ICSE Board

Class X Mathematics

Board Question Paper 2014

(Two and a half hours)

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the Question Paper.

The time given at the head of this Paper is the time allowed for writing the answers.

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*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) Ranbir borrows Rs. 20,000 at 12% per annum compound interest. If he repays Rs. 8400 at the end of the first year and Rs. 9680 at the end of the second year, find the amount of loan outstanding at the beginning of the third year. [3]

(b) Find the values of x, which satisfy the inequation

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2, x \in W$. Graph the solution set on the number line. [3]

(c) A die has 6 faces marked by the given numbers as shown below:

1	2	3	-1	-2	-3

The die is thrown once. What is the probability of getting:

(i) a positive integer.

(ii) an integer greater than -3.

(iii) the smallest integer. [4]

Question 2

(a) Find x,y if $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ -2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$. [3]

(b) Shahrukh opened a 'Recurring Deposit' account in a bank and deposited Rs. 800 per month for 1 ½ years. If he received Rs. 15,084 at the time of maturity, find the rate of interest per annum. [3]

(c) Calculate the ratio in which the line joining $A(-4, 2)$ and $B(3, 6)$ is divided by a point $P(x, 3)$. Also find (i) x (ii) Length of AP. [4]

Question 3

(a) Without using trigonometric tables, evaluate

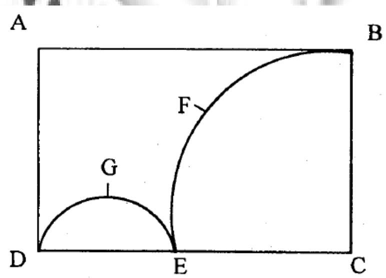
$\sin^2 34° + \sin^2 56° + 2\tan 18° \tan 72° - \cot^2 30°$ [3]

(b) Using the Remainder and Factor Theorem, factorise the following polynomial:

$x^3 + 10x^2 - 37x + 26$ [3]

(c) In the figure given below, ABCD is a rectangle. AB = 14 cm, BC = 7 cm.

From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle.(Take $\pi = 22/7$) [4]



Question 4

(a) The numbers 6, 8, 10, 12, 13 and x are arranged in an ascending order.

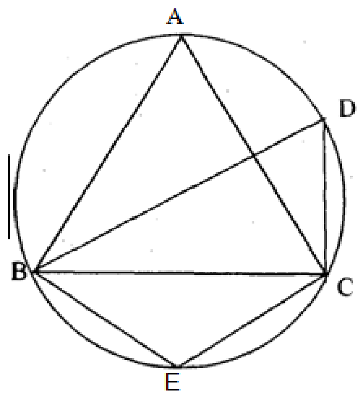
If the mean of the observations is equal to the median, find the value of x. [3]

(b) In the figure, $m\angle DBC = 58°$. BD is the diameter of the circle. Calculate: [3]

(i) $m\angle BDC$

(ii) $m\angle BEC$

(iii) $m\angle BAC$



(c) Use graph paper to answer the following questions. (Take 2 cm = 1 unit on both axes)

(i) Plot the points $A(-4, 2)$ and $B(2, 4)$

(ii) A' is the image of A when reflected at the y-axis. Plot it on the graph paper and write the co-ordinates of A'.

(iii) B' is the image of B when reflected on the line AA'. Write the co-ordinates of B'.

(iv) Write the geometric name of the figure ABA'B'.

(v) Name a line of symmetry of the figure formed. [4]

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) A shopkeeper bought a washing machine at a discount of 20% from a wholesaler, the printed price of the washing machine being`18,000. The shopkeeper sells it to a consumer at a discount of 10% on the printed price. If the rate of sales tax is 8%, find:

(i) the VAT paid by the shopkeeper.

(ii) the total amount that the consumer pays for the washing machine. [3]

(b) If $\frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$ then find the value of :

(i) x: y

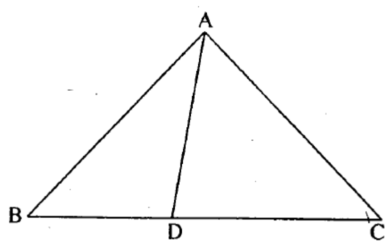
(ii) $\frac{x^3+y^3}{x^3-y^3}$ [3]

(c) In ΔABC, $\angle ABC = \angle DAC$. AB = 8 cm, AC = 4 cm, AD = 5 cm.

(i) Prove that ΔACD is similar to ΔBCA.

(ii) Find BC and CD

(iii) Find- area of Δ ACD : area of Δ ABC [4]



Question 6

(a) The value of 'a' for which of the following points A(a, 3), B (2, 1) and C(5, a) are collinear. Hence find the equation of the line. [3]

(b) Salman invests a sum of money in 50 shares, paying 15% dividend quoted at 20% premium. If his annual dividend is 600, calculate:

(i) the number of shares he bought.

(ii) his total investment.

(iii) the rate of return on his investment. [3]

(c) The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate:

(i) the radius of the sphere.

(ii) the number of cones recast. (Take $\pi = 22/17$) [4]

Question 7

(a) Calculate the mean of the distribution given below using the short cut method.

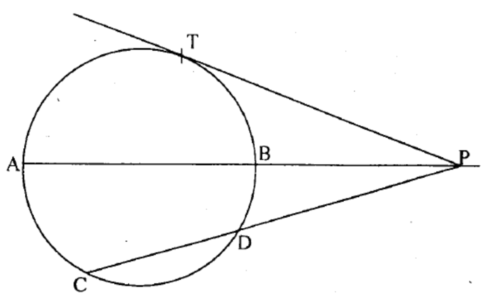
Marks	11-20	21-	31-40	41-50	51-60	61-70	71-80
No. of Students	2	6	10	12	9	7	4

[3]

(b) In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm. Find:

(i) AB.

(ii) the length of tangent PT. [3]



(c) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$.

Find $A^2 + AC - 5B$ [4]

Question 8

(a) The compound interest, calculated yearly, on a certain sum of money for the second year is 1320 and for the third year is 1452. Calculate the rate of interest and the original sum of money. [3]

(b) Construct a ΔABC with BC = 6.5 cm, AB = 5.5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle. [3]

(c) (Use a graph paper for this question.) The daily pocket expenses of 200 students in a school are given below:

Pocket expenses (in Rs.)	Number of students (Frequency)
0-5	10
5-10	14
10-15	28
15-20	42
20-25	50
25-30	30
30-35	14
35-40	12

Draw a histogram representing the above distribution and estimate the mode from the graph. [4]

Question 9

(a) If $(x - 9) : (3x + 6)$ is the duplicate ratio of 4 : 9, find the value of x. [3]

(b) Solve for x using the quadratic formula. Write your answer corrected to two significant figures. $(x - 1)^2 - 3x + 4 = 0$ [3]

(c) A page from the 'Savings Bank' account of Priyanka is given below:

Date	Particulars	Amount withdrawn (Rs.)	Amount deposited (Rs.)	Balance (Rs.)
03/04/2006	B/F			4000.00
05/04/2006	By cash		2000.00	6000.00
18/04/2006	By cheque		6000.00	12000.00
25/05/2006	By cheque	5000.00		7000.00
30/05/2006	By cash		3000.00	10000.00
20/07/2006	By self	4000.00		6000.00
10/09/2006	By cash		2000.00	8000.00
19/09/2006	To cheque	1000.00		7000.00

If the interest earned by Priyanka for the period ending September, 2006 is Rs. 175, the find the rate of interest. [4]

Question 10

(a) A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. [4]

(b) The marks obtained by 100 students in a Mathematics test are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.
Use a scale of 2 cm = 10 units on both axes.

Use the ogive to estimate the:

(i) Median.

(ii) Lower quartile.

(iii) Number of students who obtained more than 85% marks in the test.

(iv) Number of students who did not pass in the test if the pass percentage was 35.

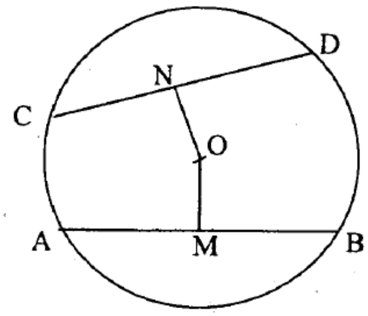
[6]

Question 11

(a) In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD.
AB = 24 cm, OM= 5 cm, ON= 12 cm. Find the:

(i) radius of the circle.

(ii) length of chord CD. [3]



(b) Prove the identity

$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$ [3]

(c) An aeroplane at an altitude of 250 m observes the angle of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer corrected to the nearest whole number. [4]

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Class X Mathematics
Board Paper 2014 Solution
(Two and a half hours)

SECTION A

1.

(a)

Given that Ranbir borrows Rs.20000

at 12% compound interest.

For the first year,

Interest $\text{I}=\dfrac{20000\times 1\times 12}{100}=\text{Rs.}2400$

Thus,amount after one year = Rs.20000+Rs.2400 = Rs.22400

$\therefore$ Money repaid = Rs.8400

$\therefore$ Balance= Rs.22400 $-$ Rs.8400 $=$ 14000

For the second year,

Interest $\text{I}=\dfrac{14000\times 1\times 12}{100}=\text{Rs.}1680$

Thus the amount =Rs.14000+ Rs.1680 = Rs.15680

Ranbir paid Rs. 9680 in the second year.

$\therefore$ The loan outstanding at the beginning of the third year

= Rs.15680 $-$ Rs. 9680 = Rs.6000

(b)

We need to find the values of x, such that

x satisfies the inequation $-2\frac{5}{6}<\frac{1}{2}-\frac{2x}{3}\leq 2, x\in W$

Consider the given inequation:

$-2\frac{5}{6}<\frac{1}{2}-\frac{2x}{3}\leq 2$

$\Rightarrow \frac{-17}{6}<\frac{3-4x}{6}\leq\frac{12}{6}$

$\Rightarrow \frac{17}{6}>\frac{4x-3}{6}\geq\frac{-12}{6}$

$\Rightarrow 17>4x-3\geq -12$

$\Rightarrow -12\leq 4x-3<17$

$\Rightarrow -12+3\leq 4x-3+3<17+3$

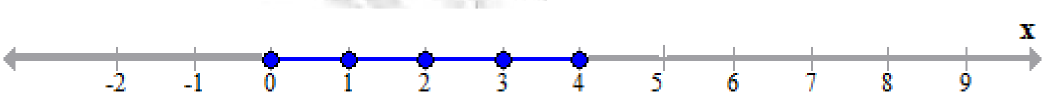
$\Rightarrow -9\leq 4x<20$

$\Rightarrow -\frac{9}{4}\leq\frac{4x}{4}<\frac{20}{4}$

$\Rightarrow -\frac{9}{4}\leq x<5$

Since $x\in W$, the Required solution set= $\{0,1,2,3,4\}$

And the required line is



(c)

Given that the die has 6 faces marked
by the given numbers as below:

$\boxed{3}\ \boxed{2}\ \boxed{1}\ \boxed{-1}\ \boxed{-2}\ \boxed{-3}$

(i)

Let us find the probability of getting a positive integer.
When a die is rolled, the total number of possible
outcomes =6

For getting a positive integer, the favourable outcomes
are:1,2,3

$\Rightarrow$ Number of favourable outcomes = 3

$\Rightarrow$ Required probability=$\frac{3}{6}=\frac{1}{2}$

(ii) Let us find the probability of getting an integer greater than -3.
When a die is rolled, the total number of possible
outcomes =6
For getting getting an integer greater than -3, the favourable outcomes
are:$-2,-1,1,2,3$

$\Rightarrow$ Number of favourable outcomes = 5

$\Rightarrow$ Required probability=$\frac{5}{6}$

(iii) Let us find the probability of getting a smallest integer
When a die is rolled, the total number of possible
outcomes =6
For getting getting getting a smallest integer, the favourable outcomes
are:-3

$\Rightarrow$ Number of favourable outcomes = 1

$\Rightarrow$ Required probability=$\frac{1}{6}$

2.

(a)

Consider the following equation:

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix}+3\begin{bmatrix} -2 \\ 1 \end{bmatrix}=2\begin{bmatrix} y \\ 3 \end{bmatrix}$$

Multiplying and adding the corresponding elements of the matrices, we have

$$\Rightarrow (-2)(-1)+0(2x)+3(-2)=2y$$
$$\Rightarrow 2-6=2y$$
$$\Rightarrow -4=2y$$
$$\Rightarrow y=-2$$

Similarly,

$$3(-1)+1(2x)+3(1)=2(3)$$
$$\Rightarrow -3+2x+3=6$$
$$\Rightarrow 2x=6$$
$$\Rightarrow x=3$$

Thus, the values of x and y are: 3, −2

(b)

Shahrukh deposited Rs.800 per month for $n=1\frac{1}{2}$ years

Since $1\frac{1}{2}$ years = 18 months,

Total money deposited $=18\times 800=$ Rs.14400

Given that the maturity value = Rs.15084

$\therefore$ Interest = Maturity Value − Total sum deposited

$=15084-14400$

$=684$

We know that Interest

$$I = P \times \frac{n(n+1)}{2\times 12}\times\frac{r}{100}$$

$$\Rightarrow 684=800\times\frac{18(18+1)}{2\times 12}\times\frac{r}{100}$$

$$\Rightarrow \frac{684\times 2\times 12\times 100}{800\times 18\times 19}=r$$

$$\Rightarrow r=6\%$$

(c)

Let $A(-4,2)$ and $B(3,6)$ be two points.

Let $P(x,3)$ be the point which divides the line joining the line segment in the ratio k:1

Thus, we have

$$\frac{3k-4}{k+1}=x; \qquad \frac{6k+2}{k+1}=3$$

For

$$6k+2=3(k+1)$$

$$\Rightarrow 6k+2=3k+3$$

$$\Rightarrow 3k=3-2$$

$$\Rightarrow 3k=1$$

$$\Rightarrow k=\frac{1}{3}$$

Now consider the equation,

$$\frac{3k-4}{k+1}=x$$

Substituting the value of k in the above equation we have,

$$\frac{3\times\frac{1}{3}-4}{\frac{1}{3}+1}=x$$

$$\Rightarrow \frac{-3}{\frac{4}{3}}=x$$

$$\Rightarrow \frac{-9}{4}=x$$

Therefore, coordinate of P is $\left(-\frac{9}{4},3\right)$

Now let us find the distance AP:

$$AP=\sqrt{\left(\frac{-9}{4}+4\right)^2+(3-2)^2}$$

$$\Rightarrow AP=\sqrt{\frac{49}{16}+1}$$

$$\Rightarrow AP=\sqrt{\frac{49+16}{16}}$$

$$\Rightarrow AP=\sqrt{\frac{65}{16}}=\frac{\sqrt{65}}{4} \text{ units}$$

3.

(a) Consider the expression $\sin^2 34^\circ + \sin^2 56^\circ + 2\tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$:

$$\sin^2 34^\circ + \sin^2 56^\circ + 2\tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$$
$$= \sin^2 34^\circ + \sin^2\left(90^\circ - 56^\circ\right) + 2\tan 18^\circ \tan\left(90^\circ - 72^\circ\right) - \cot^2 30^\circ$$

$$= \sin^2 34^\circ + \cos^2 34^\circ + 2\tan 18^\circ \cot 18^\circ - \cot^2 30^\circ$$
$$= \left(\sin^2 34^\circ + \cos^2 34^\circ\right) + 2\tan 18^\circ \times \frac{1}{\tan 18^\circ} - \cot^2 30^\circ$$
$$= 1 + 2\times 1 - \left(\sqrt{3}\right)^2$$
$$= 1 + 2 - 3$$
$$= 3 - 3$$
$$= 0$$

(b)

By remainder Theorem,

For x=1, the value of the given expression is the remainder.

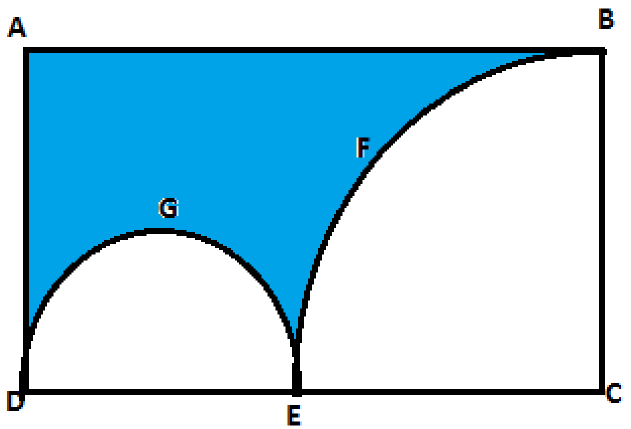
$$x^3 + 10x^2 - 37x + 26$$
$$= (1)^3 + 10(1)^2 - 37(1) + 26$$
$$= 1 + 10 - 37 + 26$$
$$= 37 - 37$$
$$= 0$$
$$\Rightarrow x - 1 \text{ is a factor of } x^3 + 10x^2 - 37x + 26$$

$$\begin{array}{r|l} & x^2+11x-26 \\ \hline x-1 & x^3+10x^2-37x+26 \\ & \underline{x^3-x^2} \\ & \quad 11x^2-37x \\ & \quad \underline{11x^2-11x} \\ & \qquad -26x+26 \\ & \qquad \underline{-26x+26} \\ & \qquad\quad 0 \end{array}$$

Thus, by factor theorem,

$$\Rightarrow x^3 + 10x^2 - 37x + 26 = (x-1)\left(x^2 + 11x - 26\right)$$
$$= (x-1)\left(x^2 + 13x - 2x - 26\right)$$
$$= (x-1)\left(x(x+13) - 2(x+13)\right)$$
$$\Rightarrow x^3 + 10x^2 - 37x + 26 = (x-1)(x+13)(x-2)$$

(c) Considering the given figure:



Given dimensions of the rectangle: AB=14 cm and BC=7 cm

Thus the radius of the quarter circle is 7 cm

Area of the quarter circle is = $\frac{1}{4}\times\frac{22}{7}\times7^2$ sq. cm

$\Rightarrow$ Area of the quarter circle = $\frac{77}{2}$ sq. cm

Since EC = 7cm and DC = 14 cm, we have,

DE = DC $-\mathrm{EC}=14-7=7$ cm

Therefore, radius of the semi circle is $=\frac{7}{2}$ cm

Thus the area of the semi circle is $=\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2$ sq. cm

$\Rightarrow$ Area of the semi circle $=\frac{77}{4}$ sq. cm

Area of the rectangle is=AB$\times$BC=14$\times$7=98 sq.cm

Thus, the required area=Area$(\mathrm{ABCD})-[\mathrm{Area}(\mathrm{BCEF})+\mathrm{Area}(\mathrm{DGE})]$

$$=98-\frac{77}{2}-\frac{77}{4}=40.25 \text{ sq.cm}$$

4.

(a)

Consider the given set of numbers: 6,8,10,12,13,x

There are six numbers and six is even.

Thus the median of the given numbers is $\dfrac{\frac{N}{2}\text{th term}+\left(\frac{N}{2}+1\right)\text{th term}}{2}$

$$\Rightarrow \text{Median}=\frac{\frac{6}{2}\text{th term}+\left(\frac{6}{2}+1\right)\text{th term}}{2}$$

$$\Rightarrow \text{Median}=\frac{\text{3rd term}+\text{4th term}}{2}$$

$$\Rightarrow \text{Median}=\frac{10+12}{2}$$

$$\Rightarrow \text{Median}=\frac{22}{2}=11$$

Given that the mean of 6,8,10,12,13,x is median of 6,8,10,12,13,x

Thus, we have

$$\frac{6+8+10+12+13+x}{6}=11$$

$\Rightarrow 6+8+10+12+13+x = 66$

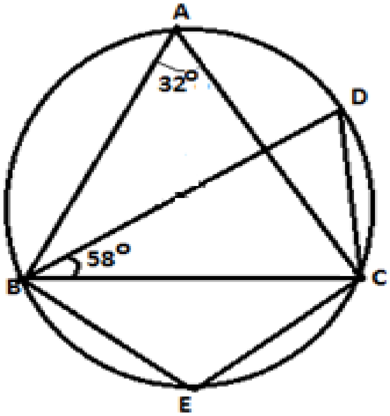
$\Rightarrow 49+x = 66$

$\Rightarrow x = 66-49$

$\Rightarrow x=17$

(b)

Consider the following figure.



Given that BD is a diameter of the circle.

The angle in a semi circle is a right angle.

$\therefore \angle BCD = 90^\circ$

Also given that $\angle DBC = 58^\circ$

Consider the triangle ΔBDC:

By angle sum property, we have

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$\Rightarrow 58^\circ + 90^\circ + \angle BDC = 180^\circ$

$\Rightarrow 148^\circ + \angle BDC = 180^\circ$

$\Rightarrow \angle BDC = 180^\circ - 148^\circ$

$\Rightarrow \angle BDC = 32^\circ$

Angles in the same segment are equal.

Thus, $\angle BDC = 32^\circ \Rightarrow \angle BAC = 32^\circ$

Now, □BACE is a cyclic quadrilateral,

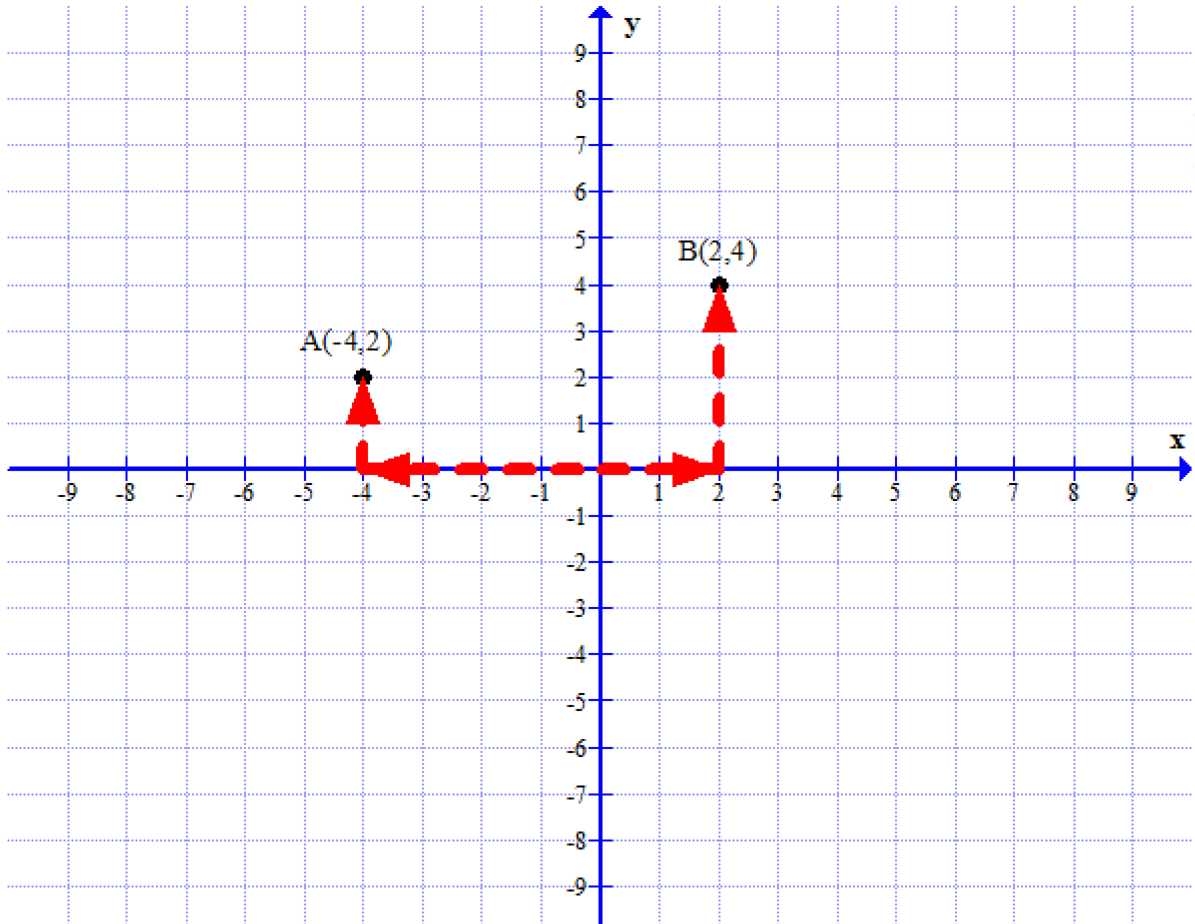
m∠BAC + m∠BEC = 180°

⇒ 32 + m∠BEC = 180°

⇒m∠BEC = 180° - 32 = 148°

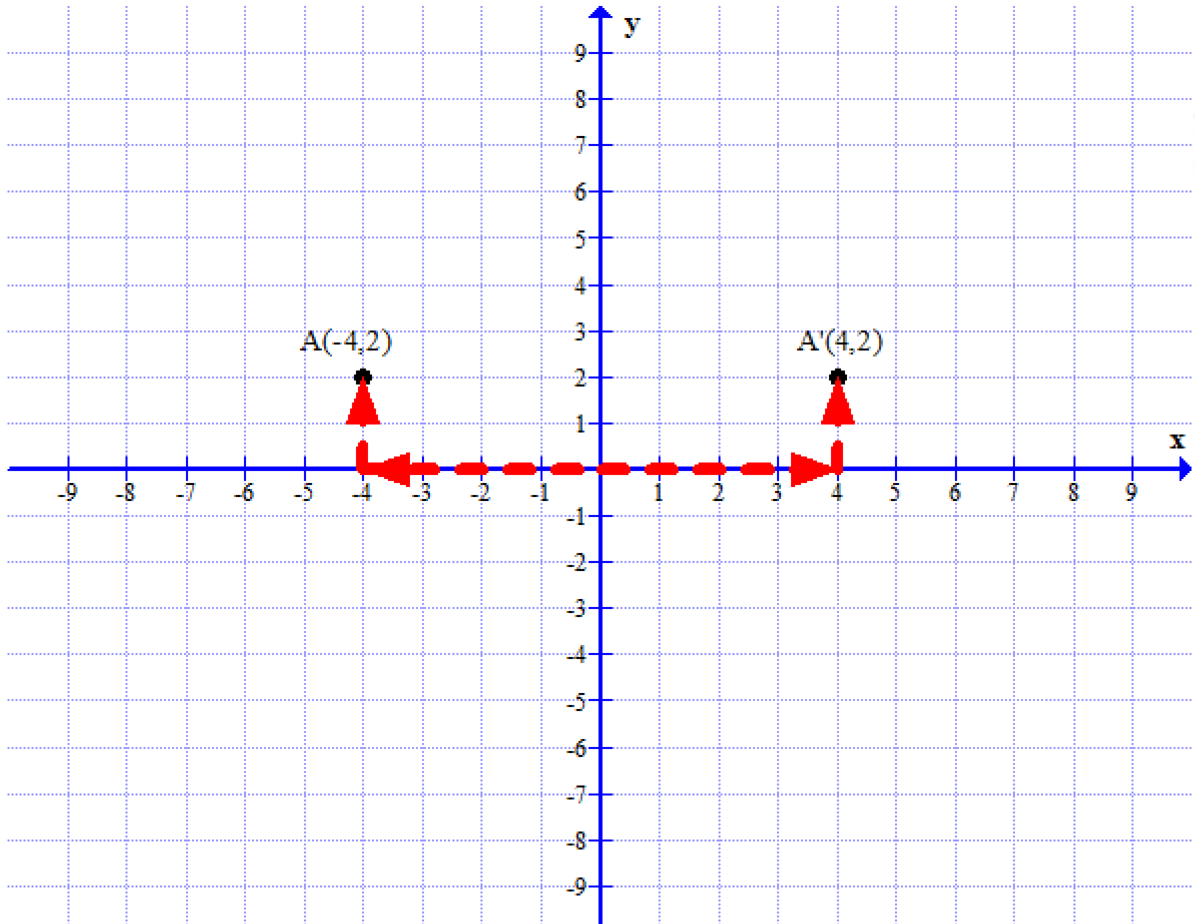
(c) Consider 1 unit on the graph to be 2 cm.

(i)To plot the point A(-4,2), move 4 units along the negative x-axis. Then move 2 units along the positive y axis.
To plot the point B(2,4), move 2 units along the positive x-axis. Then move 4 units along the positive y axis.



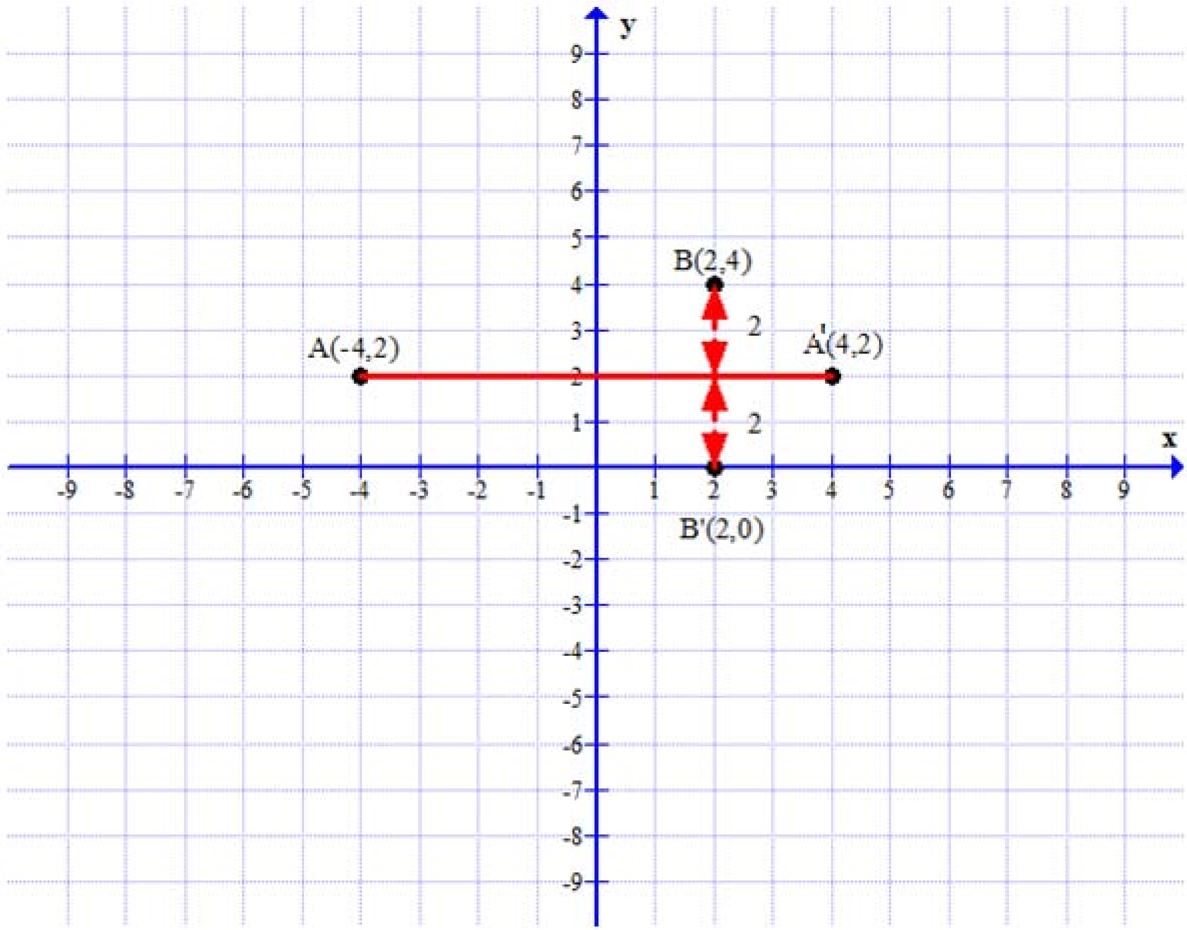
(ii) The y-axis acts as a line of symmetry between A and A'. Thus, perpendicular distance of A from y-axis = perpendicular distance of A' from y-axis. Thus, the y-coordinate of A' will be same as A, and the x-coordinate of A' will be the negative of the x-coordinate of A.

Thus, the coordinates of A' will be (4,2). Plot these in the same way as was done in (i).

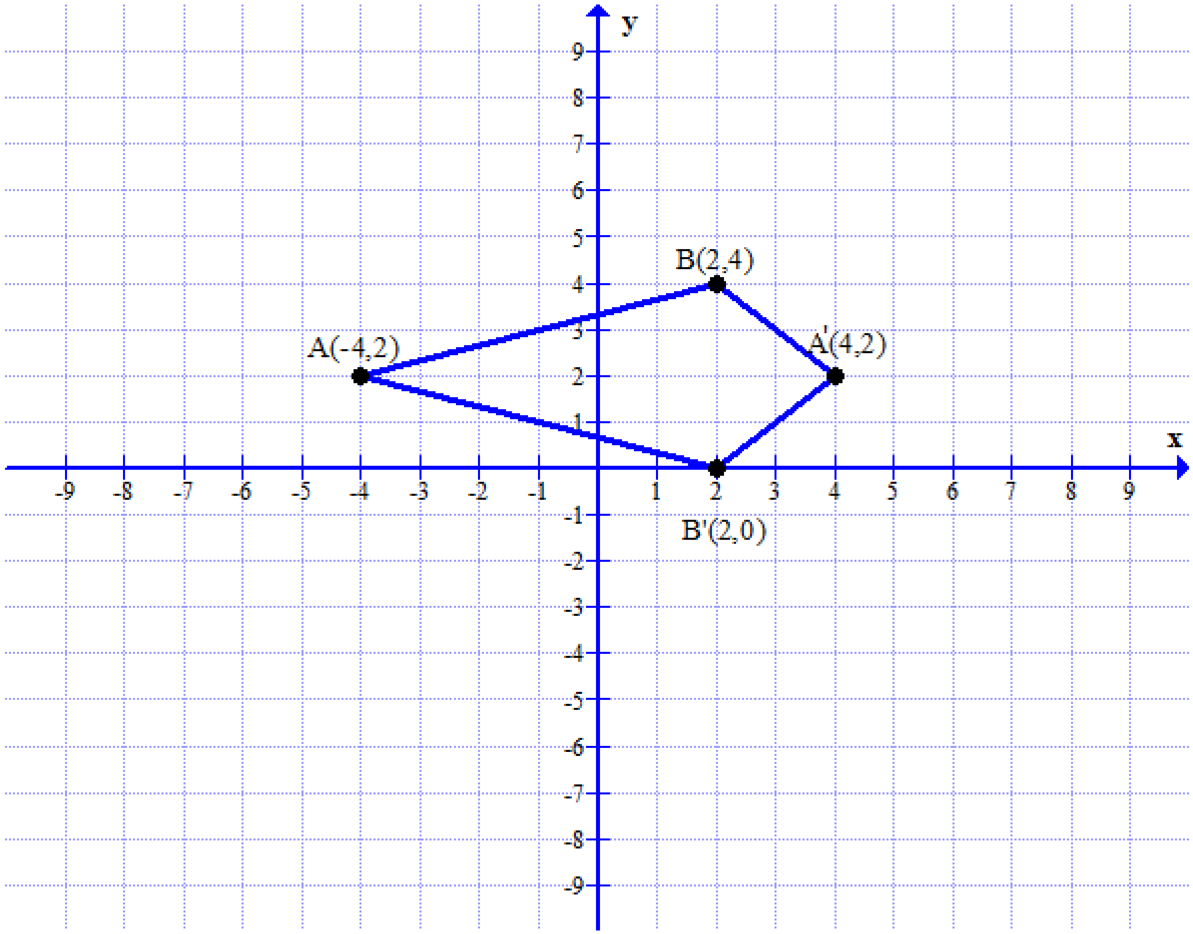


(iii) The line AA’ acts as a line of symmetry between B and B’. Thus, perpendicular distance of B from AA’ = perpendicular distance of B’ from AA’. Thus, the x-coordinate of B’ will be same as B, and the y-coordinate of B’ will be the same distance away from AA’ as B is.

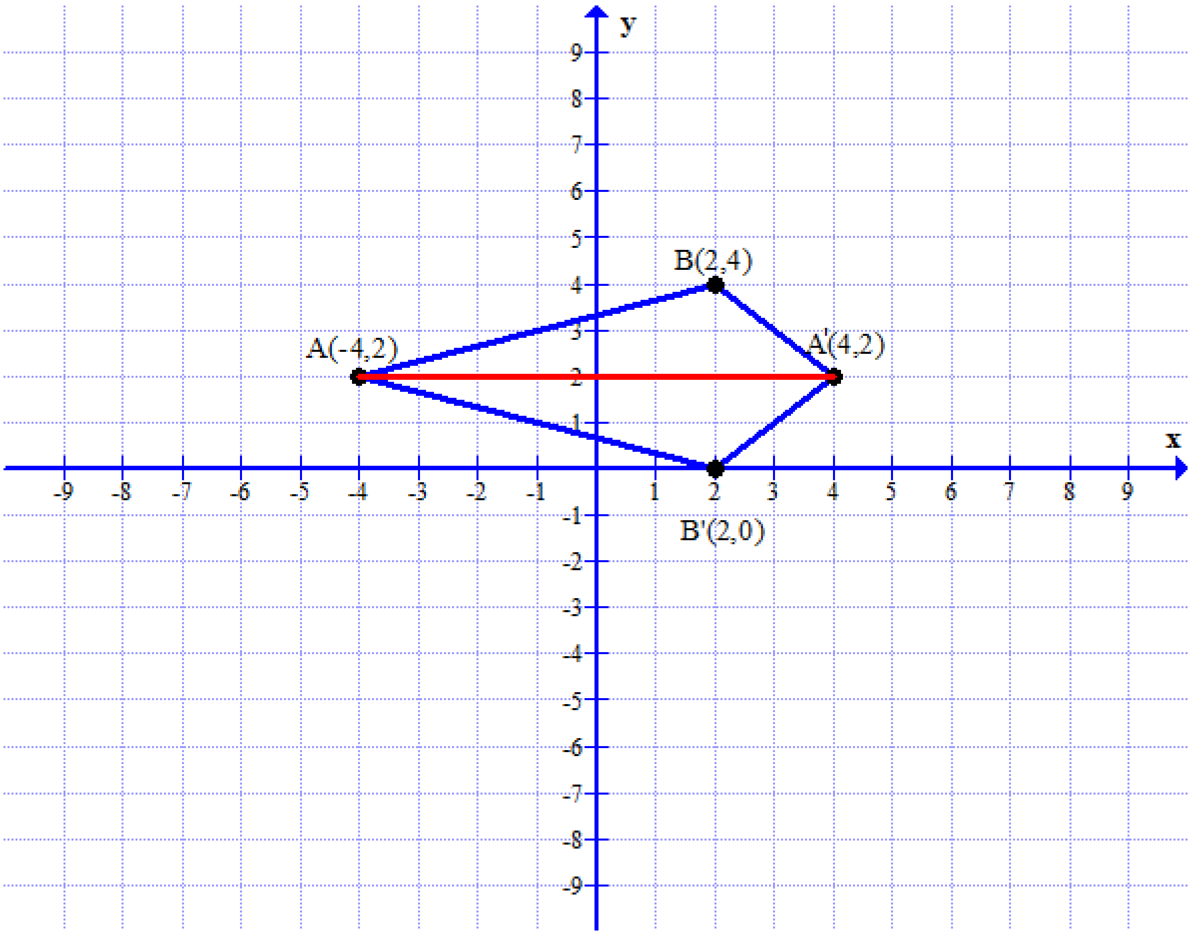
Thus, the coordinates of B’ will be (2,0). Plot these in the same way as was done in (i)



(iv) It can be observed that the figure that is formed by all the 4 points has 4 sides and thus, is a quadrilateral. Since the four sides can be grouped into two pairs of equal-length sides that are adjacent to each other, it is a kite.



(v) A line of symmetry is a line which creates a mirror image on both sides. Thus, in the image, line AA’ is the line of symmetry.



SECTION B (40 Marks)

Attempt any four questions from this section

5.

(a)

Printed price of the washing machine = Rs. 18,000

Discount = 20% of 18,000 = $\frac{20}{100}\times18000=3600$

Thus, sale price for the wholesaler = 18000 – 3600 = Rs. 14,400

Sales tax paid by shopkeeper = 8% of 14,400 = $\frac{8}{100}\times14400=1152$

The shopkeeper sells the washing machine for 10%disount on printed price.
Thus, the shopkeeper sells the washing machine to the customer at the price :

$18000-\frac{10}{100}\times18000=18000-1800=16200$

Thus, the tax charged by the shopkeeper = 8% of 16,200 = $\frac{8}{100}\times16200=1296$

i. Thus, VAT paid by the shopkeeper = Tax charged – Tax paid = 1296 – 1152 = Rs. 144
ii. Total amount that the customer pays for the washing machine is:
Price at which the shopkeeper sells the washing machine + Tax charged by the shopkeeper = 16,200 + 1296 = Rs. 17,496

(b) It is given that:

$$\frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$$

Applying componendo and dividendo,

$$\frac{x^2+y^2+x^2-y^2}{x^2+y^2-x^2+y^2}=\frac{17+8}{17-8}$$

$$\Rightarrow\frac{2x^2}{2y^2}=\frac{25}{9}$$

$$\Rightarrow\frac{x}{y}=\sqrt{\frac{25}{9}}$$

$$\Rightarrow\frac{x}{y}=\pm\frac{5}{3}$$

i. Thus, $\frac{x}{y}=\pm\frac{5}{3}$

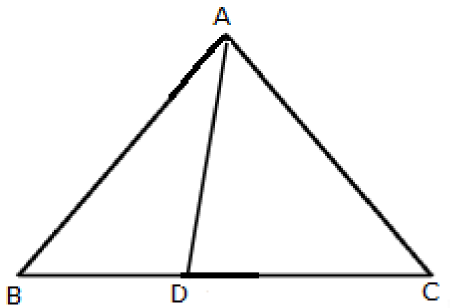
ii. Now, consider $\frac{x^3}{y^3}=\left(\frac{x}{y}\right)^3=\pm\left(\frac{5}{3}\right)^3=\pm\frac{125}{27}$

Again, applying componendo and dividendo,

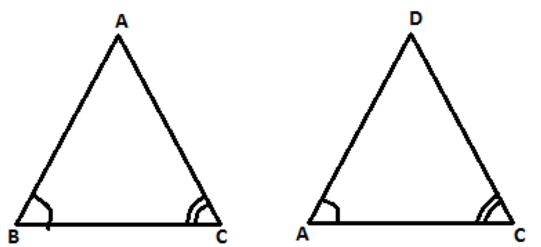
$$\frac{x^3+y^3}{x^3-y^3}=\frac{125+27}{125-27} \text{ or } \frac{-125+27}{-125-27}$$

$$\Rightarrow \frac{x^3+y^3}{x^3-y^3}=\frac{152}{98}=\frac{76}{49} \text{or } \frac{49}{76}, \text{ depending upon the sign of x and y}$$

(c) Consider the given triangle



Given that $\angle ABC=\angle DAC$.



(i) Consider the triangles ΔABC and ΔDAC.

$\angle ABC=\angle DAC$ [given]

$\angle C=\angle C$ [common]

By AA-Similarity, $\Delta ABC \sim \Delta DAC$.

(ii) Hence the corresponding sides are proportional.

$$\frac{AB}{DA}=\frac{AC}{DC}=\frac{BC}{AC}$$

$$\Rightarrow \frac{8}{5}=\frac{4}{DC}$$

$$\Rightarrow DC=\frac{4\times 5}{8}$$

$$\Rightarrow DC=\frac{5}{2} \text{ cm}=2.5\text{cm}$$

$$\because \frac{AB}{DA}=\frac{BC}{AC}$$

$$\Rightarrow \frac{8}{5} = \frac{BC}{4}$$

$$\Rightarrow BC = \frac{8 \times 4}{5}$$

$$\Rightarrow BC = \frac{32}{5} \text{ cm} = 6.4\text{cm}$$

(iii) We need to find the ratios of the area of the triangles ΔABC and ΔDAC.

Since the triangles ΔABC and ΔDAC are similar triangles, we have

$$\frac{AB}{DA} = \frac{AC}{DC} = \frac{BC}{AC}$$

If two similar triangles have sides in the ratio, a:b,

then their areas are in the ratio $a^2 : b^2$

$$\text{Thus, } \frac{\text{Area}(\Delta ACD)}{\text{Area}(\Delta ABC)} = \frac{AB^2}{DA^2} = \frac{AC^2}{DC^2} = \frac{BC^2}{AC^2}$$

$$\text{So, } \frac{\text{Area}(\Delta ACD)}{\text{Area}(\Delta ABC)} = \frac{4^2}{8^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta ACD)}{\text{Area}(\Delta ABC)} = \frac{16}{64} = 1:4$$

6.

(a) If 3 points are collinear, the slope between any 2 points is the same. Thus, for A(a,3), B(2,1) and C(5,a) to be collinear, the slope between A and B and between B and C should be the same.

$$\frac{1-3}{2-a}=\frac{a-1}{5-2}$$
$$\Rightarrow \frac{-2}{2-a}=\frac{a-1}{3}$$
$$\Rightarrow \frac{2}{a-2}=\frac{a-1}{3}$$
$$\Rightarrow 6=(a-2)(a-1)$$
$$\Rightarrow a^2-3a+2=6$$
$$\Rightarrow a^2-3a-4=0$$
$$\Rightarrow a=-1 \text{ or } 4$$

Rejecting a = -1 as it does not satisfy the equation, we have a = 4

Thus, slope of BC :

$$\frac{a-1}{5-2}=\frac{4-1}{3}=\frac{3}{3}=1=m$$

Thus, the equation of the line can be :

$$\frac{y-1}{x-2}=1$$
$$\Rightarrow y-x=-1$$
$$\Rightarrow x-y=1$$

(b) Given :

Nominal Value (NV) of each share = Rs. 50

Since the shares are quoted at 20% premium, the market value of each share is:

Market Value (MV) of each share = $50+\frac{20}{100}\times 50=\text{Rs.}60$

(i) Dividend = Number of shares × dividend percentage × NV

Let n be the number of shares.

Thus,

$600=n\times 15\%\times 50$

$\Rightarrow 600=n\times\frac{15}{100}\times 50$

$\Rightarrow n=\frac{600\times 2}{15}$

$\Rightarrow n=80$

(ii) Total investment = n X MV

Thus, total investment is: 80 × 60 = 4800

(iii) Rate of interest = $\frac{\text{Dividend}}{\text{Total Investment}}\times 100$

$=\frac{600}{4800}\times 100$

$=12.5\%$

(c)

(i) Total surface area of the sphere = $4\pi r^2$, where r is the radius of the sphere.

Thus,

$4\pi r^2=2464\text{ cm}^2$

$\Rightarrow 4\times\frac{22}{7}\times r^2=2464$

$\Rightarrow r^2=196$

$\Rightarrow r=14\text{ cm}$

(ii) Since the sphere is melted and recast into cones,

Volume of a sphere = n × Volume of a cone, where n is the number of cones.

$\frac{4}{3}\times\pi\times r^3=n\times\frac{1}{3}\times\pi\times r_c^2\times h_c$, where r_c and h_c are the radius and height of the cone.

Thus,

$\frac{4}{3}\times\pi\times(14)^3=n\times\frac{1}{3}\times\pi\times(3.5)^2\times(7)$

$\Rightarrow 4\times(14)^3=n\times(3.5)^2\times(7)$

$\Rightarrow n=128$

7.

(a) Let A be the assumed mean and d be the deviation of x from the assumed mean.
Let A = 40.
d = x – A

Marks (C.I.)	No. of students (Frequency f)	Mid-point of C.I. (x)	d = x – A A = 45.5	f X d
11-20	2	15.5	-30	-60
21-30	6	25.5	-20	-120
31-40	10	35.5	-10	-100
41-50	12	45.5	0	0
51-60	9	55.5	10	90
61-70	7	65.5	20	140
71-80	4	75.5	30	120
	Total f = 50			Total f_d=70

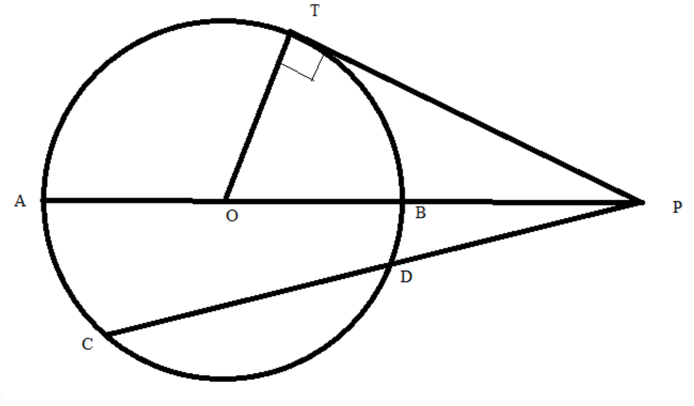
$$\text{Mean} = A + \frac{\text{Total } f_d}{\text{Total } f}$$

$$\Rightarrow \text{Mean} = 45.5 + \frac{70}{50}$$

$$\Rightarrow \text{Mean} = 45.5 + 1.4$$

$$\therefore \text{Mean} = 46.9$$

(b) Theorem used: Product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.



(i) As chord CD and tangent at point T intersect each other at P,
$PC \times PD = PT^2$ ..... (i)
AB is the diameter and tangent at point T intersect each other at P,
$PA \times PB = PT^2$ ....(ii)
From (i) and (ii), $PC \times PD = PA \times PB$ ... (iii)

Given: PD = 5cm, CD = 7.8 cm
PA = PB + AB = 4 + AB, and PC = PD + CD = 12.8 cm
Subs. these values in (iii),
12.8 × 5 = (4 + AB) × 4

$$\Rightarrow 4 + AB = \frac{12.8 \times 5}{4}$$

$\Rightarrow$ 4 + AB = 16
$\Rightarrow$ AB = 12cm

(ii)

$$PC \times PD = PT^2$$

$$\Rightarrow PT^2 = 12.8 \times 5 = 64$$

$$\Rightarrow PT = 8 \text{ cm}$$

Thus, the length of the tangent is 8 cm.

(c) Given :

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$$

Thus,

$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$$

$$5B = 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

Thus,

$$A^2 + AC - 5B$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$$

8.

(a) C. I. for the third year = Rs. 1452.
C. I. for the second year = Rs. 1320
S.I. on Rs. 1320 for one year = Rs. 1452 – Rs. 1320 = Rs. 132

$\therefore$ Rate of Interest = $\dfrac{132\times100}{1320}=10\%$

Let P be the original sum of money and r be the rate of interest.
Amount after 2 years – Amount after one year = C.I. for second year.

$$P\left(1+\frac{10}{100}\right)^2-P\left(1+\frac{10}{100}\right)=1320$$

$$\Rightarrow P\left[\left(\frac{110}{100}\right)^2-\left(\frac{110}{100}\right)\right]=1320$$

$$\Rightarrow P\left[\left(\frac{11}{10}\right)^2-\frac{11}{10}\right]=1320$$

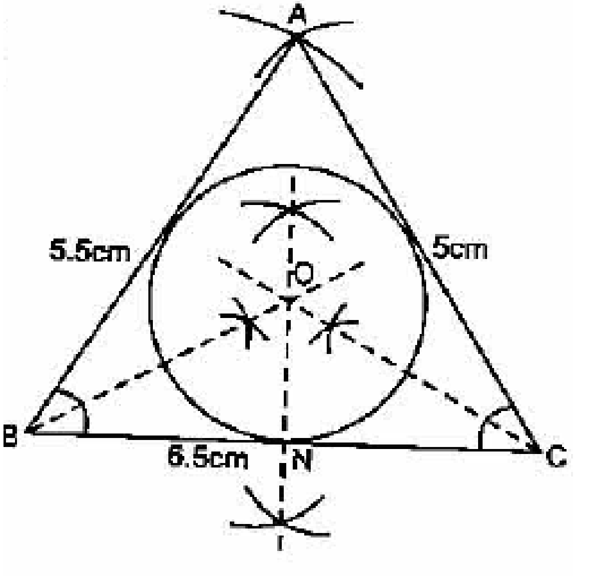
$$\Rightarrow P\left[\frac{121}{100}-\frac{11}{10}\right]=1320$$

$$\Rightarrow P=\frac{1320\times100}{11}=\text{Rs.}12{,}000$$

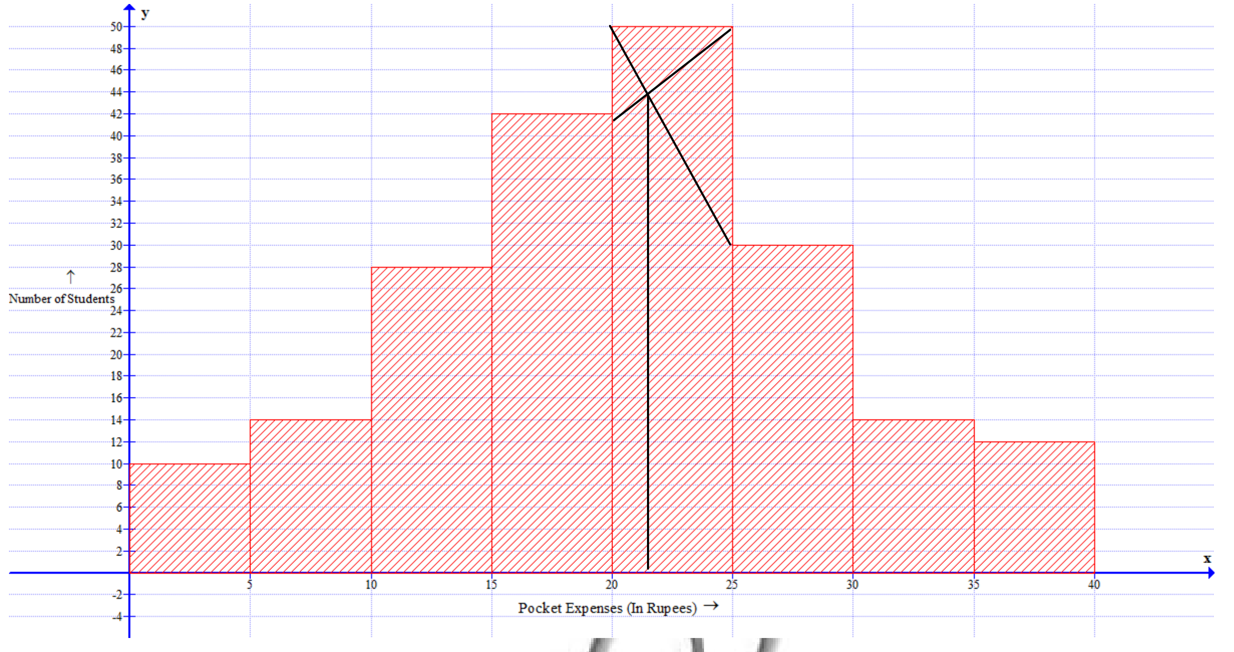
Thus, rate of interest is 10% and original sum of money is Rs.12,000.

(b) The incircle of the triangle is drawn with the joining point of all angular bisectors as the center.

- Construct a ΔABC with the given data.
- Draw the angle bisector of $\angle B$ and $\angle C$. Let these bisectors cut at O.
- Taking O as centre. Draw a incircle which touches all the sides of the ΔABC
- From O draw a perpendicular to side BC which cut at N.
- Measure On which is required radius of the incircle. ON = 1.5cm



(c)



Mode = 21

9.

(a)Duplicate ratio of a:b is $a^2 : b^2$

It is given that the duplicate ratio of $(x-9):(3x+6)=4:9$

Thus,

$(x-9)^2 : (3x+6)^2 = 4:9$

$$\Rightarrow \frac{x-9}{3x+6} = \frac{4^2}{9^2}$$

$$\Rightarrow \frac{x-9}{3x+6} = \frac{16}{81}$$

$$\Rightarrow 81x - 729 = 48x + 96$$

$$\Rightarrow 81x - 48x = 96+729$$

$$\Rightarrow 33x = 825$$

$$\Rightarrow x = \frac{825}{33} = 25$$

(b)

The given quadratic equation is :

$(x-1)^2 - 3x + 4 = 0$

$\Rightarrow x^2 - 2x + 1 - 3x + 4 = 0$

$\Rightarrow x^2 - 5x + 5 = 0$

The roots of the quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the given equation,
a = 1, b = -5, c=5

Thus, the roots of the equation are :

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

$$x = \frac{5 + \sqrt{5}}{2} \text{ or } x = \frac{5 - \sqrt{5}}{2}$$

$$x = 3.618 \text{ or } x = 1.382$$

(c) Qualifying principal for various months:

Month	Principal
April	6000
May	7000
June	10000
July	6000
August	6000
September	7000
Total	42000

Here, P= Rs.42,000

Let R% be the rate of interest and T=$\frac{1}{12}$ year

Given that the interest , I=Rs.175

Thus, we have

$$I=\frac{P\times R\times T}{100}$$

$$\Rightarrow 175=\frac{42000\times R\times 1}{100\times 12}$$

$$\Rightarrow \frac{175\times 100\times 12}{42000}=R$$

$$\Rightarrow R=5\%$$

Thus the rate of interest is 5%

10.

(a)

Let the digit at the tens place be 'a' and at units place be 'b'.
The two-digit so formed will be 10a+b.

According to given conditions, product of its digits is 6.
Thus,

$a \times b = 6$

$\Rightarrow a = \frac{6}{b}$ ....(1)

9 is added to the number = 10a+b+9

It is given that this new value is equal to the value of the reversed number.
If the digits are reversed, the new number formed = 10b+a.

Thus,

$10a + b + 9 = 10b + a$

$\Rightarrow 9a - 9b = 9$

$\Rightarrow a - b = 1$

Substitute (1) in the above equation

Thus,

$\Rightarrow a - \frac{6}{a} = 1$

$\Rightarrow a^2 - a - 6 = 0$

Thus,

$a = -3$ or 2

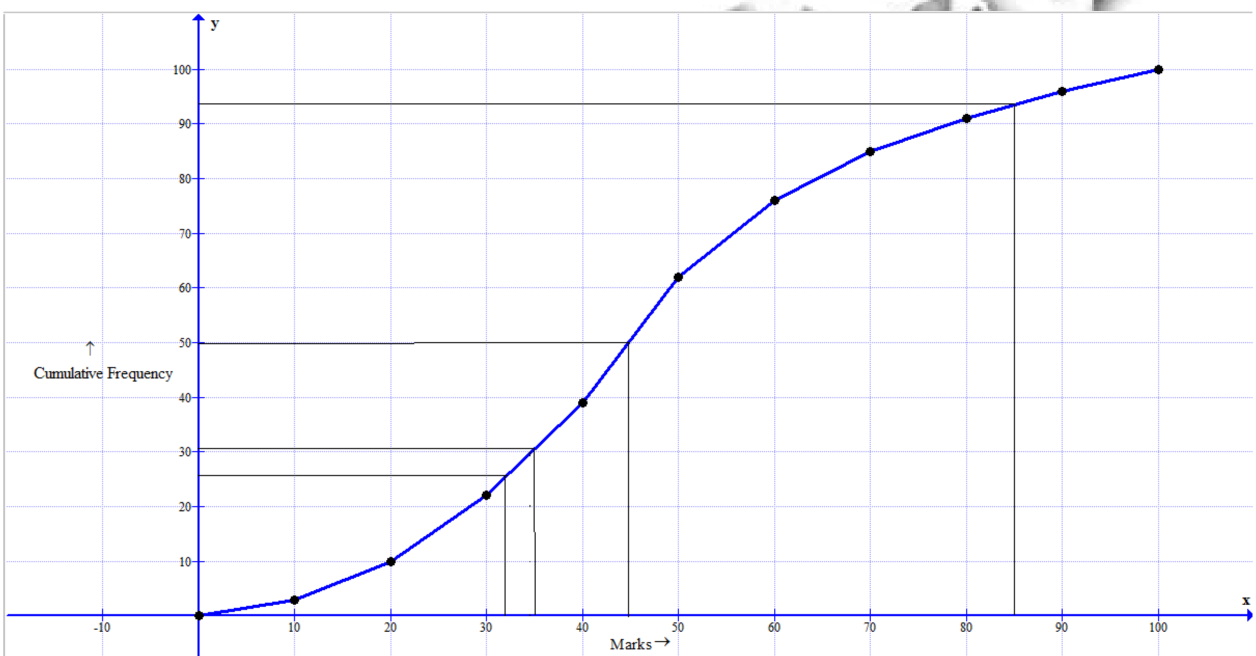
Since a digit cannot be negative, a = 2.

$b = \frac{6}{a} = \frac{6}{2} = 3$

Thus, the required number is : 10a + b = 23

(b) Draw the cumulative frequency table.

Marks	Number of Students (Frequency)	Cumulative Frequency
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100



Scale : x-axis : 1 unit = 10marks
y-axis : 1 unit = 10 students

(i) Median = $\left(\frac{N}{2}\right)^{th}$ term $= \left(\frac{100}{2}\right)^{th}$ term $= 50^{th}$ term

Draw a horizontal line through mark 50 on y-axis. The, draw a vertical line from the point it cuts on the graph. The point this line touches the x-axis is the median. Thus, median = 45

(ii) Lower quartile = $\left(\frac{N}{4}\right)^{th}$ term $= \left(\frac{100}{4}\right)^{th}$ term $= 25^{th}$ term

Draw a horizontal line through mark 25 on y-axis. The, draw a vertical line from the point it cuts on the graph. The point this line touches the x-axis is the lower quartile.

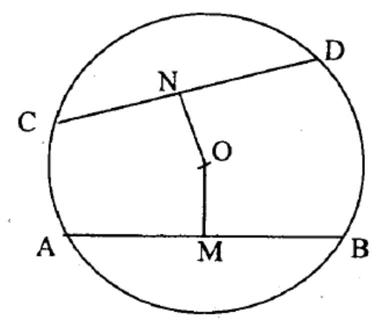
Thus, lower quartile = 31

(iii) Draw a vertical line through mark 85 on x-axis. The, draw a horizontal line from the point it cuts on the graph.
The point this line touches the y-axis is the number of students who obtained less than 85% marks =93
Thus, number of students who obtained more than 85% marks =7

(iv) Draw a vertical line through mark 35 on x-axis. The, draw a horizontal line from the point it cuts on the graph.
The point this line touches the y-axis is the number of students who obtained less than 35% marks =21

11.

(a) A line from center to a chord that is perpendicular to it, bisects it.
It is given that AB = 24 cm
Thus, MB = 12 cm



(i) Applying Pythagoras theorem for ΔOMB,

$OM^2 + MB^2 = OB^2$

$5^2 + 12^2 = OB^2$

$OB = 13$

Thus, radius of the circle = 13 cm.

(ii) Similarly, applying Pythagoras theorem for ΔOND,

$ON^2 + ND^2 = OD^2$

OD is the radius of the circle

$12^2 + ND^2 = 13^2$

$ND = 5$

A line from center to a chord that is perpendicular to it, bisects it.
ND = 5 cm
Thus, CD = 10 cm

(b) Consider LHS:

$$(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)$$

$$=(\sin\theta+\cos\theta)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$=(\sin\theta+\cos\theta)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$=\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}$$

$$=\frac{\sin\theta}{\cos\theta\sin\theta}+\frac{\cos\theta}{\cos\theta\sin\theta}$$

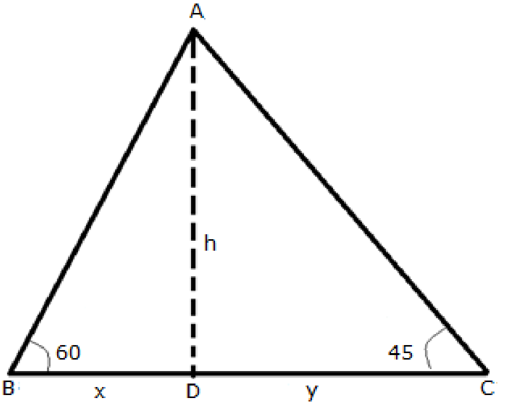
$$=\frac{1}{\cos\theta}+\frac{1}{\sin\theta}$$

$$=\sec\theta+\cos ec\theta$$

=RHS

Thus, L.H.S. = R.H.S.

(c) Let A be the position of the airplane and let BC be the river. Let D be the point in BC just below the airplane.



For ΔADC,

$$\tan 45^\circ=\frac{h}{y}$$

$$1=\frac{250}{y}$$

$$y=250$$

For ΔADB,

$$\tan 60^\circ$$

$$=\frac{AD}{DB}$$

$$=\frac{h}{x}$$

$$=\frac{250}{x}$$

$$\Rightarrow x=\frac{250}{\tan 60^\circ}=\frac{250}{\sqrt{3}}\text{ m}$$

Thus, the width of the river = DB + DC = $250+\dfrac{250}{\sqrt{3}}=394$ m