## General Aptitude

## Q. No. 1 - 5 Carry One Mark Each

1. Find the missing sequence in the letter series below:

A, CD, GHI,?, UVWXY
(A) LMN
(B) MNO
(C) MNOP
(D) NOPQ

Answer: (C)
Exp:

2. Choose the correct verb to fill in the blank below:

Let us $\qquad$ _.
(A) Introvert
(B) alternate
(C) atheist
(D) altruist

## Answer: (B)

3. Choose the most appropriate word from the options given below to complete the following sentence?
If the athlete had wanted to come first in the race, he $\qquad$ several hours every day.
(A) Should practice
(B) Should have practised
(C) Practised
(D) Should be practicing

Answer: (B)
Exp: For condition regarding something which already happened, should have practiced is the correct choice.
4. Choose the most suitable one word substitute for the following expression Connotation of a road or way
(A) Pertinacious
(B) Viaticum
(C) Clandestine
(D) Ravenous

Answer: (A)
Exp: No word is relevant. Least irrelevant word is pertinacious.
5. If $x>y>I$, which of the following must be true?
(i) $\ln x>\ln y$
(ii) $\mathrm{e}^{\mathrm{x}}>\mathrm{e}^{\mathrm{y}}$
(iii) $\mathrm{y}^{\mathrm{x}}>\mathrm{x}^{\mathrm{y}}$
(iv) $\cos x>\cos y$
(A) (i) and (ii)
(B) (i) and (iii)
(C) (iii) and (iv)
(D) (ii) and (iv)

Answer: (A)
Exp: For whole numbers, greater the value greater will be its log. Same logic for power of e.

## Q. No. 6 - 10 Carry Two Marks Each

6. From a circular sheet of paper of radius 30 cm , a sector of $10 \%$ area is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is $\qquad$ _.
Answer: 13.08
Exp: $90 \%$ of area of sheet $=$ Cross sectional area of cone

$$
\begin{aligned}
& \Rightarrow 0.9 \times \pi \times 30 \times 30=\pi \times \mathrm{r}_{1} \times 30 \\
& \Rightarrow 27 \mathrm{~cm}=\mathrm{r}_{1} \\
& \begin{aligned}
\therefore \text { height of the cone } & =\sqrt{30^{2}-27^{2}} \\
& =13.08 \mathrm{~cm}
\end{aligned}
\end{aligned}
$$

7. In the following question, the first and the last sentence of the passage are in order and numbered 1 and 6 . The rest of the passage is split into 4 parts and numbered as $2,3,4$, and 5. These 4 parts are not arranged in proper order. Read the sentences and arrange them in a logical sequence to make a passage and choose the correct sequence from the given options.
8. One Diwali, the family rises early in the morning.
9. The whole family, including the young and the old enjoy doing this,
10. Children let off fireworks later in the night with their friends.
11. At sunset, the lamps are lit and the family performs various rituals
12. Father, mother, and children visit relatives and exchange gifts and sweets.
13. Houses look so pretty with lighted lamps all around.
(A) $2,5,3,4$
(B) 5, 2, 4, 3
(C) 3, 5, 4, 2
(D) 4, 5, 2, 3

Answer: (B)
8. Ms. X will be in Bagdogra from 01/05/2014 to 20/05/2014 and from 22/05/2014 to $31 / 05 / 2014$. On the morning of $21 / 05 / 204$, she will reach Kochi via Mumbai
Which one of the statements below is logically valid and can be inferred from the above sentences?
(A) Ms. X will be in Kochi for one day, only in May
(B) Ms. X will be in Kochi for only one day in May
(C) Ms. X will be only in Kochi for one day in May
(D) Only Ms. X will be in Kochi for one day in May.

Answer: (A)
Exp: Second sentence says that Ms. X reaches Kochi on 21/05/2014. Also she has to be in Bagdogora on 22/05/2014.
$\therefore \quad$ She stays in Kochi for only one day in may.
9. $\quad \log \tan 1^{\circ}+\log \tan 2^{\circ}+\ldots \ldots+\log \tan 89^{\circ}$ is $\qquad$
(A) 1
(B) $1 / \sqrt{2}$
(C) 0
(D) -1

Answer: (C)
Exp: $\quad \log \tan 1^{\circ}+\log \tan 89^{\circ}=\log \left(\tan 1^{\circ} \times \tan 89^{\circ}\right)$

$$
\begin{aligned}
& =\log \left(\tan 1^{\circ} \times \cot 1^{\circ}\right) \\
& =\log 1 \\
& =0
\end{aligned}
$$

Using the same logic total sum is ' 0 '.
10. Ram and Shyam shared a secret and promised to each other that it would remain between them. Ram expressed himself in one of the following ways as given in the choices below. Identify the correct way as per standard English.
(A) It would remain between you and me.
(B) It would remain between I and you
(C) It would remain between you and I
(D) It would remain with me.

Answer:
(A)


Q. No. 1-25 Carry One Mark Each

1. A coaxial cable is made of two brass conductors. The spacing between the conductors is filled with Teflon $\left(\varepsilon_{r}=2.1, \tan \delta=0\right)$. Which one of the following circuits can represent the lumped element model of a small piece of this cable having length $\Delta z$ ?


## Answer: (B)

Exp: Loss tangent $\tan \delta=0=\frac{\sigma}{\omega \in}$

$$
\sigma=0
$$

$\mathrm{G} \rightarrow$ Conductivity of the dielectric material
So, $\sigma=0=\mathrm{G}$
2. The phase margin (in degrees) of the system $G(s)=\frac{10}{s(s+10)}$ is $\qquad$ .

Answer: 84.32
3. In the circuit shown, diodes $D_{1}, D_{2}$ and $D_{3}$ are ideal, and the inputs $E_{1}, E_{2}$ and $E_{3}$ are ' 0 V ' for logic ' 0 ' and ' 10 V ' for logic ' 1 '. What logic gate does the circuit represent?

(A) 3 input OR gate
(B) 3 input NOR gate
(C) 3 input AND gate
(D) 3 input XOR gate

## Answer: (C)

Exp: Case (i) : If any input is logic 0 (i.e., 0 V ) then corresponding diode is "ON" and due to ideal diode output voltage $\mathrm{V}_{\mathrm{o}}=0$ as well as if there is any input logic 1 (i.e., 10V) corresponding diode will be OFF.
Case (ii) : If all the inputs are high (i.e., 10V) then all the diodes are R.B (OFF) and output voltage $\mathrm{V}_{\mathrm{o}}=10 \mathrm{~V}$.
So, it is a positive logic 3-inputs AND gate.
4. In the circuit shown in the figure, the BJT has a current gain $(\beta)$ of 50 . For an emitter base voltage $V_{E B}=600 \mathrm{mV}$, the emitter collector voltage $V_{E C}$ (in Volts) is $\qquad$ _.


Answer:
2.04

Exp: $\quad V_{E B}=0.7 \mathrm{~V}$
$\mathrm{I}_{\mathrm{B}}=0.0383 \mathrm{~mA}$
$\mathrm{I}_{\mathrm{C}}=1.916 \mathrm{~mA}$
$\mathrm{V}_{\mathrm{EC}}=3-\mathrm{I}_{\mathrm{C}} \mathrm{R}_{\mathrm{C}}=3-(1.916 \times 0.5)=2.04 \mathrm{~V}$
5. The contour on the $x-y$ plane, where the partial derivative of $x^{2}+y^{2}$ with respect to $y$ is equal to the partial derivative of $6 y+4 x$ with respect to $x$, is
(A) $y=2$
(B) $x=2$
(C) $x+y=4$
(D) $x-y=0$

Answer: (A)
Exp: The partial derivative of $x^{2}+y^{2}$ with respect to $y$ is $0+2 y \Rightarrow 2 y$.
The partial derivative of $6 y+4 x$ with respect $x$ is $0+4=4$.
Given that both are equal.

$$
\Rightarrow 2 \mathrm{y}=4 \Rightarrow \mathrm{y}=2
$$

6. 


(A) $\sec ^{2} x \quad$ (B) $\cos 4 x$ erin(C) 1 (A) UCOES (D) 0

## Answer:

> (A)

Exp: $\quad A=\left[\begin{array}{cc}1 & \operatorname{Tan} \mathrm{x} \\ -\operatorname{Tan} \mathrm{x} & 1\end{array}\right]$

$$
\begin{aligned}
& A^{T}=\left[\begin{array}{lr}
1 & -\operatorname{Tan} x \\
\operatorname{Tan} x & 1
\end{array}\right] \\
& |A|=1+\operatorname{Tan}^{2} x=\operatorname{Sec}^{2} x \\
& A^{-1}=\frac{1}{\sec ^{2} x}\left[\begin{array}{ll}
1 & -\operatorname{Tan} x \\
\operatorname{Tan} x & 1
\end{array}\right]
\end{aligned}
$$

$$
A^{T} A^{-1}=\left[\begin{array}{lc}
1 & -\operatorname{Tan} x \\
\operatorname{Tan} x & 1
\end{array}\right] \frac{1}{\sec ^{2} x}\left[\begin{array}{lc}
1 & -\operatorname{Tan} x \\
\operatorname{Tan} x & 1
\end{array}\right]
$$

$$
=\frac{1}{\sec ^{2} x}\left[\begin{array}{ll}
1-\operatorname{Tan}^{2} x & -\operatorname{Tan} x-\operatorname{Tan} x \\
\operatorname{Tax}+\operatorname{Tan} x & -\operatorname{Tan}^{2} x+1
\end{array}\right]
$$

$$
=\frac{1}{\sec ^{2} x}\left[\begin{array}{ll}
1-\operatorname{Tan}^{2} x & -2 \operatorname{Tan}^{2} x \\
2 \operatorname{Tan} x & 1-\operatorname{Tan}^{2} x
\end{array}\right]
$$

$$
\left|A^{T} A^{-1}\right|=\frac{1}{\sec ^{2} x}\left[\left(1-\operatorname{Tan}^{2} x\right)^{2}+4 \operatorname{Tan}^{2} x\right]
$$

$$
=\frac{1}{\sec ^{2}}\left[1+\operatorname{Tan}^{4} x-2 \operatorname{Tan}^{2} x+4 \operatorname{Tan}^{2} x\right]
$$

$$
\begin{aligned}
& =\frac{1}{\sec ^{2} x}\left[1+\operatorname{Tan}^{4} x+2 \operatorname{Tan}^{2} x\right] \\
& =\frac{1}{\sec ^{2} x}\left(1+\operatorname{Tan}^{2} x\right)^{2} \\
& =\frac{1}{\sec ^{2} x}\left(\sec ^{2} x\right)^{2}=\sec ^{2} x
\end{aligned}
$$

7. In the circuit shown, the voltage Vx (in Volts) is $\qquad$ .


Answer: 8
Exp:


Apply KCL at point P

$$
\begin{aligned}
& \frac{\mathrm{V}_{\mathrm{x}}}{20}+\frac{\mathrm{V}_{\mathrm{x}}-0.25 \mathrm{~V}_{\mathrm{x}}}{10}+0.5 \mathrm{~V}_{\mathrm{x}}=5 \\
& \mathrm{~V}_{\mathrm{x}}\left(\frac{1}{20}+\frac{0.75}{10}+0.5\right)=5 \\
& \mathrm{~V}_{\mathrm{x}}\left(\frac{5}{8}\right)=5 \Rightarrow \mathrm{~V}_{\mathrm{x}}=8 \mathrm{~V}
\end{aligned}
$$

8. Which one of the following 8085 microprocessor programs correctly calculates the product of two 8 -bit numbers stored in registers $B$ and $C$ ?(Options)
(A) MVI A, 00 H
JNZ LOOP
CMP C
LOOP DCR B

HLT
(C) MVI A, 00 H

LOOP ADD C
(B) MVI, A, 00H

CMP C
LOOP DCR B
JNZ LOOP
HLT
(D) MVI A, 00 H

ADD C

DCR B
JNZ LOOP
HLT

JNZ LOOP
LOOP INR B
HLT

## Answer: (C)

Exp: MVI A, $00 \mathrm{H} \leftarrow$ Load accumulator by 00 H
Loop: $\quad \mathrm{ADDC} \leftarrow$ Add the content of accumulator with content of C register and store result in accumulator. This will continue till B register reaches to 004.
DCRB
JNZ LOOP

## HLT

So, repetitive addition of a number as many times will give the product of these two numbers.
9. Consider the function $g(t)=e^{-t} \sin (2 \pi t) u(t)$ where $u(t)$ is the unit step function. The area under $g(t)$ is $\qquad$ —.
Answer: 0.155
$\begin{array}{ll}\text { Exp: } & \mathrm{g}(\mathrm{t})=\mathrm{e}^{-\mathrm{t}} \sin (2 \pi \mathrm{t}) \cdot \mathrm{u}(\mathrm{t}) \\ & \text { Let } \mathrm{y}(\mathrm{t})=\sin (2 \pi \mathrm{t}) \cdot \mathrm{u}(\mathrm{t}) \\ & \text { then } \mathrm{Y}(\mathrm{s})=\frac{2 \pi}{\mathrm{~s}^{2}+(2 \pi)^{2}}\end{array}$

$$
\begin{aligned}
& \mathrm{g}(\mathrm{t})=\mathrm{e}^{-\mathrm{t}} \mathrm{y}(\mathrm{t}) \\
& \Rightarrow \mathrm{G}(\mathrm{~s})=\mathrm{Y}(\mathrm{~s}+1) \\
& \mathrm{G}(\mathrm{~s})=\frac{2 \pi}{(\mathrm{~s}+1)^{2}+(2 \pi)^{2}} \\
& \mathrm{G}(\mathrm{~s})=\int_{-\infty}^{\infty} \mathrm{g}(\mathrm{t}) \cdot \mathrm{e}^{-\mathrm{st}} \mathrm{dt} \\
& \mathrm{G}(0)=\int_{-\infty}^{\infty} \mathrm{g}(\mathrm{t}) \cdot \mathrm{dt} \\
& \Rightarrow \text { Area under } \mathrm{g}(\mathrm{t})=\frac{2 \pi}{1+(2 \pi)^{2}}=\frac{6.28}{40.438}=0.155
\end{aligned}
$$

10. In the circuit shown using an ideal opamp, the $3-\mathrm{dB}$ cut-off frequency (in Hz ) is $\qquad$ -


Answer: 159.15

Exp: $\quad f_{3 \mathrm{~dB}}=\frac{1}{2 \pi \mathrm{RC}}$

$$
=\frac{1}{2 \pi \times 10 \times 10^{3} \times 0.1 \times 10^{-6}}=159.15 \mathrm{~Hz}
$$

11. The modulation scheme commonly used for transmission from GSM mobile terminals is
(A) 4-QAM
(B) 16-PSK
(C) Walsh-Hadamard orthogonal codes
(D) Gaussian Minimum Shift Keying (GMSK)

Answer: (D)
12. Which one of the following processes is preferred to from the gate dielectric $\left(\mathrm{SiO}_{2}\right)$ of MOSFETs?
(A) Sputtering
(B) Molecular beam epitaxy
(C) Wet oxidation
(D) Dry oxidation

Answer: (D)
13. Consider the Bode plot shown in figure. Assume that all the poles and zeros are real valued.

The value of $f_{H}-f_{L}($ in Hz$)$ is $\qquad$ .
Answer: 8970
Exp: $\quad 40=\frac{40-0}{\log _{10}(300)-\log _{10}\left(f_{L}\right)}$
$\log _{10}\left(\frac{300}{f_{L}}\right)=1$
$300=10 \mathrm{f}_{\mathrm{L}}$
$\mathrm{f}_{\mathrm{L}}=30 \mathrm{~Hz}$

$$
\begin{align*}
& -40=\frac{0-40}{\log _{10} \mathrm{f}_{\mathrm{H}}-\log (900)}  \tag{i}\\
& \log _{10}\left(\frac{\mathrm{f}_{\mathrm{H}}}{900}\right)=1 \\
& \mathrm{f}_{\mathrm{H}}=900 \times 10=9000 \quad \ldots \ldots \ldots . .  \tag{ii}\\
& \mathrm{f}_{\mathrm{H}}-\mathrm{f}_{\mathrm{L}}=9000-30=8970 \mathrm{~Hz}
\end{align*}
$$

14. In the circuit shown, assume that diodes $D_{1}$ and $D_{2}$ are ideal. In the steady-state condition the average voltage $V_{a b}$ (in Volts) across the $0.5 \mu \mathrm{~F}$ capacitor is $\qquad$ -.


Answer: 100
15. The transfer function of a first order controller is given as $G_{C}(s)=\frac{K(s+a)}{s+b}$ where, $K, a$ and $b$ are positive real numbers. The condition for this controller to act as a phase lead compensator is
(A) $a<b$
(B) $a>b$
(C) $\mathrm{K}<\mathrm{ab}$
(D) $\mathrm{K}>\mathrm{ab}$

Answer: (A)

16. A message signal $m(t)=A_{m} \sin \left(2 \pi f_{m} t\right)$ is used to modulate the phase of a carrier $A_{c}$ $\cos \left(2 \pi f_{c} t\right)$ to get the modulated signal $y(t)=A_{c} \cos \left(2 \pi f_{c} t+m(t)\right)$. The bandwidth of $y(t)$
(A) depends on $A_{m}$ but not on $f_{m}$
(B) depends on $f_{m}$ but not on $A_{m}$
(C) depends on both $A_{m}$ and $f_{m}$
(D) does not depends on $A_{m}$ or $f_{m}$

## Answer: (C)

Exp: $\quad y(t)=A_{c} \cos \left[2 \pi f_{c} t+m(t)\right]$
$\mathrm{m}(\mathrm{t})=\mathrm{A}_{\mathrm{m}} \sin \left(2 \pi \mathrm{f}_{\mathrm{m}} \mathrm{t}\right)$
Since $y(t)$ is phase modulated signal,
$\phi(\mathrm{t})=2 \pi \mathrm{f}_{\mathrm{c}} \mathrm{t}+\mathrm{m}(\mathrm{t})$
Bandwidth $=2\left[\Delta \mathrm{f}+\mathrm{f}_{\mathrm{m}}\right]$
$\Delta \mathrm{f}=\frac{1}{2 \pi} \frac{\mathrm{~d}}{\mathrm{dt}} \mathrm{m}(\mathrm{t})$
$\Rightarrow \Delta f$ depends on $A_{m}$ as well as $f_{m}$. Thus Bandwidth depends on both $A_{m}$ and $f_{m}$.
17. The directivity of an antenna array can be increased by adding more antenna elements, as a larger number of elements
(A) improves the radiation efficiency
(B) increases the effective area of the antenna
(C) results in a better impedance matching
(D) allows more power to be transmitted by the antenna

Answer: (B)
Exp: $D=\frac{4 \pi}{\lambda^{2}} \mathrm{Ae}$
$\mathrm{D} \uparrow \rightarrow \mathrm{Ae} \uparrow$
18. For the circuit shown in the figure, the Thevenin equivalent voltage (in Volts) across terminals $a-b$ is $\qquad$ .


Answer: 10
Exp:


Apply nodal equation at point P
$\mathrm{V}_{+\mathrm{n}}\left(\frac{1}{3}+\frac{1}{6}\right)=\frac{12}{3}+1$
$\frac{\mathrm{V}_{+\mathrm{n}}}{2}=5 \Rightarrow \mathrm{~V}_{+\mathrm{n}}=10 \mathrm{~V}$
19. The impulse response of an LTI system can be obtained by
(A) differentiating the unit ramp response
(B) differentiating the unit step response
(C) integrating the unit ramp response
(D) integrating the unit step response

Answer: (B)
Exp: Let $\mathrm{h}(\mathrm{t})$ be the impulse response of the system

$y(t)$ is unit step response of the system

$$
\mathrm{y}(\mathrm{t})=\int_{-\infty}^{\mathrm{t}} \mathrm{~h}(\tau) \mathrm{d} \tau
$$

If we need to get $\mathrm{h}(\mathrm{t})$, then we have to differentiate $\mathrm{y}(\mathrm{t})$.
Thus differentiating the unit-step response gives impulse response for LTI system.
20. Consider a four point moving average filter defined by the equation $y[n]=\sum_{i=0}^{5} \alpha_{i}[n-i]$ The condition on the filter coefficients that results in a null at zero frequency is
(A) $\alpha_{1}=\alpha_{2}=0 ; \alpha_{0}=-\alpha_{3}$
(B) $\alpha_{1}=\alpha_{2}=1 ; \alpha_{0}=-\alpha_{3}$
(C) $\alpha_{0}=\alpha_{3}=0 ; \alpha_{1}=\alpha_{2}$
(D) $\alpha_{1}=\alpha_{2}=0 ; \alpha_{0}=\alpha_{3}$

## Answer: (A)

Exp: Given $y[n]=\sum_{i=0}^{3} \alpha_{i} x(n-i)$
$\Rightarrow \mathrm{y}[\mathrm{n}]=\alpha_{0} \mathrm{x}[\mathrm{n}]+\alpha_{1} \mathrm{x}[\mathrm{n}-1]+\alpha_{2} \mathrm{x}[\mathrm{n}-2]+\alpha_{3} \mathrm{x}[\mathrm{n}-3]$
Getting a null at zero frequency implies that given filter can be high pass filter but it cannot be low pass filter.
High pass filter is possible if we have negative coefficients.
Let say, $\alpha_{1}=\alpha_{2}=0, \alpha_{0}=-\alpha_{3}$
$\Rightarrow \mathrm{y}[\mathrm{n}]=-\alpha_{3} \mathrm{x}[\mathrm{n}]+\alpha_{3} \mathrm{x}[\mathrm{n}-3]$
$H(z)=-\alpha_{3}\left[1-z^{-3}\right]$
$\Rightarrow \mathrm{H}\left(\mathrm{e}^{\mathrm{j} \Omega}\right)=-\alpha_{3}\left[1-\mathrm{e}^{-\mathrm{j} 3 \Omega}\right]$

$=-\alpha_{3} 2 \cdot \sin \frac{3 \Omega}{2} \cdot e^{-j \frac{3 \Omega}{2}} \cdot e^{j \frac{\pi}{2}}$
$\left.\Rightarrow \mathrm{H}\left(\mathrm{e}^{\mathrm{j} \Omega}\right)\right|_{\Omega=0}=0$
In other cases it in not possible.
21. If $C$ is a circle of radius $r$ with centre $z_{0}$, in the complex z-plane and if $n$ is a non-zero integer, then $\oint \frac{d z}{\left(z-z_{0}\right)^{n+1}}$ equals
(A) $2 \pi n j$
(B) 0
(C) $\frac{n j}{2 \pi}$
(D) $2 \pi n$

Answer: (B)
Exp: By Cauchy's Integral formula,

$$
\begin{aligned}
& \oint_{c} \frac{\mathrm{f}(\mathrm{z})}{\left(\mathrm{z}-\mathrm{z}_{0}\right)^{\mathrm{n+1}}} \mathrm{dz}=\frac{2 \pi \mathrm{i} . \mathrm{f}^{\mathrm{n}}\left(\mathrm{z}_{0}\right)}{\mathrm{n}!} \\
& \oint_{\mathrm{c}} \frac{\mathrm{dz}}{\left(\mathrm{z}-\mathrm{z}_{0}\right)^{\mathrm{n}+1}}=\frac{2 \pi \mathrm{i}}{\mathrm{n}!} \times 0=0 \quad\binom{\because \mathrm{f}(\mathrm{z})=1}{\mathrm{f}^{\mathrm{n}}(\mathrm{z})=0 \text { at any } \mathrm{z}_{0}}
\end{aligned}
$$

22. At very high frequencies, the peak output voltage $\mathrm{V}_{0}$ (in Volts) is $\qquad$ .


Answer: 0.5
Exp: For capacitor $Z_{C}=\frac{1}{j \omega C}$
Very high frequency means $\omega \rightarrow \infty \Rightarrow \mathrm{Z}_{\mathrm{C}} \rightarrow 0$
So, all capacitors are replaced by short circuit.


By voltage division, $\mathrm{V}_{0}=\frac{\mathrm{V}_{\mathrm{i}}}{2}$
$\mathrm{V}_{0}=\frac{1 \cdot \sin \omega \mathrm{t}}{2}=0.5 \sin \omega \mathrm{t}$
Thus, Peak voltage $=0.5$
23. If the base width in a bipolar junction transistor is doubled, which one of the following statements will be TRUE?
(A) Current gain will increase
(B) Unity gain frequency will increase
(C) Emitter base junction capacitance will increase
(D) Early voltage will increase

Answer: (D)
Exp: $\quad W_{B}$ doubled (increased) $\rightarrow$ early effect is still present but its effect less severe relative to previous $\mathrm{W}_{\mathrm{B}}$. Slope $\mathrm{I}_{\mathrm{C}} \mathrm{Vs} \mathrm{V}_{\mathrm{CE}}$ decreases
24. The value of $\sum_{n-0}^{\infty} n\left(\frac{1}{2}\right)^{n}$ is $\qquad$ -.

Answer: 2
Exp: Given that $\sum_{\mathrm{n}=0}^{\infty} \mathrm{n}\left(\frac{1}{2}\right)^{\mathrm{n}}$
$=0+1 \cdot \frac{1}{2}+2\left(\frac{1}{2}\right)^{2}+3\left(\frac{1}{2}\right)^{3}+4\left(\frac{1}{2}\right)^{4}+\ldots \ldots$
$=\frac{1}{2}\left[1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^{2}+4\left(\frac{1}{2}\right)^{3}+\ldots \ldots \ldots\right]$
$=\frac{1}{2}\left[1-\frac{1}{2}\right]^{-2} \quad\left(\because(1-\mathrm{x})^{-2}=1+2 \mathrm{x}+3 \mathrm{x}^{2}+4 \mathrm{x}^{3}+\ldots.\right)$
$=\frac{1}{2}\left(\frac{1}{2}\right)^{-2}=\left(\frac{1}{2}\right)^{-1}=2$
25. The circuit shown consists of J-K flip-flops, each with an active low asynchronous reset ( $\overline{\mathrm{R}_{\mathrm{d}}}$ input). The counter corresponding to this circuit is

(A) a modulo- 5 binary up counter
(B) a modulo-6 binary down counter
(C) a modulo- 5 binary down counter
(D) a modulo-6 binary up counter

## Answer: (A)

Exp: Analysis:

1. Clock is taken from normal output and it is -ve edge triggering. So, it is UP-counter.
2. Input of the NAND-gate is taken from $Q_{2}$ and $Q_{0}$. So $Q_{2}=1$ and $Q_{0}=1$.
3. To find the modulus

$$
\left(\mathrm{Q}_{2}, \mathrm{Q}_{1}, \mathrm{Q}_{0}\right)=\left(\begin{array}{lll}
1 & 0 & 1
\end{array}\right)
$$

So, it is MOD - 5 binary UP-counter.

## Q. No. 26 - 55 Carry Two Marks Each

26. A 200 m long transmission line having parameters shown in the figure is terminated into a load $\mathrm{R}_{\mathrm{L}}$. The line is connected to a 400 V source having source resistance $\mathrm{R}_{\mathrm{S}}$ through a

[^0]switch which is closed at $t=0$. The transient response of the circuit at the input of the line $(z=0)$ is also drawn in the figure. The value of $\mathrm{R}_{\mathrm{L}}($ in $\Omega)$ is $\qquad$


## Answer: 30

Exp:


Given $\mathrm{V}(\mathrm{t}=2 \mu \mathrm{~s}, \mathrm{Z}=0)=62.5$

$$
\begin{gathered}
62.5=\mathrm{V}(\mathrm{t}=0, \mathrm{z}=0)+\mathrm{V}(\mathrm{t}=1, \mathrm{z}=0)+\mathrm{V}(\mathrm{t}=2, \mathrm{z}=0) \\
62.5=100+\sqrt{\mathrm{R}}(100)+\sqrt{\mathrm{R}} \mid \mathrm{S}(100) \\
\sqrt{\mathrm{R}}=\frac{\mathrm{R}_{\mathrm{L}}-50}{\mathrm{R}_{\mathrm{L}}+50}, \overline{\mathrm{~S}}=\frac{1}{2} \\
\mathrm{So}, \sqrt{\mathrm{R}}=\frac{-1}{4}, \\
\mathrm{R}_{\mathrm{L}}=30 \Omega
\end{gathered}
$$

27. A coaxial capacitor of inner radius 1 mm and outer radius 5 mm has a capacitance per unit length of $172 \mathrm{pF} / \mathrm{m}$. If the ratio of outer radius to inner is doubled, the capacitance per unit length (in $\mathrm{pF} / \mathrm{m}$ ) is $\qquad$ .
Answer: 120.22
$\operatorname{Exp}: C=\frac{2 \pi \in l}{\ln (\mathrm{~b} / \mathrm{a})}$
$\frac{\mathrm{C}}{1}=\frac{2 \pi \epsilon}{\ln (\mathrm{~b} / \mathrm{a})}=\mathrm{C}_{1}$
$\frac{\mathrm{C}_{1}}{\mathrm{C}_{2}}=\frac{\ln \left(\mathrm{b}_{2} / \mathrm{a}_{2}\right)}{\ln \left(\mathrm{b}_{1} / \mathrm{a}_{1}\right)}$
$\frac{172 \mathrm{pF}}{\mathrm{C}_{2}}=\frac{\ln (10 / 1)}{\ln (5)}$
$\mathrm{C}_{2}=\frac{\ln (5)}{\ln (10)} 172 \mathrm{pF}$
$\mathrm{C}_{2}=120.22 \mathrm{pF}$
28. A universal logic gate can implement any Boolean function by connecting sufficient number of them appropriately. Three gates are shown.


Which one of the following statements is TRUE?
(A) Gate 1 is a universal gate.
(B) Gate 2 is a universal gate.
(C) Gate 3 is a universal gate.
(D) None of the gates shown is a universal gate.

Answer:
Exp: Only NAND and NOR are universal gate, but in the question other gates are mentioned.
29. The Newton-Raphson method is used to solve the equation $f(x)=x^{3}-5 x^{2}+6 x-8=0$.

Taking the initial guess as $x=5$, the solution obtained at the end of the first iteration is $\qquad$
Answer: 4.2903
Exp: $\quad f(x)=x^{3}-5 x^{2}+6 x-8$
$\mathrm{x}_{0}=5$
$f^{\prime}(x)=3 x^{2}-10 x+6$
By Newton-Raphson method.

$$
\begin{aligned}
\mathrm{x}_{1}=\mathrm{x}_{0}-\frac{\mathrm{f}\left(\mathrm{x}_{0}\right)}{\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right)} & =5-\frac{\mathrm{f}(5)}{\mathrm{f}^{\prime}(5)} \\
& =5-\frac{22}{31} \\
& =5-0.7097 \\
& =4.2903
\end{aligned}
$$

30. A random binary wave $y(t)$ is given by $\mathrm{y}(\mathrm{t})=\sum_{\mathrm{n}=-\infty}^{\infty} \mathrm{X}_{\mathrm{n}} \mathrm{p}(\mathrm{t}-\mathrm{nT}-\phi)$.
where $p(t)=u(t)-u(t-T), u(t)$ is the unit step function and $\varphi$ is an independent random variable with uniform distribution in $[0, T]$. The sequence $\left\{\mathrm{X}_{\mathrm{n}}\right\}$ consists of independent and identically distributed binary valued random variables with $\mathrm{P}\left\{\mathrm{X}_{\mathrm{n}}=+1\right\}=\mathrm{P}\left\{\mathrm{X}_{\mathrm{n}}=-1\right\}$ $=0.5$ for each n .
The value of the auto correlation $R_{y y}\left(\frac{3 T}{4}\right) \triangleq E\left[y(t) y\left(t-\frac{3 T}{4}\right)\right]$ equals $\qquad$ .
Answer: 0.25
Exp: $\quad y(t)=\sum_{n=-\infty}^{\infty} X_{n} P(t-n T-\phi)$

$$
R_{y y(z)}=\left[1-\frac{|\tau|}{T}\right]
$$

Derivation of above autocorrelation function can be found in any book dealing with random process.
[B.P. Lathi, Simon, Haykin, Schaum series].

$$
\mathrm{R}_{\mathrm{yy}}\left(\frac{3 \mathrm{~T}}{4}\right)=\left[1-\frac{3 \pi / 4}{\pi}\right]
$$


31. A three bit pseudo random number generator is shown. Initially the value of output $\mathrm{Y}=$ $\mathrm{Y}_{2} \mathrm{Y}_{1} \mathrm{Y}_{0}$ is set to 111. The value of output Y after three clock cycles is

(A) 000
(B) 001
(C) 010
(D) 100

Answer: (D)
Exp:

| $\mathrm{D}_{2}\left(\mathrm{Q}_{1} \oplus \mathrm{Q}_{0}\right)$ | $\mathrm{D}_{1}\left(\mathrm{Q}_{2}\right)$ | $\mathrm{D}_{0}\left(\mathrm{Q}_{1}\right)$ | $\mathrm{Y}_{2}\left(\mathrm{Q}_{2}\right)$ | $\mathrm{Q}_{1}\left(\mathrm{Y}_{1}\right)$ | $\mathrm{Q}_{0} \mathrm{Y}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| [ - | - | - | 1 | 1 | 1 |
| $1^{\text {st }}$ clock $\quad \square 0$ | 1 | 1 | 0 | 1 | 1 |
| $2^{\text {nd }}$ clock $\xrightarrow{\square} 0$ | 0 | 1 | 0 | 0 | 1 |
| $3^{\text {rd }}$ clock $\longrightarrow 1$ | 0 | 0 | 1 | 0 | 0 |

After three clock pulses output $\mathrm{Y}_{2} \mathrm{Y}_{1} \mathrm{Y}_{0}=100$
32. In the circuit shown, assume that the opamp is ideal. If the gain $\left(\mathrm{V}_{\mathrm{o}} / \mathrm{V}_{\text {in }}\right)$ is- 12, the value of $R($ in $\mathrm{k} \Omega)$ is $\qquad$


## Answer:

Exp:


Again apply nodal analysis at node $\mathrm{V}_{\mathrm{x}}$
$\frac{0-\mathrm{V}_{\mathrm{x}}}{10 \mathrm{k}}=\frac{\mathrm{V}_{\mathrm{x}}-0}{\mathrm{R}}+\frac{\mathrm{V}_{\mathrm{x}}-\mathrm{V}_{0}}{10 \mathrm{k}}$ $\qquad$
Put the value of $V_{x}$ from equation (1) in equation (2) we get

$$
\mathrm{R}=1 \mathrm{k} \Omega
$$

33. Two sequences $x_{1}[n]$ and $x_{2}[n]$ have the same energy. Suppose $x_{1}[n]=\alpha 0.5^{n} u[n]$, where $a$ is a positive real number and $u[n]$ is the unit step sequence. Assume

$$
\mathrm{x}_{2}[\mathrm{n}]=\left\{\begin{array}{c}
\sqrt{1.5} \text { for } \mathrm{n}=0,1 \\
0 \text { otherwise }
\end{array}\right.
$$

Then the value of $a$ is $\qquad$ .

Answer:
2.25

Exp: $\quad x_{1}[n]=\alpha(0.5)^{n} u[n]$
$\mathrm{x}_{2}[\mathrm{n}]=\{\sqrt{1.5}, \sqrt{1.5}, 0, \ldots .$.
$\uparrow$

Energy of $x_{1}[n]=\sum_{n=0}^{\infty} x_{1}^{2}[n]$

$$
\begin{aligned}
& =\sum_{n=0}^{\infty} \alpha \cdot(0.5)^{2 n} \\
& =\alpha\left[1+\frac{1}{4}+\frac{1}{16}+\ldots\right] \\
& =\alpha\left[\frac{1}{1-\frac{1}{4}}\right]=\alpha \frac{4}{3}
\end{aligned}
$$

Energy of $\mathrm{x}_{2}[\mathrm{n}]=(\sqrt{1.5})^{2}+(\sqrt{1.5})^{2}=3$
It is given that energy of $x_{1}[n]$ is same as energy of $x_{2}[n]$.
$\Rightarrow \alpha \cdot \frac{4}{3}=3$
$\Rightarrow \alpha=\frac{9}{4}=2.25$
34. The ABCD parameters of the following 2-port network are

(A) $\left[\begin{array}{cc}3.5+\mathrm{j} 2 & 20.5 \\ 20.5 & 3.5-\mathrm{j} 2\end{array}\right]$
(B) $\left[\begin{array}{cc}3.5+\mathrm{j} 2 & 0.5 \\ 0.5 & 3.5-\mathrm{j} 2\end{array}\right]$
(C) $\left[\begin{array}{cc}10 & 2+\mathrm{j} 0 \\ 2+\mathrm{j} 0 & 10\end{array}\right]$
(D) $\left[\begin{array}{cc}7+\mathrm{j} 4 & 0.5 \\ 30.5 & 7-\mathrm{j} 4\end{array}\right]$

Answer: (B)
Exp: For the standard ' $T$ ' network, obtain the Z-matrix first and then convert it into T-matrix

$$
\begin{aligned}
Z & =\left[\begin{array}{cc}
7+\mathrm{j} 4 & 2 \\
2 & 7-\mathrm{j} 4
\end{array}\right] \\
\Delta \mathrm{Z} & =[(7+\mathrm{j} 4)(7-\mathrm{j} 4)]-4 \\
& =49+16-4=61 \\
A & =\frac{Z_{11}}{\mathrm{Z}_{21}}=\frac{7+\mathrm{j} 4}{2}=3.5+\mathrm{j} 2 \\
B & =\frac{\Delta Z}{Z_{21}}=\frac{61}{2}=30.5 \\
C & =\frac{1}{Z_{21}}=\frac{1}{2}=0.5
\end{aligned}
$$

$D=\frac{Z_{22}}{Z_{21}}=\frac{7-j 4}{2}=3.5-j 2$
$\mathrm{T}=\left[\begin{array}{cc}3.5+\mathrm{j} 2 & 30.5 \\ 0.5 & 3.5-\mathrm{j} 2\end{array}\right]$
35. A network is described by the state model as

$$
\begin{aligned}
& \dot{x}_{1}=2 x_{1}-x_{2}+3 u \\
& \dot{x}_{2}=-4 x_{2}-u \\
& y=3 x_{1}-2 x_{2}
\end{aligned}
$$

The transfer function $H(s)\left(=\frac{Y(s)}{U(s)}\right)$ is
(A) $\frac{11 \mathrm{~s}+35}{(\mathrm{~s}-2)(\mathrm{s}+4)}$
(B) $\frac{11 s-35}{(s-2)(s+4)}$
(C) $\frac{11 s+38}{(s-2)(s+4)}$
(D) $\frac{11 s-38}{(s-2)(s+4)}$

Answer: (A)
Exp:

$$
\begin{aligned}
& \mathrm{A}=\left[\begin{array}{ll}
2 & -1 \\
0 & -4
\end{array}\right] \mathrm{B}=\left[\begin{array}{c}
3 \\
-1
\end{array}\right] \\
& \mathrm{C}=\left[\begin{array}{ll}
3 & -2
\end{array}\right] \\
& \mathrm{H}(\mathrm{~s})=\mathrm{C}(\mathrm{sI}-\mathrm{A})^{-1} \mathrm{~B} \\
& =\left[\begin{array}{ll}
3 & -2
\end{array}\right]\left[\begin{array}{cc}
\mathrm{s}-2 & 1 \\
0 & \mathrm{~s}+4
\end{array}\right]\left[\begin{array}{c}
3 \\
-1
\end{array}\right] \\
& =\left[\begin{array}{ll}
3 & -2
\end{array}\right] \frac{1}{(\mathrm{~s}-2)(\mathrm{s}+4)}\left[\begin{array}{cc}
\mathrm{s}+4 & -1 \\
0 & \mathrm{~s}-2
\end{array}\right]\left[\begin{array}{c}
3 \\
-1
\end{array}\right] \\
& =\left[\begin{array}{ll}
3 & -2
\end{array}\right] \frac{1}{\mathrm{~s}^{2}+2 \mathrm{~s}-8}\left[\begin{array}{c}
3 \mathrm{~s}+12+1 \\
-\mathrm{s}+2
\end{array}\right] \\
& =\frac{95+39+25-4}{(\mathrm{~s}-2)(\mathrm{s}+4)} \\
& =\frac{11 \mathrm{~s}+35}{(\mathrm{~s}-2)(\mathrm{s}+4)}
\end{aligned}
$$

36. In the circuit shown, the current $I$ flowing through the $50 \Omega$ resistor will be zero if the value of capacitor $C$ (in $\mu \mathrm{F}$ ) is $\qquad$ -


## Answer: 20

Exp: Convert the network into phasor domain


If $\mathrm{I}=0, \mathrm{~V}_{\mathrm{p}}=\mathrm{V}_{\mathrm{i}}$,
$V_{i}\left(\frac{1}{j 5}+\frac{1}{j\left(5-\frac{1}{5000 C}\right)}\right)=0$
$\Rightarrow \frac{1}{j 5}=\frac{-1}{j\left(5-\frac{1}{5000 C}\right)}=0$
$\frac{1}{5}=\frac{1}{\frac{1}{5000 \mathrm{C}}-5}$
$\Rightarrow 5=\frac{1}{5000 \mathrm{C}}-5$
$\Rightarrow \frac{1}{5000 \mathrm{C}}=10$
$\Rightarrow \mathrm{C}=\frac{1}{5 \times 10^{4}}=20 \mu \mathrm{~F}$
37. A realization of a stable discrete time system is shown in figure. If the system is excited by a unit step sequence input $x[n]$, the response $y[n]$ is

(A) $4\left(-\frac{1}{3}\right)^{\mathrm{n}} \mathrm{u}(\mathrm{n})-5\left(-\frac{2}{3}\right)^{\mathrm{n}} \mathrm{u}[\mathrm{n}]$
(B) $5\left(-\frac{2}{3}\right)^{\mathrm{n}} \mathrm{u}(\mathrm{n})-3\left(-\frac{1}{3}\right)^{\mathrm{n}} \mathrm{u}[\mathrm{n}]$
(C) $5\left(\frac{1}{3}\right)^{\mathrm{n}} \mathrm{u}(\mathrm{n})-5\left(\frac{2}{3}\right)^{\mathrm{n}} \mathrm{u}[\mathrm{n}]$
(D) $5\left(\frac{2}{3}\right)^{\mathrm{n}} \mathrm{u}(\mathrm{n})-5\left(\frac{1}{3}\right)^{\mathrm{n}} \mathrm{u}[\mathrm{n}]$

Answer: (C)
Exp: $\quad$ [n]


From the graph

$$
\begin{aligned}
& \mathrm{v}[\mathrm{n}]=\mathrm{x}[\mathrm{n}]+\mathrm{v}[\mathrm{n}-1]-\frac{2}{9} \mathrm{v}[\mathrm{n}-2] \\
& \mathrm{y}[\mathrm{n}]=-\frac{5}{3} \mathrm{v}[\mathrm{n}-1]+\frac{5}{3} \mathrm{v}[\mathrm{n}-2] \\
& \mathrm{V}(\mathrm{z})=\left[1-\mathrm{z}^{-1}+\frac{2}{9} \mathrm{z}^{-2}\right]=\mathrm{X}(\mathrm{z}) \\
& \Rightarrow \frac{\mathrm{V}(\mathrm{z})}{\mathrm{X}(\mathrm{z})}=\frac{1}{1-\mathrm{z}^{-1}+\frac{2}{9} \mathrm{z}^{-2}} \rightarrow(\mathrm{H} \\
& \frac{\mathrm{Y}(\mathrm{z})}{\mathrm{V}(\mathrm{z})}=\frac{-5}{3} \mathrm{z}^{-1}+\frac{5}{3} \mathrm{z}^{-2} \rightarrow(2)
\end{aligned}
$$

Multiplying (1) and (2) we get

$$
\Rightarrow \frac{\mathrm{Y}(\mathrm{z})}{\mathrm{X}(\mathrm{z})}=\frac{-\frac{5}{3} \mathrm{z}^{-1}\left[1-\mathrm{z}^{-1}\right]}{1-\mathrm{z}^{-1}+\frac{2}{9} \mathrm{z}^{-2}}
$$

For unit step response, $\mathrm{X}(\mathrm{z})=\frac{1}{1-\mathrm{z}^{-1}}$

$$
\begin{aligned}
\Rightarrow Y(z) & =\frac{-\frac{5}{3} z^{-1}}{1-z^{-1}+\frac{2}{9} z^{-2}} \\
& =\frac{A}{1-\frac{1}{3} z^{-1}}+\frac{B}{1-\frac{2}{3} z^{-1}}
\end{aligned}
$$

On solving,
$\mathrm{A}=5$; $\mathrm{B}=-5$
$\Rightarrow y[n]=5\left(\frac{1}{3}\right)^{\mathrm{n}} \mathrm{u}[\mathrm{n}]-5\left(\frac{2}{3}\right)^{\mathrm{n}} \mathrm{u}[\mathrm{n}]$
38. The complex envelope of the bandpass signal $x(t)=\sqrt{2}\left(\frac{\sin (\pi t / 5)}{\pi t / 5}\right) \sin \left(\pi t-\frac{\pi}{4}\right)$, centered about $\mathrm{f}=\frac{1}{2} \mathrm{~Hz}$, is
(A) $\left(\frac{\sin (\pi t / 5)}{\pi t / 5}\right) \mathrm{e}^{\mathrm{j} \frac{\pi}{4}}$
(B) $\left(\frac{\sin (\pi \mathrm{t} / 5)}{\pi \mathrm{t} / 5}\right) \mathrm{e}^{-\mathrm{j} \frac{\pi}{4}}$
(C) $\sqrt{2}\left(\frac{\sin (\pi \mathrm{t} / 5)}{\pi \mathrm{t} / 5}\right) \mathrm{e}^{\mathrm{j} \frac{\pi}{4}}$
(D) $\sqrt{2}\left(\frac{\sin (\pi \mathrm{t} / 5)}{\pi \mathrm{t} / 5}\right) \mathrm{e}^{\mathrm{j} \frac{\pi}{4}}$

Answer: (C)
Exp: $\quad \mathrm{x}(\mathrm{t})=-\sqrt{2}\left(\frac{\sin (\pi \mathrm{t} / 5)}{\pi \mathrm{t} / 5}\right) \sin \left(\pi \mathrm{t}-\frac{\pi}{4}\right)$
we can write above expression as

[Low pass representation of Bandpass signals]
$\mathrm{X}_{\mathrm{c}}(\mathrm{t})=\frac{\sin (\pi \mathrm{t} / 5)}{(\pi \mathrm{t} / 5)}, \mathrm{X}_{\mathrm{s}}(\mathrm{t})=\frac{\sin (\pi \mathrm{t} / 5)}{(\pi \mathrm{t} / 5)}$
$x_{c e}(t)$ is the complex envelope of $x(t)$

$$
\begin{aligned}
\mathrm{x}_{\mathrm{ce}}(\mathrm{t}) & =\mathrm{x}_{\mathrm{c}}(\mathrm{t})+j \mathrm{x}_{\mathrm{s}}(\mathrm{t}) \\
& =\frac{\sin (\pi \mathrm{t} / 5)}{\pi \mathrm{t} / 5}[1+\mathrm{j}]=\frac{\sqrt{2} \sin (\pi \mathrm{t} / 5)}{(\pi \mathrm{t} / 5)} \mathrm{e}^{\mathrm{j} \pi / 4}
\end{aligned}
$$

39. In the circuit shown, assume that the diodes $D_{1}$ and $D_{2}$ are ideal. The average value of voltage $V_{a b}$ (in Volts), across terminals ' $a$ ' and ' $b$ ' is $\qquad$ .


## Answer: 5

40. Consider the differential equation

$$
\frac{\mathrm{d}^{2} \mathrm{x}(\mathrm{t})}{\mathrm{dt}^{2}}+3 \frac{\mathrm{dx}(\mathrm{t})}{\mathrm{dt}}+2 \mathrm{x}(\mathrm{t})=0
$$

Given $x(0)=20$ and $x(1)=10 / e$, where $e=2.71$. the value of $x(2)$ is $\qquad$
Answer: 0.8556
Exp: Given $\frac{\mathrm{d}^{2} \mathrm{x}(\mathrm{t})}{\mathrm{dt}^{2}}+\frac{3 \mathrm{dx}(\mathrm{t})}{\mathrm{dt}}+2 \mathrm{x}(\mathrm{t})=0$

$$
\begin{aligned}
& x(0)=20 \\
& x(1)=\frac{10}{e} \\
& x(2)=----
\end{aligned}
$$

Auxillary equation is

$$
\begin{aligned}
& m^{2}+3 m+2=0 \\
& m=-1,-2
\end{aligned}
$$

Complementary solution $x_{e}=c_{1} e^{-t}+c_{2} e^{-2 t}$


From (a) $c_{1}=20-c_{2}$
Now $10=\left(20-c_{2}\right)+\mathrm{c}_{2} \mathrm{e}^{-1}$
$10=\mathrm{c}_{2}\left(\mathrm{e}^{-1}-1\right)+20$
$c_{2}=\frac{10-20}{\mathrm{e}^{-1}-1}=-\frac{10}{\mathrm{e}^{-1}-1}=\frac{10}{1-\mathrm{e}^{-1}}=\frac{10 \mathrm{e}}{\mathrm{e}-1}$
$\Rightarrow c_{1}=20-\frac{10 e}{e-1}=\frac{20 e-20-10 e}{e-1}=\frac{10 e-20}{e-1}$
$x(t)=\frac{10 e-20}{e-1} e^{-t}+\frac{10 e}{e-1} e^{-2 t}$
$x(2)=\left(\frac{10 e-20}{e-1}\right) e^{-2}+\left(\frac{10 e}{e-1}\right) e^{-4}=0.8556$
41. Let $\tilde{\mathrm{x}}[\mathrm{n}]=1+\cos \left(\frac{\pi n}{8}\right)$ be periodic signal with period 16. Its DFS coefficient are defined by $\alpha_{k}=\frac{1}{16} \sum_{x=0}^{15} \tilde{\mathrm{x}}[\mathrm{n}] \exp \left(-\mathrm{j} \frac{\pi}{8} \mathrm{kn}\right)$ for all $k$. The value of the coefficient $\alpha_{31}$ is $\qquad$ .
Answer: 0.5

Exp: $\quad x[n]=1+\cos \left(\frac{\pi}{8} n\right)$
$\mathrm{N}=16$
$x[n]=1+\frac{1}{2} e^{j \frac{2 \pi n}{16}}+\frac{1}{2} e^{-j \frac{2 \pi n}{16}}$
$\mathrm{a}_{-1}=\frac{1}{2}, \mathrm{a}_{1}=\frac{1}{2}, \mathrm{a}_{0}=1$
$\mathrm{a}_{1}=\mathrm{a}_{-1+16} \Rightarrow \mathrm{a}_{-1}=\mathrm{a}_{15}=\frac{1}{2}$
$\Rightarrow \mathrm{a}_{0}=1, \mathrm{a}_{1}=\frac{1}{\mathrm{~L}}, \mathrm{a}_{2}$ to $\mathrm{a}_{14}=0, \mathrm{a}_{15}=\frac{1}{2}$
DFS coefficients are also periodic with period 16.
$a_{31}=a_{16+15}$
$\mathrm{a}_{31}=\mathrm{a}_{15}$
$\Rightarrow \mathrm{a}_{31}=\frac{1}{2}$
42. A fair die with faces $\{1,2,3,4,5,6\}$ is thrown repeatedly till ' 3 ' is observed for the first time. Let $X$ denote the number of times the die is thrown. The expected value of $X$ is

Answer:


Exp: Probability of getting $3=\frac{1}{6}$
Probability of not getting $3=1-\frac{1}{6}=\frac{5}{6}$
If dice thrown repeatedly till first 3 observed first time then

$$
\begin{aligned}
\mathrm{E}(\mathrm{x}) & =\frac{1}{6}+2\left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)+3\left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)+\ldots \\
& =\frac{1}{6}\left[1+2\left(\frac{5}{6}\right)^{2}+3\left(\frac{5}{6}\right)^{4}+\ldots\right] \\
& =\frac{1}{6}\left[1+2\left(\frac{5}{6}\right)^{2}+3\left(\left(\frac{5}{6}\right)^{2}\right)^{2}+\ldots\right] \\
& =\frac{1}{6}\left[1-\left(\frac{5}{6}\right)^{2}\right]^{-2} \\
& =\frac{1}{6}\left[\frac{11}{36}\right]^{-2}=\frac{1}{6} \times \frac{(36)^{2}}{(11)^{2}}=1.7851
\end{aligned}
$$

43. The electric field profile in the depletion region of a $p-n$ junction in equilibrium is shown in the figure. Which one of the following statements is NOT TRUE?

(A) The left side of the junction is n-type and the right side is p-type
(B) Both the n-type and p-type depletion regions are uniformly doped
(C) The potential difference across the depletion region is 700 mV
(D) If the p-type region has a doping concentration of $10^{15} \mathrm{~cm}^{-3}$, then the doping concentration in the n-type region will be $10^{16} \mathrm{~cm}^{-3}$

## Answer: (C)

Exp: Built in potential

$$
\begin{aligned}
\psi_{\mathrm{O}} & =\frac{1}{2} \times\left(10^{6} \mathrm{~V} / \mathrm{m}\right) \times\left(1.1 \times 10^{-6} \mathrm{~m}\right) \\
& =0.55 \text { volts }
\end{aligned}
$$

But in Question (option C) is given as 700 mV .
44. A vector field $D=2 \rho^{2} \mathrm{a}_{\rho}+\mathrm{Z}_{\mathrm{z}}$ exists inside a cylindrical region enclosed by the surfaces $\rho=1, z=0$ and $z=5$. Let $S$ be the surface bounding this cylindrical region. The surface integral of this field on $S\left(\oiint_{s} D . d s\right)$ is $\qquad$ -.

Answer: 78.53
Exp: $\quad D=2 \rho^{2} a \rho+z_{z}$

$$
\begin{aligned}
& \oint_{S} D . d s=\int_{V}(\nabla \cdot D) d v \\
& \nabla . \mathrm{D}=\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \mathrm{D}_{\rho}\right)+\frac{1}{\rho} \frac{\partial \mathrm{D} \phi}{\partial \phi}+\frac{\partial \mathrm{D}_{\mathrm{z}}}{\partial \mathrm{z}} \\
& =\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho 2 \rho^{2}\right)+0+1 \\
& =\frac{1}{\rho} 2(3) \rho^{2}+1 \\
& =6 \rho+1 \\
& \int_{V}(\nabla \cdot D) d v=\int_{\rho=0}^{1} \int_{\phi=0}^{2 \pi} \int_{z=0}^{5}(6 \rho+1) \rho d \rho d \phi d z \\
& =\left.\left(\frac{6 \rho^{3}}{3}+\frac{\rho^{2}}{2}\right)\right|_{0} ^{1}(2 \pi)(5) \\
& =\left(2+\frac{1}{2}\right) 10 \pi \\
& \int_{v}(\nabla . D) d v=78.53
\end{aligned}
$$

45. An npn BJT having reverse saturation current $I_{s}=10^{-15} \mathrm{~A}$ is biased in the forward active region with $\mathrm{V}_{\mathrm{BE}}=700 \mathrm{mV}$. The thermal voltage $\left(\mathrm{V}_{\mathrm{T}}\right)$ is 25 mV and the current gain $(\beta)$ may vary from 50 to 150 due to manufacturing variations. The maximum emitter current (in $\mu \mathrm{A}$ ) is $\qquad$ -.
Answer: 1475
Exp: $\quad I_{B}=\frac{I_{C}}{\beta}=\frac{I_{S}}{\beta} e^{\mathrm{VBE}_{B E} / V_{\mathrm{T}}}$

$$
\begin{aligned}
\mathrm{I}_{\mathrm{E}} & =(\beta+1) \mathrm{I}_{\mathrm{B}} \\
& =\frac{\beta+1}{\beta} \mathrm{I}_{\mathrm{S}} \cdot \mathrm{e}^{\mathrm{V}_{\mathrm{BE}} / V_{\mathrm{T}}} \\
& =(1.02)\left(10^{-9} \times 10^{-6}\right) \mathrm{e}^{\frac{700 \times 10^{-3}}{25 \times 10^{-3}}} \\
& =1475 \mu \mathrm{~A}
\end{aligned}
$$

46. Consider the 3 m long lossless air-filled transmission line shown in the figure. It has a characteristic impedance of $120 \pi \Omega$, is terminated by a short circuit, and is excited with a frequency of 37.5 MHz . What is the nature of the input impedance $\left(Z_{\text {in }}\right)$ ?

(A) Open
(B) Short
(C) Inductive
(D) Capacitive

Answer: (D)
Exp: $Z_{\text {in }}=\mathrm{JZ}_{0} \tan \beta l$

$$
\begin{array}{rl|l}
\beta l & =\frac{2 \pi}{\lambda} \cdot l & \lambda=\frac{3 \times 10^{8}}{37.5 \times 10^{6}} \\
& =\frac{2 \pi}{8}(3) & \begin{array}{c}
=8 \\
\end{array} \\
& =\frac{3 \pi}{4} &
\end{array}
$$

Short circuited line
$0<\beta l<\frac{\pi}{2} \rightarrow$ Inductor
$\frac{\pi}{2}<\beta l<\pi \rightarrow$ Capacitor
47. The current in an enhancement mode NMOS transistor biased in saturation mode was measured to be 1 mA at a drain-source voltage of 5 V . When the drain-source voltage was increased to 6 V while keeping gate-source voltage same, the drain current increased to 1.02 mA . Assume that drain to source saturation voltages is much smaller than the applied drain-source voltage. The channel length modulation parameter $\lambda$ (in $\mathrm{V}^{-1}$ ) is
$\qquad$ —.

[^1]
## Answer: 0.022

Exp: NMOS SATURATION
$\mathrm{I}_{\mathrm{D}}=1 \mathrm{~mA} @ \mathrm{~V}_{\mathrm{DS}}=5 \mathrm{~V}$
$\mathrm{I}_{\mathrm{D}}=1.02 \mathrm{~mA} @ \mathrm{~V}_{\mathrm{DS}}=6 \mathrm{~V}$
$\mathrm{V}_{\mathrm{DSat}} \ll \mathrm{V}_{\mathrm{DS}}$
$\mathrm{I}_{\mathrm{D}}=\mathrm{k}\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}\right)^{2}\left(1+\lambda \mathrm{V}_{\mathrm{DS}}\right)$
$\mathrm{I}_{\mathrm{D}}=\mathrm{k}^{\prime}\left(1+\lambda \mathrm{V}_{\mathrm{DS}}\right)$
$10^{-3}=\mathrm{k}^{\prime}(1+5 \lambda)$
$1.02 \times 10^{-3}=\mathrm{k}^{\prime}(1+6 \lambda)$
$1.02=\frac{1+6 \lambda}{1+5 \lambda} \Rightarrow 1.02+5.1 \lambda=1+6 \lambda$
$0.02=0.9 \lambda$
$\lambda=0.022 \mathrm{~V}^{-1}$
48. For the system shown in figure, $s=-2.75$ lies on the root locus if $K$ is $\qquad$ .


Exp: $\quad G(s) H(s)=\frac{10 k(s+3)}{(s+2)}$

$$
=\frac{\mathrm{k}^{\prime}(\mathrm{s}+3)}{(\mathrm{s}+2)}
$$

For k'
$\mathrm{k}^{\prime}=\frac{\text { Poles length }}{\text { zero length }}=\frac{0.75}{0.25}=3$
$10 \mathrm{k}=3$
$\mathrm{k}=0.3$
49. An SR latch is implemented using TTL gates as shown in the figure. The set and reset pulse inputs are provided using the push-button switches. It is observed that the circuit fails to work as desired. The SR latch can be made functional by changing

(A) NOR gates to NAND gates
(B) inverters to buffers
(C) NOR gates to NAND gates and inverters to buffers
(D) 5 V to ground

## Answer: (D)

50. The variance of the random variable $X$ with probability density function $f(x)=\frac{1}{2}|x| e^{-|x|}$ is $\qquad$
Answer: 6
Exp: Given that $f(x)=\frac{1}{2}|x| e^{-|x|}$ is probability density function of random variable $X$.

$$
\begin{aligned}
& V(x)=E\left(x^{2}\right)-\{E(x)\}^{2} \\
& \begin{aligned}
E(x) & =\int_{-\infty}^{\infty} x f(x) d x=\int_{-\infty}^{\infty} \frac{1}{2}|x| e^{|x|} x d x \\
& =0(\because \text { the function is odd })
\end{aligned}
\end{aligned}
$$

$$
\mathrm{E}(\mathrm{x})^{2}=\int_{-\infty}^{\infty} \mathrm{x}^{2} \mathrm{f}(\mathrm{x}) \mathrm{dx}
$$

$$
\begin{aligned}
& =\int_{-\infty}^{\infty} \mathrm{x}^{2} \frac{1}{2}|\mathrm{x}| \mathrm{e}^{-|\mathrm{x}|} \mathrm{dx} \\
& =\frac{2}{3} \int_{2}^{\infty} \mathrm{x}^{3} \mathrm{e}^{-\mathrm{x}} \mathrm{dx}(\because \text { functionis even }) \text { eer\|nO SUCCOSS} \\
& =3!=6
\end{aligned}
$$

51. Consider a continuous-time signal defined as

$$
\mathrm{x}(\mathrm{t})=\left(\frac{\sin (\mathrm{at} / 2)}{(\pi \mathrm{t} / 2)}\right) * \sum_{\mathrm{n}=-\infty}^{\infty} \delta(\mathrm{t}-10 \mathrm{n})
$$

Where '*' denotes the convolution operation and t is in seconds. The Nyquist sampling rate (in samples / sec) for $x(t)$ is $\qquad$ -.
Answer: 0.4
Exp: $\quad x(t)=\frac{\sin (\pi t / 2)}{(\pi t / 2)} * \sum_{n=-\infty}^{\infty} \delta(t-10 n)$
Convolution in time domain becomes multiplication in frequency domain.

$$
\begin{aligned}
& \frac{1}{10} \sum_{\mathrm{n}=-\infty}^{\infty} \delta\left(\mathrm{f}-\mathrm{kf}_{\mathrm{s}}\right) \\
& \mathrm{F}_{\mathrm{s}}=\frac{1}{\mathrm{~T}_{\mathrm{s}}}=0.1
\end{aligned}
$$



Multiplication in frequency domain will result maximum frequency is 0.2 .


Thus Nyquist rate $=0.4 \mathrm{samples} / \mathrm{sec}$
52. In the circuit shown, the both the enhancement mode NMOS transistors have the following characteristics: $\mathrm{k}_{\mathrm{n}}=\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}(\mathrm{W} / \mathrm{L})=1 \mathrm{~mA} / \mathrm{V}^{2} ; \mathrm{V}_{\mathrm{TN}}=1 \mathrm{~V}$. Assume that the channel length modulation parameter $\lambda$ is zero and body is shorted to source. The minimum supply voltage $V_{D D}$ (in volts) needed to ensure that transistor $\mathrm{M}_{1}$ operates in saturation mode of operation is $\qquad$ —.
Answer: 3
Exp: Lower transistor $\left(\mathrm{M}_{1}\right)$ to work in saturation
$\mathrm{V}_{\mathrm{DS} 1} \geq \mathrm{V}_{\mathrm{GS} 1}-\mathrm{V}_{+}$
So, for minimum $V_{D D}$
$\mathrm{V}_{\mathrm{DS} 1}=\mathrm{V}_{\mathrm{GS} 1}-\mathrm{V}_{+}$
$\mathrm{V}_{\mathrm{DS} 1}=2-1=1 \mathrm{~V}$
$\mathrm{V}_{\mathrm{DS} 1}=\mathrm{V}_{\mathrm{D} 1}-\mathrm{V}_{\mathrm{S} 1}$
$1 \mathrm{~V}=\mathrm{V}_{\mathrm{D} 1}-0$
$\therefore \mathrm{V}_{\mathrm{D} 1}=1 \mathrm{~V}$

and $\mathrm{I}_{\mathrm{D} 1}=\mathrm{K}^{\prime}\left(\mathrm{V}_{\mathrm{GS} 1}-\mathrm{V}_{+}\right)^{2}$

$$
\mathrm{I}_{\mathrm{D} 1}=\frac{1 \mathrm{~mA}}{\mathrm{~V}^{2}} \times(2-1)^{2}=1 \mathrm{~mA}
$$

Now transistor $\mathrm{M}_{2}, \mathrm{~V}_{\mathrm{DG}}=0 \mathrm{~V}$
So, it will work into saturation region and same current will flow

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{D} 2}=\mathrm{I}_{\mathrm{D} 1}=\mathrm{K}^{\prime}\left(\mathrm{V}_{\mathrm{GS} 2}-\mathrm{V}_{+}\right)^{2} \\
& 1 \mathrm{~mA}=1 \mathrm{~mA} / \mathrm{V}^{2} \times\left(\mathrm{V}_{\mathrm{DD}}-1-1\right)^{2}\left(\because \mathrm{~V}_{\mathrm{S} 2}=\mathrm{V}_{\mathrm{D} 1}\right) \\
& \therefore \mathrm{V}_{\mathrm{DD}}=3 \mathrm{~V}
\end{aligned}
$$

53. The position control of a $D C$ servo-motor is given in the figure. The values of the parameters are $K_{T}=1 \mathrm{~N}-\mathrm{m} \mathrm{A}, \mathrm{R}_{\mathrm{a}}=1 \Omega, \mathrm{~L}_{\mathrm{a}}=0.1 \mathrm{H} . \mathrm{J}=5 \mathrm{~kg}-\mathrm{m}^{2}, \mathrm{~B}=1 \mathrm{~N}-\mathrm{m}(\mathrm{rad} / \mathrm{sec})$ and $\mathrm{K}_{\mathrm{b}}=1 \mathrm{~V} /(\mathrm{rad} / \mathrm{sec})$. The steady-state position response (in radians) due to unit impulse disturbance torque $\mathrm{T}_{\mathrm{d}}$ is $\qquad$ -.


Answer: -0.5
Exp: $\quad T_{d}(s)=1$

$$
\theta(s)=-\frac{1}{s\left[(J s+B)+\frac{K_{b} K_{T}}{R_{a}+L_{a} s}\right]}
$$

Steady State Value is $\lim \mathrm{s} \theta(\mathrm{s})=-0.5$
54. The characteristic equation of an LTI system is given by $F(s)=s^{5}+2 s^{4}+3 s^{3}+6 s^{2}-4 s-$ $8=0$. The number of roots that lie strictly in the left half s-plane is $\qquad$
Answer:
: 2
Exp:
$s^{5}+2 s^{4}+3 s^{3}+6 s^{2}-4 s-8=0$

$$
\begin{array}{l|llll}
S^{5} & 1 & 3 & -4 & \\
S^{4} & 2 & 6 & -8 & \\
S^{3} & 8 & 12 & 0 & \\
S^{2} & 3 & -8 & 0 & \rightarrow \text { as } \in \rightarrow 0 \\
S^{1} & -9.33 & 0 & 0 & \\
S^{0} & -8 & &
\end{array}
$$

$2 s^{4}+6 s^{2}-8=0$
Let $x=s^{2}$, then
$2 x^{2}+6 x-8 \Rightarrow x=1,-4$
$s^{2}=1,-4 \Rightarrow s= \pm 1, \pm 2 j$
Number of roots lies on RHS $\rightarrow \mathrm{s}=1$
There are only two poles left on LHS.
55. Suppose $x[n]$ is an absolutely summable discrete-time signal. Its $z$-transform is a rational function with two poles and two zeroes. The poles are at $z= \pm 2 j$. Which one of the following statements is TRUE for the signal $x[n]$ ?
(A) It is a finite duration signal.
(B) It is a causal signal.
(C) It is a non-causal signal.
(D) It is a periodic signal

## Answer: (C)

Exp: Since $x[n]$ in absolutely summable thus its ROC must include unit circle.


Thus ROC must be inside the circling radius 2.
$\mathrm{x}[\mathrm{n}]$ must be a non-causal signal.
(B) GATEFORUM


[^0]:    © All rights reserved by GATE Forum Educational Services Pvt. Ltd. No part of this booklet may be reproduced or utilized in any form without the written permission. Discuss this questions paper at www.gatementor.com.

[^1]:    © All rights reserved by GATE Forum Educational Services Pvt. Ltd. No part of this booklet may be reproduced or utilized in any form without the written permission. Discuss this questions paper at www.gatementor.com.

