

Sample Paper-05 (unsolved)

Mathematics

Class - XII

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- All questions are compulsory.
- The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- Use of calculators is not permitted.

Section A

- If $A^{-1} = \begin{bmatrix} 3 & 4 \\ -2 & 8 \end{bmatrix}$, find $(A^T)^{-1}$.
- If $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{3}$ and angle between \vec{a} and \vec{b} is 45° , find $\vec{a} \cdot \vec{b}$.
- Draw the graph of $f(x) = \begin{cases} \frac{|x|}{x}; x \neq 0 \\ 0; x = 0 \end{cases}$.
- Find the value of λ , so that $\begin{bmatrix} 7 & 1 \\ 2 & \lambda \end{bmatrix}$ may be singular.
- Write the domain of $\tan^{-1}(x)$.
- Find B s.t. $AB=I$, $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$.

Section B

- Using properties of determinants, prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2 + b^2 + c^2)(a+b+c)(b-c)(c-a)(a-b)$$

8. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{3\sin x + 4\cos x}{5}\right)$.
9. Show that an onto function $f : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ is always 1-1.
10. Find the particular solution of $\frac{dy}{dx} + 2y \tan x = \sin x$, $y\left(\frac{\pi}{3}\right) = 0$
11. Prove that $\tan^{-1}(2x) - \tan^{-1}(3x) = \frac{\pi}{4}$.
12. A line makes angles $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube, prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$
13. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that (i) both are black (ii) both are red?
14. Determine k s.t. the function f defined by :
- $$f(x) = \begin{cases} kx+1, & x \leq 5 \\ 3x-5, & x > 5 \end{cases} \text{ is continuous at } x=5.$$
15. Show that $y = \frac{4\sin \theta}{2 + \cos \theta} - \theta$ is an increasing function of θ in $[0, \frac{\pi}{2}]$.
16. Define dot product of vectors and find the value of λ such that $\vec{a} \perp \vec{b}$ where $\vec{a} = 3i + 3j + \lambda k$ and $\vec{b} = \lambda i - j + 4k$.
17. Integrate $\int x^{2n-1} \sin x^n dx$.
18. Find the equation of the plane passing through the points $(2,3,4), (5,6,7), (1,0,0)$.
19. Find the area of triangle with vertices $(2,3,4), (5,1,6), (7,-1,3)$.

Section C

20. Solve the following system of equations:

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

21. Show that the height of a closed cylinder of given volume and minimum surface area is equal to its diameter.
22. A manufacturer produces two types of steel trunks. He has machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machine A and B can work for 18 and 15 hours at most in a day. He earns a profit of Rs. 30 on the first trunk and a profit of Rs. 25 per trunk on the second trunk. How many trunks of each type must be made each day to maximize his profit.
23. Find the area of the smaller region enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$.
24. A Company has two plants to manufacture cycles. The first plant manufactures 60% of the cycles and the second plant manufactures 40%. 80% of the cycles of the first plant are rated of standard quality and 90% of the cycles of the second plant are rated of standard quality. A cycle is picked up at random and it is found to be of standard quality. Find the probability that it comes from the second plant.
25. If $(x-a)^2 + (y-b)^2 = c^2$, for some constant $c > 0$, prove that $\frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.
26. Evaluate $\int \frac{1}{\sin x(2 + \cos x)} dx$