DATE : 25/05/2014

CODE **7**

Aakash

Medical | IIT-JEE | Foundations

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Time : 3 hrs.

Answers & Solutions

for

JEE (Advanced)-2014

Max. Marks: 180

PAPER - 1 (Code - 7)

INSTRUCTIONS

Question Paper Format

The question paper consists of **three parts** (Physics, Chemistry and Mathematics). Each part consists of two sections.

Section 1 contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE are correct**.

Section 2 contains **10 questions**. The answer to each of the questions is a single-digit integer, ranging from 0 to 9 (both inclusive).

Marking Scheme

For each question in **Section 1**, you will be awarded **3 marks** if you darken all the bubble(s) corresponding to the correct answer(s) and **zero mark** if no bubbles are darkened. **No negative** marks will be awarded for incorrect answers in this section.

For each question in **Section 2**, you will be awarded **3 marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. **No negative** marks will be awarded for incorrect answer in this section.

PART–I : PHYSICS

SECTION - 1 : (One or More Than One Options Correct Type)

This section contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE are correct**.

1. A light source, which emits two wavelengths λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) Form the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes for λ_1 is greater than λ_2

Answer (A, B, C)

Hint : $\beta = \frac{\lambda D}{d} \Rightarrow \beta_2 > \beta_1 \because \lambda_2 > \lambda_1$

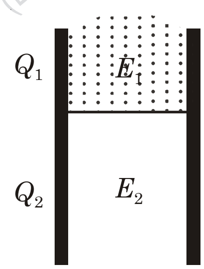
Also, $m = \frac{y}{\beta} \Rightarrow m_1 > m_2$

3rd maxima of λ_2 lies at $3(600 \text{ nm}) \times \frac{D}{d} = (1800 \text{ nm})\frac{D}{d}$

5th minima of λ_1 lies at $(2\times5-1),(400)\times\frac{D}{2d} = (1800 \text{ nm})\frac{D}{d}$

Angular separation is $\frac{\beta}{D} = \frac{\lambda}{d} \Rightarrow$ It is more for λ_2.

2. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

Answer (A, D)

Hint : $C = C_1 + C_2$

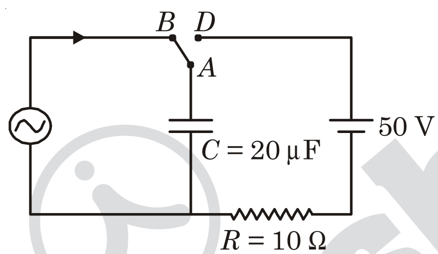
$$C_1 = \frac{K\varepsilon_0 A/3}{d},\ C_2 = \frac{\varepsilon_0 2A/3}{d}$$

$$\Rightarrow C = \frac{(K+2)\varepsilon_0 A}{3d}$$

$$\Rightarrow \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = \frac{V}{d}$, where V is potential difference between the plates.

3. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500$ rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu$F, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s).



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

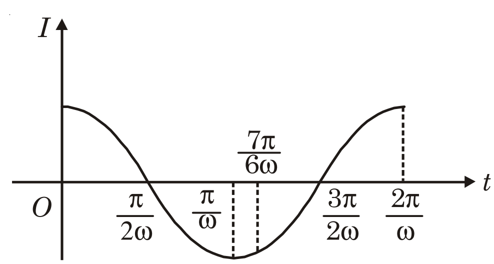
(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(C) Immediately after A is connected to D, the current in R is 10 A

(D) $Q = 2 \times 10^{-3}$ C

Answer (C, D)

Hint : The variation of current is shown below



Between $t = 0$ to $\frac{7\pi}{6\omega}$, charge will be maximum at $\frac{\pi}{2\omega}$

$$Q = \int_0^{\pi/2\omega} I_0 \cos\omega t\, dt = \frac{I_0}{\omega}(\sin\omega t)_0^{\pi/2\omega} = \frac{I_0}{\omega} = \frac{1}{500} = 2\times10^{-3} C$$

At $t = \frac{7\pi}{6\omega}$, sense of current will be opposite to initial sense i.e. anticlockwise.

At $t = \frac{7\pi}{6\omega}$, the charge on upper plate is

$$\int_0^{7\pi/6\omega} I_0 \cos\omega t dt = \frac{I_0}{\omega}[\sin\omega t]_0^{7\pi/6\omega}$$

$$= \frac{1}{500} \times \sin\left(\frac{7\pi}{6}\right) = -\frac{1}{1000} = -10^{-3}\text{C}$$

Applying KVL immediately after switch is shifted to D,

$$+50 + \frac{10^{-3}}{20 \times 10^{-6}} - i \times 10 = 0$$

Final charge on C after shifting the switch, $Q' = CV = 20 \times 10^{-6} \times 50 = 10^{-3}$ C.

So, total charge flown from battery = 2×10^{-3} C.

4. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$

(D) $E_2(r_0/2) = 4E_3(r_0/2)$

Answer (C)

Hint : $E_1(r_0) = \frac{Q}{4\pi\varepsilon_0 r^2}$, $E_1\left(\frac{r_0}{2}\right) = \frac{4Q}{4\pi\varepsilon_0 r^2}$

$$E_2(r_0) = \frac{\lambda}{2\pi\varepsilon_0 r}, E_2\left(\frac{r_0}{2}\right) = \frac{2\lambda}{2\pi\varepsilon_0 r}$$

$$E_3(r_0) = E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\varepsilon_0}$$

Now, $E_1(r_0) = E_2(r_0) = E_3(r_0)$

$$\Rightarrow \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow Q = 2\pi r^2\sigma$$

$$\Rightarrow \frac{\lambda}{\pi r} = \sigma \text{ or } r = \frac{\lambda}{\sigma\pi}$$

$$E_1\left(\frac{r_0}{2}\right) = 4E_1(r_0) = 2E_2\left(\frac{r_0}{8}\right)$$

Also, $E_2\left(\frac{r_0}{2}\right) = 2E_3(r_0) = 2E_3\left(\frac{r_0}{2}\right)$

Only C is correct.

5. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(**Useful information :** $\sqrt{167RT} = 640\ J^{1/2}mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

Answer (D)

Hint : Minimum length $= \frac{\lambda}{4}$

$\Rightarrow \lambda = 4l$

Now, $v = f\lambda = (244) \times 4 \times l$

as $l = 0.350 \pm 0.005$

$\Rightarrow$ v lies between 336.7 m/s to 346.5 m/s

Now, $v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}}$, here M is molecular mass in gram

$= \sqrt{100\gamma RT} \times \sqrt{\frac{10}{M}}$.

For monoatomic gas $\gamma = 1.67$ $\Rightarrow v = 640 \times \sqrt{\frac{10}{M}}$

For diatomic gas, $\gamma = 1.4 \Rightarrow v = 590 \times \sqrt{\frac{10}{M}}$

$\therefore\ v_{Ne} = 640 \times \frac{7}{10} = 448$ m/s

$v_{Ar} = 640 \times \frac{17}{32} = 340$ m/s

$v_{O_2} = 590 \times \frac{9}{16} = 331.8$ m/s

$v_{N_2} = 590 \times \frac{3}{5} = 354$ m/s

$\therefore$ Only possible answer is Argon.

6. One end of a taut string of length 3 m along the x-axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is(are)

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

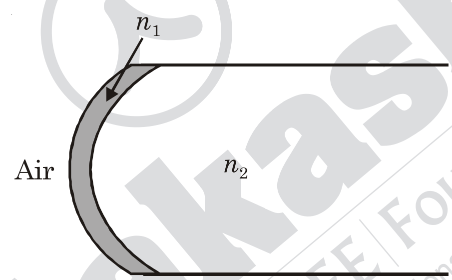
Answer (A, C, D)

Hint : There should be a node at $x = 0$ and antinode at $x = 3$ m. Also, $v = \frac{\omega}{k} = 100$ m/s.

$\therefore$ $y = 0$ at $x = 0$ and $y = \pm A$ at $x = 3$ m.

Only A, C and D are satisfy the condition.

7. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



(A) $|f_1| = 3R$

(B) $|f_1| = 2.8R$

(C) $|f_2| = 2R$

(D) $|f_2| = 1.4R$

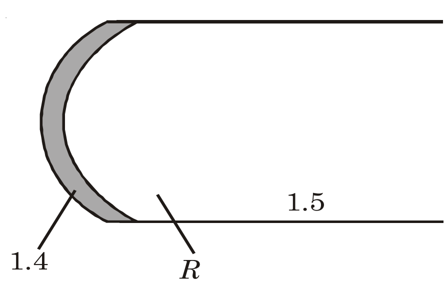
Answer (A, C)

Hint : As thickness of film is uniform, the effective power of the film is zero.

$\therefore$ We can find the answer just by considering glass-air interface.

In case-1, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Gives $\frac{1.5}{f_1} - 0 = \frac{1.5-1}{R} \Rightarrow f_1 = 3R$



In case-2, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Gives $\frac{1}{f_2} - 0 = \frac{1-1.5}{-R}$

$\Rightarrow f_2 = 2R$

8. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?

(A) 4 if wires are in parallel

(B) 2 if wires are in series

(C) 1 if wires are in series

(D) 0.5 if wires are in parallel

Answer (B, D)

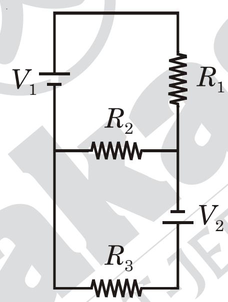
Hint : $t = \dfrac{H}{P} = \dfrac{HR}{V^2}$

$\Rightarrow \; t \propto R.$ $\qquad R = \rho l / A,\ R' = \dfrac{\rho l}{4A}$ (as $d' = 2d$)

When wires are in series, $R_1 = R' + R' = 2R' = \dfrac{R}{2} \Rightarrow t' = \dfrac{t}{2} = 2$ min

When wires are in parallel, $R_2 = \dfrac{R'}{2} = \dfrac{R}{8} \Rightarrow t' = \dfrac{t}{8} = 0.5$ min

9. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$

(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$

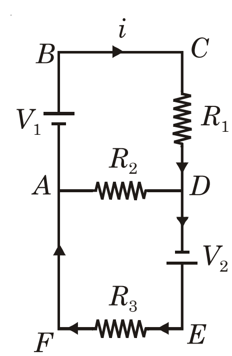
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

Answer (A, B, D)

Hint : Using KVL, in *ABCDEFA*, we get

$-iR_1 + V_2 - iR_3 + V_1 = 0$

$\Rightarrow i = \dfrac{V_1 + V_2}{R_1 + R_3}$



Using KVL in *ABCDA*,

$0 + V_1 - iR_1 = 0 \Rightarrow i = \dfrac{V_1}{R_1}$

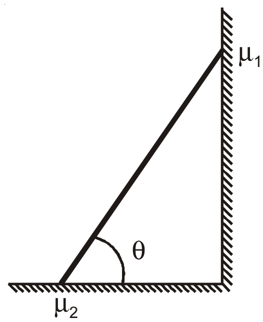
$\Rightarrow \dfrac{V_1}{R_1} = \dfrac{V_1 + V_2}{R_1 + R_3}$

$\Rightarrow V_1R_1 + V_1R_3 = V_1R_1 + V_2R_1$

$\Rightarrow \dfrac{V_1}{R_1} = \dfrac{V_2}{R_3}$

Now possible answers are A, B, D.

10. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0, \mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0, \mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0, \mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0, \mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

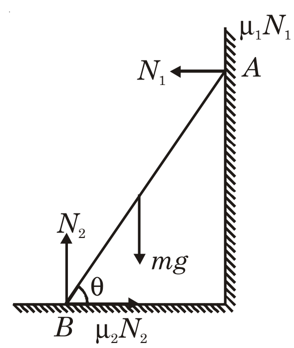
Answer (C, D)

Hint : μ_2 can never be zero for maximum equilibrium.

When $\mu_1 = 0$ we have

$N_1 = \mu_2 N_2$ .... (i)

$N_2 = m_2 g$ .... (ii)



$$\tau_B = 0 \Rightarrow mg\frac{L}{2}\cos\theta = N_1 L\sin\theta$$

$$\Rightarrow N_1 = \frac{mg\cot\theta}{2}$$

$$\Rightarrow N_1 \tan\theta = \frac{mg}{2}$$

When $\mu_1 \neq 0$ we have

$\mu_1 N_1 + N_2 = mg$ ... (i)

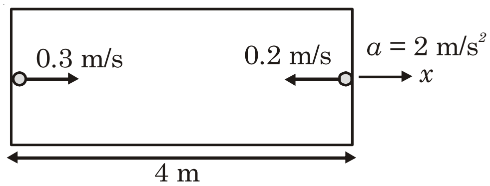
$\mu_2 N_2 = N_1$ ... (ii)

$$\Rightarrow N_2 = \frac{mg}{1+\mu_1\mu_2}$$

SECTION - 2 : (One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (*both inclusive*).

11. A rocket is moving in a gravity free space with a constant acceleration of 2 m/s^2 along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of 0.3 m/s relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of 0.2 m/s from its right end relative to the rocket. The time in seconds when the two balls hit each other is



Answer (8)

Hint :

$$S_1 = 0.2t + \frac{1}{2}\times 2\times t^2$$

$$S_2 = 0.3t - \frac{1}{2}\times 2\times t^2$$

$S_1 + S_2 = 4$

$0.5t = 4$

$t = 8$

12. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 – 30 V. If connected to a $\frac{2n}{249}\,\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is

Answer (5)

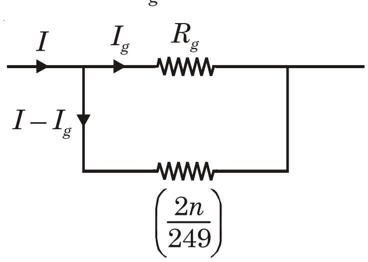
Hint : $I_g = 0.006$ A, $R = 4990\ \Omega$

$V = 30 \qquad V = I_g(R_g + R)$

$30 = 0.006(R_g + 4990)$

$$\frac{30\times 1000}{6} = R_g + 4990$$

$R_g = 10\ \Omega$



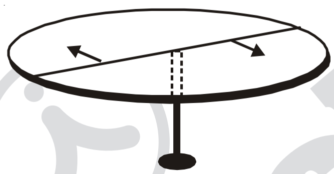
$$0.006 \times 10 = (1.5 - 0.006) \times \frac{2n}{249}$$

$$\Rightarrow \frac{0.06}{1.5} = \frac{2n}{249}$$

$$2n = \frac{0.06 \times 249}{1.494} = 10$$

$$\Rightarrow \boxed{n = 5}$$

13. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



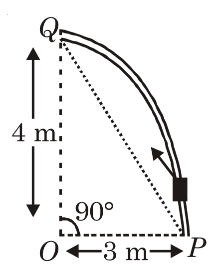
Answer (4)

Hint : $2mvr = I\omega$

$$\Rightarrow 2 \times 0.05 \times 9 \times 0.25 = \frac{0.45 \times (0.5)^2}{2} \times \omega$$

$$\Rightarrow \omega = \frac{2 \times 0.05 \times 9 \times 0.25 \times 2}{0.45 \times 0.5 \times 0.5} = 4$$

14. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



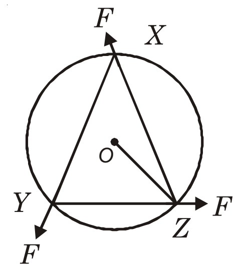
Answer (5)

Hint : $W_g = -mgh = -1 \times 10 \times 4 = -40$ J

$W_f = F \times d = 18 \times 5 = +90$ J

$\therefore$ KE = +90 – 40 = 50 = 10n $\Rightarrow$ n = 5

15. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is

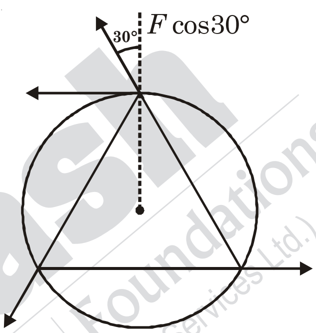


Answer (2)

Hint : $\tau = 3 \times (F\sin 30) \times r$

$$= 3\times0.5\times\frac{1}{2}\times0.5$$

$$= 3\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$$



$$I=\frac{MR^2}{2}=\frac{1.5\times0.5\times0.5}{2}=\frac{15}{10}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{16}$$

$$\tau = I\alpha \;\Rightarrow\; \alpha=\frac{\frac{3}{8}}{\frac{3}{16}}=2$$

$$\omega = \omega_0 + \alpha t = 2 \times 1 = 2$$

16. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1}=3$, the value of $\frac{R_1}{R_2}$ is

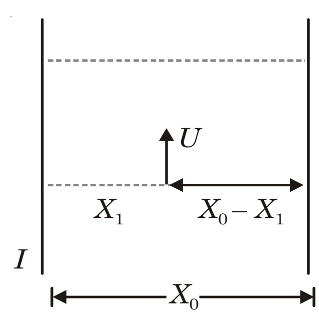
Answer (3)

Hint :

$$B_1=\frac{\mu_0 I}{2\pi X_1}-\frac{\mu_0 I}{2\pi(X_0-X_1)}$$

$$=\frac{\mu_0 I}{2\pi}\left[\frac{1}{X_1}-\frac{1}{X_0-X_1}\right]$$

$$=\frac{\mu_0 I}{2\pi}\left[\frac{X_0-X_1-X_1}{X_1(X_0-X_1)}\right]$$



$$B_1 = \frac{\mu_0 I}{2\pi}\left[\frac{X_0 - 2X_1}{X_1(X_0 - X_1)}\right]$$

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{1}{X_1} + \frac{1}{X_0 - X_1}\right] = \frac{\mu_0 I}{2\pi}\left[\frac{X_0}{X_1(X_0 - X_1)}\right]$$

$$\frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{X_0}{X_0 - 2X_1} = \frac{\frac{X_0}{X_1}}{\frac{X_0}{X_1} - 2} = \frac{3}{3-2} = 3$$

17. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways-engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

Answer (3)

Hint : $dB = \rho^a s^b f^c$

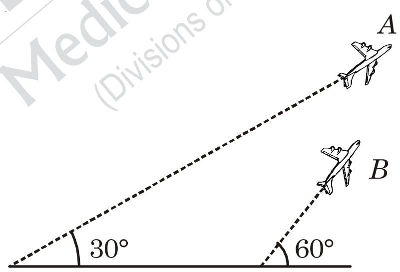
$$M^0L^1T^0 = M^aL^{-3a} \times M^bT^{-3b}T^{-c}$$
$$= M^{a+b}L^{-3a}T^{-3b-c}$$

$a + b = 0,\ -3a = 1$

$$\Rightarrow \quad a = -\frac{1}{3},\ b = \frac{1}{3}$$

$\therefore\ n = 3$

18. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ m/s. At time t = 0 s, an observer in A finds B at a distance of 500 m. The observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



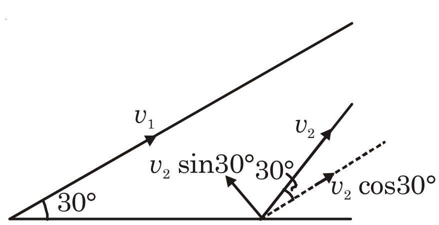
Answer (5)

Hint :

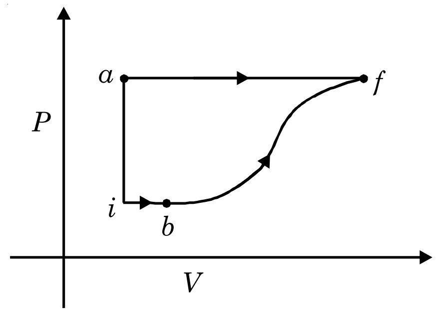
$$v_1 = v_2 \cos 30°$$

$$\Rightarrow 100\sqrt{3} = v_2 \times \frac{\sqrt{3}}{2} \Rightarrow v_2 = 200 \text{ m/s}$$

$$\therefore\ t_0 = \frac{d}{v_2 \sin 30°} = \frac{500}{200 \times \frac{1}{2}} = 5 \text{ s}$$



19. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf}/Q_{ib} is



Answer (2)

Hint : $W_{ia} = 0$ $\quad W_{af} = 200$ J, $U_i = 100$ J, $U_b = 200$ J

$W_{ib} = 50$ J

$W_{bf} = 100$ J

$Q_{iaf} = 500$ J

$W_{iaf} = 200$ J

$U_f - U_i = 300$ J

$\boxed{U_f = 400\text{ J}}$

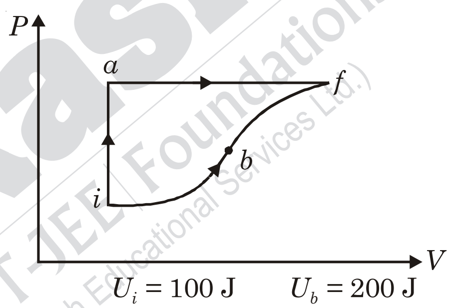
$Q_{ib} = U_{ib} + W_{ib}$

$= (200 - 100) + 50 = 150$

$Q_{bf} = U_{bf} + W_{bf}$

$= (400 - 200) + 100 = 300$

$$\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$



20. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

Answer (4)

Hint : $Y = \dfrac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}}$ $\qquad \Delta\ell = 25\times10^{-50}$ m

$$Y = \frac{F}{A}\cdot\frac{\ell}{\Delta\ell}$$

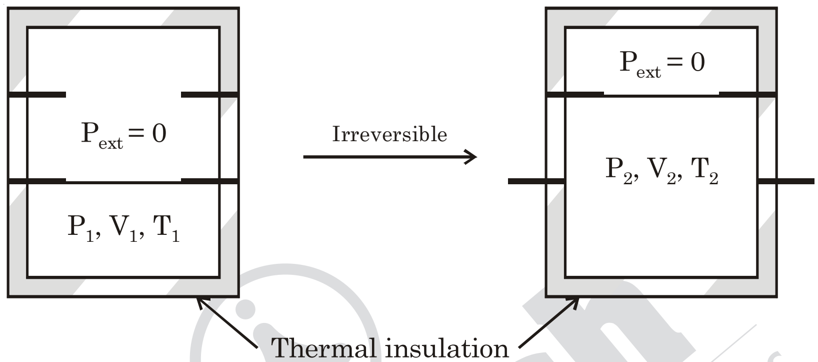
$$\frac{\Delta Y}{Y}\times100 = \frac{10^{-5}}{25\times10^{-5}}\times100 = 4\%$$

PART–II : CHEMISTRY

SECTION - 1 : (One or More Than One Options Correct Type)

This section contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE are correct**.

21. An ideal gas in a thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2, respectively. For this expansion,



(A) $q = 0$

(B) $T_2 = T_1$

(C) $P_2V_2 = P_1V_1$

(D) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

Answer (A, B, C)

Hint : Work against zero external pressure is zero q = 0 due to insulated boundary

So, $q = 0$

$\Delta U = 0$

$\Delta T = 0$

$\therefore T_2 = T_1$

and $P_2V_2 = P_1V_1$

22. Hydrogen bonding plays a central role in the following phenomena

(A) Ice floats in water

(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions

(C) Formic acid is more acidic than acetic acid

(D) Dimerisation of acetic acid in benzene

Answer (A, B, D)

Hint : In ice water molecules are excessively H-bonded giving a cage-like structure which is lighter than water. Primary amines are more basic than tertiary amines in aqueous solution mainly due to H-bonding with water molecules. Dimerisation of acetic acid in benzene is due to intermolecular H-bonding.

23. Upon heating with Cu_2S, the reagent(s) that give copper metal is/are

(A) $CuFeS_2$

(B) CuO

(C) Cu_2O

(D) $CuSO_4$

Answer (B, C, D)

Hint : $Cu_2S + 2Cu_2O \xrightarrow{\Delta} 6Cu + SO_2$

Since, $CuSO_4$ as well as CuO both on heating produces Cu_2O as shown below so, they also produce Cu on heating with Cu_2S.

$$2CuO \xrightarrow{\Delta} Cu_2O + \frac{1}{2}O_2$$

$$CuSO_4 \xrightarrow{\Delta} CuO + SO_2 + \frac{1}{2}O_2$$

24. The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are

(A) Tert-butanol and 2-methylpropan-2-ol

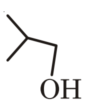
(B) Tert-butanol and 1, 1-dimethylethan-1-ol

(C) n-butanol and butan-1-ol

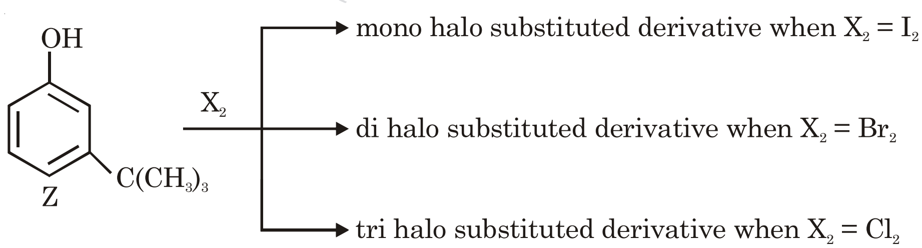
(D) Isobutyl alcohol and 2-methylpropan-1-ol

Answer (A, C, D)

Hint :

	Common name	IUPAC name
(A) 	tert-butanol	2-methylpropanol
(C) 	n-butanol	butan-1-ol
(D) 	isobutyl alcohol	2-methyl butan-1-ol

25. The reactivity of compound Z with different halogens under appropriate conditions is given below :



The observed pattern of electrophilic substitution can be explained by

(A) The steric effect of the halogen

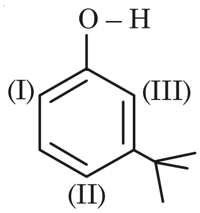
(B) The steric effect of the *tert*-butyl group

(C) The electronic effect of the phenolic group

(D) The electronic effect of the *tert*-butyl group

Answer (A, B, C)

Hint :



(I), (II) and (III) are probable positions as per electronic effect of OH group where electrophile can attack and due to steric hindrance at (II) and (III) positions bulky electrophiles are not preferred like I only preferred at position I, Bromine at positions II and I while chlorine at all three positions.

26. The pair(s) of reagents that yield paramagnetic species is/are

(A) Na and excess of NH_3

(B) K and excess of O_2

(C) Cu and dilute HNO_3

(D) O_2 and 2-ethylanthraquinol

Answer (A, B, C)

Hint : Dilute solution of Na in liquid ammonia is paramagnetic

$$K + \underset{\text{(excess)}}{O_2} \longrightarrow KO_2$$

O_2^- is paramagnetic due to unpaired electron in antibonding orbital

$$3Cu + \underset{\text{(diluted)}}{8HNO_3} \longrightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$$

NO is paramagnetic due to unpaired electrons on "N".

27. The correct statement(s) for orthoboric acid is/are

(A) It behaves as a weak acid in water due to self ionization

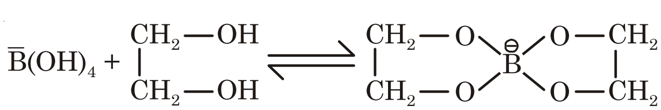
(B) Acidity of its aqueous solution increases upon addition of ethylene glycol

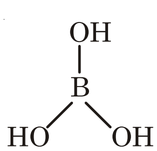
(C) It has a three dimensional structure due to hydrogen bonding

(D) It is a weak electrolyte in water

Answer (B, D)

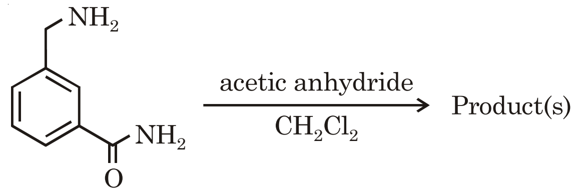
Hint : $H_3BO_3 + H_2O \rightleftharpoons \overline{B}(OH)_4 + H^+$

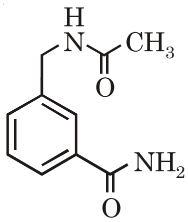


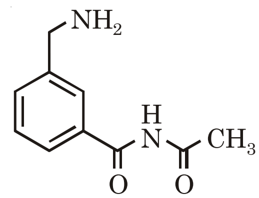


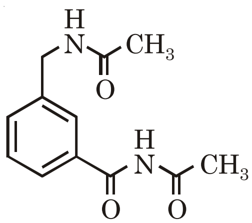
In boric acid, planar H_3BO_3 units are joined by hydrogen bonds to give layered structure.

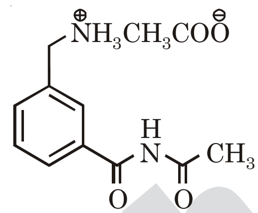
28. In the reaction shown below, the major product(s) formed is/are



(A)  + CH_3COOH

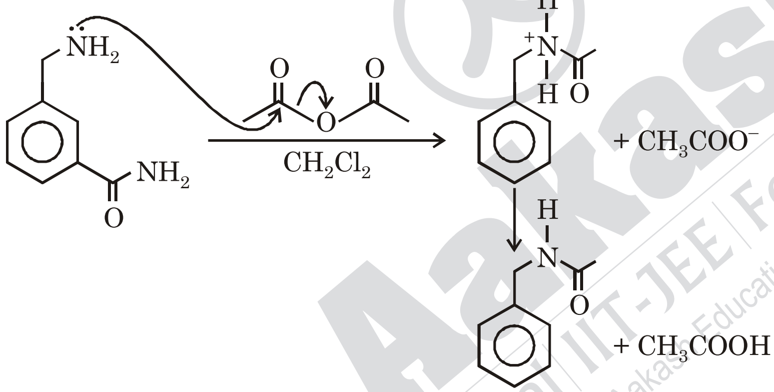
(B)  + CH_3COOH

(C)  + H_2O

(D) 

Answer (A)

Hint :



29. In a galvanic cell, the salt bridge

(A) Does not participate chemically in the cell reaction

(B) Stops the diffusion of ions from one electrode to another

(C) Is necessary for the occurrence of the cell reaction

(D) Ensures mixing of the two electrolytic solutions

Answer (A, B)

Hint : In a galvanic cell, the salt bridge does not participate in the cell reaction, stops diffusion of ions from one electrode to another and is not necessary for the occurrence of the cell reaction.

30. For the reaction

$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$

The correct statements(s) in the balanced equation is/are

(A) Stoichiometric coefficient of HSO_4^- is 6

(B) Iodide is oxidized

(C) Sulphur is reduced

(D) H_2O is one of the products

Answer (A, B, D)

Hint : Balanced chemical equation is

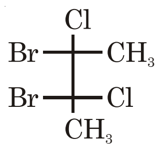
$$6I^- + ClO_3^- + 6H_2SO_4 \longrightarrow Cl^- + 6HSO_4^- + 3I_2 + 3H_2O$$

Here, H_2O is produced and I^- is oxidized.

SECTION - 2 : (One Integer Value Correct Type)

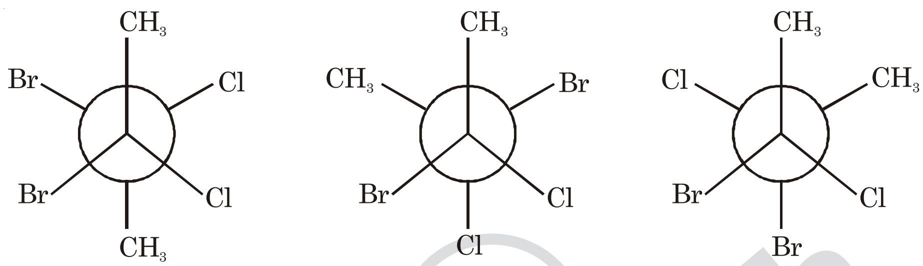
This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (*both inclusive*).

31. The total number(s) of **stable** conformers with **non-zero** dipole moment for the following compound is (are)



Answer (3)

Hint : Following conformers are stable with non-zero dipole moment



32. Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3 and SnS_2, the total number of **BLACK** coloured sulphides is

Answer (7)

Hint : Black sulphides are

PbS, CuS, HgS, NiS, CoS, Bi_2S_3 and Ag_2S

MnS is buff

SnS_2 is yellow

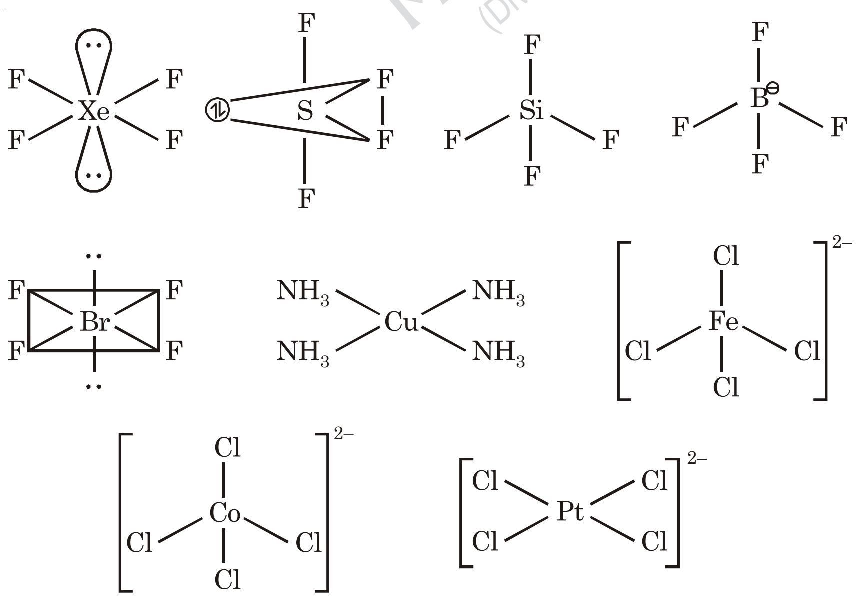
33. A list of species having the formula XZ_4 is given below.

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$.

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

Answer (4)

Hint :



34. Consider all possible isomeric ketones, including stereoisomers of MW = 100. All these isomers are independently reacted with $NaBH_4$ (**NOTE** : stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is/are

Answer (5)

Hint : (I) $\overset{1}{C}H_3-\overset{2}{C}H_2-\overset{3}{C}H_2-\overset{4}{C}H_2-\underset{5}{C}(=O)-\underset{6}{C}H_3$

(II) $CH_3-CH_2-CH_2-C(=O)-CH_2-CH_3$

(III) $CH_3-CH_2-CH(CH_3)-C(=O)-CH_3$

It has chiral centre

(IV) $CH_3-C(CH_3)_2-C(=O)-CH_3$

(V) $CH_3-CH(CH_3)-C(=O)-CH_2-CH_3$

(VI) $CH_3-CH(CH_3)-CH_2-C(=O)-CH_3$

Only (III) form diastereomers on addition reaction so, desired ketones as per addition reaction is 5.

35. Consider the following list of reagents :

Acidified $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$, HNO_3, and $Na_2S_2O_3$.

The total number of reagents that can oxidise aqueous iodide to iodine is

Answer (7)

Hint : $K_2Cr_2O_7 + I^- + H^+ \to I_2$

$CuSO_4 + I^- \to I_2$

$H_2O_2 + I^- \to I_2$

$Cl_2 + I^- \to I_2$

$O_3 + I^- \to I_2$

$FeCl_3 + I^- \to I_2$

$HNO_3 + I^- \to I_2$

36. A compound H_2X with molar weight of 80 g is dissolved in a solvent having density of 0.4 g ml^{-1}. Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is

Answer (8)

Hint : $m = \dfrac{w_2 \times 1000}{m_2 \times w_1}$

1 mL solvent having mass 0.4 g.

1000 mL solvent having mass 400 g

1000 mL solution contain 3.2 × 80 g solute = 256 g

$\therefore \quad m = \dfrac{256 \times 1000}{80 \times 400} = 8$

37. In an atom, the total number of electrons having quantum numbers n = 4, $|m_l| = 1$ and $m_s = -\dfrac{1}{2}$ is

Answer (6)

Hint : $|m_l| = 1$

means m_l can be +1 and –1

So, for n = 4 six orbitals are possible and bear six electrons with $s = -\dfrac{1}{2}$

38. MX_2 dissociates into M^{2+} and X^- ions in an aqueous solution, with a degree of dissociation (α) of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is

Answer (2)

Hint : α = 0.5

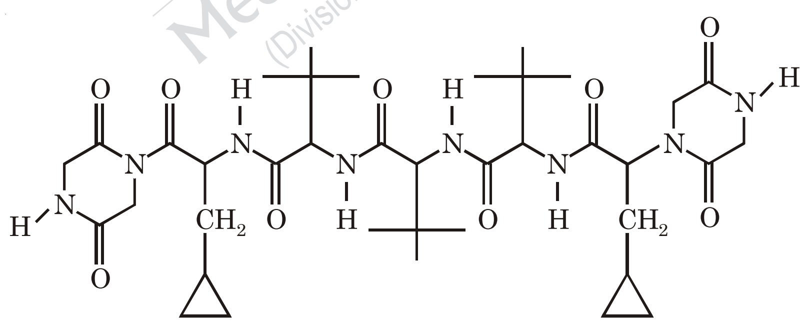
$$\underset{(1-\alpha)}{MX_2} \rightleftharpoons \underset{\alpha}{M^{+2}} + \underset{2\alpha}{2X^-}$$

$i = \dfrac{1 - \alpha + \alpha + 2\alpha}{1}$

i = 1 + 2α

i = 1 + 2 × 0.5 = 2

39. The total number of distinct naturally occurring amino acids obtained by complete acidic hydrolysis of the peptide shown below is



Answer (1)

Hint : On hydrolysis only glycine is formed as natural amino acid.

40. If the value of Avogadro number is 6.023×10^{23} mol^{-1} and the value of Boltzmann constant is 1.380×10^{-23} J K^{-1}, then the number of significant digits in the calculated value of the universal gas constant is

Answer (4)

Hint : $6.023 \times 10^{23} \times 1.380 \times 10^{-23} = 8.312$

It has four significant figure.

PART–III : MATHEMATICS

SECTION - 1 : (One or More Than One Options Correct Type)

This section contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE are correct**.

41. Let $f : (0, \infty) \to \mathbb{R}$ be given by $f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$. Then

(A) $f(x)$ is monotonically increasing on $[1, \infty)$

(B) $f(x)$ is monotonically decreasing on $(0, 1)$

(C) $f(x)+f\left(\frac{1}{x}\right)=0$, for all $x \in (0, \infty)$

(D) $f(2^x)$ is an odd function of x on $\mathbb{R}$

Answer (A, C, D)

Hint : $f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$

$$f'(x) = \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x} > 0$$

Also, $f(x)+f\left(\frac{1}{x}\right) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t} + \int_{x}^{\frac{1}{x}} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t} = 0$

and so, $f(2^x)+f(2^{-x})=0$

i.e., (A, C, D) is correct answer.

42. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N$ and $M^2 = N^2$, then

(A) Determinant of $(M^2 + MN^2)$ is 0

(B) There is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(C) Determinant of $(M^2 + MN^2) \geq 1$

(D) For a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

Answer (A, B)

Hint : $M \neq N^2 \quad \Rightarrow M - N^2 \neq 0$

$M^2 - N^4 = 0 \quad \Rightarrow (M - N^2)(M + N^2) + N^2M - MN^2 = 0$

as $MN = NM$

$\Rightarrow MN^2 = NMN$

$= NNM$

$= N^2M$

So, $(M - N^2)(M + N^2) = 0$

So either $M + N^2 = 0$ or $M - N^2$ and $M + N^2$ both are singular.

So, there exist a 3×3 non-zero matrix U i.e., $M - N^2$ such that

$(M + N^2)U = 0 \quad \Rightarrow \quad (M^2 + MN^2)U = 0$

Also, $|M^2 + MN^2| = |M||M + N^2| = 0$

43. Let $a \in \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be given by

$f(x) = x^5 - 5x + a$.

Then

(A) $f(x)$ has three real roots if $a > 4$

(B) $f(x)$ has only one real root if $a > 4$

(C) $f(x)$ has three real roots if $a < -4$

(D) $f(x)$ has three real roots if $-4 < a < 4$

Answer (B, D)

Hint : $f(x) = x^5 - 5x + a$

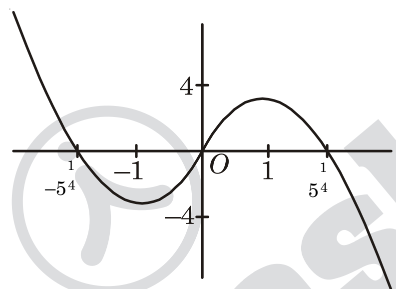
if $f(x) = 0 \quad \Rightarrow \quad a = 5x - x^5 = g(x)$

$g(x) = 0 \Rightarrow x = 0,\ 5^{\frac{1}{4}},\ -5^{\frac{1}{4}}$

$g'(x) = 0 \Rightarrow 5 - 5x^4 = 0 \Rightarrow x = 1, -1$

$g(-1) = -4$

$g(1) = 4$



If $a \in (-4, 4) \Rightarrow f(x) = 0$ has 3 real roots

if $a > 4$ or $a < -4 \Rightarrow f(x) = 0$ has only 1 real root.

44. From a point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x$, $z = 1$ and $y = -x$, $z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$

(B) 1

(C) –1

(D) $-\sqrt{2}$

Answer (C)

Hint : $\dfrac{x}{1} = \dfrac{y}{1} = \dfrac{z-1}{0} = K$ ...(i)

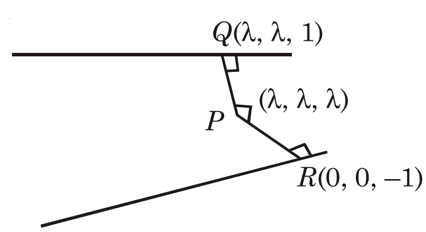
$\Rightarrow \quad Q : (K, K, 1)$

$\dfrac{x}{-1} = \dfrac{y}{1} = \dfrac{z+1}{0} = m$ ...(ii)

$\Rightarrow \quad R : (-m, m, -1)$

$\because \quad PQ$ is perpendicular to line (i)

$\therefore \quad (\lambda - K) + (\lambda - K) + 0 = 0 \quad \Rightarrow \quad K = \lambda$



PR is perpendicular to line (ii)

$\therefore \quad -1(-m + \lambda) + (m - \lambda) + 0 = 0$

$\Rightarrow \quad m = 0$

Also line PQ is perpendicular to PR

$\Rightarrow \quad 0 + 0 + (\lambda - 1)(\lambda + 1) = 0$

$\Rightarrow \quad \lambda = \pm 1$

$\lambda = 1$ will be rejected as P will coincide with Q.

45. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(A) The first column of M is the transpose of the second row of M

(B) The second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) The product of entries in the main diagonal of M is not the square of an integer

Answer (C, D)

Hint : Let $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, where $a, b, c \in I$

for invertible matrix, $\det(M) \neq 0$ $\Rightarrow ac - b^2 \neq 0$

i.e. $ac \neq b^2$

So, options (C) & (D) are satisfies the above condition.

46. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

(A) $\vec{b} = (\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$

(B) $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$

(C) $\vec{a}\cdot\vec{b} = -(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$

(D) $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

Answer (A, B, C)

Hint : Given that $\vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{z} = \vec{z}.\vec{x} = 1$ and $\vec{a} = \lambda(\vec{x}\times(\vec{y}\times\vec{z})), \vec{b} = \mu(\vec{y}\times(\vec{z}\times\vec{x}))$

$= \lambda(\vec{y}-\vec{z}) \qquad = \mu(\vec{z}-\vec{x})$

$\therefore \quad \vec{a}\cdot\vec{y} = \lambda(2-1) = \lambda, \vec{b}\cdot\vec{z} = \mu$

$(\vec{a}\cdot\vec{y})(\vec{y}-\vec{z}) = \lambda(\vec{y}-\vec{z}) = \vec{a}$ (Option B is correct)

$(\vec{b}\cdot\vec{z})(\vec{z}-\vec{x}) = \mu(\vec{z}-\vec{x}) = \vec{b}$ (Option A is correct)

$(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z}) = \lambda\mu$

and $\vec{a}\cdot\vec{b} = \lambda\mu(\vec{y}-\vec{z})\cdot(\vec{z}-\vec{x})$

$= \lambda\mu(1 - 1 - 2 + 1)$

$= -\lambda\mu$

$= -(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$ (Option C is correct)

$(\vec{a}\cdot\vec{y})(\vec{z}-\vec{y}) = \lambda(\vec{z}-\vec{y}) \neq \vec{a}$

$\therefore$ (A, B, C) are correct.

47. For every pair of continuous functions $f, g : [0, 1] \to \mathbb{R}$ such that

$\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are) :

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$

(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

Answer (A, D)

Hint : Let f and g be maximum at c_1 and c_2 respectively, $c_1, c_2 \in (0, 1)$.

Let $h(x) = f(x) - g(x)$

Now $h(c_1) = f(c_1) - g(c_1) = +\text{ve}$

and $h(c_2) = f(c_2) - g(c_2) = -\text{ve}$

$\therefore$ $h(x) = 0$ has at least one root in (c_1, c_2)

$\therefore$ $f(x) = g(x)$ for some $x = c \in (c_1, c_2)$

$\therefore$ $f(c) = g(c)$ for some $c \in (0, 1)$

Clearly (A, D) are correct

48. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : \mathbb{R} \to \mathbb{R}$ be defined as

$$g(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t)\,dt & \text{if } a \le x \le b, \\ \int_a^b f(t)dt & \text{if } x > b \end{cases}. \text{ Then}$$

(A) $g(x)$ is continuous but not differentiable at a

(B) $g(x)$ is differentiable on $\mathbb{R}$

(C) $g(x)$ is continuous but not differentiable at b

(D) $g(x)$ is continuous and differentiable at either a or b but not both

Answer (A, C)

Hint : $g(a^-) = 0$, $g(a) = \int_a^a f(t)dt = 0$, $g(a^+) = \lim\limits_{h\to 0}\int_a^{a+h} f(t)dt = 0$

$$g(b^-) = \lim_{h\to 0}\int_a^{b-h} f(t)dt = \int_a^b f(t)dt$$

$$g(b) = \int_a^b f(t)dt = g(b^+)$$

Hence $g(x)$ is continuous at $x = a$ as well as $x = b$

Now, $$g'(a^-) = \lim_{h\to 0}\frac{g(a-h)-g(a)}{-h} = 0$$

$$g'(a^+) = \lim_{h\to 0}\frac{g(a+h)-g(a)}{h} = \lim_{h\to 0}\frac{\int_a^{a+h} f(t)dt - 0}{h} = \lim_{h\to 0}\frac{f(a+h)}{1} = f(a)\ (\neq 0)$$

Hence $g(x)$ is not differentiable at $x = a$.

$$g'(b^-) = \lim_{h\to 0}\frac{g(b-h)-g(b)}{-h} = \lim_{h\to 0}\frac{\int_a^{b-h} f(t)dt - \int_a^b f(t)dt}{-h} = \lim_{h\to 0}\frac{f(b-h)}{1} = f(b)(\neq 0)$$

$$g'(b^+) = \lim_{h\to 0}\frac{g(b+h)-g(b)}{h} = \lim_{h\to 0}\frac{\int_a^b f(t)dt - \int_a^b f(t)dt}{h} = 0$$

Hence $g(x)$ is not differentiable at $x = b$.

49. Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to\mathbb{R}$ be given by $f(x) = (\log(\sec x + \tan x))^3$. Then

(A) $f(x)$ is an odd function

(B) $f(x)$ is a one-one function

(C) $f(x)$ is an onto function

(D) $f(x)$ is an even function

Answer (A, B, C)

Hint : $f(x) = (\log(\sec x + \tan x))^3$

$f(-x) = (\log(\sec x - \tan x))^3$

$= \log\left(\frac{1}{\sec x+\tan x}\right)^3$

$= -f(x)$

$\therefore$ f is odd.

Also $f'(x) = 3(\log(\sec x + \tan x))^2 \cdot \frac{(\sec x\tan x+\sec^2 x)}{\sec x+\tan x}$

$= 3\sec x \cdot (\log(\sec x + \tan x))^2 > 0$ in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ f is increasing on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ f is one-one

$\lim\limits_{x\to\frac{\pi}{2}^-}(\log(\sec x+\tan x))^3\to\infty$ and $\lim\limits_{x\to-\frac{\pi}{2}^+}(\log(\sec x+\tan x))^3\to-\infty$

$\therefore$ Range is $\mathbb{R}$.

50. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) Radius of S is 8

(B) Radius of S is 7

(C) Centre of S is (–7, 1)

(D) Centre of S is (–8, 1)

Answer (B, C)

Hint : Let the circle be $x^2 + y^2 + 2gx + 2fy + C = 0$.

It is orthogonal with $(x - 1)^2 + y^2 = 16$

$\therefore$ $2(-g + 0) = -15 + C \quad\Rightarrow\quad -2g = -15 + C$

It is also orthogonal with $x^2 + y^2 = 1$

$\therefore$ $0 = -1 + C \quad\Rightarrow\quad C = 1$

$\therefore$ $g = 7$

This circle passes through (0, 1)

$\therefore$ $1 + 2f + 1 = 0 \quad\Rightarrow\quad f = -1$

$\therefore$ The circle is $x^2 + y^2 + 14x - 2y + 1 = 0$

$(x + 7)^2 + (y - 1)^2 = 49$

$\therefore$ centre : (–7, 1) and radius : 7

SECTION - 2 : (One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (*both inclusive*).

51. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Answer (5)

Hint : Number of line joining adjacent points = n

$n = {}^nC_2 - n$

$2n = {}^nC_2$

$2n = \dfrac{n.(n-1)}{2}$

$n = 0$ or $n = 5$

52. The value of $\int_0^1 4x^3 \left\{\dfrac{d^2}{dx^2}(1-x^2)^5\right\} dx$

is

Answer (2)

Hint : Using by parts

$$\left.4x^2.\frac{d(1-x^2)^5}{dx}\right]_0^1 - \int_0^1 d(1-x^2)^5.12x^2dx$$

$$= \left.-12x^2.(1-x^2)^5\right]_0^1 + \int_0^1 24x(1-x^2)^5dx$$

$1 - x^2 = t$

$-2xdx = dt$

$$= -\int_1^0 12t^5 dt$$

$= 2$

53. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

Answer (7)

Hint : $n_1 + n_2 + n_3 + n_4 + n_5 = 20$

Maximum of $n_5 = 10$. $n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 4$

$n_5 = 09$. $n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 5$

$n_5 = 08$. $n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 6$

or $n_3 = 4, n_4 = 5$

$n_5 = 07$. $n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6$

or $n_2 = 3, n_3 = 4, n_4 = 5$

$n_5 = 06$. $n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5$

Total number of ways = 7.

54. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

Answer (4)

Hint : $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$

Now $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2 = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix}$

$$= 1\left(1-\frac{1}{4}\right)-\frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)+\frac{1}{2}\left(\frac{1}{4}-\frac{1}{2}\right)$$

$$= \frac{3}{4}-\frac{1}{8}-\frac{1}{8} = \frac{1}{2}$$

$\therefore\ \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = \pm\frac{1}{\sqrt{2}}$

Now, $\vec{a}.(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}) = p+\frac{q}{2}+\frac{r}{2}$

$$\pm\frac{1}{\sqrt{2}} = p+\frac{q}{2}+\frac{r}{2}$$

$2p + q + r = \pm\sqrt{2}$ ....(i)

$\vec{b}.(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}) = \frac{p}{2}+q+\frac{r}{2}$

$\Rightarrow\ p + 2q + r = 0$ ....(ii)

$\vec{c}.(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}) = \frac{p}{2}+\frac{q}{2}+r$

$p + q + 2r = \pm\sqrt{2}$ ....(iii)

Now,

$p = r = -q$

$p = r = \pm\frac{1}{\sqrt{2}}$, $q = \mp\frac{1}{\sqrt{2}}$

$$\frac{p^2+2q^2+r^2}{q^2} = 4$$

55. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is

Answer (8)

Hint : $(y - x^5)^2 = x(1 + x^2)^2$

$$2(y - x^5)\left(\frac{dy}{dx} - 5x^4\right) = (1 + x^2)^2 + 2x(1 + x^2).2x$$

$x = 1,\ y = 3$

$$2(3-1)\left(\frac{dy}{dx} - 5\right) = (1+1)^2 + 4(1+1)$$

$$4\left(\frac{dy}{dx} - 5\right) = 12$$

$$\frac{dy}{dx} - 5 = 3$$

$$\frac{dy}{dx} = 8$$

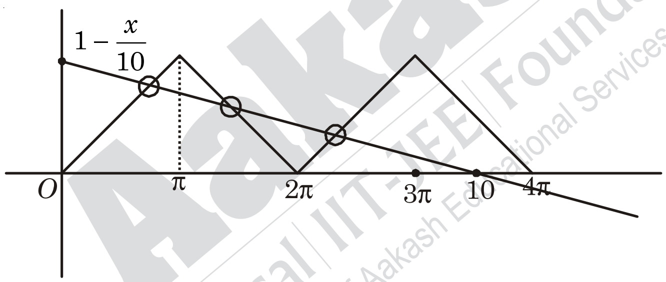
56. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation

$$f(x) = \frac{10 - x}{10}$$

is

Answer (3)

Hint : $f : [0, 4\pi] \to [0, \pi],\ f(x) = \cos^{-1} \cos x$



$$f(x) = 1 - \frac{x}{10}$$

57. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2 + a - 14}{a + 1}$ is

Answer (4)

Hint : $a,\ ar,\ ar^2,\ 0 > 1$ $\quad r$ is integer

$$\frac{a + b + c}{3} = b + 2$$

$a + ar + ar^2 = 3(ar + 2)$

$a + ar + ar^2 = 3ar + 6$

$ar^2 - 2ar + a - 6 = 0$

$a(r - 1)^2 = 6$

$a = 6,\ r = 2$

$$\text{So } \frac{a^2 + a - 14}{a + 1} = \frac{6^2 + 6 - 14}{6 + 1} = \frac{28}{7} = 4$$

58. The largest value of the non-negative integer a for which

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$

is

Answer (2)

Hint : $\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}$

$$\lim_{x\to 1}\left\{\frac{a(1-x)+\sin(x-1)}{(x-1)+\sin(x-1)}\right\}^{1+\sqrt{x}}$$

$$\lim_{x\to 1}\left\{\frac{-a+\frac{\sin(x-1)}{x-1}}{1+\frac{\sin(x-1)}{x-1}}\right\}^{(1+\sqrt{x})}$$

$$\left\{\frac{-a+1}{2}\right\}^2=\frac{1}{4}$$

$$\frac{-a+1}{2}=\pm\frac{1}{2}$$

$-a + 1 = \pm 1 \quad \Rightarrow \quad a = 0,\ a = 2$

$\therefore \quad a = 2$

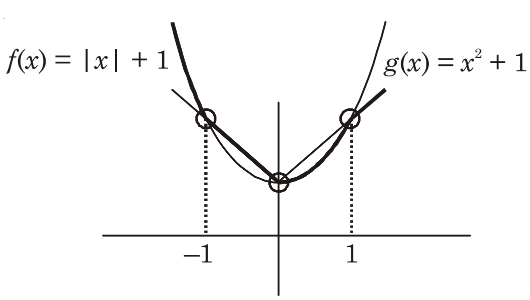
59. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h:\mathbb{R}\to\mathbb{R}$ by

$$h(x)=\begin{cases}\max\ \{f(x),g(x)\} & \text{if } x\le 0,\\ \min\ \{f(x),g(x)\} & \text{if } x>0.\end{cases}$$

The number of points at which $h(x)$ is not differentiable is

Answer (3)

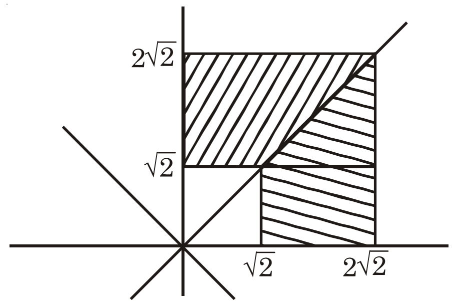
Hint :



60. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

Answer (6)

Hint : $x > 0,\ y > 0,\ 2 \le \left|\frac{x-y}{\sqrt{2}}\right| + \left|\frac{x+y}{\sqrt{2}}\right| \le 4$



case (1), $x > y$ $\quad 2 \le \frac{x-y}{\sqrt{2}} + \frac{x+y}{\sqrt{2}} \le 4$

$$2 \le \frac{2x}{\sqrt{2}} \le 4$$

$$1 \le \frac{x}{\sqrt{2}} \le 2$$

$$\sqrt{2} \le x \le 2\sqrt{2} \qquad \text{.....(i)}$$

Similarly $y > x$

$$\sqrt{2} \le y \le 2\sqrt{2} \qquad \text{....(ii)}$$

Required area = area of shaded region

$$= 2\left(\sqrt{2}\times\sqrt{2} + \frac{1}{2}\times\sqrt{2}\times\sqrt{2}\right)$$

$$= 2(2 + 1) = 6 \text{ sq. unit.}$$

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