Reg. No. :

Question Paper Code : 53187

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2010

Fourth Semester

Computer Science and Engineering

MA 2262 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008)

(Normal tables be permitted in the examination Hall)

Time : Three hours | Maximum : 100 Marks

Answer ALL questions

PART A — (10 × 2 = 20 Marks)

1. If a random variable X has the distribution function $F(X) = \begin{cases} 1 - e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0, \end{cases}$ where α is the parameter, then find $P(1 \leq X \leq 2)$.

2. Every week the average number of wrong-number phone calls received by a certain mail order house is seven. What is the probability that they will receive two wrong calls tomorrow?

3. If there is no linear correlation between two random variables X and Y, then what can you say about the regression lines?

4. Let the joint pdf of the random variable (X, Y) be given by $f(x, y) = 4xye^{-(x^2+y^2)}$; $x > 0$ and $y > 0$. Are X and Y independent? Why or why not?

5. Examine whether the Poisson process $\{x(t)\}$ is stationary or not.

6. When is a Markov chain, called homogeneous?

7. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 mins between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 mins. Find the average number of persons waiting in the system.

8. Draw the state transition rate diagram of an M/M/c queueing model.

9. What do you mean by bottleneck of a network?

10. Consider a service facility with two sequential stations with respective service rates of 3/min and 4/min. The arrival rate is 2/min. What is the average service time of the system, if the system could be approximated by a two stage Tandem queue?

PART B — ($5 \times 16 = 80$ Marks)

11. (a) (i) The distribution function of a random variable X is given by $F(X) = 1-(1+x)e^{-x}$; $x \ge 0$. Find the density function, mean and variance of X. (8)

(ii) A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is 'p'. Find the value of 'p' so that the probability that an odd number of tosses required is equal to 0.6. Can you find a value of 'p' so that the probability is 0.5 that an odd number of tosses are required? (8)

Or

(b) (i) If X is a random variable with a continuous distribution function $F(X)$, prove that $Y = F(X)$ has a uniform distribution in $(0,1)$. Further if $f(X) = \begin{cases} \frac{1}{2}(x-1); & 1 \le x \le 3 \\ 0; & \text{otherwise} \end{cases}$, find the range of Y corresponding to the range $1.1 \le x \le 2.9$. (8)

(ii) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$.

What is the probability that the repair time exceeds $2h$? What is the conditional probability that a repair takes at least $10h$ given that its duration exceeds $9h$? (8)

12. (a) (i) Given $f(x, y) = cx(x-y)$, $0 < x < 2$, $-x < y < x$ and '0' elsewhere. Evaluate 'c' and find $f_X(x)$ and $f_Y(y)$. (8)

(ii) Compute the coefficient of correlation between X and Y using the following data : (8)

X :	1	3	5	7	8	10
Y :	8	12	15	17	18	20

Or

(b) (i) For two random variables X and Y with the same mean, the two regression equations are $y = ax + b$ and $x = cy + d$. Find the common mean, ratio of the standard deviations and also show that $\frac{b}{d} = \frac{1-a}{1-c}$. (8)

(ii) If $X_1, X_2, \ldots. X_n$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + \ldots + X_n$ and $n = 75$. (8)

13. (a) (i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2/min, find the probability that the interval between 2 consecutive arrivals is more than 1 min, between 1 and 2 mins, and 4 mins or less. (8)

(ii) An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follow a highly distorted signal, with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted. (8)

Or

(b) (i) Suppose that a mouse is moving inside the maze shown in the adjacent figure from one cell to another, in search of food. When at a cell, the mouse will move to one of the adjoining cells randomly. For $n \geq 0$, X_n be the cell number the mouse will visit after having changed cells 'n' times. Is $\{X_n; n = 0,1,....\}$ a Markov chain? If so, write its state space and transition probability matrix. (8)

1	4	7
2	5	8
3	6	9

(ii) The following is the transition probability matrix of a Markov chain with state space $\{0,1,2,3,4\}$. Specify the classes, and determine which classes are transient and which are recurrent. Give reasons. (8)

$$P = \begin{pmatrix} 2/5 & 0 & 0 & 3/5 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 0 & 1/3 & 0 & 2/3 \end{pmatrix}$$

14. (a) If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and it takes exactly 1.5 min to reach the correct seat after purchasing the ticket,

(i) Can he expect to be seated for the start of the picture?

(ii) What is the probability that he will be seated for the start of the picture?

(iii) How early must he arrive in order to be 99% sure of being seated for the start of the picture? (16)

Or

(b) There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour,

(i) What fraction of the time all the typists will be busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter has to spend for waiting and for being typed?

15. (a) Derive Pollaczek-Khinchin formula of M/G/1 queue. (16)

Or

(b) Write short notes on the following :

(i) Queue networks (4)

(ii) Series queues (4)

(iii) Open networks (4)

(iv) Closed networks. (4)
