EAMCET 2014

ENGINEERING

Question Paper with Solutions

CODE-A

MATHS

1. If R is the set of all real numbers and if $f: R-\{2\} \to R$ is defined by $f(x)=\frac{2+x}{2-x}$ for $x \in R-\{2\}$, then the range of f is

1) R 2) $R-\{1\}$ 3) $R-\{-1\}$ 4) $R-\{-2\}$

Key: 3

Sol: $\frac{y}{1}=\frac{2+x}{2-x}$

$2+x=2y-xy$

$x+xy=2(y-1)$

$x(1+y)=2(y-1)$

$2=2\left[\frac{(y-1)}{y+1}\right]$

$\therefore$ Range $=R-\{-1\}$

2. Let Q be the set of all rational numbers in $[0,1]$ and $f:[0,1]\to[0,1]$ be defined by

$$f(x)=\begin{cases} x & \text{for } x \in Q \\ 1-x & \text{for } x \notin Q \end{cases}.$$

Then the set $S=\{x \in [0,1] : (fof)(x)=x\}$ is equal to

1) Q 2) $[0,1]-Q$ 3) $(0,1)$ 4) $[0,1]$

Key: 4

Sol: $f=f^{-1}$

3. $\sum_{k=1}^{2n+1}(-1)^{k-1}.k^2=$

1) $(n+1)(2n+1)$ 2) $(n+1)(2n-1)$ 3) $(n-1)(2n+1)$ 4) $(n-1)(2n-1)$

Key: 1

Sol: $.\sum_{k=1}^{3}(-1)^{k-1}k^2$

$=1+(-1)^{2-1}(4)+(-1)^{3-1}3^2$

$=1-4+9=6$

$2.3=6$

4. If a, b, c and d are real numbers such that $a^2+b^2+c^2=d^2=1$ and if $A=\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$ then

$A^{-1}=$

1) $\begin{bmatrix} a-ib & c+id \\ -c+id & a+ib \end{bmatrix}$ 2) $\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$ 3) $\begin{bmatrix} a+ib & c+id \\ c-id & a-ib \end{bmatrix}$ 4) $\begin{bmatrix} a+ib & -c-id \\ c-id & a-ib \end{bmatrix}$

Key: 2

Sol: . $|A| = \dfrac{1}{(a+ib)(a-ib)+(c-id)(c+id)}$

$= \dfrac{1}{a^2+b^2+c^2+d^2} = 1$

$A^{-1}\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$

5. **If the matrix** $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & \alpha \end{bmatrix}$ **is of rank 3, then** $\alpha =$

1) 5 2) 4 3) 1 4) -5

Key: 1

Sol: $|A| \neq 0$ then $\alpha = 5$

6. **If** $k > 1$**, and the determinant of the matrix** A^2**, where** $A = \begin{bmatrix} k & k\alpha & \alpha \\ 0 & \alpha & k\alpha \\ 0 & 0 & k \end{bmatrix}$**, is** k^2 **then** $|\alpha| =$

1) k 2) k^2 3) $\dfrac{1}{k}$ 4) $\dfrac{1}{k^2}$

Key: 3

Sol: $|A| = k(k\alpha - 0) = k^2\alpha$

$\begin{bmatrix} k & k\alpha & \alpha \\ 0 & \alpha & k\alpha \\ 0 & 0 & k \end{bmatrix}\begin{bmatrix} k & k\alpha & \alpha \\ 0 & \alpha & k\alpha \\ 0 & 0 & k \end{bmatrix}$

7. **The number of solutions for** $z^3 + \bar{z} = 0$ **is**

1) 1 2) 2 3) 3 4) 5

Key: 4

Sol: $z^3 + \bar{z} = 0$

$(x+iy)^3 = -x+iy$

$x^3 = i^3y^3 + 3x^2iy + 3xi^2y^2 = -x+iy$

$x^3 - iy^3 + 3x^2iy - 3xy^2 = -x+iy$

$\left(x^3 - 3xy^2\right) + i\left(3x^2y - y^3\right) = -x+iy$

$x^3 - 3xy^2 = -x$

$x^2 - 3y^2 = -1$

$x^3 - 3\left(3x^2 - 1\right) = -1$

$x^2 - 9x^2 + 3 + 1 = 0$

$-8x^2 + 4 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{2}$

$3x^2y - y^3 = y$

$3x^2 - y^2 = 1$

$y^2 = 3x^2 - 1$

$x = \frac{1}{\sqrt{2}}$

$y^2 = \frac{3}{2} - 1 = \frac{1}{2}$

$y = \pm\frac{1}{\sqrt{2}}$

$x = -\frac{1}{\sqrt{2}}$

$y^2 = \frac{3}{2} - 1 = \frac{1}{2}$

$y = \pm\frac{1}{\sqrt{2}}$

8. The least positive integer n for which $(1+i)^n = (1-i)^n$ holds is

1) 2 2) 4 3) 6 4) 8

Key: 2

Sol: $(1+i)^n = \left(\sqrt{2}\right)^n \left(\frac{1}{\sqrt{2}} + i - \frac{1}{\sqrt{2}}\right)^n = \left(\sqrt{2}\right)^n \text{cis}\frac{n\pi}{4}$

$= 2^{\frac{n}{2}} \text{ cis}\left(-\frac{n\pi}{4}\right)$

$\text{cis}\frac{n\pi}{4} = \text{cis}\left(\frac{-n\pi}{4}\right)$

$2i\sin\frac{n\pi}{4} = 0$

$\sin\frac{n\pi}{4} = 0$

$n = 4$

9. If $x = p + q, y = pw + qw^2$ and $z = pw^2 + qw$ where w is a complex cube root of unity then $xyz =$

1) $p^2 - pq + q^2$ 2) $1 + p^3 + q^3$ 3) $p^3 - q^3$ 4) $p^3 + q^3$

Key:4

Sol: $p = q = 1$

$x = 2$

$y = w + w^2$

$z = w^2 + w$

$xyz = 2(-1)(-1) = 2$

10. **If** $Z_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$ **for** $r = 1, 2, 3,$ **..... then** $Z_1Z_2Z_3.....\infty =$

1) 1 2) 2 3) -1 4) -2

Key:10

Sol: $z_r = \text{cis}\left(\frac{\pi}{2^r}\right)$

$z_1.z_2...........\infty = \text{cis}\left(\frac{\pi}{2} + \frac{\pi}{2^2} + ...\infty\right)$

$\text{cis}\pi = -1$

11. **If** x_1 **and** x_2 **are the real roots of the equation** $x^2 - kx + c = 0$ **then the distance betwen the points** $A(x_1, 0)$ **and** $B(x_1, 0)$ **is**

1) $\sqrt{k^2 - c}$ 2) $\sqrt{c - k^2}$ 3) $\sqrt{k^2 - 4c}$ 4) $\sqrt{k^2 + 4c}$

Key: 3

Sol: $x_1 + x_2 = k$

$x_1x_2 = c$

$|x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1x_2}$

$= \sqrt{k^2 - 4c}$

12. **If** x **is real, then the minimum value of** $y = \frac{x^2 - x + 1}{x^2 + x + 1}$ **is**

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) 2 4) 3

Key: 1

Sol: .$\frac{y}{1} = \frac{x^2 - x + 1}{x^2 + x + 1}$

$x^2 - x + 1 = x^2y + xy + y$

$(1-y)x^2(-1-y)x + (1-y) = 0$

$(1+y)^2 - 4(1-y)^2 > 0$

$1 + y^2 + 2y - 4(1 + y^2 - 2y) > 0$

$1 + y^2 + 2y - 4 - 4y^2 + 8y \geq 0$

$-3y^2 + 10y - 3 \geq 0$

$3y^2 - 10y + 3 < 0$

$3y^2 - 9y - y + 3 < 0$

$3y(y-3) - 1(y-3) \le 0$

$y \in \left[\frac{1}{3}, 3\right]$

$\therefore 3$

13. If p and q are distinct prime numbers and if the equation $x^2 - px + q = 0$ has positive integers as its roots then the roots of the equation are

1) 2, 3 2) 1, 2 3) 3, 1 4) 1, -1

Key: 2

Sol: .Sum of the roots $\alpha + \beta = p$

Product of the roots $\alpha\beta = q$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, x = 2$

14. The cubic eqation whose roots are the squares of the roots of $x^3 - 2x^2 + 10x - 8 = 0$ is

1) $x^3 + 8x^2 + 68x - 64 = 0$ 2) $x^3 + 16x^2 - 68x - 64 = 0$

3) $x^3 - 16x^2 + 68x - 64 = 0$ 4) $x^3 + 16x^2 + 68x - 64 = 0$

Key: 3

Sol: .Let $y = \sqrt{x}$

$x\sqrt{x} - 2x + 10\sqrt{x} - 8 = 0$

$\sqrt{x}(x+10) = (2x+8)$

$\Rightarrow x(x+10)^2 = (2x+8)^2$

$\Rightarrow x(x^2 + 20x + 100) = 4x^2 + 64 + 32x$

$\Rightarrow x^3 + 20x^2 + 100x - 4x^2 - 32x - 64 = 0$

$\Rightarrow x^3 + 16x^2 + 68x - 64 = 0$

15. Out of thirty points in a plane, eight of them are collinear. The number of straight lines that can be formed by joining these points is

1) 540 2) 408 3) 348 4) 296

Key: 2

Sol: $30c_2 - 8c_2 + 1$

$= \frac{30 \times 29}{2} - \frac{8 \times 7}{2} + 1$

$= 15 \times 29 - 28 + 1$

$= 435 - 28 + 1$

$= 436 - 28$

$= 408$

16. **If n is an integer with $0 \le n \le 11$ then the minimum value of $n!(11-n)!$ is attained when a value of $n =$**

1) 5 2) 7 3) 9 4) 11

Key: 1

Sol: . $^{11}C_n = \dfrac{11!}{(11-n)!n!}$

$\text{minimum value} = \dfrac{^nC_{n-1}}{2}$

$^{11}C_n = {}^{11}C_5$

$\therefore\ n = 5$

17. **If** $(a+bx)^{-3} = \dfrac{1}{27} + \dfrac{1}{3}x + ...$ **then the ordered pair** $(a,b) =$

1) $\left(1, \dfrac{1}{3}\right)$ 2) $(3,9)$ 3) $(3,-9)$ 4) $(3,-27)$

Key: 3

Sol: . $\dfrac{1}{a^3}\left[1+\dfrac{b}{a}x\right]^{-3} = \dfrac{1}{27} + \dfrac{x}{3} + ...$

$\Rightarrow \dfrac{1}{a^3}\left[1-\dfrac{3b}{a}x+......\right] = \dfrac{1}{27} + \dfrac{x}{3}$

$\Rightarrow \dfrac{1}{a^3} = \dfrac{1}{27} = a = 3$

$-\dfrac{3b}{a^4}x = \dfrac{x}{3} = -\dfrac{3b}{27} = \dfrac{1}{3}\quad b = -9$

$\therefore (3,-9)$

18. **The term independent of x in the expansion of** $\left(\sqrt{x} - \dfrac{2}{\sqrt{x}}\right)^{18}$ **is**

1) $\binom{18}{9}2^{12}$ 2) $\binom{18}{6}2^{6}$ 3) $\binom{18}{6}2^{8}$ 4) $-\binom{18}{9}2^{9}$

Key:4

Sol: . $\left(\sqrt{x} - \dfrac{2}{\sqrt{x}}\right)^{18}$

independent term exist in T_{r+1}

where $r = \dfrac{np}{p+q} = \dfrac{18\times\frac{1}{2}}{1} = 9$

$T_{9+1} = 18C_9(-2)^9$

19. $\frac{2x^3+x^2-5}{x^4-25}=\frac{Ax+B}{x^2-5}+\frac{Cx+1}{x^2+5}\Rightarrow(A,B,C)=$

1) $(1,1,0)$ 2) $(1,0,1)$ 3) $(1,2,1)$ 4) $(1,1,1)$

Key: 2

Sol: . $\frac{2x^3+x^2-5}{x^4-25}=\frac{(Ax+B)(x^2+y^2)+(cx+1)(x^2-5)}{x^4-25}$

$A+C=2\to(1)$

$5A-5C=0\ \to(2)$

$A+C=2$
$\underline{A-C=0}$
$2A=2, A=1,\ C=1$

Compare x^2

$B+1=1$ $B=0$

$(1,0,1)$

20. **If** $\cos x=\tan y, \cot y=\tan z$ **and** $\cot z=\tan x$**; then** $\sin x=$

1) $\frac{\sqrt{5}-1}{4}$ 2) $\frac{\sqrt{5}+1}{2}$ 3) $\frac{\sqrt{5}-1}{2}$ 4) $\frac{\sqrt{5}+1}{4}$

Key: 3

Sol: . $\cos x=\tan y=\frac{1}{\cot y}=\frac{1}{\tan z}$

$\cos x=\tan x$

$\cos x-\tan x=0$

$\cos x-\frac{\sin x}{\cos x}=0$

$\cos x-\sin x=0$

$1-\sin x-\sin x=0$

$\sin x+\sin x-1=0$

$\sin x=\frac{-1+\sqrt{1+4}}{2}=\frac{\sqrt{5}-1}{2}$

21. $\tan 81^0-\tan 63^0-\tan 27^0+\tan 9^0=$

1) 0   2) 2   3) 4   4) 6

Key: 3

Sol: . $(\cot 9^0+\tan 9^0)-(\cot 27^0+\tan 27^0)$

$=\frac{2}{\sin 18^0}-\frac{2}{\sin(54^0)}=4$

22. If x and y are acute angles such that $\cos x+\cos y=\frac{3}{2}$ and $\sin x+\sin y=\frac{3}{4}$ then $\sin(x+y)=$

1) $\frac{3}{4}$ 2) $\frac{3}{5}$ 3) $\frac{4}{5}$ 4) $\frac{2}{5}$

Key: 3

Sol: . $\frac{\sin x+\sin y}{\cos x+\cos y}=\frac{\frac{3}{4}}{\frac{3}{2}}=\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$

$$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}=\frac{1}{2}$$

$$\tan\left(\frac{x+y}{2}\right)=\frac{1}{2}$$

$$\sin(x+y)=\frac{2\tan\left(\frac{x+y}{2}\right)}{1+\tan^2\left(\frac{x+y}{2}\right)}$$

$$=\frac{2\times\frac{1}{2}}{1+\frac{1}{4}}=\frac{1}{\left(\frac{5}{4}\right)}=\frac{4}{5}$$

23. The sum of the solutions in $(0,2\pi)$ of the equation $\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)=\frac{1}{4}$ is

1) π 2) 2π 3) 3π 4) 4π

Key: 2

Sol: . $\frac{1}{4}\cos(3x)=\frac{1}{4}$

$\cos 3x=1$

$\Rightarrow \cos 3x=\cos(2n\pi)$

$3x=2n\pi$

$x=\frac{2n\pi}{3}$

Solutions $\frac{2\pi}{3},\frac{4\pi}{3}$

Sum $= =\frac{6\pi}{3}=2\pi$

24. **If** $x>0, y>0, z>0, xy+yz+zx<1$ **and if** $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$ **then** $x+y+z=$

1) xyz 2) $3xyz$ 3) $\sqrt{xyz}$ 4) 0

Key: 1

Sol: .Given $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$

$x+y=z=xyz$

25. $\operatorname{sech}^{-1}\left(\frac{1}{2}\right)-\operatorname{cosech}^{-1}\left(\frac{3}{4}\right)=$

1) $\log_e\left(\frac{1+\sqrt{3}}{3}\right)$ 2) $\log_e\left(\frac{2+\sqrt{3}}{3}\right)$ 3) $\log_e\left(\frac{2-\sqrt{3}}{3}\right)$ 4) $\log_e\left(3\left(2+\sqrt{3}\right)\right)$

Key: 2

Sol: . $\cosh^{-1}(2)-\sin^{-1}\left(\frac{4}{3}\right)$

$\log\left(2+\sqrt{3}\right)-\log\left(\frac{4}{3}+\sqrt{\frac{16}{9}+1}\right)$

$\log\left(2+\sqrt{3}\right)-\log\left(\frac{4}{3}+\frac{5}{3}\right)$

$=\log\left(2+\sqrt{3}\right)\log 3$

$=\log\left(\frac{2+\sqrt{3}}{3}\right)$

26. **If any** $\Delta ABC, \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}=$

1) $\cos^2 A$ 2) $\cos^2 B$ 3) $\sin^2 A$ 4) $\sin^2 B$

Key: 3

Sol: . $\frac{(25)2(s-a)\times 2(s-b)\times 2(s-c)}{4b^2c^2}$

$=\frac{8s(s-a)(s-b)(s-c)}{4b^2c^2}$

$=\frac{8\Delta^2}{4b^2c^2}$ $\Delta=\frac{1}{2}bc\sin A$

$=\frac{8\times\frac{1}{2}b^2c^2\sin^2 A}{4b^2c}$

$=\sin^2 A$

27. **If the angles of triangle are in the ratio** $1:1:4$ **then the ratio of the perimeter of the triangle to its largest side is**

1) $3:2$ 2) $\sqrt{3}+2:\sqrt{2}$ 3) $\sqrt{3}+2:\sqrt{3}$ 4) $\sqrt{2}+2:\sqrt{3}$

Key: 2

Sol: . $\alpha+\alpha+4\alpha=180^0$

$6\alpha=180^0$

$\alpha=30^0$

$30^0,30^0,120^0$

$a=2R\sin 30^0=R$

$b=2R\sin 30^0=R$

$c=2R\sin\left(120^0\right)=\sqrt{3}R$

$2S:C=2R+\sqrt{3}R:\sqrt{3}R$

$=2+\sqrt{3}:\sqrt{3}$

28. If in a triangle ABC, $r_1=2, r_2=3$ and $r_3=6$ then $a=$

1) 1 2) 2 3) 3 4) 4

Key: 3

Sol: . $\frac{1}{a}=\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}=1$

$\Delta=1$

$\Delta=\sqrt{r_1r_2r_3r}=6$

$S=\frac{\Delta}{a}=6$

$\Delta_1=\frac{\Delta}{S-a}\Rightarrow a=3$

29. Three non-zero non-collinear vectors $\bar{a},\bar{b},\bar{c}$ are such that $\bar{a}+3\bar{b}$ is collinear with $\bar{c}$, while $3\bar{b}+2\bar{c}$ is collinear with $\bar{a}$. Then $3\bar{a}+2\bar{c}=$

1) $2\bar{a}$ 2) $3\bar{b}$ 3) $4\bar{c}$ 4) $\bar{0}$

Key: 4

Sol: . $a+3\bar{b}=\lambda\left(\bar{c}\right)\to(1)$

$3\bar{b}+2\bar{c}=\mu\left(\bar{a}\right)\to(2)$

$\bar{a}+3\bar{b}+2\bar{c}=\lambda\bar{c}+2\bar{c}=(\lambda+2)\bar{c}$

$\bar{a}+\mu\bar{a}=\bar{a}+3\bar{b}+2\bar{c}$

$(\lambda+2)\bar{c}=(1+\mu)\bar{a}$

$\lambda+2=0,\mu+1=0$

$\lambda=-2,\mu=-1$; $\Rightarrow\bar{a}+3\bar{b}+2\bar{c}=\bar{0}$

30. If $\bar{a},\bar{b}$ and $\bar{c}$ are non-coplanar vectors and if $\bar{d}$ is such that $\bar{d}=\frac{1}{x}\left(\bar{a}+\bar{b}+\bar{c}\right)$ and $\bar{a}=\frac{1}{y}\left(\bar{b}+\bar{c}+\bar{d}\right)$ where x and y are non-zero real numbers, then $\frac{1}{xy}\left(\bar{a}+\bar{b}+\bar{c}+\bar{d}\right)=$

1) $-\bar{a}$ 2) $\bar{0}$ 3) $2\bar{a}$ 4) $3\bar{c}$

Key: 2

Sol: By eliminating $\bar{d}$

Find x & y

31. The angle between the lines $\bar{r}=\left(2\bar{i}-3\bar{j}+\bar{k}\right)+\lambda\left(\bar{i}+4\bar{j}+3\bar{k}\right)$ and $\bar{r}=\left(\bar{i}-\bar{j}+2\bar{k}\right)+\mu\left(\bar{i}+2\bar{j}-3\bar{k}\right)$ is

1) $\cos^{-1}\left(\frac{9}{\sqrt{91}}\right)$ 2) $\cos^{-1}\left(\frac{7}{\sqrt{84}}\right)$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Key: 4

Sol: . $\cos = \dfrac{1+8-9}{\sqrt{1+16+9}\sqrt{1+4+9}} = 0$

$\theta = \dfrac{\pi}{2}$

32. If $\bar{a}, \bar{b}$ and $\bar{c}$ are vectors with magnitudes 2, 3 and 4 respectively then the best upper bound of $\left|\bar{a}-\bar{b}\right|^2 + \left|\bar{b}-\bar{c}\right|^2 + \left|\bar{c}-\bar{a}\right|^2$ among the given values is

1) 97 2) 87 3) 90 4) 93

Key:2

Sol: $\left|\bar{a}\right| = 2, \left|\bar{b}\right| = 3, \left|\bar{c}\right| = 4$

$2\left(\left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 + \left|\bar{c}\right|^2 - \left(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}\right)\right)$

$= 2\left(29 - \{6\cos A + 12.\cos B + 8\cos C\}\right)$

$= 2\left[29 - (26)\right]$

33. If x, y, z are non-zero real numbers, $\bar{a} = x\bar{i} + 2\bar{j}, \bar{b} = y\bar{j} + 3\bar{k}$ and $\bar{c} = x\bar{i} + y\bar{j} + z\bar{k}$ are such that $\bar{a}\times\bar{b} = 2\bar{i} - 3\bar{j} + \bar{k}$ then $\left[\bar{a}\,\bar{b}\,\bar{c}\right] =$

1) 10 2) 9 3) 6 4) 3

Key: 2

Sol: . $\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x & 2 & 0 \\ 0 & y & 3 \end{vmatrix}$

$2\bar{i} - 3\bar{j} + \bar{k} = 6\bar{i} + 3x\bar{j} + \bar{k}(xy)$

$2 = 6, x = 1, y = 1$

$\left[\bar{a}\ \bar{b}\ \bar{c}\right] = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 1 & 1 & 6 \end{vmatrix}$

$= 1(6-3) - 2(-3)$

$= 3 + 6 = 9$

34. The shortest distance between the skew lines $\bar{r} = \left(\bar{i} + 2\bar{j} + 3\bar{k}\right) + t\left(\bar{i} + 3\bar{j} + 2\bar{k}\right)$ and $\bar{r} = \left(4\bar{i} + 5\bar{j} + 6\bar{k}\right) + t\left(2\bar{i} + 3\bar{j} + \bar{k}\right)$ is

1) 3 2) $2\sqrt{3}$ 3) $\sqrt{3}$ 4) $\sqrt{6}$

Key: 3

Sol: . $\dfrac{\left[\bar{a}-\bar{c}\ \ \bar{b}\ \ \bar{d}\right]}{\left|\bar{b}\times\bar{d}\right|}$

$$\bar{b}\times\bar{d}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 3 & 2\\ 2 & 3 & 1\end{vmatrix}$$

$=\bar{i}(-3)-\bar{j}(-3)+\bar{k}(-3)$

$=-3\bar{i}+3\bar{j}-3\bar{k}$

$\left|\bar{b}\times\bar{d}\right|=\sqrt{9+9+9}=\sqrt{27}-3\sqrt{3}$

$\left[\bar{a}-\bar{c}\ \ \bar{b}\ \ \bar{d}\right]=\left(-3\bar{i}-3\bar{j}-3\bar{k}\right).\left(-3\bar{i}+3\bar{j}-3\bar{k}\right)$

$=9-9+9=9$

$\therefore \text{S.D.}=\dfrac{9}{3\sqrt{3}}=\dfrac{3}{\sqrt{3}}=\sqrt{3}$

35. **The mean of four observations is 3. If the sum of the squares of these observations is 48 then their standard deviation is**

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) $\sqrt{5}$ 4) $\sqrt{7}$

Key: 2

Sol: $x_1+x_2+x_3+x_4=12$ -----(1)

$x_1^2+x_2^2+x_3^3+x_4^4=48$

$\text{S.D.}=\sqrt{\dfrac{\Sigma x_i^2}{n}-\mu^2}$

$=\sqrt{\dfrac{48}{4}-9}$

$=\sqrt{12-9}$

$=\sqrt{3}$

36. **If** $x_1, x_2, ...x_n$ **are n observations such that** $\sum_{i=1}^{n} x_i^2=400$ **and** $\sum_{i=1}^{n} x_i=80$ **then the least value of n is**

1) 12 2) 15 3) 16 4) 18

Key:3

Sol: Variance ≥ 0 and mean > variance

variance $=\Sigma\dfrac{Xi^2}{n}-\bar{x}^2$, mean $=\Sigma\dfrac{xi}{n}$

37. **If A, B and C are mutually exclusive and exhaustive events of a random experiment such that** $P(B)=\dfrac{3}{2}P(A)$ **and** $P(C)=\dfrac{1}{2}P(B)$ **then** $P(A\cup C)=$

1) $\dfrac{3}{13}$ 2) $\dfrac{6}{13}$ 3) $\dfrac{7}{13}$ 4) $\dfrac{10}{13}$

Key:3

Sol: $P(A)+P(B)=P(C)=1$

38. **A six-faced unbaised die is thrown twice and the sum of the numbers appearing on the upper face is observed to be 7. The probability that the number 3 has appeared at least once is**

1) $\dfrac{1}{2}$ 2) $\dfrac{1}{3}$ 3) $\dfrac{1}{4}$ 4) $\dfrac{1}{5}$

Key:2

Sol: Use conditional probability

$$P\left(\frac{A}{B}\right)=P\left(\frac{A\cap B}{B}\right)$$

39. A candidate takes three tests in succession and the probability of passing the first test is p. The probability of passing each succeeding test is p or $\frac{p}{2}$ according as he passes or fails in the preceding one. The candidate is selected if he ;passes at least two tests. The probability that the candidate is selected is

1) $p(2-p)$ 2) $p+p^2+p^3$ 3) $p^2(1-p)$ 4) $p^2(2-p)$

Key:4

Sol: Required events can occured in the following mutually exclusive ways.

SSS, SSF, SFS, FSS

40. A random variable X has the probability distribution given below. Its variance is

X	1	2	3	4	5
$P(X=x)$	K	2k	3k	2k	k

1) $\frac{4}{3}$ 2) $\frac{5}{3}$ 3) $\frac{10}{3}$ 4) $\frac{16}{3}$

Key:1

Sol: Variance = $\Sigma xi^2 P\left(x=xi^2\right)-\left(\bar{x}\right)^2$

41. If the mean and variance of a binomial variate X are 8 and 4 respectively then $P(X<3)=$

1) $\frac{137}{2^{16}}$ 2) $\frac{697}{2^{16}}$ 3) $\frac{265}{2^{16}}$ 4) $\frac{265}{2^{15}}$

Key:1

Sol: $npq=4$, $np=8$

$q=\frac{1}{2}, p=\frac{1}{2}$

$n=16$

$P(x<3)=\frac{{}^{16}C_0+{}^{16}C_1+{}^{16}C_2}{2^{16}}$

42. The locus of the centroid of the triangle with vertices at $(a\cos\theta, a\sin\theta),(b\sin\theta,-b\cos\theta)$ and $(1,0)$ is (Here θ is a parameter)

1) $(3x-1)^2+9y^2=a^2-b^2$ 2) $(3x-1)^2+9y^2=a^2+b^2$

3) $(3x+1)^2+9y^2=a^2-b^2$ 4) $(3x+1)^2+9y^2=a^2+b^2$

Key:2

Sol: $a\cos\theta+b\sin\theta+1=3x$,

$a\sin\theta-b\cos\theta=3y$

eliminate θ.

43. **The point $P(1,3)$ undergoes the following transformations successively**
(i) Reflection with respect to the line $y=x$
(ii) Translation through 3 units along the positive direction of the X-axis
(iii) Rotation through an angle of $\frac{\pi}{6}$ about the origin in the clockwise direction
The final position of the point P is

1) $\left(\frac{7}{\sqrt{2}},\frac{-5}{\sqrt{2}}\right)$ 2) $\left(\frac{6+\sqrt{3}}{2},\frac{1-6\sqrt{3}}{2}\right)$ 3) $\left(\frac{6\sqrt{3}-1}{2},\frac{6+\sqrt{3}}{2}\right)$ 4) $\left(\frac{6\sqrt{3}+1}{2},\frac{\sqrt{3}-6}{2}\right)$

Key:3

Sol: Image of $(1,3)$ w.r.t. x-axis is $(3,1)$

Translation through 3 units along the positive directiion of the X-axis is $(6,1)$

$$\left(r\cos\left(\theta-\frac{\pi}{6}\right),r\sin\left(\theta-\frac{\pi}{6}\right)\right)$$

$$r=\sqrt{37},\cos\theta=\frac{6}{\sqrt{37}},\sin\theta=\frac{1}{\sqrt{37}}$$

44. **The equation of a straight line, perpendicular to $3x-4y=6$ and forming a triangle of area 6 squares units with coordinate axes, is**

1) $4x+3y=12$ **2)** $4x+3y+24=0$ **3)** $3x+4y=12$ **4)** $x-2y=6$

Key:1

Sol: Perpendicular line is $4x+3y+k=0$

$$=\frac{1}{2}\frac{|c^2|}{|ab|}=6$$

45. **If the image of $\left(-\frac{7}{5},-\frac{6}{5}\right)$ in a line is $(1,2)$, then the equation of the line is**

1) $3x-y=0$ 2) $4x-y=0$ 3) $3x+4y=1$ 4) $4x+3y=1$

Key:3

Sol: Find perpendicular bisector of line segment joining of two points

46. **If a line l passes through $(k,2k),(3k,3k)$ and $(3,1), k\neq 0$, then the distance from the origin to the line l is**

1) $\frac{4}{\sqrt{5}}$ 2) $\frac{3}{\sqrt{5}}$ 3) $\frac{2}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{5}}$

Key:4

Sol: .Find 'K' using colliniarity

47. **The area (in square units) of the triangle formed by the lines $x^2-3xy+y^2=0$ and $x+y+1=0$**

1) $\frac{\sqrt{3}}{2}$ 2) $5\sqrt{2}$ 3) $\frac{1}{2\sqrt{5}}$ 4) $\frac{2}{\sqrt{3}}$

Key:3

Sol: Area $=\left|\dfrac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|$

48. If $x^2+\alpha y^2+2\beta y=a^2$ represents a pair of perpendicular lines, then $\beta=$

1) a 2) 2a 3) 3a 4) 4a

Key:1

Sol: $\alpha+1=0$

$\Delta=0$

49. A circle with centre at (2, 4) is such that the line $x+y+2=0$ cuts a chord of length 6. The radius of the circle is

1) $\sqrt{11}$ 2) $\sqrt{21}$ 3) $\sqrt{31}$ 4) $\sqrt{41}$

Key:4

Sol: Length of Chord = $2\sqrt{r^2-d^2}=6$

$d=$perpendicular distance from centre to chord.

50. The point at which the circles $x^2+y^2-4x-4y+7=0$ and $x^2+y^2-4x-4y+7=0$ touch each other is

1) $\left(\frac{2}{5},\frac{5}{6}\right)$ 2) $\left(\frac{14}{5},\frac{13}{6}\right)$ 3) $\left(\frac{12}{5},2+\frac{\sqrt{21}}{5}\right)$ 4) $\left(\frac{13}{5},\frac{14}{5}\right)$

Key:2

Sol: Point of contact divides c_1c_2 in the ratio $r_1:r_2$ internally

51. The condition for the lines $lx+my+n=0$ and $l_1x+m_1y+n_1=0$ to be conjugate with aspect to the circle $x^2+y^2=r^2$ is

1) $r^2(ll_1-mm_1)=nn_1$ 2) $r^2(ll_1+mm_1)+nn_1=0$

3) $r^2(lm_1+l_1m)=nn_1$ 4) $r^2(ll_1+mm_1)=nn_1$

Key:4

Sol: Conceptual

52. The length of the common chord of the two circles $x^2+y^2-4y=0$ and $x^2+y^2-8x-4y+11=0$ is

1) $\frac{\sqrt{11}}{2}$ 2) $\sqrt{135}$ 3) $\frac{\sqrt{135}}{4}$ 4) $\frac{\sqrt{145}}{4}$

Key:3

Sol: Common chord $S-S^1=0$

then apply $2\sqrt{r^2-d^2}$

53. The locus of the centre of the circle which cuts the circle $x^2+y^2-20x+4=0$ orthogonally and touches the line x = 2 is

1) $y^2=4x$ 2) $y^2=16x$ 3) $x^2=4y$ 4) $x^2=16y$

Key:2

Sol: $d^2 = r_1^2 + r_2^2$

$r_1 = 96,\ r_2 = |x_1 - 2|$

$\therefore d = \sqrt{(x_1 - 10)^2 + y_1^2}$

54. If a normal chord at a point t on the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then t =

1) $\sqrt{2}$ 2) 2 3) $\sqrt{3}$ 4) 1

Key:1

Sol: $t_1 t_2 = -4$

$t_2 = -t_1 - \frac{2}{t_1}$

55. The slopes of the focal chords of the parabola $y^2 = 32x$ which are tangents tot he circle $x^2 + y^2 = 4$ are

1) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ 2) $\frac{1}{\sqrt{15}}, \frac{-1}{\sqrt{15}}$ 3) $\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}$ 4) $\frac{1}{2}, \frac{-1}{2}$

Key:2

Sol: $2x - (t_1 + t_2)y - 2a = 0$

where $a = 8$

apply tangency conditions

56. If tangents are drawn from any point on the circle $x^2 + y^2 = 25$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ then the angle between the tangents is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $2\frac{\pi}{3}$

Key:3

Sol: $x^2 + y^2 = 25$ is the director circle

57. An ellipse passing through $(4\sqrt{2}, 2\sqrt{6})$ has foci at (–4, 0) and (4, 0). Its eccentricity is

1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{2}$

Key:1

Sol: $SS^1 = 2ae$

$SC + S'C = 2a$

58. A hyperbola pases through a focus of the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$. its transverse and conjugate axes coincide respectively with the major amd ,ompr axes of the ellipse. The product of eccentricities is 1. Then the equation of the hyperbola is

1) $\frac{x^2}{169} - \frac{y^2}{25} = 1$ 2) $\frac{x^2}{144} - \frac{y^2}{25} = 1$ 3) $\frac{x^2}{25} - \frac{y^2}{9} = 1$ 4) $\frac{x^2}{144} - \frac{y^2}{9} = 1$

Key:2

Sol: $e_1 = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$

$e_2 = \sqrt{1 + \frac{9}{144}} = \frac{13}{12}$

$e_1 . e_2 = 1$

59. If the line joining A(1, 3, 4) and B is divided by the point (–2, 3, 5) in the ratio 1:3, then B is

1) (–11, 3, –8) 2) (–8, 12, 20) 3) (13, 6, –13) 4) (–11, 3, 8)

Key: 4

Sol: A ———————— B

$\lambda = x_1; x_2 - \lambda = 1:3$

$(-2-1); x_2 + 2 = 1:3$

$\frac{-3}{x_2 + 2} = \frac{1}{3}$

$x_2 + 2 = -9$

$x_2 = -11$

$\frac{\beta - y_1}{y_2 - \beta} = \frac{1}{3}$

$\frac{3-3}{y_2 - 3}$

$y_2 = 3$

$\left(-11, 3, \frac{r - Z_1}{Z_2 - r}\right) = \frac{1}{3}$

$\frac{5-4}{Z_2 - 5} = \frac{1}{3}$

$3 = Z_2 - 5$

$Z_2 = 8$

60. If the direction cosines of two lines are given by $l + m + n = 0$ and $l^2 - 5m^2 + n^2 = 0$ then the angle between them is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Key:1

Sol: . $l = -m - n$

$(m+n)^2 - 5m^2 + n^2 = 0$

$m^2 + n^2 + mn - 5m^2 + n^2 = 0$

$-4m^2 + 2mn + 2n^2 = 0$

$2m^2 - mn - n^2 = 0$

$2m^2 - 2mn + mn - n^2 = 0$

$2m(m-n) + n(m-n) = 0$

$m = n$

$l = -2n$

$l : m : n = -2n; n; n$

$= 2, 1, 1$

$m = \frac{n}{2}$

$l = -\frac{n}{2} - n = \frac{-3}{2}n$

$l : m : n$

$= \frac{-3n}{2} : \frac{n}{2} : n$

$= \frac{-3}{2}, \frac{1}{2}, 1$

$-2\left(\frac{-3}{2}\right) + \frac{1}{2} + 1$

$$\cos\theta = \frac{3 + \frac{1}{2} + 1}{\sqrt{6}\sqrt{\frac{9}{4} + \frac{1}{4} + 1}}$$

$$= \frac{\frac{9}{2}}{\sqrt{6}\frac{\sqrt{15}}{2}} = \frac{9}{3\sqrt{10}}$$

61. If A (3, 4, 5), B(4, 6, 3), C(–1, 2, 4) and D(1, 0, 5) are such that the angle between the lines $\overleftrightarrow{DC}$ and $\overleftrightarrow{AB}$ is θ then cos θ =

1) $\frac{2}{9}$ 2) $\frac{4}{9}$ 3) $\frac{5}{9}$ 4) $\frac{7}{9}$

Key: 2

Sol: . DC = (2, –2, 1)

AB = (1, 2, –2)

$$\cos\theta = \frac{|2-4-2|}{3\times 3} = \frac{4}{9}$$

62. $\lim_{x\to 0} \frac{\sqrt{1+x^2} - \sqrt{1-x+x^2}}{3^x - 1} =$

1) $\log 9$ 2) $\frac{1}{\log 9}$ 3) $\log 3$ 4) $\frac{1}{\log 3}$

Key: 2

Sol: . $\lim_{x\to 0}\dfrac{\dfrac{\sqrt{1+x^2}}{x}-\dfrac{\sqrt{1-x+x^2}}{x}}{\left(\dfrac{3x-1}{x}\right)}$

$$=\frac{1}{\log 3}\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1-x+x^2}}{x}$$

$$=\frac{1}{\log 3}\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{1-x+x^2}}(2n-1)$$

$$=\frac{l}{\log 3}\left[0+\frac{1}{2\sqrt{1}}\right]$$

$$=\frac{1}{\log 9}$$

63. If f:[–2, 2] $\rightarrow$ R is defiend by

$$f(x)=\begin{cases}\dfrac{\sqrt{1+cx}-\sqrt{1-cx}}{x} & \text{for} \quad -2\le x<0\\ \dfrac{x+3}{x=1} & \text{for} \quad 0\le x\le 2\end{cases}$$

is continuous on [–2, 2], then c =

1) 3 2) $\frac{3}{2}$ 3) $\frac{3}{\sqrt{2}}$ 4) $\frac{2}{\sqrt{3}}$

Key: 3

Sol: . $f(0-)=\underset{x\to 0}{\text{Lt}}\dfrac{1}{2\sqrt{1+cx}}c-\dfrac{1}{2\sqrt{1-cx}}(-c)$

$=\frac{c}{2}+\frac{c}{2}=c$; $f(0+)=3$

$C=3$

64. If $f(x)=x\tan^{-1}x$ **then** $\lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1}=$

1) $\frac{\pi}{4}$ 2) $\frac{\pi+1}{4}$ 3) $\frac{\pi+2}{4}$ 4) $\frac{\pi+3}{4}$

Key:3

Sol: .Apply L - hospital rule

65. $y=\tan^{-1}\left(\dfrac{\sqrt{1+a^2x^2}-1}{ax}\right)\Rightarrow\left(1+a^2x^2\right)y''+2a^2xy'=$

1) a^2 2) $2a^2$ 3) 0 4) $-2a^2$

Key:3

Sol: Put $ax=\tan\theta$

66. **If** $f(x)=\frac{x}{1+x}$ **and** $g(x)=f(f(x))$ **then** $g'(x)=$

1) $\frac{1}{(x+1)^2}$ 2) $\frac{1}{x^2}$ 3) $\frac{1}{(2x+1)^2}$ 4) $\frac{1}{(2x+3)^2}$

Key:3

Sol: $g(x)=\frac{\frac{x}{1+x}}{1+\frac{x}{1+x}}=\frac{x}{1+2x}$

67. **If the curves** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **and** $\frac{x^2}{25}+\frac{y^2}{16}=1$ **cut each other othogonally, then** $a^2-b^2=$

1) 400 2) 75 3) 41 4) 9

Key: 4

Sol: Product of tangents slopes = -1

68. **The condition that** $f(x)=ax^3+bx^2+cx=d$ **has no exterme value is**

1) $b^2=4ac$ 2) $b^2=3ac$ 3) $b^2<3ac$ 4) $b^2>3ac$

Key:3

Sol: $\frac{3ax^2+bx+c}{b^2-4ac<0}$

$(2b)^2-4(3a)c<0$

$4b^2-12ac<0$

$b^2-3ac<0 \Rightarrow b^2<3ac$

69. **If there is an error of** $\pm 0.04\text{cm}$ **in the measurement of the diameter of a sphere then the approximate percentage error in its volume, when the radius is 10 cm, is**

1) ± 0.06 2) ± 0.006 3) ± 0.6 4) ± 1.2

Key:4

Sol: . $v=\frac{4}{3}\pi r^3$

70. **The value of c in the Lagrange's mean - value theorem for** $f(x)\sqrt{x-2}$ **in the itnerval [2, 6] is**

1) $\frac{5}{2}$ 2) 3 3) 4 4) $\frac{9}{2}$

Key:2

Sol: $f'(c)=\frac{f(b)-f(a)}{b-a}$

71. $\int\frac{dx}{\sqrt{\sin^3 x\cos x}}=g(x)+c\Rightarrow g(x)=$

1) $\frac{-2}{\sqrt{\tan x}}$ 2) $\frac{2}{\sqrt{\cot x}}$ 3) $\frac{2}{\sqrt{\tan x}}$ 4) $\frac{-2}{\sqrt{\cot x}}$

Key: 1

Sol: . $\int \frac{dx}{\sqrt{\tan_x^3 \cos_x^4}} = \int \frac{\sec^2 xdy}{\sqrt{\tan_x^3}}$ Put. $\tan x = t$

72. If $\int \frac{dx}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}} = \frac{A\sqrt{x}}{\sqrt{1-x}} + \frac{B}{\sqrt{1-x}} + C$ **, where is a real constant then A + B =**

1) 0 2) 1 3) 2 4) 3

Key: 1

Sol: .Put $\sqrt{x} = t$

73. For any integer $n \geq 2$**, let** $I_n = \int \tan^n xdx$**. If** $I_n = \frac{1}{a}\tan^{n-1} x - bI_{n-2}$ for $n \geq 2$, **then the ordered pair (a, b) =**

1) $\left(n-1\frac{n-2}{n-1}\right)$ 2) (n, 1) 3) (n–1, 1) 4) $\left(n-1, \frac{n-1}{n-2}\right)$

Key: 3

Sol: .In $= \int \tan_x^{n-2}\left(\sec_n^2 - 1\right)$

$= \frac{1}{n-1}\tan_x^{n-1} - I_{n-2}$

74. If $\int \frac{\left(x^2-1\right)}{(x+1)^2\sqrt{x\left(x^2+x=1\right)}}dx = A\tan^{-1}\left(\sqrt{\frac{x^2+x+1}{x}}\right) + c,$ **in which c is a constant then A =**

1) 3 2) 2 3) 1 4) $\frac{1}{2}$

Key: 2

Sol: .divide nr and dr by x^2

put $x + \frac{1}{x} = t^2$

75. By the definition of the definite integral, the value of $\lim_{n\to\infty}\left(\frac{1^4}{1^5+n^5} + \frac{2^4}{2^5+n^5} + \frac{3^4}{3^5+n^5} + ...+ \frac{n^4}{n^5+n^5}\right)$ is

1) $\frac{1}{5}\log 2$ 2) $\frac{1}{4}\log 2$ 3) $\frac{1}{3}\log 2$ 4) $\log 2$

Key: 1

Sol: . $\int_0^1 \frac{x^4}{1+x^5}dx$

76. $\int_0^{\pi/6} \cos^4 3\theta \sin^2 6\theta \, d\theta =$

1) $\frac{5}{192}$ 2) $\frac{5\pi}{256}$ 3) $\frac{5\pi}{192}$ 4) $\frac{\pi}{96}$

Key: 3

Sol: .Put $3\theta = t$ and aply (1)
use reduction formula

77. The area (in square units) of the region bounded by $x = -1$, $x = 2$, $y = x^2 + 1$ and $y = 2x - 2$ is

1) 7 2) 8 3) 9 4) 10

Key: 3

Sol: . $\int_{-1}^{2} (x^2+1) - (2x-2)\,dx$

78. The differential equation of the family of parabolas with vertex at (0, –1) and having axis along the y-axis is

1) $xy' + y + 1 = 0$ 2) $xy' - 2y - 2 = 0$ 3) $xy' - y - 1 = 0$ 4) $yy' + 2xy + 1 = 0$

Key: 2

Sol: . $x^2 = k(y+1); /x^2 = C_1(y+1)$ Eliminate C_1

79. The solution of $x\frac{dy}{dx} = y + xe^{y/x}$ with $y(1) = 0$ is

1) $e^{-y/x} = \log x$ 2) $e^{-y/x} + 2\log x = 1$ 3) $e^{-y/x} + \log x = 1$ 4) $e^{y/x} + \log x = 1$

Key: 3

Sol: .Put $y = vx$

80. The solution of $\cos y + (x \sin y - 1)\frac{dy}{dx} = 0$ is

1) $\tan y - \sec y = cx$ 2) $\tan y + \sec y = cx$ 3) $x \sec y + \tan y = c$ 4) $x \sec y = \tan y + c$

Key: 4

Sol: .Reduce it in the form $\frac{dx}{dy} + Px = q$

PHYSICS

81. Match the following (Take the relative strength of the strongest fundamental forces in nature as one)

A **Fundamental forces in nature**	B **Relative strength**
(a) Strong nuclear force	(e) 10^{-2}
(b) Weak nuclear force	(f) 1
(c) Electromagnetic force	(g) 10^{10}
(d) Gravitational force	(h) 10^{-13}
	(i) 10^{-19}

1) (a) - (f), (b) - (h), (c) - (e), (d) - (h)
2) (a) - (f), (b) - (h), (c) - (e), (d) - (i)
3) (a) - (f), (b) - (e), (c) - (h), (d) - (i)
4) (a) - (f), (b) - (i), (c) - (e), (d) - (h)

Key: 2

Sol: a - f ; b - h; c - e; d - i

82. **If C the velocity of light, h Planck's constant and G Gravitational constant are taken as fundamental quantities, then the dimensional formula of mass is**

1) $h^{1/2}C^{1/2}G^{-1/2}$ 2) $h^{-1/2}C^{1/2}G^{-1/2}$ 3) $h^{-1/2}C^{-1/2}G^{-1/2}$ 4) $h^{-1/2}G^{-1/2}C^{0}$

Key: 1

Sol: $c^a h^b G^c = M$

$$\left(LT^{-1}\right)^a \left(ML^2T^{-1}\right)^b \left(M^{-1}L^3T^{-2}\right)^c = M$$

b - c = 1

a + 2b + 3c = 0

-a - b - 2c = 0

$\therefore a = 1/2,\ b = 1/2,\ c = -1/2$

83. **A person walks along a straight road from his house to a market 2.5 kms away with a speed of 5 km/hr and instantly turns back and reaches his house with a speed of 7.5 kms/hr. The average speed of the person during the time interval 0 to 50 minutes is (in m/sec)**

1) $\frac{5}{3}$ 2) $\frac{5}{6}$ 3) $\frac{1}{3}$ 4) $4\frac{2}{3}$

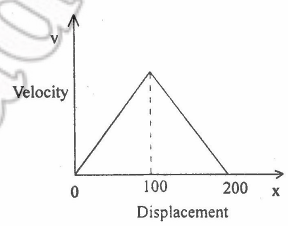
Key: 1

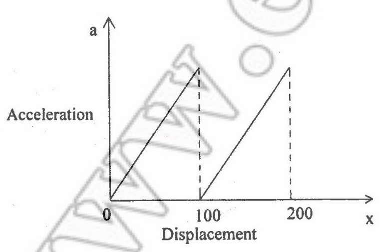
Sol: $t_1 = \frac{2.5}{5} = \frac{1}{2}$ hrs

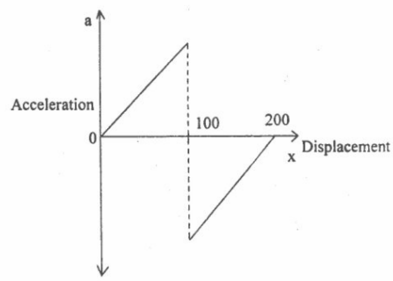
$$t_2 = \frac{2.5}{7.5} = \frac{1}{3} \text{ hrs} = 20 \text{ min.}$$

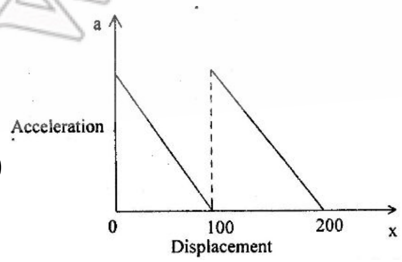
$$v = \frac{5 \times 1000}{50 \times 60} = \frac{5}{3} \text{ m/s}$$

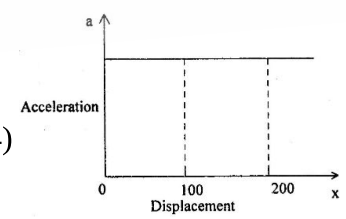
84. **Velocity (v) versus displacement (x) plot of a body moving along a straight line is as shown in the graph. The corresponding plot of acceleration (a) as a function of displacement (x) is**



1)


2)


3)


4)


Key: 2

Sol: v = kx

$$\frac{dv}{dt} = kv = k^2x$$

$$a = k^2x$$

$$v = -kx + v_0$$

$$\frac{dv}{dt} = -kv = -k(-kx + v_0)$$

$$a = k^2x - kv_0$$

85. The path of a projectile is given by the equation y = ax - bx², where a and b are constants and x and y are respectively horizontal and vertical distance of projectile from the point of projection. The maximum height attained by the projectile and the angle of projection are respectively

1) $\frac{b^2}{2a},\ \tan^{-1}(b)$ 2) $\frac{a^2}{b},\ \tan^{-1}(2b)$ 3) $\frac{a^2}{4b},\ \tan^{-1}(a)$ 4) $\frac{2a^2}{b},\ \tan^{-1}(a)$

Key: 3

Sol: $\tan\theta = a$

$$\theta = \tan^{-1}a$$

$$\tan\theta = a; \quad \frac{g}{2u^2\cos^2\theta} = b$$

$$\frac{a^2}{b} = \frac{\tan^2\theta}{g} \times 2u^2\cos^2\theta = \frac{\sin^2\theta}{g} \times 2u^2 = \left(\frac{u^2\sin^2\theta}{2g}\right) \times 4$$

$$\frac{a^2}{4b} = H$$

86. A body is projected at an angle θ so that its range is maximum. If T is the time of flight then the value of maximum range is (acceleration due to gravity = g)

1) $\frac{gT}{2}$ 2) $\frac{gT^2}{2}$ 3) $\frac{g^2T^2}{2}$ 4) $\frac{g^2T}{2}$

Key: 2

Sol: As range is maximum $\theta = 45^0$

$$T = \frac{2u\sin\theta}{g} = \frac{2u\sin 45^0}{g} = \frac{\sqrt{2}u}{g}$$

$$= \frac{1}{g}\left(\frac{Tg}{\sqrt{2}}\right)^2 = \frac{1}{g} \times \frac{T^2g^2}{2}$$

$$R = \frac{T^2g}{2}$$

87. A mass M kg is suspended by a weightless string. The horizontal force required to hold the mass at 60⁰ with the vertical is

1) $Mg\sqrt{3}$ 2) $Mg\left(\sqrt{3}+1\right)$ 3) $\frac{Mg}{\sqrt{3}}$ 4) Mg

Key: 1

Sol: $F = Mg\tan\theta \Rightarrow F = Mg\sqrt{3}$

88. The force required to move a body up a rough inclined plane is double the force required to prevent the body from sliding down the plane. The coefficient of friction when the angle of inclination of the plane is 60^0 is

1) $\frac{1}{\sqrt{2}}$ 2) $\frac{1}{\sqrt{3}}$ 3) $\frac{1}{2}$ 4) $\frac{1}{3}$

Key: 2

Sol: $F_{up} = 2(F_{down})$

$mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$

$\Rightarrow \mu = \frac{1}{3}\tan\theta = \frac{1}{3}\tan 60^0 = \frac{1}{\sqrt{3}}$

89. A cannon shell fired breaks into two equal parts at its highest point. One part retraces the path to the cannon with kinetic energy E_1 and kinetic energy of the second part is E_2. Relation betweeen the E_1 and E_2 is

1) $E_2 = E_1$ 2) $E_2 = 4E_1$ 3) $E_2 = 9E_1$ 4) $E_2 = 15E_1$

Key: 3

Sol: .At highest point $mu\cos\theta = -\frac{m}{2}u\cos\theta + \frac{m}{2}v$

$\frac{3m}{2}u\cos\theta = \frac{m}{2}v \Rightarrow v = 3u\cos\theta$

$E_1 = \frac{1}{2}\times\frac{m}{2}\times u^2\cos^2\theta = \frac{mu^2\cos^2\theta}{4}$

$E_2 = \frac{1}{2}\times\frac{m}{2}\times 9u^2\cos^2\theta = \frac{9mu^2\cos^2\theta}{4}$

$\Rightarrow E_2 = 9E_1$

90. A bus moving on a level road with a velocity V can be stopped at a distance of x, by the application of a retarding force F. The load on the bus is increased by 25% by boarding the passengers. Now, if the bus is moving with the same speed and if the same retarding force is applied, the distance travelled by the bus before it stops is,

1) x 2) 5x 3) 2.5x 4) 1.25x

Key: 4

Sol: . $v^2 - u^2 = 2as = 2\left(\frac{F}{m}\right)s$

$-u^2 = -2\left(\frac{F}{m}\right)s$

$s \propto m \Rightarrow \frac{s_1}{s_2} = \frac{m_1}{m_2} \Rightarrow \frac{x}{s_2} = \frac{m}{\frac{5}{4}m} \Rightarrow s_2 = \frac{5x}{4} = 1.25x$

91. A wheel which is initially at rest is subjected to a constant angular acceleration about its axis. It rotates through an angle of 15^0 in time t secs. The increase in angle through which it rotates in the next 2t secs is

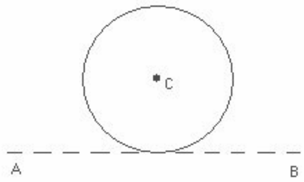
1) 120^0 2) 30^0 3) 45^0 4) 90^0

Key: 1

Sol: . $15 = \frac{1}{2}\alpha t^2$

$\Delta\theta = \frac{1}{2}\ (\alpha)9t^2 - \frac{1}{2}(\alpha)t^2 = 15\times 9 - 15 = 120^0$

92. A thin wire of length l **having linear density** ρ **is bent into a circular loop with C as its centre, as shown in figure. The moment of inertia of the loop about the line AB is**



1) $\frac{\rho l^3}{16\pi^2}$ 2) $\frac{\rho l^3}{8\pi^2}$ 3) $\frac{3\rho l^3}{8\pi^2}$ 4) $\frac{5\rho l^3}{16\pi^2}$

Key: 3

Sol: . $\frac{3mR^2}{2} = \frac{3}{2}(\rho l)\left(\frac{l}{2\pi}\right)^2 = \frac{3}{8}\times\frac{\rho l^3}{\pi^2} \quad (2\pi r = l)$

93. The ratio between kinetic and potential energies of a body excuting simple harmonic motion, whe it is at a distance of $\frac{1}{N}$ **of its amplitude from the mean position is**

1) $\frac{1}{N^2}$ 2) N^2 3) $N^2 - 1$ 4) $N^2 + 1$

Key: 3

Sol: . $\dfrac{\frac{1}{2}k\left(A^2 - x^2\right)}{\frac{1}{2}k\ x^2} = \dfrac{A^2 - \frac{A^2}{N^2}}{\frac{A^2}{N^2}} = N^2 - 1$

94. A satellite is revolving very close to a planet of density ρ **. The period of revoluton of satellite is**

1) $\sqrt{\frac{3\pi}{2\rho G}}$ 2) $\sqrt{\frac{3\pi}{\rho G}}$ 3) $\sqrt{\frac{3\pi G}{\rho}}$ 4) $\sqrt{\frac{3\pi\rho}{G}}$

Key: 2

Sol: . $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{R}{\frac{4}{3}\pi\,\rho\,G\,R}} = \sqrt{\frac{3\pi}{\rho G}}$

95. Two wires of the same material and length but diameters in the ratio 1 : 2 are stretched by the same force. The elastic potential energy per unit volume for the two wires when stretched by the same force will be in the ratio

1) 1 : 1 2) 2 : 1 3) 4 : 1 4) 16 : 1

Key: 4

Sol: . $\frac{1}{2}\times\frac{F}{A}\times\frac{1}{l}\times\frac{Fl}{YA} \propto \frac{1}{r^4} \Rightarrow \left(\frac{2}{1}\right)^4 = \frac{16}{1}$

96. When a big drop of water is formed from n small drops of water, the energy loss is 3E, where E is the energy of the bigger drop. If R is the radius of the bigger drop and r is the radius of the smaller drop, then number of of smaller drops (n) is

1) $\frac{4R}{r}$ 2) $\frac{2R^2}{r}$ 3) $\frac{4R^2}{r^2}$ 4) $\frac{4R}{r^2}$

Key: 3

Sol: . $n \times 4\pi r^2 \times T - 4\pi R^2 \times T = 3 \times 4\pi R^2 \times T$

$$n = \frac{4R^2}{r^2}$$

97. A steam at 100^{0}C is passed into 1 kg of water contained in a calorimeter of water equivalent 0.2 kg at 9^{0}C, till the temperature of the calorimeter and water in it is increased to 90^{0}C. The mass of steam condensed in kg is nearly (sp. heat of water = 1 cal/g-^{0}C, Latent heat of vaporisation = 540 cal/g)

1) 0.18 2) 0.27 3) 0.54 4) 0.81

Key: 1

Sol: . $m \times 540 + m \times 1 \times 10 = 1200 \times 1 \times 81$

$$m = \frac{1200 \times 81}{550} = 176.7\,g \approx 0.18\ kg$$

98. A very small hole in an electric furnace is used for heating metals. The hole nearly acts as a black body. The area of the hole is 200 mm^2. To keep a metal at 727^{0}C, heat energy flowing through this hole per sec, in joules is $(\sigma = 5.67 \times 10^{-8} Wm^{-2}k^{-4})$.

1) 2.268 2) 1.134 3) 11.34 4) 22.68

Key: 3

Sol: . $P = \sigma A T^4 = 5.67 \times 10^{-8} \times 200 \times 10^{-6} \times (10^3)^4 = 11.34$

99. Five moles of Hydrogen initially at STP is compressed adiabatically so that its temperature becomes 673 K. The increase in internal energy of the gas, in Kilo Joules is (R = 8.3 J/mole-K; $\gamma = 1.4$ **for diatomic gas)**

1) 21.55 2) 41.50 3) 65.55 4) 80.5

Key: 2

Sol: . $\Delta U = n\frac{R}{\gamma - 1} dT \quad = 5 \times \frac{8.3}{0.4} \times 400 = 41.50$

100. The volume of one mole of the gas is changed from V to 2V at constant pressure P. If γ is the ratio of specific heats of the gas, change in internal energy of the gas is

1) $\frac{R}{\gamma - 1}$ 2) PV 3) $\frac{PV}{\gamma - 1}$ 4) $\frac{r.PV}{\gamma - 1}$

Key: 3

Sol: . $\Delta U = n\left(\frac{R}{\gamma - 1}\right)\Delta T = \frac{P\Delta V}{\gamma - 1} = \frac{PV}{\gamma - 1}$

101. A closed pipe is suddenly opened and changed to an open pipe of same length. The fundamental frequency of the resulting open pipe is less than of 3rd harmomic of the earlier closed pipe by 55 Hz. Then, the value of fundamental frequency of the closed pipe is

1) 110 Hz 2) 55 Hz 3) 220 Hz 4) 165 Hz

Key: 2

Sol: $\frac{v}{2l}=3\times\frac{v}{4l}-55$

$55=\frac{3v}{4l}-\frac{v}{2l}=\frac{(3-2)v}{4l}$

$\frac{v}{4l}=55\,Hz$

102. A convex lens has its radii of curvature equal. The focal length of the lens is f. If it is divided vertically into two identical plano-convex lenses by cutting it, then the focal length of the plano-convex lens is (μ = the refractive index of the material of the lens)

1) $\frac{f}{2}$ 2) 2f 3) $(\mu-1)f$ 4) f

Key: 2

Sol: $\frac{1}{f}=(\mu-1)\frac{2}{R}$

$\frac{\mu-1}{R}=\frac{1}{2f}$

$\therefore$ focal length = 2f

103. A thin converging lens of focal length f = 25 cm forms the image of an object on a screen placed at a distance of 75 cm from the lens. The screen is moved closer to the lens by a distance of 25cm. The distance through which the object has shifted so that its image on the screen is sharp again is

1) 16.25 cm 2) 12.5 cm 3) 13.5 cm 4) 37.5 cm

Key: 2

Sol: case(1)

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$\frac{1}{25}=\frac{1}{75}-\frac{1}{x}$

$\frac{1}{x}=\frac{1}{75}-\frac{1}{25}=\frac{1-3}{75}$

$x=-\frac{75}{2}=-37.5\text{ cm}$

case(2)

$\frac{1}{25}=\frac{1}{50}-\frac{1}{x}$

$\frac{1}{x}=\frac{1}{50}-\frac{1}{25}=\frac{1-2}{50}$

$x=-50\text{ cm}$

50-37.5=12.5 cm

104. In a double slit interference experiment, the fringe width obtained with a light of wavelength $5900\,\overset{0}{A}$ was 1.2 mm for parallel narrow slits placed 2 mm apart. In this arrangement, if the slit separation is increased by one-and-half times the previous value, the fringe width is

1) 0.8 mm 2) 1.8 mm 3) 1.6 mm 4) 0.9 mm

Key: 1

Sol: $\beta = \frac{\lambda D}{d}$

$\frac{\beta_1}{\beta_2} = \frac{d_2}{d_1}$ **=1.5**

$\beta_2 = \frac{1.2}{1.5} = \frac{4}{5} = 0.8\text{mm}$

105. A charge Q is divided into two charges q and Q - q. The value of q such that the force between them is maximum is

1) $\frac{3Q}{4}$ 2) $\frac{Q}{2}$ 3) $\frac{Q}{3}$ 4) Q

Key: 2

Sol: . $q_1 = Q \quad \& \quad q_2 = Q - q$

$\frac{dF}{dQ} = 0$

$\frac{d}{dQ}\left(\frac{1}{4\pi \in_0} Q(Q-q)\right) = 0$

$\therefore q_1 = q_2 = \frac{Q}{2}$

106. Two concentric hollow spherical shells have radii r and R (R>>r). A charge Q is distributed on them such that the surface charge densities are equal. The electric potential at the centre is

1) $\frac{Q(R^2+r^2)}{4\pi \in_0 (R+r)}$ 2) $\frac{Q}{R+r}$ 3) 0 4) $\frac{Q(R+r)}{4\pi \in_0 (R^2+r^2)}$

Key: 4

Sol: . $\sigma = \frac{Q}{4\pi(r^2+R^2)}$

$V = \frac{1}{4\pi \in_0}\left(\frac{\sigma \times 4\pi r^2}{r} + \frac{\sigma \times 4\pi R^2}{R}\right) = \frac{\sigma}{\in_0}(r+R) = \frac{Q(r+R)}{4\pi \in_0 (r^2+R^2)}$

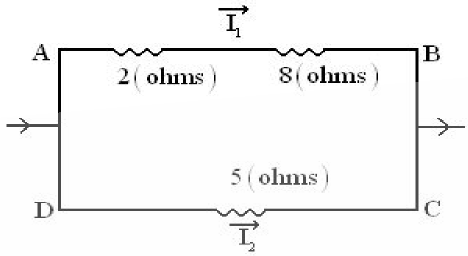
107. Wires A and B have resistivities ρ_A and ρ_B, $(\rho_B = 2\rho_A)$ and have lengths l_A and l_B. If the diameter of the wire B is twice that of A and the two wires have same resistance, then $\frac{l_B}{l_A}$ is

1) 1 2) 1/2 3) 1/4 4) 2

Key: 4

Sol: . $\left(\frac{\rho l}{A}\right)_A = \left(\frac{\rho l}{A}\right)_B \Rightarrow \frac{l_B}{l_A} = \frac{\rho_A}{\rho_B} \times \frac{A_B}{A_A} = \frac{1}{2} \times \frac{4}{1} = 2$

108. In the circuit shown, the heat produced in 5 ohms resistance due to current through it is 50 J/s. Then the heat generated /second in 2 ohms resistance is



1) 4 J/s 2) 9 J/s 3) 10 J/s 4) 5 J/s

Key: 4

Sol: $i_1 = \frac{i \times 5}{5+10} = \frac{i}{3}$

$i_2 = \frac{i \times 10}{15} = \frac{2}{3}i$

$\frac{P_1}{P_2} = \frac{i_1^2 R_1}{i_2^2 R_2}$

$\frac{P_1}{50} = \left(\frac{i_1}{i_2}\right)^2 . \frac{2}{5} = \left(\frac{1}{2}\right)^2 . \frac{2}{5}$

$\frac{P_1}{50} = \frac{1}{10}$

$P_1 = 5\,J/s$

109. A steady current flows in a long wire. It is bent into a circular loop of one turn and the magnetic field at the centre of the coil is B. If the same wire is bent into a circular loop of n turns, the magnetic field at the centre of the coil is

1) nB 2) nB^2 3) n^2B 4) B/n

Key: 3

Sol: $B = \frac{\mu_0 ni}{2r}$ $n_1 = 1$, $n_2 = n$, $2\pi r_1 = n \times 2\pi r_2$ $\Rightarrow \frac{r_1}{r_2} = \frac{n}{1}$

$\frac{B_1}{B_2} = \frac{n_1}{n_2} . \frac{r_1}{r_2}$

$\frac{B}{B_2} = \frac{1}{n} . \frac{1}{n}$

$B_2 = n^2 B$

110. An electrically charged particle enters into a uniform magnetic induction field in a direction perpendicular to the field with a velocity V. Then, it travels

1) with force in the direction of the field

2) in a circular path with a radius directly proportional to V^2

3) in a circular path with a radius directly proportional to its velocity

4) in a straight line without acceleration

Key : 3

Sol: $Bvq = \frac{mv^2}{r}$

$Bq = \frac{mv}{r}$

$r \alpha v$

111. At a certain place, the angle of dip is 60° and the horizontal component of earth's magnetic field (B_H) is $0.8\times10^{-4}T$. The earth's overall magnetic field is

1) $1.6\times10^{-3}T$ 2) $1.5\times10^{-3}T$ 3) $1.6\times10^{-4}T$ 4) $1.5\times10^{-4}T$

Key: 3

Sol: $B_H = B\cos\theta$

$0.8\times10^{-4} = B\cos\theta$

$B = 1.6\times10^{-4}$

112. A coil of wire of radius r has 600 turns and a self inductance of 108mH. The self inductance of a coil with same radius and 500 turns is

1) 75 mH 2) 108 mH 3) 90 mH 4) 80 mH

Key 3

Ans: $L \alpha N^2$

$\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2$

$\frac{108}{L_2} = \left(\frac{600}{500}\right)^2$

$L_2 = 108\times\frac{25}{36}$

$L = 75\,mH$

113. A capacitor of $50\mu F$ is connected to a power source V=220 sin 50t (V in volt, t in second). The value of rms current (in Amperes)

1) 0.55 A 2) $\sqrt{2}$ 3) $\frac{(0.55)}{\sqrt{2}}A$ 4) $\frac{\sqrt{2}}{0.55}A$

Key: 3

Sol: $C = 50\,\mu F$

$X_C = \frac{1}{\omega C} = \frac{1}{50\times50\times10^{-6}}\Omega$

$i_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\left(\frac{1}{\omega C}\right)} = \omega C\,V_{rms}$

$= 50\times50\times10^{-6}\times\frac{V_0}{\sqrt{2}} = 25\times10^{-4}\times\frac{220}{\sqrt{2}} = \frac{25\times22\times10^{-3}}{\sqrt{2}}$

$= \frac{550\times10^{-3}}{\sqrt{2}}$

$$i_{rms} = \frac{0.55}{\sqrt{2}} A$$

114. The electric field for an electromagnetic wave in free space is $\vec{E} = \vec{i}\,30\cos(kz - 5\times10^8 t)$ where magnitude of E is in V/m. The magnitude of wave vector, k is (velocity of em wave in free space= $3\times10^8 m/s$)

1) $3\,rad m^{-1}$ 2) $1.66\,rad\,m^{-1}$ 3) $0.83\,rad\,m^{-1}$ 4) $0.46\,rad\,m^{-1}$

Key: 2

Sol: $\vec{E} = \vec{i}\,30\cos\left(kz - 5\times10^8 t\right)$

$C = 3\times10^8$

$\upsilon\lambda = 3\times10^8$

$\frac{\omega}{2\pi}.\lambda = 3\times10^8$

$\lambda = \frac{2\pi\times3\times10^8}{\omega}$

$= \frac{2\pi\times3\times10^8}{5\times10^8} = \frac{6\pi}{5} m$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6\pi}\times5$

$k = \frac{5}{3} m$

k= 1.66m

115. The energy of a photon is equal to the kinetic energy of a proton. If λ_1 is the de Broglie wavelength of a proton, λ_2 the wavelength associated with the photon, and if the energy fo the photon is E, then (λ_1/λ_2) is proportional to

1) $E^{1/2}$ 2) E^2 3) E 4) E^4

Key :1

Sol: $K.E_{proton} = E_{photon}$

$(proton)\frac{h}{mv} = \lambda_1$

$(Photon) E = h\upsilon = \frac{hC}{\lambda_2}$

$\lambda_2 = \frac{hC}{E}$

$P = \sqrt{2mE}$

$\lambda_1 = \frac{h}{\sqrt{2mE}}$

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{h}{\sqrt{2mE}}}{\left(\frac{hC}{E}\right)}$$

$$= \frac{1}{C\sqrt{2m}}\frac{E}{\sqrt{E}}$$

$$\frac{\lambda_1}{\lambda_2}\alpha\sqrt{E}$$

116. The radius of the first orbit of hydrogen is r_H, and the energy in the ground state is -13.6 eV. Considering a μ^- particle with a mass 207 m_e revolving round a proton as in Hydrogen atom, the energy and radius of proton and μ^- combination respectively in the first orbit are (assume nucleus to be stationary)

1) $-207\times13.6\,eV, 207\,r_H$

2) $\frac{-13.6}{207}eV, \frac{r_H}{207}$

3) $\frac{-13.6}{207}eV, 207\ r_H$

4) $-13.6\times207\,eV, \frac{r_H}{207}$

Key: 4

Sol: $r \propto \frac{1}{m}$ $\qquad E \propto m$

117. If the radius of a nucleus with mass number 125 is 1.5 Fermi, then radius of a nucleus with mass number 64 is

1) 0.96 Fermi 2) 1.92 Fermi 3) 1.2 Fermi 4) 0.48 Fermi

Key: 3

Sol: $R \propto A^{1/3} \Rightarrow \frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} \Rightarrow R_2 = \left(\frac{64}{125}\right)^{1/3} \times 1.5$

$R_2 = \left(\frac{4}{5}\right)\times1.5 = 1.2\ \text{ferm.}$

118. A crystal of intrinsic silicon at room temperature has a carrier concentration of $1.6\times10^{16}/m^3$. If the donor concentration level is $4.8\times10^{20}/m^3$, then the concentration of holes in the semiconductor is

1) $4\times10^{11}/m^3$ 2) $4\times10^{12}/m^3$ 3) $5.3\times10^{11}/m^3$ 4) $53\times10^{12}/m^3$

Key: 3

Sol: $n^2 = n_e.n_h \Rightarrow n_h = \frac{n^2}{n_e}$

$= \frac{2.56\times10^{32}}{4.8\times10^{20}} = 5.3\times10^{11}/m^3$

119. The output characteristics of an n-p-n transistor represent, [I_C Collector current, V_{CE}= potential difference between collector and emitter, I_B= Base current, V_{BB}= Voltage given to base, V_{BE}= the potential differnce between base and emitter]

1) changes in I_C with changes in V_{CE} (I_B=constant)
2) changes in I_B with changes in V_{CE}
3) changes in I_C as V_{BE} is changed
4) changes in I_C as I_B and V_{BB} are changed

Key: 1

Sol: Graph between I_C and V_{CE} when I_B=constant

120. A T.V transmitting Antenna is 128 m tall. If the receiving Antenna is at the ground level, the maximum distance between them for satisfactory communication in L.O.S. mode is (Radius of the earth = $6.4\times10^6\,\text{m}$**)**

1) $\frac{128}{\sqrt{10}}\text{km}$ 2) $128\times\sqrt{10}\text{km}$ 3) $\frac{64}{\sqrt{10}}\text{km}$ 4) $64\times\sqrt{10}\text{km}$

Key: 1

$$h=\sqrt{2Rh_T}=\sqrt{2\times6.4\times10^6\times128}$$

$$=\sqrt{128\times128\times10^5}$$

$$=\frac{128\times10^3}{\sqrt{10}}\text{m}==\frac{128}{\sqrt{10}}\text{km}$$

CHEMISTRY

121. In an atom the order of increasing energy of electrons with quantum numbers

(i) $n=4, l=1$ **(ii)** $n=4, l=0$ **(iii)** $n=3, l=2$ **and** **(iv)** $n=3, l=1$ **is**

(1) (ii)$<$(iv)$<$(i)$<$(iii) (2) (i)$<$(iii)$<$(ii)$<$(iv)

(3) (iv)$<$(ii)$<$(iii)$<$(i) (4) (iii)$<$(i)$<$(iv)$<$(ii)

Key: 3

Sol: Applying $(n+l)$ rule

122. The number of angular and radial nodes of 4d orbital respectively are

(1) 1, 2 (2) 3, 0 (3) 2, 1 (4) 3, 1

Key: 3

Sol: Number of radial nodes $=(n-l-1)=(4-2-1)=1$

Number of angular nodes $=l=2$

123. The oxidation state and covalency of Al **in** $\left[AlCl(H_2O)_5\right]^{2+}$ **are respectively**

(1) +3, 6 (2) +2, 6 (3) +3, 3 (4) +6, 6

Key: 1

Sol: $\left[AlCl(H_2O)_5\right]^{2+}$

$x+(-1)+5(0)=+2\Rightarrow x=+3$

Covalency $\Rightarrow$ Cl=1, H_2O=5 $\Rightarrow$ Total=1+5=6 .

124. The increasing order of the atomic radius of Si, S, Na, Mg, Al **is**

(1) $Na < Al < Mg < S < Si$ (2) $Na < Mg < Si < Al < S$

(3) $Na < Mg < Al < Si < S$ (4) $S < Si < Al < Mg < Na$

Key: 4

Sol: In periode Left to right atomic size decreases as z-effective increases

125. The number of electrons in the valence shell of the central atom of a molecule is 8. The molecule is

(1) BeH_2 (2) SCl_2 (3) SF_6 (4) BCl_3

Key: 2

Sol: SCl_2 Number of lone pairs on 'S' $= \frac{6-2\times1}{2} = 2$

$\therefore$ Total number of pairs $= 2\,B.P. + 2L.P = 8e^-$.

126. Which one of the following has longest covalent bond distance ?

(1) C - H (2) C - N (3) C - O (4) C - C

Key: 4

Sol: Conceptual .

127. The ratio of rates of diffusion of gases X and Y is $1:5$ **and that of Y and Z is** $1:6$. **The ratio of rates of diffusion of Z and X is**

(1) $1:6$ (2) $30:1$ (3) $6:1$ (4) $1:30$

Key: 2

Sol: $\frac{r_x}{r_y} = \frac{\sqrt{M_y}}{\sqrt{M_x}} = \frac{1}{5} \longrightarrow (1)$

$$\frac{r_y}{r_z} = \frac{\sqrt{M_z}}{\sqrt{M_y}} = \frac{1}{6} \longrightarrow (2)$$

$$\frac{r_z}{r_x} = \frac{\sqrt{M_x}}{\sqrt{M_z}} = eq^n\,(1) \times eq^n\,(2)$$

128. The molecular interactions responsible for hydrogen bonding in HF

(1) dipole - dipole (2) dipole - induced dipole (3) ion - dipole (4) ion - induced dipole

Key: 1

Sol: Conceptual .

129. $KMnO_4$ **reacts with** KI **in basic medium to form** I_2 **and** MnO_2. **When 250 mL of 0.1** $M\,KI$ **solution is mixed with 250 mL of** $0.02\,M\,KMnO_4$ **in basic medium, what is the number of moles of** I_2 **formed ?**

(1) 0.0075 (2) 0.005 (3) 0.01 (4) 0.015

Key: 1

Sol:

n-factor = 1

$$MnO_4^- + I^- \longrightarrow MnO_2 + I_2$$

n-factor = 3

Number of milli equivalent of $MnO_4^- = 0.02 \times 3 \times 250 = 15$

Number of milli equivalentof $I^- = 0.1\times1\times250 = 25$.

$\therefore$ Number of milli equivalents of I_2 formed

=n- factor X number of milli moles

Number of milli moles of I_2 form $= \frac{15}{1000}$ moles

130. The oxide of a metal contains 40% of oxygen. The valency of metal is 2. What is the atomic weight of the metal ?

(1) 12 (2) 40 (3) 36 (4) 24

Key: 1

Sol: Conceptual

131. The temperature in K at which $\Delta G = 0$, for a given reaction with $\Delta H = -20.5\,kJ\,mol^{-1}$ and $\Delta S = -50.0\,JK^{-1}\,mol^{-1}$ is

(1) 410 (2) 2.44 (3) -2.44 (4) -410

Key: 1

Sol: $0 = -20.5\times10^3 - (-50)\times T$

$\therefore\ T = 410$

132. In a reaction $A+B \rightleftharpoons C+D$, 40% of B has reacted at equilibrium, when 1 mol of A was heated with 1 mole of B in a 10 litre closed vessel. The value of K_c is

(1) 0.18 (2) 0.22 (3) 0.36 (4) 0.44

Key: 4

Sol: $A+B \rightleftharpoons C+D$

t=0 1 1 0 0

t = equilibrium

$(1-0.4)\ (1-0.4)\ 0.4\ 0.4$

$\therefore K_c = \frac{0.4\times0.4}{0.6\times0.6} = 0.44$

133. If the ionic product of $Ni(OH)_2$ is 1.9×10^{-15}, the molar solubility of $Ni(OH)_2$ in $1.0\,M$ NaOH

(1) $1.9\times10^{-13}M$ (2) $1.9\times10^{-15}M$ (3) $1.9\times10^{-14}M$ (4) $1.9\times10^{-18}M$

Key: 2

Sol: $S = \frac{K_{sp}}{C^2} = \frac{1.9\times10^{-15}}{(1)^2} = 1.9\times10^{-15}$

134. Temporary hardness of water is removed in Clark's process by adding

(1) Calgon (2) Borax (3) Lime (4) Caustic Soda

Key: 3

Sol: Conceptual

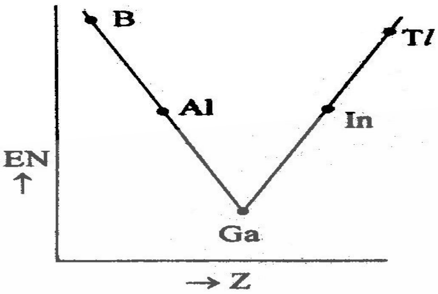
135. KO_2 exhibits paramagnetic behaviour. This is due to the paramagnetic nature of

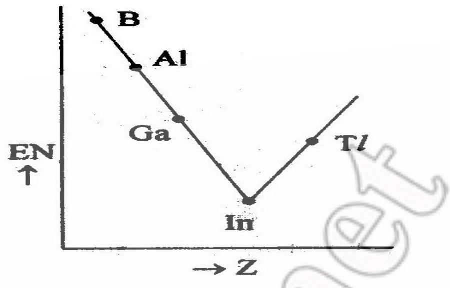
(1) K^+ (2) O_2 (3) O_2^- (4) KO^-

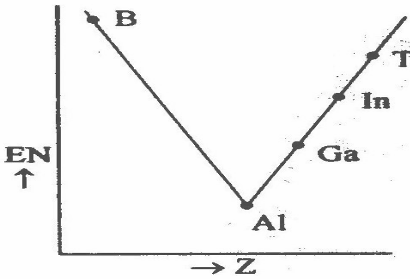
Key: 3

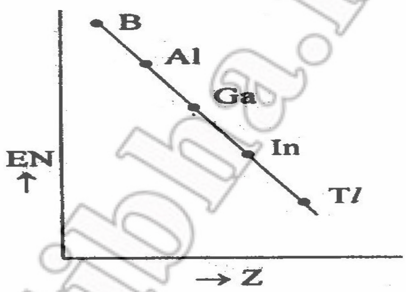
Sol: Conceptual

136. Which one of the following correctly represents the variation of electronegativity (EN) with atomic number (Z) of group 13 elements ?

(1) 

(2) 

(3) 

(4) 

Key: 3
Sol: Conceptual

137. Which one of the following elements reacts with steam ?
(1) Ge (2) Si (3) Sn (4) C
Key: 3
Sol: Conceptual

138. What are X and Y in the following reaction ?

$$CF_2Cl_2 \xrightarrow{uv} X+Y$$

(1) C_2F_4, Cl_2 (2) $CFCl_2, F$ (3) $:CCl_2, F_2$ (4) CF_2Cl, Ci
Key: 4
Sol: Conceptual

139. What are the shapes of ethyne and methane ?
(1) tetrahedral and trigonal planar (2) linear and tetrahedral
(3) trigonal planar and linear (4) square planar and linear
Key: 2
Sol: Conceptual

140. What is Z in the following reaction ?

$$CH_3\text{-}CH_2\text{-}CO_2^-Na^+ \xrightarrow[\Delta]{NaOH/CaO} Z$$

(1) n-butane (2) ethane (3) ethyne (4) propane
Key: 2

Sol: $CH_3\text{-}CH_2\text{-}CO_2^-Na^+ \xrightarrow[\Delta]{NaOH/CaO} CH_3\text{-}CH_3$
De carboxilation in presence of sodalime

141. Which one of the following gives sooty flame on combustion ?
(1) CH_4 (2) C_2H_6 (3) C_6H_6 (4) C_2H_4
Key: 3
Sol: Conceptual

142. Which one of the following elements on doping with germanium, make it a p-type semiconductor

(1) Sb (2) As (3) Ga (4) Bi

Key: 1

Sol: Conceptual

143. The molar mass of a solute X in g mol^{-1}, if its 1% solution is isotonic with a 5% solution of cane sugar (molar mass $=342\, g\, mol^{-1}$), is

(1) 34.2 (2) 136.2 (3) 171.2 (4) 68.4

Key: 4

Sol: Osmatic pressure of $x =$ Osmatic pressure of cane sugar

$$\frac{1}{M}\times\frac{1000}{100}\times RT=\frac{5}{342}\times\frac{1000}{100}\times RT$$

M=68.4

144. Vapour pressure in mm Hg of 0.1 mole of urea in 180 g of water at 25^0C is (The vapour pressure of water at 25^0C is 24 mm Hg)

(1) 20.76 (2) 23.76 (3) 24.76 (4) 2.376

Key: 2

Sol: $P_s = P_0 \times$ mole fraction of urea

$$P_s = 24\times\frac{0.1}{0.1+10} = 2.376 \Rightarrow 24-0.24 = 23.76$$

145. At 298 K the molar conductivities at infinite dilution $\left(\wedge_m^0\right)$ of NH_4Cl, KOH and KCl are $152.8, 272.6$ and $149.8\, S\, cm^2\, mol^{-1}$ respectively. The $\wedge_m^0$ of NH_4OH in S $cm^2 mol^{-1}$ and % dissociation of 0.01 $M\, NH_4OH$ with $\wedge_m = 25.1\, S\, cm^2\, mol^{-1}$ at the same temperature are

(1) 275.6, 9.1 (2) 269.6, 9.6 (3) 30, 84 (4) 275.6, 0.91

Key: 1

Sol: $\wedge_m^0 NH_4OH = \wedge_m^0\left(NH_4Cl + KOH\right) - \wedge_m^0\left(KCl\right)$

$$= 152.8 + 272.6 - 149.8$$
$$= 275.6$$

$$\alpha = \frac{\wedge_m}{\wedge_m^0} = \frac{25.1}{275.6} = 9.1$$

146. In a first order reaction the concentration of the reactant decreases from 0.6 M to 0.3 M in 15 minutes. The time taken for the concentration to change from $0.1M$ to $0.025M$ in minutes is

(1) 12 (2) 30 (3) 3 (4) 1.2

Key: 2

Sol: $t_{1/2} = 15\, min$

$$\therefore t = \frac{2.303}{0.693}\times 15\log\frac{(0.1)}{(0.25)} = 30$$

147. Assertion (A) : Van der Waals' are responsible for chemisorption
Reason (R) : High temperature is favourable for chemisorption
The correct answer is

(1) (A) and (R) are correct and (R) is the correct explanation of (A)
(2) (A) and (R) are correct but (R) is not the correct explanation of (A)
(3) (A) is correct but (R) is not correct
(4) (A) is not correct but (R) is correct

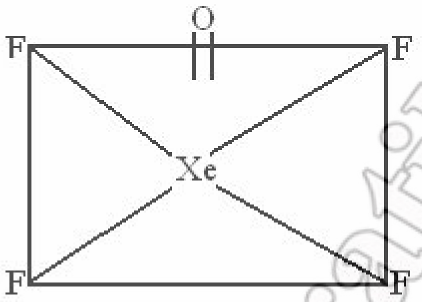
Key: 4
Sol: Conceptual

148. What is the role of limestone during the extraction of iron from haematite ore ?
(1) oxidizing agent (2) reducing agent (3) flux (4) leaching agent
Key: 3
Sol: Conceptual

149. The charring of sugar takes place when treated with concentrated H_2SO_4. What is the type of reaction involved in it ?
(1) Hydrolysis reaction (2) Addition reaction
(3) Disproportionation reaction (4) Dehydration reaction
Key: 4
Sol: Conceptual

150. The structure of $XeOF_4$ is
(1) Square planar (2) Square pyramidal (3) Pyramidal (4) Trigonal bipyramidal
Key: 2

Sol: Structure of $XeOF_4$



151. Which one of the following ions has same number of unpaired electrons as those present in V^{3+} ion ?
(1) Ni^{2+} (2) Mn^{2+} (3) Cr^{3+} (4) Fe^{3+}
Key: 1
Sol: $V^{+3} = 3d^2 4s^0$
$Ni^{+} = 3d^8 4s^0$

152 Match the following

List - I	List - II
(A) sp^3	**(I)** $\left[Co(NH_3)_6\right]^{3+}$
(B) dsp^3	**(II)** $\left[Ni(Co)_4\right]$
(C) sp^3d^2	**(III)** $\left[Pt(NH_3)_2 Cl_2\right]$
(D) d^2sp^3	**(IV)** $\left[CoF_6\right]^{3-}$
	(V) $\left[Fe(Co)_5\right]$

	(A)	(B)	(C)	(D)
1)	(V)	(II)	(IV)	(III)
2)	(II)	(III)	(IV)	(I)
3)	(II)	(III)	(I)	(V)
4)	(III)	(II)	(IV)	(I)

Key: 2
Sol: Conceptual

153. Identify the corpolymer from the following:

1) $\left[CF_2-CF_2\right]_n$

2) $\left[CH_2-\underset{\displaystyle Cl}{\underset{|}{C}}=CH-CH_2\right]_n$

3) $\left[CH_2-\underset{\displaystyle Cl}{\underset{|}{C}}H\right]_n$

4) $\left[CH_2-CH=CH-CH_2-\underset{\displaystyle C_6H_5}{\underset{|}{C}}H-CH_2\right]_n$

Key: 4

Sol: Conceptual

154. Lactose is a disaccharide of ________

1) $\beta-D-$Glucose and $\beta-D-$Galactose
2) $\alpha-D-$Glucose and $\beta-D-$Ribose
3) $\alpha-D-$Glucose and $\beta-D-$Galactose
4) $\alpha-D-$Glucose and $\alpha-D-$Fructose

Key: 1

Sol: Conceptual

155. What are the substances which mimic the natural chemical messengers?

1) Antagonists 2) Agonists 3) Receptors 4) Antibiotics

Key: 2

Sol: Conceptual

156. Which one of the following is more readily hydrolysed by S_N1 mechanism?

1) $C_6H_5CH_2Br$ 2) $C_6H_5CH(CH_3)Br$ 3) $(C_6H_5)_2CHBr$ 4) $(C_6H_5)_2C(CH_3)Br$

Key: 4

Sol: Conceptual

157. $C_6H_5-O-CH_2CH_3 \xrightarrow[\Delta]{HI} Y+Z$ **Identify Y and Z in the above reaction:**

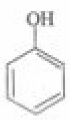
	Y	**Z**
1)	C_2H_5I	C_6H_5CHO
2)	C_6H_5I	H_3CCH_2OH
3)	C_6H_5OH	H_3CCH_2I
4)	C_6H_5OH	H_3CCH_3

Key: 3

Sol: $C_6H_5-O-CH_2CH_3 \xrightarrow[\Delta]{HI} C_6H_5OH + H_3C-CH_2-I$

158. $X \xrightarrow{Y}$ Benzoquinone **Indetify X and Y in the above reaction:**

X Y

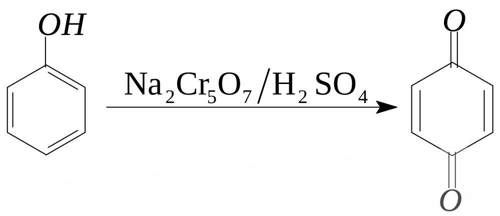
1)  $Na_2Cr_2O_7 / H_2SO_4$

2)  $Na_2Cr_2O_7 / H_2SO_4$

3)  Zn

4)  Zn

Key: 1

Sol: 

159. $H_3CMgBr + CO_2 \xrightarrow{\text{Dry ether}} Y \xrightarrow{H_3O^{\ominus}} Z$ **Identify Z from the following:**

1) Acetic acid 2) Propanic acid 3) Methyl acetate 4) Ethyl acetate

Key: 1

Sol: $CH_3 - Mg - Br + CO_2 \xrightarrow{\text{dry ether}} CH_3 - \overset{O}{\overset{\|}{C}} - O^- \xrightarrow{H_3O^+} CH_3 - \overset{O}{\overset{\|}{C}} - OH$

160. What is Z in the following reaction sequence?

$$C_6H_5NH_2 \xrightarrow[\substack{ii) H_3PO_2 + H_2O \\ iii) CO, HCl; \text{anhydrous } AlCl_3/CuCl}]{i) NaNO_2 + HCl/273K} Z$$

1) C_6H_5OH 2) C_6H_5CHO 3) C_6H_6 4) $C_6H_5CO_2H$

Key: 2

Sol: $C_6H_5NH_2 \xrightarrow{NaNO_2, HCl, 275K} C_6H_5\text{-}\overset{+}{N_2}Cl^- \xrightarrow{H_3PO_2 + H_2O} C_6H_6 \xrightarrow[\text{Anhydrous } AlCl_3/CuCl_2]{CO, HCl} C_6H_5\text{-}CHO$

* * *