0\right] c^2 = m_0 c^2$$

or $$\frac{m_0 c^2}{\sqrt{1 - (v^2/c^2)}} = 2 m_0 c^2$$

or $$\frac{1}{1 - (v^2/c^2)} = 4$$

or $$4v^2/c^2 = 3$$

$$\therefore \quad v = \frac{\sqrt{3}c}{2}$$

36. If a charge particle of mass m and charge q is accelerated through a potential difference V. and E is the energy acquired by the particle, then

$$E = qV$$

If v is velocity of particle, then

$$E = \frac{1}{2} mv^2$$

or $$v = \sqrt{\left(\frac{2E}{m}\right)}$$

Now, de-Broglie wavelength of particle is

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{(2E/m)}}$$

Substituting the value of E, we get

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For electron, $$\lambda_e = \frac{h}{\sqrt{2m_e eV}}$$

For proton, $$\lambda_p = \frac{h}{\sqrt{2m_p eV}}$$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\left(\frac{m_p}{m_e}\right)}$$

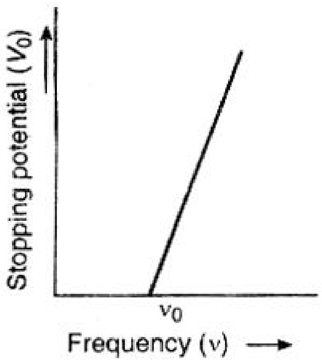
37. The particles are moving with velocities $0.8\,c$ and $-0.4\,c$ in the laboratory frame, say S' frame. Let S be a reference frame in which the particle with velocity $-0.4\,c$ is at rest. Then, the velocity of S' (laboratory) relative to S is $v = 0.4\,c$. Therefore, the particle which in S' has velocity $u' = +0.8\,c$ has a velocity in S given by

$$u = \frac{u' + v}{1 + \dfrac{u'v}{c^2}}$$

$$= \frac{0.8\,c + 0.4\,c}{1 + \dfrac{(0.8\,c)(0.4\,c)}{c^2}}$$

$$= \frac{1.2\,c}{1 + 0.32} = 0.9\,c$$

38. The emission of photoelectron takes place only, when the frequency of the incident light is above a certain critical value, characteristic of that metal. The critical value of frequency is known as the threshold frequency for the metal of the emitting electrode.



Suppose that when light of certain frequency is incident over a metal surface, the photo-electrons are emitted. To take photoelectric current zero, a particular value of stopping potential will be needed. If we go on reducing the frequency of incident light, the value of stopping potential will also go on decreasing. At certain value of frequency ν_0, the photoelectric current will become zero, even when no retarding potential is applied.

This frequency ν_0 corresponds to the threshold for the metal surface. The emission of photoelectrons does not take place, till frequency of incident light is below this value.

39. Experimental measurements show that volume of a nucleus is proportional to its mass number A. If R is the radius of the nucleus assumed to be spherical, then its

volume $\left(\frac{4}{3}\pi R^3\right) \propto A$

or $\quad R \propto A^{1/3}$

or $\quad R = R_0 A^{1/3}$

where R_0 is an empirical constant whose value is found to be 1.1×10^{-15} m.

40. Radiocarbon dating relies on a simple natural phenomenon. As the earth's upper is bombarded by cosmic radiation, atmospheric nitrogen is broken down into an unstable isotope of carbon-carbon 14 (C-14).

The unstable isotope is brought to earth by atmospheric activity, such as storms, and becomes fixed in the biosphere. Because it reacts identically to C-12 and C-13, C-14 attached to complex organic molecules through photosynthesis in plants and becomes their molecular makeup. Animals eating those plants in turn absorb carbon-14 as welll as stable isotopes. This process of ingesting C-14 continues as long as the plant or animal remains alive.

The C-14 within an organism is continually decaying into stable carbon isotopes, but organism is absorbing more C-14 during its life, the ratio of C-14 to C-12 remains about same as the ratio in the atmosphere. When the organism dies, the ratio of C-14 within its carcass begins to gradually decrease.

Chemistry

41. Frenkel defects arises when an ion is missing from its normal position and occupies an interstitial site between the lattice points.

42. The 8 : 8 type of packing is present in cesium chloride (CsCl). In this structure each Cs^+ ion is surrounded by $8Cl^-$ ions and each Cl^- ion is also surrounded by $8Cs^+$ ions.

43. When a solid melts, its entropy (S) increases because on changes from solid to liquid disorder or randomness of molecules increases.

44. The Gibbs-Helmholtz equation is as :

$$G = H + T\left(\frac{\partial G}{\partial T}\right)_P$$

Dividing above equation by T^2

$$\frac{G}{T^2} = \frac{H}{T^2} + \frac{1}{T}\left(\frac{\partial G}{\partial T}\right)_P$$

This on rearrangement becomes

$$\left[\frac{\partial(G/T)}{\partial T}\right]_P = -\frac{H}{T^2}$$

$$H = -T^2\left[\frac{\partial(G/T)}{\partial T}\right]_P$$

where H = enthalpy

45. The condition for spontaneity in an isothermal process is that ΔG should be negative.

$\because \quad \Delta G = \Delta A + P \cdot \Delta V$

$$\Delta G = \Delta A + W \quad (\because W = P \cdot \Delta V)$$

$\therefore$ For spontaneity in an isothermal process:

$$\Delta A + W < 0$$

46. $$2C(s) + 2O_2(g) \longrightarrow 2CO_2(g); \quad \Delta H = -787 \text{ kJ} \quad ...(i)$$

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l); \quad \Delta H = -286 \text{ kJ} \quad ...(ii)$$

$$C_2H_2(g) + 2\frac{1}{2}O_2(g) \longrightarrow 2CO_2(g) + H_2O(l); \quad \Delta H = -1310 \text{ kJ} \quad ...(iii)$$

Add the Eqs. (i) and (ii) and subtract Eq. (iii) from them.

$$2C + H_2 \longrightarrow C_2H_2 - 787 - 286 + 1310$$
$$2C + H_2 \longrightarrow C_2H_2 - 1073 + 1310$$
$$2C + H_2 \longrightarrow C_2H_2 + 237$$

Hence, the heat of formation of acetylene is + 237 kJ.

47. $$NH_4Cl(s) \longrightarrow NH_4^+(aq) + Cl^-(aq) \quad \Delta H = +3.5 \text{ kcal/mol}$$

This is the endothermic reaction hence, increasing the temperature will shift the equilibrium to the right.

48. Arrhenius equation is written as:

$$K = Ae^{-E_a/RT}$$

Taking logarithm, above equation may be written as :

$$\ln K = \ln A - \frac{E_a}{R} \times \frac{1}{T}$$

$$\therefore \quad \ln K_1 = \ln A - \frac{E_a}{R} \times \frac{1}{T_1} \quad ...(i)$$

$$\ln K_2 = \ln A - \frac{E_a}{R} \times \frac{1}{T_2} \quad ...(ii)$$

Subtracting the Eq. (i) from Eq. (ii)

$$\ln K_2 - \ln K_1 = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln \frac{K_2}{K_1} = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

49.

H_2	+	I_2	$\rightleftharpoons$	$2HI$
1		1		0
(1 − 0.8) = 0.2		(1 − 0.8) = 0.2		2 × 0.8 = 1.6

$$K_c = \frac{[HI]^2}{[H_2][I_2]} = \frac{1.6 \times 1.6}{0.2 \times 0.2}$$

$$K_c = 64$$

50. $$H_2(g) + I_2(g) \longrightarrow 2HI(g)$$

The equilibrium constant (K_p) changes with the change in temperature.

Note : Variation of equilibrium constant with temperature can be express as:

$$\log \frac{K_2}{K_1} = \frac{\Delta H}{2.303\,R}\left[\frac{T_2 - T_1}{T_1 \cdot T_2}\right]$$

51. $W = 60$ g

$i = 5$ A

$$\text{Equivalent weight of Ca} = \frac{\text{atomic weight}}{\text{valency}} = \frac{40}{2} = 20$$

According to first law of Faraday electrolysis:

$$W = Zit = \frac{\text{equivalent weight}}{96500} \times i \times t$$

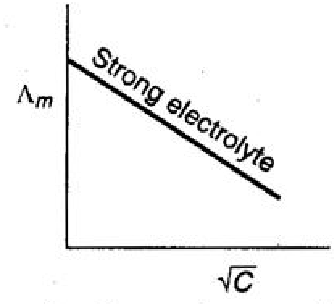
$$\therefore \quad 60 = \frac{20}{96500} \times 5 \times t$$

$$t = \frac{96500 \times 60}{20 \times 5} \text{ s}$$

$$= \frac{96500 \times 60}{20 \times 5 \times 60 \times 60} \text{ h}$$

$$= 16.08 \text{ h.}$$

52. For strong electrolytes the plot of molar conductance (Λ_m) *vs* $\sqrt{C}$ is linear.



Variation of molar conductance (Λ_m) with $\sqrt{C}$ for strong electrolyte.

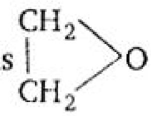
53. From Kohlrausch's law :

$$\Lambda_m^\infty = \nu_+\lambda_+^\infty + \nu_-\lambda_-^\infty$$

For $CaCl_2$:

$$\Lambda^{\infty}_m (CaCl_2) = \lambda^{\infty}_{Ca^{2+}} + 2\lambda^{\infty}_{Cl^-}$$
$$= 118.88 \times 10^{-4} + 2 \times 77.33 \times 10^{-4}$$
$$= 118.88 \times 10^{-4} + 154.66 \times 10^{-4}$$
$$= 273.54 \times 10^{-4} \text{ m}^2 \text{ mho mol}^{-1}$$

54. The substances which have lower reduction potentials are stronger reducing agent. The reduction potential of zinc is lowest among these hence, it is the strongest reducing agent.

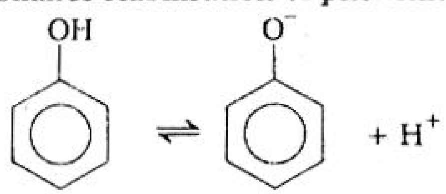
55. The structural formula of epoxide is 

It consists three membered ring with two carbon and one oxygen.

56. In the Grignard reaction magnesium metal forms an organometallic bond.

$$RX + Mg \xrightarrow{\text{Dry ether}} \underset{\text{Grignard reagent}}{R{-}Mg{-}X}$$

57. Phenol is more acidic than ethanol due to resonance stabilisation of phenoxide ion.



phenol phenoxide ion

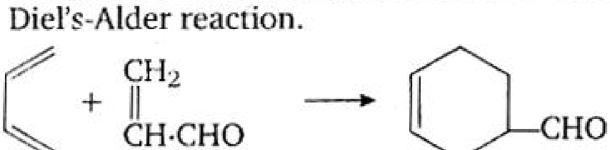
58. Aldol condensation is given by acetaldehyde due to the presence of α-hydrogen atom.

$$CH_3CHO + H \cdot CH_2CHO \xrightarrow{\text{Dil. NaOH}} \underset{\text{aldol}}{CH_3{-}\underset{OH}{\underset{|}{CH}}{-}CH_2{-}CHO}$$

59. $\underset{6}{H_3C}{-}\underset{5}{CH_2}{-}\underset{4}{\overset{O}{\overset{||}{C}}}{-}\underset{3}{CH_2}{-}\underset{2}{CH_2}{-}\underset{1}{\overset{O}{\overset{||}{C}}}{-}OCH_3$

methyl-4-oxohexanoate

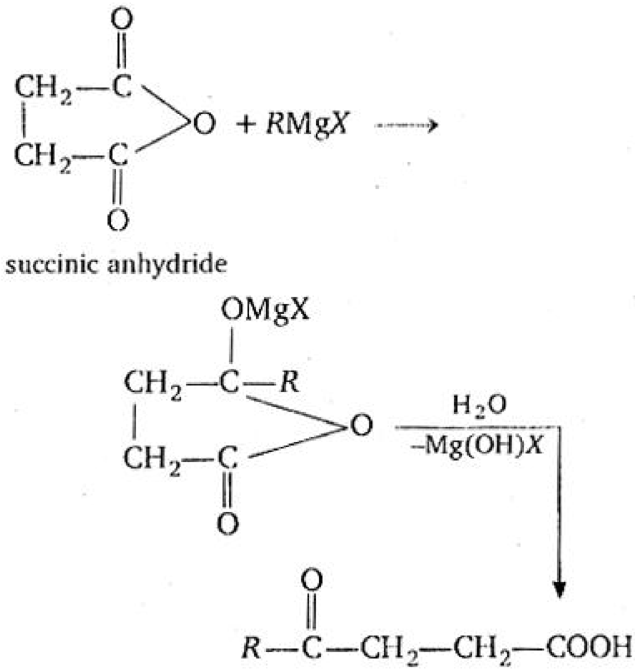
60. The addition of α, β-unsaturated carbonyl compound, with conjugated diene is called Diel's-Alder reaction.



diene acrolein (dienophile) adduct

61. In the Rosenmund's reaction the acid chlorides are converted to corresponding aldehydes by catalytic reaction. This reaction is carried in the presence of palladium deposited over barium sulphate.

$$\underset{\text{acetyl chloride}}{CH_3{-}\overset{O}{\overset{||}{C}}{-}Cl} + H_2 \xrightarrow{\text{Pd, BaSO}_4} \underset{\text{acetaldehyde}}{CH_3CHO} + HCl$$

62. 

succinic anhydride

63. (+) and (–) Tartaric acid does not possess any element of symmetry.

64. The strength of carboxylic acid depends upon the nature of the electron withdrawing halogen atom. Greater the electron withdrawing influence of the halogen atom stronger will be the acid. The electron withdrawing effect of the halogen decreases as:

$$F > Cl > Br > I$$

Hence, $CH_2(I) \cdot COOH$ is the weakest acid among these.

65. Urea on slow heating gives biuret.

$$H_2N \cdot CONH \boxed{H + H_2N} CONH_2 \xrightarrow{\Delta} \underset{\text{biuret}}{H_2NCONH \cdot CONH_2} + \underset{\text{ammonia}}{NH_3}$$

66. Transesterification is the process of conversion of one ester to another ester.

$$R-\overset{O}{\overset{\|}{C}}-OR' + R''-OH \underset{\text{Reflux}}{\overset{H^+,\ R''ONa}{\rightleftharpoons}} R-\overset{O}{\overset{\|}{C}}-OR'' + R'OH$$

ester

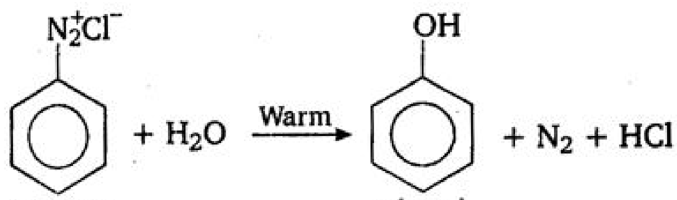
$$\underset{n\text{-butyl acetate}}{CH_3COOC_4H_9} + C_2H_5OH \underset{\text{Reflux}}{\overset{H^+,\ C_2H_5ONa}{\rightleftharpoons}} \underset{\text{ethyl acetate}}{CH_3COOC_2H_5} + C_4H_9OH$$

67. The order of base strength of amines in aqueous solution is as:

$$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N$$

Tertiary amine is less basic than primary and secondary amine due to steric effect of three methyl group present on nitrogen atom.

68. When aqueous solution of benzene diazonium chloride is boiled, it gives phenol.



benzene diazonium chloride — phenol

69. Carbylamine reaction is given by aliphatic and aromatic primary amine hence, it can be used for the distinguish of primary amine with secondary and tertiary amine. In this reaction a primary amine reacts with chloroform and alcoholic KOH to give poisonous substance isocyanide.

$$\underset{\text{primary amine}}{RNH_2} + CHCl_3 + 3KOH(alc) \xrightarrow{\Delta} \underset{\text{alkyl isocyanide}}{RNC} + 3KCl + 3H_2O$$

70. Benzaldehyde reacts with ammonia to form hydrobenzamide.

$$\begin{matrix} C_6H_5CH{=}O \\ C_6H_5CH{=}O \end{matrix} + \begin{matrix} H_2N{-}H \\ H_2N{-}H \end{matrix} + O{=}CH{-}C_6H_5 \longrightarrow \begin{matrix} C_6H_5CH{=}N \\ C_6H_5CH{=}N \end{matrix}\!\!>CH\cdot C_6H_5 + 3H_2O$$

Hydrobenzamide

71. Key Idea :

No. of hybrid orbital $= \frac{1}{2}$ [No. of e^- in V-shell of atom + No. of monovalent atoms – charge on cation + charge on anion]

No. of hybrid orbital	2	3	4	5	6	7
Type of hybridisation	sp	sp^2	sp^3	sp^3d	sp^3d^2	sp^3d^3

Hybridisation in $TeCl_4$:

No. of hybrid orbital $= \frac{1}{2}[6 + 4 + 0 + 0] = 5$

Hence, $TeCl_4$ shows sp^3d hybridisation.

72. $HgCl_2$ compound is easily volatile. They are insoluble in water and soluble in acids.

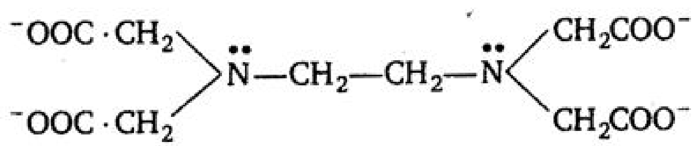
73. $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{90}_{36}Kr + ^{144}_{56}Ba + 2\,^{1}_{0}n$

Note : In the nuclear reaction atomic number and mass are conserved.

74. AgCl dissolves in the solution of NH_3 but not in water because Ag^+ forms a complex ion with NH_3.

$$AgCl + 2NH_3 \longrightarrow \underset{\text{diamine silver (I) chloride}}{[Ag(NH_3)_2]Cl}$$

75. Ethylenediamine tetra-acetic acid is a hexadentate ligand because it has six donor centers.

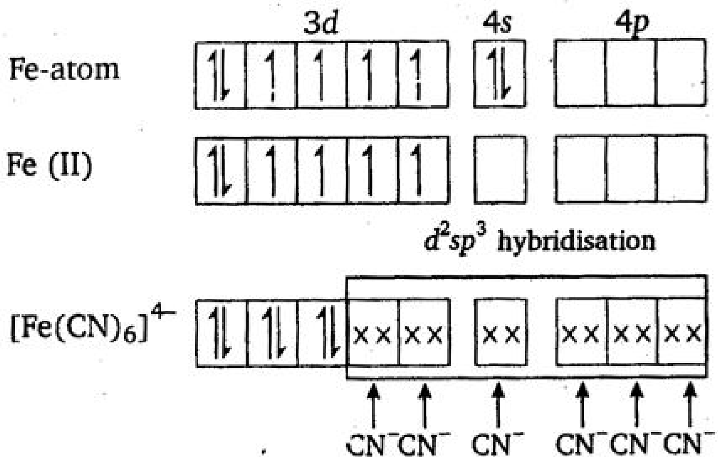


ethylene diamine tetra acetate ion

76. A coordinate bond is a dative covalent bond in which two atoms form bond and one of them provides both electrons.

$$X{:} + Y \longrightarrow X{:}Y \quad \text{or} \quad X \longrightarrow Y$$

77. Ferrocyanide ion $[Fe(CN)_6]^{4-}$ is diamagnetic in nature hence $K_4[Fe(CN)_6]$ complex has zero magnetic moment.



78. The IUPAC name of $[Ni(PPh_3)_2Cl_2]^{2+}$ is dichloro bis (triphenylphosphine) nickel (II)

79. $VOSO_4$ is paramagnetic as well as coloured compound.

The oxidation state of vanadium in $VOSO_4$ is +4.

$$V\,[Z = 23] = [Ar]\,3d^3 4s^2$$

$$V^{4+}\,[Z = 23] = [Ar]\,3d^1 4s^0$$

It has one unpaired electron hence, it is paramagnetic in nature.

80. On an X-ray diffraction photograph the intensity of the spots depends on the electron density of the atoms/ions.

Mathematics

81. $\because\ \alpha = \dfrac{3\pi}{4}$

and we know that $f'(3) = \tan\alpha$

$$= \tan\frac{3\pi}{4}$$

$$= -1$$

82. $\because\quad f(x) = x^2 e^{-2x}$

$$\therefore\quad f'(x) = 2x\,e^{-2x} - 2x^2 e^{-2x}$$

$$= 2x(1-x)e^{-2x}$$

Put $f'(x) = 0$ for maxima or minima, we get

$$2x(1-x)e^{-2x} = 0$$

$$x = 0, 1$$

Now, $f''(x) = 2x(-1)e^{-2x} + 2(1-x)e^{-2x} - 2\cdot 2x(1-x)e^{-2x}$

$$f''(0) = 0 + 2e^0 = 2$$

and $\quad f''(1) = -2e^{-2} + 0 - 0 = -\dfrac{2}{e^2} < 0$

$\therefore\ f(x)$ is maximum at $x = 1$.

Thus, maximum value of $f(x) = 1\cdot e^{-2} = \dfrac{1}{e^2}$

83. $$(x+y)\sin u = x^2y^2 \quad \ldots\text{(i)}$$

On differentiating partially w.r.t. x of Eq. (i), we get

$$(1+0)\sin u + (x+y)\cos u\,\frac{\partial u}{\partial x} = 2xy^2$$

$$\Rightarrow\quad x\sin u + (x^2+xy)\cos u\,\frac{\partial u}{\partial x} = 2x^2y^2 \quad \ldots\text{(ii)}$$

On differentiating partially w.r.t. y of Eq. (i), we get

$$(0+1)\sin u + (x+y)\cos u\,\frac{\partial u}{\partial y} = 2x^2y$$

$$\Rightarrow\quad y\sin u + (xy+y^2)\cos u\,\frac{\partial u}{\partial y} = 2x^2y^2 \quad \ldots\text{(iii)}$$

On adding Eqs. (ii) and (iii), we get

$$(x+y)\sin u + (x+y)\left\{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right\}\cos u = 4x^2y^2$$

$$\Rightarrow x^2y^2 + (x+y)\left\{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right\}\cos u = 4x^2y^2$$

$$\Rightarrow\quad (x+y)\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\cos u = 3x^2y^2$$

$$\Rightarrow\quad (x+y)\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\cos u = 3(x+y)\sin u$$

$$\Rightarrow \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$$

84. Given that, $x = t^2 + 1$ and $y = t^2 - t - 6$

$$\therefore \quad \frac{dx}{dt} = 2t \quad \text{and} \quad \frac{dy}{dt} = 2t - 1$$

Now, $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t-1}{2t}$

when, it meet x-axis, then

$$t^2 - t - 6 = 0$$

$$\Rightarrow \quad (t+2)(t-3) = 0$$

$$\Rightarrow \quad t = 3, -2$$

$$\therefore \left(\frac{dy}{dx}\right)_{\text{at } t=3} = \frac{6-1}{6} = \frac{5}{6} = m_1 \text{ (say)}$$

$$\text{and} \quad \left(\frac{dy}{dx}\right)_{\text{at } t=-2} = \frac{5}{4} = m_2 \text{ (say)}$$

$$\text{Required angle} = \pm\tan^{-1}\left\{\left|\frac{\frac{5}{6} - \frac{5}{4}}{1 + \frac{25}{24}}\right|\right\}$$

$$= \pm\tan^{-1}\left\{\frac{10}{49}\right\}$$

85. $\int_1^4 |x-3|\, dx$

$$= \int_1^3 (3-x)\, dx + \int_3^4 (x-3)\, dx$$

$$= \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$$

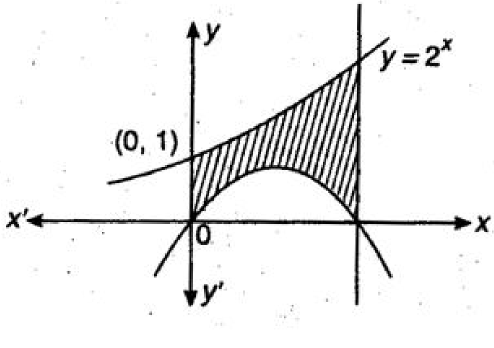
$$= \left(9 - \frac{9}{2}\right) - \left(3 - \frac{1}{2}\right) + \left(\frac{16}{2} - 12\right) - \left(\frac{9}{2} - 9\right)$$

$$= \frac{9}{2} - \frac{5}{2} - 4 + \frac{9}{2} = 9 - 4 - \frac{5}{2}$$

$$= 5 - \frac{5}{2} = \frac{5}{2}.$$

86. Required area

$$= \int_0^2 [2^x - (2x - x^2)]\, dx$$



$$= \int_0^2 (2^x - 2x + x^2)\, dx$$

$$= \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$$

$$= \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2}$$

$$= \left(\frac{3}{\log 2} - \frac{4}{3}\right) \text{sq unit}$$

87. $\displaystyle\int_0^\infty \frac{dx}{(a^2 + x^2)} = \left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_0^\infty$

$$= \frac{1}{a}\{\tan^{-1}(\infty) - \tan^{-1}(0)\}$$

$$= \frac{1}{a}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{2a}$$

88. $\displaystyle\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$

$$= \int e^x \frac{(1 + x^2 - 2x)}{(1+x^2)^2}\, dx$$

$$= \int e^x \left(\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}\right) dx$$

$$= e^x \cdot \frac{1}{1+x^2} + \int \frac{2x\, e^x}{(1+x^2)^2}\, dx$$

$$- \int \frac{2x\, e^x}{(1+x^2)^2}\, dx$$

$$= \frac{e^x}{1+x^2} + c$$

89. $x\sin\left(\dfrac{y}{x}\right) dy = \left[y\sin\left(\dfrac{y}{x}\right) - x\right] dx$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} = \frac{\frac{y}{x}\sin\left(\frac{y}{x}\right) - 1}{\sin\left(\frac{y}{x}\right)}$$

Let $\dfrac{y}{x} = u$ and $\dfrac{dy}{dx} = x\dfrac{du}{dx} + u$

$$\therefore \quad x\frac{du}{dx} + u = \frac{u\sin u - 1}{\sin u}$$

$$\Rightarrow \quad x\frac{du}{dx} = \frac{u\sin u - 1 - u\sin u}{\sin u}$$

$$\Rightarrow \quad -\sin u\, du = \frac{1}{x}\, dx$$

On integrating both sides, we get

$$\cos u = \log x + c$$

$$\Rightarrow \quad \cos\left(\frac{y}{x}\right) = \log x + c$$

$$\because \quad y(1) = \frac{\pi}{2}$$

$$\therefore \quad \cos\frac{\pi}{2} = \log 1 + c \Rightarrow c = 0$$

Thus, $\cos\left(\frac{y}{x}\right) = \log x$

90. The equation of the family of circles of radius r is

$$(x-a)^2 + (y-b)^2 = r^2 \quad \ldots(i)$$

where a and b are arbitrary constants.

On differentiating Eq. (i) w.r.t. x, we get

$$2(x-a) + 2(y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (x-a) + (y-b)\frac{dy}{dx} = 0 \quad \ldots(ii)$$

On differentiating Eq. (ii) w.r.t. x, we get

$$1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$\Rightarrow \quad (y-b) = -\frac{1+(dy/dx)^2}{d^2y/dx^2} \quad \ldots(iii)$$

On putting the value of $(y-b)$ in Eq. (ii), we get

$$(x-a) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}} \quad \ldots(iv)$$

On putting the value of $(y-b)$ and $(x-a)$, in Eq. (i), we get

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2\left(\frac{dy}{dx}\right)^2}{(d^2y/dx^2)^2} + \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}{(d^2y/dx^2)^2} = r^2$$

$$\Rightarrow \quad \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left[\frac{d^2y}{dx^2}\right]^2$$

This is the required differential equation.

91. The given equation can be written as

$$(D^2 + 2D + 1)y = 2e^{3x}, \text{ where } \frac{d}{dx} \equiv D$$

Here, $F(D) = D^2 + 2D + 1$ and $Q = 2e^{3x}$

The auxiliary equation is

$$m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0$$

$$\Rightarrow \quad m = -1, -1$$

$\therefore$ The CF $= (c_1 + c_2x)e^{-x}$

and $$\text{PI} = \frac{1}{F(D)}2e^{3x} = 2\cdot\frac{1}{D^2+2D+1}\cdot e^{3x}$$

$$= 2\cdot\frac{e^{3x}}{9+6+1} = \frac{e^{3x}}{8}$$

Hence, the complete solution is

$$y = \text{CF} + \text{PI}$$

$$\Rightarrow \quad y = (c_1 + c_2x)e^{-x} + \frac{e^{3x}}{8}.$$

92. $$y\, dx + (x - y^3)\, dy = 0$$

Here, $M = y$ and $N = x - y^3$. We have,

$$\frac{\partial M}{\partial y} = 1 \text{ and } \frac{\partial N}{\partial x} = 1, \text{ i.e., } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Hence, the given equation is exact.

Integrating M, *i.e.*, y w.r.t. x, treating y as a constant, we get xy.

Again, the only terms in N which do not contain x are $-y^3$.

Integrating $(-y^3)$ w.r.t. y we obtain $-\frac{y^4}{4}$.

Hence, the solution of the given differential equation is

$$xy - \frac{y^4}{4} + c = 0$$

$$\Rightarrow \quad y^4 = 4xy + c$$

93. Required number of integral solution of

$$x_1 + x_2 + x_3 = 0 = {}^{15+3-1}C_{3-1}$$

$$= {}^{17}C_2$$

94. $\because \quad A = \{1, 2, 3, \ldots, n\}$ and $B = \{a, b, c\}$

$\therefore \quad n(A) = n$ and $n(B) = 3$

Number of onto function from A to B

$$= \sum_{r=1}^{3} (-1)^{3-r}\, {}^3C_r\, r^n$$

$$= (-1)^2\ {}^3C_1(1)^n + (-1)^1\ {}^3C_2(2)^n + (-1)^0\ {}^3C_3 3^n$$
$$= 3^n - 3\cdot 2^n + 3$$
$$= 3^n - 3(2^n - 1)$$

95. Let the total number of persons in the room $= n$.

$\therefore$ Total number of handshakes $= {}^nC_2$.

But number of handshakes $= 66$.

$$\therefore \quad \frac{n!}{2!(n-2)!} = 66$$
$$\Rightarrow \quad \frac{n(n-1)}{2} = 66$$
$$\Rightarrow \quad n^2 - n - 132 = 0$$
$$\Rightarrow \quad n^2 - 12n + 11n - 132 = 0$$
$$\Rightarrow \quad n(n-12) + 11(n-12) = 0$$
$$\Rightarrow \quad (n-12)(n+11) = 0$$
$$\Rightarrow \quad n = 12 \quad (\because n \neq -11)$$

96. The identify element for multiplication modulo 10, is 1 and $3X_{10}\,7 = 1$.
So, the inverse of 7 is 3.

97. In out of 9 tickets, 5 tickets are odd numbers and 4 tickets are even number.

$$\text{Required probability} = \left\{\frac{{}^5C_1}{{}^9C_1}\times\frac{{}^4C_1}{{}^8C_1}\times\frac{{}^4C_1}{{}^7C_1} + \frac{{}^4C_1}{{}^9C_1}\times\frac{{}^5C_1}{{}^8C_1}\times\frac{{}^3C_1}{{}^7C_1}\right\}$$
$$= \frac{5}{9}\times\frac{4}{8}\times\frac{4}{7} + \frac{4}{9}\times\frac{5}{8}\times\frac{3}{7}$$
$$= \frac{80+60}{504} = \frac{140}{504}$$
$$= \frac{5}{18}$$

98. $\because\ P(A) = \frac{1}{12}, P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$

We know that

$$P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} \Rightarrow \frac{1}{15} = \frac{P(A\cap B)}{\frac{1}{12}}$$
$$\Rightarrow \quad P(A\cap B) = \frac{1}{180}$$

Also, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$= \frac{1}{12} + \frac{5}{12} - \frac{1}{180}$$
$$= \frac{15+75-1}{180} = \frac{89}{180}$$

99. $\because \quad f(x) = \frac{x}{2} \qquad (0 \le x \le 2)$

Now, $P(X > 1.5) = \int_{1.5}^{2} \frac{x}{2}\,dx$

$$= 0.4375$$

and $P(X > 1) = \int_{1}^{2}\frac{x}{2}\,dx = 0.75$

Hence, $P\left(\frac{X>1.5}{X>1}\right) = \frac{P(X>1.5)}{P(X>1)}$

$$= \frac{0.4375}{0.75} = \frac{7}{12}$$

100. Since, $(n+1)p = \frac{101}{3}$ is not an integer.

Therefore, $P(X = r)$ is maximum when $r = \left[\frac{101}{3}\right] = 33$.

101. $\because\ A(\theta) = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

Also, $AB = I$.

$$\Rightarrow \quad B = A^{-1}$$
$$= \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$
$$= \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$
$$\Rightarrow (\sec^2\theta)B = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$
$$= A(-\theta).$$

102.
$$\begin{vmatrix} 2x+1 & 4 & 8 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0$$
$$\Rightarrow \begin{vmatrix} 2x+10 & 2x+10 & 2x+10 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0$$
$$(R_1 \to R_1 + R_2 + R_3)$$

$$\Rightarrow \quad (2x+10)\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0$$

$$\Rightarrow \quad (2x+10)\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2x-2 & 0 \\ 7 & -1 & 2x-7 \end{vmatrix} = 0$$

$(C_3 \to C_3 - C_1 \text{ and } C_2 \to C_2 - C_1)$

$$\Rightarrow \quad (2x+10)(2x-2)(2x-7) = 0$$

$$\Rightarrow \quad x = -5, 1, \frac{7}{2}$$

Hence, other roots are 1 and $\frac{7}{2}$.

103. The system of given equations are

$$Kx + 2y - z = 1 \quad \ldots(\text{i})$$

$$(K-1)y - 2z = 2 \quad \ldots(\text{ii})$$

and $$(K+2)z = 3 \quad \ldots(\text{iii})$$

This system of equations has a unique solution, if

$$\begin{vmatrix} K & 2 & -1 \\ 0 & K-1 & -2 \\ 0 & 0 & K+2 \end{vmatrix} \neq 0$$

$$\Rightarrow \quad (K+2)\begin{vmatrix} K & 2 \\ 0 & K-1 \end{vmatrix} \neq 0$$

$$\Rightarrow \quad (K+2)(K)(K-1) \neq 0$$

$$\Rightarrow \quad K \neq -2, 0, 1$$

i.e., $K = -1$, is a required answer.

104. Since, the rank of matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is 1, then

$$\begin{vmatrix} 2 & 5 \\ -4 & a-4 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2a - 8 + 20 = 0$$

$$\Rightarrow \quad 2a + 12 = 0$$

$$\Rightarrow \quad a = -6$$

105. If $b^2 - 4ac \geq 0$, then the equation $ax^4 + bx^2 + c = 0$ has all roots real, if $b < 0$, $a > 0, c > 0$

106. $\log_3 x + \log_3 \sqrt{x} + \log_3 \sqrt[4]{x} + \log_3 \sqrt[8]{x} + \ldots = 4$

$$\Rightarrow \quad \log_3 x + \frac{1}{2}\log_3 x + \frac{1}{4}\log_3 x + \frac{1}{8}\log_3 x + \ldots = 4$$

$$\Rightarrow \quad \log_3 x\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right] = 4$$

$$\Rightarrow \quad \log_3 x\left[\frac{1}{1-1/2}\right] = 8$$

$$\Rightarrow \quad \log_3 x = 4$$

$$\Rightarrow \quad x = 3^4 = 81$$

107. $$\left(x+\frac{1}{x}\right)^3 + \left(x+\frac{1}{x}\right) = 0$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2 + 1\right] = 0$$

From above it is clear that the number of real roots of given equation is 0.

108. $\because$ H is the harmonic mean between P and Q.

$$\therefore \quad H = \frac{2PQ}{P+Q}$$

$$\Rightarrow \quad \frac{H}{P} = \frac{2Q}{P+Q} \text{ and } \frac{H}{Q} = \frac{2P}{P+Q}$$

$$\therefore \quad \frac{H}{P} + \frac{H}{Q} = \frac{2Q}{P+Q} + \frac{2P}{P+Q} = \frac{2(P+Q)}{P+Q}$$

$$= 2.$$

109. $$(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}} + \vec{\mathbf{b}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$$

$$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}} + (\vec{\mathbf{b}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}}$$

$$= \vec{\mathbf{a}}$$

110. Since, the coordinates of A, B and C are $(1, 2, -1)$, $(2, 0, 3)$ and $(3, -1, 2)$, then

$$\overrightarrow{\mathbf{AB}} = (2-1)\hat{\mathbf{i}} + (0-2)\hat{\mathbf{j}} + (3+1)\hat{\mathbf{k}}$$

$$= \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

and $$\overrightarrow{\mathbf{AC}} = (3-1)\hat{\mathbf{i}} + (-1-2)\hat{\mathbf{j}} + (2+1)\hat{\mathbf{k}}$$

$$= 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$$\cos\theta = \frac{(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})}{\sqrt{1+4+16}\sqrt{4+9+9}}$$

$$= \frac{2+6+12}{\sqrt{21}\sqrt{22}} = \frac{20}{\sqrt{462}}$$

$$\Rightarrow \quad \sqrt{462}\cos\theta = 20.$$

111. Since, $\vec{a}$ and $\vec{b}$ are coplanar, therefore $\vec{a} \times \vec{b}$ is a vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. Similarly, $\vec{c} \times \vec{d}$ is a vector perpendicular to the plane containing $\vec{c}$ and $\vec{d}$. Thus, the two planes will be parallel if their normals, *i.e.*, $(\vec{a} \times \vec{b})$ and $(\vec{c} \times \vec{d})$ are parallel.

$$\Rightarrow \quad (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$$

112. Since, the diagonals of a parallelogram are $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$. Then the sides of a parallelogram are $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$

Now, $\vec{a} \times \vec{b} = (2\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} + 2\hat{j} - 3\hat{k}) = \hat{i} + 7\hat{j} + 5\hat{k}$

Area of parallelogram $= |\vec{a} \times \vec{b}| = \sqrt{1 + 49 + 25} = \sqrt{75}$

113. $\because \cos x + \cos^2 x = 1$

$$\Rightarrow \quad \cos x = 1 - \cos^2 x$$
$$= \sin^2 x$$

$\sin^{12} x + 3 \sin^{10} x + 3 \sin^8 x + \sin^6 x - 1$

$= \cos^6 x + 3 \cos^5 x + 3 \cos^4 x + \cos^3 x - 1$

$= (\cos^2 x + \cos x)^3 - 1 = 1 - 1 = 0$

114. $(\cos \alpha + i \sin \alpha)^{3/5} = [\cos 3\alpha + i \sin 3\alpha]^{1/5}$

$$= \left[\cos \frac{2n\pi + 3\alpha}{5} + i \sin \frac{2n\pi + 3\alpha}{5}\right]$$

Required product $= \left(\cos \frac{3\alpha}{5} + i \sin \frac{3\alpha}{5}\right)\left(\cos \frac{2\pi + 3\alpha}{5} + i \sin \frac{2\pi + 3\alpha}{5}\right)$

$$\times \left(\cos \frac{4\pi + 3\alpha}{5} + i \sin \frac{4\pi + 3\alpha}{5}\right)\left(\cos \frac{6\pi + 3\alpha}{5} + i \sin \frac{6\pi + 3\alpha}{5}\right)$$

$$\times \left(\cos \frac{8\pi + 3\alpha}{5} + i \sin \frac{8\pi + 3\alpha}{5}\right)$$

$$= \cos\left(\frac{3\alpha}{5} + \frac{2\pi}{5} + \frac{3\alpha}{5} + \frac{4\pi}{5} + \frac{3\alpha}{5} + \frac{6\pi}{5} + \frac{3\alpha}{5} + \frac{8\pi}{5} + \frac{3\alpha}{5}\right)$$

$$+ i \sin\left(\frac{3\alpha}{5} + \frac{2\pi}{5} + \frac{3\alpha}{5} + \frac{4\pi}{5} + \frac{3\alpha}{5} + \frac{6\pi}{5} + \frac{3\alpha}{5} + \frac{8\pi}{5} + \frac{3\alpha}{5}\right)$$

$$= \cos (4\pi + 3\alpha) + i \sin (4\pi + 3\alpha)$$

$$= \cos 3\alpha + i \sin 3\alpha$$

115. $\frac{(1+i)^2}{i(2i-1)} = \frac{1 + i^2 + 2i}{i(2i-1)} = \frac{2i}{i(2i-1)}$

$$= \frac{2(2i+1)}{4i^2 - 1} = \frac{4i + 2}{-4 - 1} = -\frac{4}{5}i - \frac{2}{5}$$

$\therefore$ Imaginary part of $\frac{(1+i)^2}{i(2i-1)} = -\frac{4}{5}$