JAYPEE INSTITUTE OF INFORMATION TECHNOLOGY, NOIDA
TEST-I (ODD SEMESTER, 2011-2012)

Course Name: Mathematics-I
Course Code: 10B11MA111

Max. Marks: 15
Max Time: 1 Hr.

Note: Attempt all questions .

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1. Test the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{2xy}{2x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$ (2)

Are partial derivatives f_x and f_y exist at the origin? If yes, justify your answer.

2. If $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = k \sin 2u\,(1 - 4\sin^2 u)$

Find the value of k, where $u = \tan^{-1}\left[\dfrac{x^3 + y^3}{x - y}\right]$. (2)

3. Find the area lying inside the circle $r = a \sin\theta$ and outside the cardioid $r = a(1 - \cos\theta)$. (2)

4. Use Taylor's formula to find quadratic approximation of $f(x, y) = \cos x \cos y$ at the origin. Estimate the error in the approximation if $|x| \leq 0.1$ and $|y| \leq 0.1$. (3)

5. The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant ? (3)

6. Use multiple integral to evaluate the mass of a solid uniformly distributed over the sphere $x^2 + y^2 + z^2 = 9$. (3)