PART - III : MATHEMATICS

SECTION–1 : (One or More Than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** are correct.

41. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \int_a^x f(t)\,dt & \text{if} \quad a \le x \le b. \\ \int_a^b f(t)\,dt & \text{if} \quad x > b \end{cases}$$

Then

(A) g(x) is continuous but not differentiable at a

(B) g(x) is differentiable on R

(C) g(x) is continuous but not differentiable at b

(D) g(x) is continuous and differentiable at either a or b but not both.

Ans. (A,C)

42. For every pair of continuous function $f, g : [0, 1] \to R$ such that

$$\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0,1]\},$$

the correct statement(s) is(are) :

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$

(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

Ans. (A,D)

43. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

Ans. (C,D)

44. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$.

If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is nonzero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

(A) $\vec{b} = (\vec{b}.\vec{z})(\vec{z} - \vec{x})$

(B) $\vec{a} = (\vec{a}.\vec{y})(\vec{y} - \vec{z})$

(C) $\vec{a}.\vec{b} = -(\vec{a}.\vec{y})(\vec{b}.\vec{z})$

(D) $\vec{a} = (\vec{a}.\vec{y})(\vec{z} - \vec{y})$

Ans. (A,B,C)

45. From a point $P(\lambda,\lambda,\lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$

Ans. (C)

46. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3 × 3 matrix U, if $(M^2 + MN^2)$ U equals the zero matrix then U is the zero matrix

Ans. (A,B)

47. Let $f : (0, \infty) \to R$ be given by

$$f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}.$$

Then

(A) $f(x)$ is monotonically increasing on $[1, \infty)$

(B) $f(x)$ is monotonically decreasing on (0, 1)

(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$

(D) $f(2^x)$ is an odd function of x on R

Ans. (A,C,D)

48. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by

$f(x) = (\log(\sec x + \tan x))^3$. Then :-

(A) $f(x)$ is an odd function

(B) $f(x)$ is a one-one function

(C) $f(x)$ is an onto function

(D) $f(x)$ is an even function

Ans. (A,B,C)

49. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then :-

(1) radius of S is 8

(B) radius of S is 7

(3) centre of S is (–7, 1)

(D) centre is S is (–8, 1)

Ans. (B,C)

50. Let $a \in R$ and let $f : R \to R$ be given by $f(x) = x^5 - 5x + a$. Then

(A) $f(x)$ has three real roots if $a > 4$

(B) $f(x)$ has only one real root if $a > 4$

(C) $f(x)$ has three real roots if $a < -4$

(D) $f(x)$ has three real roots if $-4 < a < 4$

Ans. (B,D)

SECTION–2 : (One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

51. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1,3) is

Ans. 8

52. Let $f : [0,4\pi] \to [0,\pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0,4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is

Ans. 3

53. The largest value of the non-negative integer a for which $\lim\limits_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ is

Ans. 0

54. Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if} \quad x \le 0, \\ \min\{f(x), g(x)\} & \text{if} \quad x > 0. \end{cases}$$

The number of points at which h(x) is not differentiable is

Ans. 3

55. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

Ans. 6

56. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. The the number of such distinct arrangements (n_1,n_2,n_3,n_4,n_5) is

Ans. 7

57. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx$ is

Ans. 2

58. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$.

If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$, where p,q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

Ans. 4

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59. Let a,b,c be positive integers such that $\frac{b}{a}$ is an integer. If a,b,c are in geometric progression and the arithmetic mean of a,b,c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is

Ans. 4

60. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Ans. 5