# FIRST PUC MATHEMATICS MODEL QUESTION PAPER 2013 NEW SYLLABUS - SUBJECT CODE: 35 

Time: $\mathbf{3}$ hours 15 minute
Max. Mark: 100

## Instructions:

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.
(ii) Use the graph sheet for the question on Linear inequalities in PART $D$.

## PART A

## Answer ALL the questions

$\mathbf{1 0} \times \mathbf{1}=10$

1. Given that the number of subsets of a set A is 16 . Find the number of elements in A .
2. If $\tan x=\frac{3}{4}$ and $x$ lies in the third quadrant, find $\sin x$.
3. Find the modulus of $\frac{1+i}{1-i}$.
4. Find ' $n$ ' if ${ }^{n} C_{7}={ }^{n} C_{6}$.
5. Find the $20^{\text {th }}$ term of the G.P., $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$
6. Find the distance between $3 x+4 y+5=0$ and $6 x+8 y+2=0$.
7. Given $\mathrm{f}(x)=\left\{\begin{array}{cc}\frac{x}{|x|}, & x \neq 0 \\ 2, & x=0\end{array}\right.$, find $\lim _{x \rightarrow 0^{+}} \mathrm{f}(x)$.
8. Write the negation of 'For all $a, b \in I, a-b \in I$ '.
9. A letter is chosen at random from the word "ASSASINATION". Find the probability that letter is vowel.
10. Let $A=\{2,3,4\}$ and R be a relation on A defined by

$$
\mathrm{R}=\{(x, \mathrm{y}) \mid x, \mathrm{y} \in \mathrm{~A}, x \text { divides } \mathrm{y}\}, \text { find ' } \mathrm{R} \text { '. }
$$

## PART - B

## Answer any TEN questions

$10 \times 2=20$
11. If $A$ and $B$ are two disjoint sets and $n(A)=15$ and $n(B)=10$ find $\mathrm{n}(\mathrm{A} \cup \mathrm{B}), \mathrm{n}(\mathrm{A} \cap \mathrm{B})$.
12. If $U=\{x: x \leq 10, x \in N\}, A=\{x: x \in N, x$ is prime $\}$ and $B=\{x: x \in N, x$ is even $\}$ write $A \cap B^{\prime}$ in roster form.
13. If $\mathrm{f}: \mathrm{Z} \rightarrow \mathrm{Z}$ is a linear function, defined by $\mathrm{f}=\{(1,1),(0,-1),(2,3)\}$, find $\mathrm{f}(\mathrm{x})$.
14. The minute hand of a clock is 2.1 cm long. How far does its tip move in 20 minute? (use $\pi=\frac{22}{7}$ ).
15. Find the general solution of $2 \cos ^{2} x-3 \sin x=0$.
16. Evaluate: $\lim _{x \rightarrow 3} \frac{(x-3)}{\left(x^{2}-5 x+6\right)}$
17. Coefficient of variation of distribution are 60 and the standard deviation is 21 , what is the arithmetic mean of the distribution?
18. Write the converse and contrapositive of 'If a parallelogram is a square, then it is a rhombus'.
19. In a certain lottery 10,000 tickets are sold and 10 equal prizes are awarded. What is the probability of not getting a prize if you buy one ticket.
20. In a triangle ABC with vertices $\mathrm{A}(2,3), \mathrm{B}(4,-1)$ and $\mathrm{C}(1,2)$. Find the length of altitude from the vertex $A$.
21. Represent the complex number $z=1+i$ in polar form.
22. Obtain all pairs of consecutive odd natural numbers such that in each pair both are more than 50 and their sum is less than 120.
23. A line cuts off equal intercepts on the coordinate axes. Find the angle made by the line with the positive $x$-axis.
24. If the origin is the centroid of the triangle $P Q R$ with vertices $P(2 a, 4,6)$ $Q(-4,3 b,-10)$ and $R(8,14,2 c)$ then find the values of $a, b$, $c$.

## PART - C

## Answer any TEN questions

25. Out of a group of 200 students (who know at least one language), 100 students know English, 80 students know Kannada, 70 students know Hindi. If 40 students know all the three languages, find the number of students who know exactly two languages.
26. Let $R: Z \rightarrow Z$ be a relation defined by $R=\{(a, b) \mid a, b, \in Z, a-b \in Z\}$. Show that
i) $\forall a \in Z,(a, a) \in R$
ii) $(a, b) \in R \Rightarrow(b, a) \in R$
iii) $(a, b) \in R,(b, c) \in R \Rightarrow(a, c) \in R$.
27. Prove that $(\cos x+\cos y)^{2}+(\sin x-\sin y)^{2}=4 \cos ^{2}\left(\frac{x+y}{2}\right)$.
28. Solve the equation $x^{2}+\frac{x}{\sqrt{2}}+1=0$.
29. How many words with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated if
i) 4 letters are used at a time ,
ii) All letters are used at a time,
iii) All letters are used but first letter is a vowel.
30. If $x+\mathrm{iy}=\frac{2+\mathrm{i}}{2-\mathrm{i}}$ prove that $x^{2}+\mathrm{y}^{2}=1$.
31. Find the term independent of $x$ in the expansion of $\left(\frac{3 x^{2}}{2}-\frac{1}{3 x}\right)^{6}$
32. Insert 3 arithmetic means between 8 and 24 .
33. A committee of two persons is selected from 2 men and 2 women. What is the probability that the committee will have (i) at least one man, (ii) at most one man.
34. Find the derivative of the function ' $\cos x$ ' with respect to ' $x$ ' from first principle.
35. A parabola with vertex at origin has its focus at the centre of $x^{2}+y^{2}-10 x+9=0$. Find its directrix and latus rectum.
36. In an A.P. if $\mathrm{m}^{\text {th }}$ term is ' n ' and the $\mathrm{n}^{\text {th }}$ term is ' m ', where $\mathrm{m} \neq \mathrm{n}$, find the $\mathrm{p}^{\text {th }}$ term.
37. Verify by the method of contradiction that $\sqrt{2}$ is irrational.
38. Two students Anil and Sunil appear in an examination. The probability that Anil will qualify in the examination is 0.05 and that Sunil will qualify is 0.10 . The probability that both will qualify the examination is 0.02 . Find the probability that Anil and Sunil will not qualify in the examination.

## PART D

## Answer any SIX questions

39. Define greatest integer function. Draw the graph of greatest integer function, Write the domain and range of the function.
40. Prove that $\lim _{\theta \rightarrow 0} \frac{\sin \theta}{\theta}=1$ ( $\theta$ being in radians) and hence show that $\lim _{\theta \rightarrow 0} \frac{\tan \theta}{\theta}=1$.
41. Prove by mathematical induction that $1^{3}+2^{3}+3^{3}+\ldots+\mathrm{n}^{3}=\frac{\mathrm{n}^{2}(\mathrm{n}+1)^{2}}{4} . \forall n \in N$.
42. A group consists of 7 boys and 5 girls. Find the number of ways in which a team of 5 members can be selected so as to have at least one boy and one girl.
43. For all real numbers $a, b$ and positive integer ' $n$ ' prove that,

$$
(a+b)^{n}={ }^{n} C_{0} a^{n}+{ }^{n} C_{1} a^{n-1} b+{ }^{n} C_{2} a^{n-2} b^{2}+\ldots+{ }^{n} C_{n-1} a b^{n-1}+{ }^{n} C_{n} b^{n}
$$

44. Derive an expression for the coordinates of a point that divides the line joining the points $\mathrm{A}\left(x_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ and $\mathrm{B}\left(x_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right)$ internally in the ratio $\mathrm{m}: \mathrm{n}$. Hence, find the coordinates of the midpoint of $A B$ where $A \equiv(1,2,3)$ and $B \equiv(5,6,7)$.
45. Derive a formula for the angle between two lines with slopes $m_{1}$ and $m_{2}$. Hence find the slopes of the lines which make an angle $\frac{\pi}{4}$ with the line $x-2 y+5=0$.
46. Prove that $\frac{\sin 9 x+\sin 7 x+\sin 3 x+\sin 5 x}{\cos 9 x+\cos 7 x+\cos 3 x+\cos 5 x}=\tan 6 x$
47. Solve the following system of inequalities graphically,

$$
2 x+y \geq 4, \quad x+y \leq 3, \quad 2 x-3 y \leq 6
$$

48. Find the mean deviation about the mean for the following data

| Marks obtained | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of students | 2 | 3 | 8 | 14 | 8 | 3 | 2 |

## PART-E

## Answer any ONE question

$1 \times 10=10$
49. (a) Prove geometrically that $\cos (A+B)=\cos A \cos B-\sin A \sin B$.

Hence find $\cos 75^{\circ}$.
(b) Find the sum to $n$ terms of the series $1^{2}+\left(1^{2}+2^{2}\right)+\left(1^{2}+2^{2}+3^{2}\right)+\ldots$
50. (a) Define ellipse as a set of points. Derive its equation in the form $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1.6$ (b) Find the derivative of $\frac{x^{5}-\cos x}{\sin x}$ using rules of differentiation.

## MODEL ANSWER PAPER



| 17 | Writing the formula c.v $=\frac{\sigma}{\bar{x}} \times 100$ | 1 |
| :---: | :---: | :---: |
|  | Writing arithmetic mean $=\bar{x}=35$ |  |
| 18 | Writing converse | 1 |
|  | Writing contrapostive | 1 |
| 19 | Writing probability of getting a prize $=\frac{10}{10000}=\frac{1}{1000}$ | 1 |
|  | $\text { Writing } \mathrm{p} \text { (not getting a prize) }=1-\frac{1}{1000}=\frac{999}{1000}$ | 1 |
| 20 | Getting: equation of BC is $x+y-3=0$ | 1 |
|  | Finding length of altitude from $A(2,3)=\left\|\frac{2+3-3}{\sqrt{2}}\right\|=\left\|\frac{2}{\sqrt{2}}\right\|=\sqrt{2}$ | 1 |
| 21 | Getting $r=\sqrt{2}$ OR $\quad \theta=\frac{\pi}{4}$ | 1 |
|  | Writing : polar form is $\sqrt{2}\left(\cos \frac{\pi}{4}+\mathrm{i} \sin \frac{\pi}{4}\right)$ | 1 |
| 22 | Taking the pair as $x, x+2$ and writing $x>50,2 x+2<120$ | 1 |
|  | Writing the required pair of numbers $(51,53), \quad(53,55), \quad(55,57), \quad(57,59)$ | 1 |
| 23 | Writing slope $=-1$ | 1 |
|  | Writing angle made $=135^{\circ}$. | 1 |
| 24 | Writing $\left(\frac{2 \mathrm{a}-4+8}{3}, \frac{4+3 \mathrm{~b}+14}{3}, \frac{6-10+2 \mathrm{c}}{3}\right)=(0,0)$ $\mathbf{O R} \quad \frac{2 a-4+8}{3}=0 \quad$ OR $\quad \frac{4+3 b+14}{3}=0 \quad$ OR $\quad \frac{6-10+2 c}{3}=0$ | 1 |
|  | Getting $a=-2, b=-6, c=2$ | 1 |
| 25$26$ | $\begin{aligned} & \text { Knowing the firmula } n(E \cup K \cup H) \\ & =n(E)+n(K)+n(H)-n(E \cap H)-n(H \cap K)-n(K \cap E) \\ & \quad \begin{array}{c} -n(E \cap H \cap K) \end{array} \\ & \text { OR } \quad n(E \cup K \cup H)=200, n(E \cap K \cap H)=40 . \end{aligned}$ | 1 |
|  | Getting $n(E \cap K)+n(K \cap H)+n(H \cap K)=90$. | 1 |
|  | Getting the answer $90-2 n(E \cap K \cap H)=10$ | 1 |
|  | Stating $\forall \mathrm{a} \in \mathrm{Z},(\mathrm{a}, \mathrm{a}) \in \mathrm{R}$ since $\mathrm{a}-\mathrm{a}=0 \in \mathrm{Z}$ | 1 |
|  | Writing $(a, b) \in R \quad \Rightarrow \quad(b, a) \in R$ with reason | 1 |
|  | Writing $(a, b) \in R,(b, c) \in R \Rightarrow(c, a) \in R$ with reason | 1 |
| 27 | For expanding the LHS. $\text { LHS }=\cos ^{2} x+\cos ^{2} y+2 \cos x \cos y+\sin ^{2} x+\sin ^{2} y-2 \sin x \sin y$ | 1 |
|  | Getting $1+1+2(\cos x \cdot \cos y-\sin x \cdot \sin y)$ | 1 |
|  | Getting $=4 \cos ^{2}\left(\frac{x+y}{2}\right)$ | 1 |


| 28 | Writing equation $\sqrt{2} \mathrm{x}^{2}+\mathrm{x}+\sqrt{2}=0$ | 1 |
| :---: | :---: | :---: |
|  | For using the formula for roots | 1 |
|  | Getting roots $\mathrm{x}=\frac{-1 \pm \sqrt{7} i}{2 \sqrt{2}}$. |  |
| 29 | Getting number of words with 4 letters $=\frac{6!}{2!}=360$ ways | 1 |
|  | Getting number of words containing all the letters of the words $=6!=720$ ways. | 1 |
|  | Getting the number of words having first letter as yowel is $=2!\times 5!=2 \times 120=240$ | 1 |
| 30 | Writing the conjugate $=\frac{2-\mathrm{i}}{2+\mathrm{i}}$. | 1 |
|  | For using $(x+i y)(x-i y)=x^{2}+y^{2}$ OR for simplifying $\frac{2+\mathrm{i}}{2-\mathrm{i}}$ to $\frac{3+4 \mathrm{i}}{5}$ | 1 |
|  | Getting the answer $x^{2}+y^{2}=1$. | 1 |
| 31 | Writing $\mathrm{T}_{\mathrm{r}+1}={ }^{6} \mathrm{C}_{\mathrm{r}}\left(\frac{3}{2} x^{2}\right)^{6-\mathrm{r}}\left(\frac{-1}{3 x}\right)^{\mathrm{r}}$ | 1 |
|  | Getting $\mathrm{r}=4$ | 1 |
|  | $\text { Getting answer }=\frac{5}{12}$ | 1 |
| 32 | Getting $d=4$ | 1 |
|  | Finding all the three A.M.s $=12,16,20$ (any one correct award one mark) | 2 |
| 33 | Knowing number of committees containing at least one man OR Knowing number of committees containing at most one man. | 1 |
|  | Getting the probability that committee contains at least one man | 1 |
|  | Getting the probability that committee contains at most one man | 1 |
|  | Let $\mathrm{f}(x)=\cos x$. <br> Writing $\mathrm{f}^{\prime}(x)=\lim _{\mathrm{h} \rightarrow 0} \frac{\mathrm{f}(x+\mathrm{h})-\mathrm{f}(x)}{\mathrm{h}}$ OR $=\lim _{\mathrm{h} \rightarrow 0} \frac{\cos (x+\mathrm{h})-\cos x}{\mathrm{~h}}$ | 1 |
|  | For using using formula for $\cos C+\cos D$. | 1 |
|  | Getting the answer $-\sin x$. | 1 |
| 35 | Writing the centre of the circle $\equiv(5,0)$ | 1 |
|  | Getting the equation $\mathrm{y}^{2}=20 x$ | 1 |
|  | Writing, directrix is $x=-5, \mathrm{LR}=20$ | 1 |
| 36 | Writing $a+(m-1) d=n$ and $a+(n-1) d=m$. | 1 |
|  | Solving for $d(=-1)$ | 1 |
|  | Getting $\mathrm{p}^{\text {th }}$ term $=n+m-p$ | 1 |


| 37 | Taking $\sqrt{2}=\frac{\mathrm{p}}{\mathrm{q}}$ where $\mathrm{p}, \mathrm{q} \in \mathrm{I}, \mathrm{q} \neq 0$ where $\mathrm{p}, \mathrm{q}$ have no common factor. | 1 |
| :---: | :---: | :---: |
|  | Showing p, q are even. |  |
|  | Concluding, by contradiction. | 1 |
| 38. | Let A, B denote the events that Anil, Sunil qualify in the exam Writing $\mathrm{P}(\mathrm{A})=0.05, \mathrm{P}(\mathrm{B})=0.1, \mathrm{P}(\mathrm{A} \cap \mathrm{B})=0.02$ | 1 |
|  | Stating $\mathrm{P}(\mathrm{~A} \cup \mathrm{~B})=\mathrm{P}(\mathrm{~A})+\mathrm{P}(\mathrm{~B})-\mathrm{P}(\mathrm{~A} \cap \mathrm{~B})=0.05+0.1-0.02=0.13$ | 1 |
|  | Getting $\mathrm{P}\left(\mathrm{A}^{\prime} \cap \mathrm{B}^{\prime}\right)=1-\mathrm{P}(\mathrm{A} \cup \mathrm{B})=1-0.13=0.87$ | 1 |
| 39 | Stating any one particular statement of the type $[x]=n$ when $n \leq x<(n+1)$ \{for example, $[x]=1$ when $1 \leq x<2\}$ | 1 |
|  | Drawing any one step (line segment) between two consecutive integers | 1 |
|  | Drawing three consecutive steps with punches. | 1 |
|  | Writing $R$ as the domain | 1 |
|  | Writing $Z$ as the range. | 1 |
| 40 | Figure | 1 |
|  | Stating Area of $\triangle \mathrm{OAB}<$ area of sector $\mathrm{OAB}<$ area of $\triangle \mathrm{OAC}$ | 1 |
|  | Getting $\quad \frac{1}{2} \mathrm{r}^{2} \sin \theta<\frac{1}{2} \mathrm{r}^{2} \theta<\frac{1}{2} \mathrm{r}^{2} \tan \theta$ | 1 |
|  | Getting $\underset{\theta \rightarrow 0}{\operatorname{Lt}} \frac{\sin \theta}{\theta}=1$ | 1 |
|  | Getting $\underset{\theta \rightarrow 0}{\operatorname{Lt}} \frac{\tan \theta}{\theta}=1$ | 1 |
| 41 | Taking $P(n)=1^{3}+2^{3}+\ldots \ldots+n^{3}=\frac{n^{2}(n+1)^{2}}{4}$ and showing $P(1)$ is true | 1 |
|  | Assuming $\mathrm{P}(\mathrm{m}): 1^{3}+2^{3}+\ldots .+\mathrm{m}^{3}=\frac{\mathrm{m}^{2}(\mathrm{~m}+1)^{2}}{4}$ to be true | 1 |
|  | Proving $P(m+1): 1^{3}+2^{3}+\ldots+m^{3}+(m+1)^{3}=\frac{(m+1)^{2}(m+2)^{2}}{4}$ is true | 2 |


|  | Concluding that the statement is true by induction | 1 |
| :---: | :---: | :---: |
| 42 | Writing possible nuber of choices . | 1 |
|  | Finding number of ways of selecting <br> 1 B and $4 \mathrm{G}={ }^{7} \mathrm{C}_{1} \times{ }^{5} \mathrm{C}_{4}=35$, <br> 2 B and $3 \mathrm{G}={ }^{7} \mathrm{C}_{2} \times{ }^{5} \mathrm{C}_{3}=210$, <br> 3 B and $2 \mathrm{G}={ }^{7} \mathrm{C}_{3} \times{ }^{5} \mathrm{C}_{2}=350$, <br> 4 B and $1 \mathrm{G}={ }^{7} \mathrm{C}_{4} \times{ }^{5} \mathrm{C}_{1}=175$ <br> (any one correct award one mark) | 3 |
|  | Total number of selections $=770$ | 1 |
| 43 | Taking $\mathrm{P}(\mathrm{n}):(\mathrm{a}+\mathrm{b})^{\mathrm{n}}+{ }^{\mathrm{n}} \mathrm{C}_{0} \mathrm{a}^{\mathrm{n}}+{ }^{\mathrm{n}} \mathrm{C}_{1} \mathrm{a}^{\mathrm{n}-1} \mathrm{~b}+\ldots \ldots+{ }^{\mathrm{n}} \mathrm{C}_{\mathrm{n}} \mathrm{b}^{\mathrm{n}}$ and showing $P(1)$ is true | 1 |
|  | Assuming $P(m)$ is true | 1 |
|  | Proving $\mathrm{P}(\mathrm{m}+1)$ is true | 2 |
|  | concluding P (n) is true by induction | 1 |
| 44 | Figure | 1 |
|  | Showing $\frac{m}{n}=\frac{A P}{P B}=\frac{A Q}{B R}$ | 1 |
|  | Getting $z=\frac{m z_{2}+n z_{1}}{m+n}$ | 1 |
|  | $\text { Getting point of division } \equiv\left(\frac{\mathrm{m} x_{2}+\mathrm{n} x_{1}}{\mathrm{~m}+\mathrm{n}}, \frac{\mathrm{my}_{2}+\mathrm{ny} y_{1}}{\mathrm{~m}+\mathrm{n}}, \frac{\mathrm{mz}}{2}+\mathrm{nz}_{1}\right)$ | 1 |
|  | Getting, mid point of $\mathrm{AB}=(3,4,5)$ | 1 |
|  | Figure | 1 |
|  | Writing $\mathrm{m}_{1}=\tan \theta_{1}, \mathrm{~m}_{2}=\tan \theta_{2}$ and writing $\theta_{1}=\theta_{2}+\theta$ | 1 |
|  | Getting $\tan \theta=\tan \left(\theta_{1}-\theta_{2}\right)=\frac{\tan \theta_{1}-\tan \theta_{2}}{1+\tan \theta_{1} \tan \theta_{2}}=\frac{\mathrm{m}_{1}-\mathrm{m}_{2}}{1+\mathrm{m}_{1} \mathrm{~m}_{2}}$ | 1 |
|  | Let $m$ be the required slope. | 1 |




