- **Nusselt number:**

$$Nu = \frac{hL}{k_f} = +\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

- This parameter is equal to the dimensionless temperature gradient at the surface, and it provides a measure of the convection heat transfer occurring at the surface.
- The Nusselt number is to the thermal boundary layer what the friction co-efficient is to the velocity boundary layer.
- The functional dependence of the average Nusselt number is :

$\overline{Nu}$ — f , r

- **Sherwood number:**

$$Sh = \frac{h_m L}{D_{AB}} = \left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=0}$$

- This parameter is equal to the dimensionless concentration gradient at the surface, and it provides a measure of the convection mass transfer occurring at the surface.
- The Sherwood number is to the concentration boundary layer what the Nusselt number is to the thermal boundary layer.
- Similar to average Nusselt number, the functional dependence of the average Sherwood number:

$\overline{S}$ — B , S

Physical significance of the dimensionless parameters:

Reynolds number:

- Reynolds number is defined as the ratio of inertia to viscous forces.
- Inertia force per unit area acting on the fluid mass is: —
- Viscous force per unit area acting on the fluid mass is: —
- Reynolds number $\frac{inertia\ force}{i\quad co\ force}$ ρ —
- For small Re_L , viscous forces are sufficiently large relative to inertia forces.
- With increasing Re_L ,viscous effects become progressively less important relative to inertia effects
- With increasing Re_L at a fixed location on a surface, viscous force become less influential relative to inertia forces. Hence the effects of viscosity do not penetrate as far in to the free stream, and the value of δ diminishes.

Prandtl number:

- Prandtl number is defined as the ratio of momentum diffusivity to the thermal diffusivity.
- The Prandtl number provides a measure of the relative effectiveness of momentum and energy transport by diffusion in the velocity and thermal boundary layers respectively.
- The Prandtl number of gases is near unity, in which case energy and momentum transport by diffusion are comparable.
- In a liquid metals r 1, the energy diffusion rate greatly exceeds the momentum diffusion rate. The opposite is true for oils, r 1.

Schmidt number:

- Schmidt number provides a measure of the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers, respectively.
- For convection mass transfer in laminar flows, it determines the relative velocity and concentration boundary layers thicknesses, where

$$\frac{}{_c} \quad S \quad /$$

Lewis number:

- Lewis number provides a measure of the relative effectiveness of energy and mass transport by diffusion in the thermal and concentration boundary layers, respectively.

$$\frac{}{_B} \quad \frac{S}{r}$$

- The Lewis number is a measure of the relative thermal and concentration boundary layer thickness

$$\frac{t}{c} \quad /$$

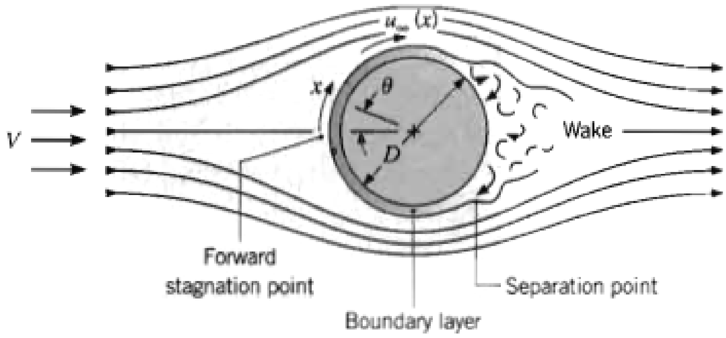
Grashof number:

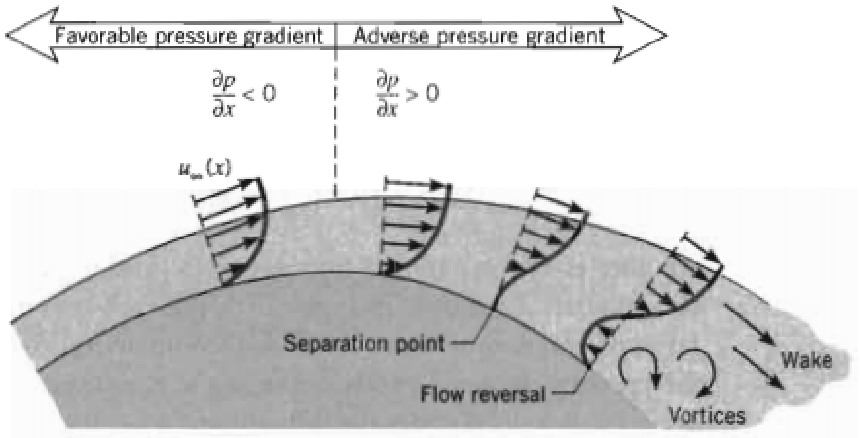
- Grashof number provides a measure of the ratio of buoyancy forces to the viscous forces in the velocity boundary layers.

$$r \quad \frac{ó}{\upsilon}$$

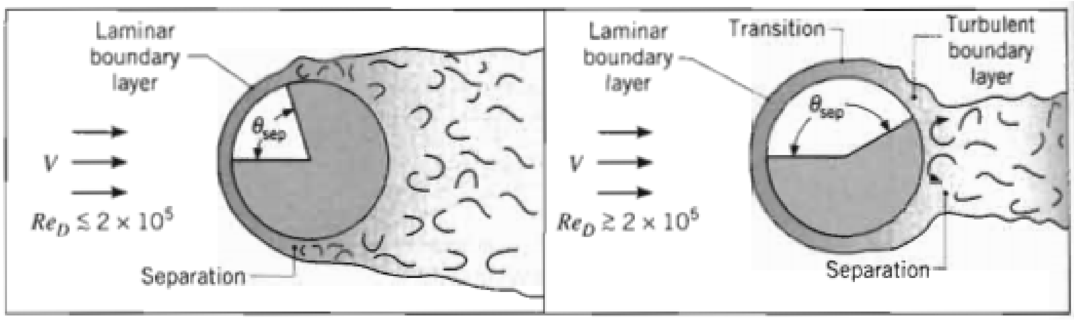
- Its role in free convection is much the same as that of the Reynolds number in forced convection.
- It is the primary variable used as a criterion for transition from laminar to turbulent boundary-layer flow.

Flow across cylinder and sphere:





- Unlike conditions for the flat plate in parallel flow, the upstream velocity V and free stream velocity differ with depending on the distance x from the stagnation point.
- At the separation point, fluid near the surface lack sufficient momentum to overcome the pressure gradient and continued downstream movement is impossible.
- The separation point is the location for which —
- Since the momentum of fluid in turbulent boundary is larger than in the laminar boundary layer, it is reasonable to expect transition to delay the occurrence of separation. If the boundary layer remains laminar, and separation occurs at . However, if , boundary layer transition occurs and separation is delayed to
- The drag force acting on the cylinder has two components: one of which is due to the boundary surface shear stress (friction drag). The other component is due to a pressure difference in the flow direction resulting from formation of the wake (pressure drag).

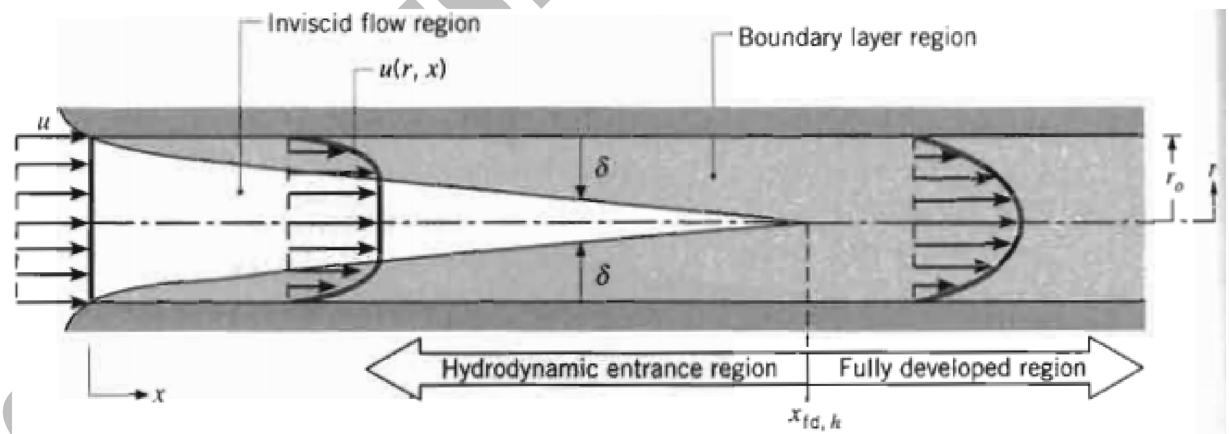


- For the separation effects are negligible and conditions are dominated by friction drag.
- Drag coefficient may be defined as:

$$C_d = \frac{F_d}{A_f \left(\frac{\rho V^2}{2} \right)}$$

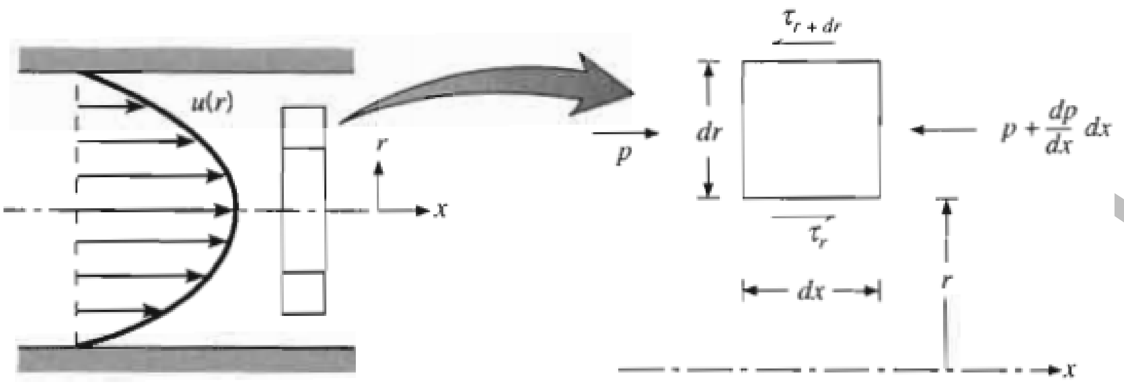
is the cylinder frontal area (the area projected perpendicular to the free stream velocity).

Flow inside tube:



- When the fluid makes contact with the surface, viscous effects become important and a boundary layer develops with increasing x. this development occurs at the expense of a shrinking inviscid flow region and concludes with boundary layer merger at the center line. Following this merger, viscous effects extends over the entire cross section and the velocity profile no longer changes with increasing x. the flow is then said to be **fully developed flow** and the distance from the entrance at which this condition is achieved is termed the **hydrodynamics entry length**.

Velocity profile in the fully developed region:



- For the annular differential element, force balance will be reduced to:

$$-\frac{d}{dr}(r\tau_r)=r\frac{dp}{dx} \quad (7)$$

- From Newton's law of viscosity
- —
- Use this in to equation (7), and after integration, we have

$$u(r)=\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{4}+C_1\ln r+C_2$$

- Boundary conditions:

  1)

  2) —

- Equation of the velocity profile is:

$$u(r)=-\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r_o^2}{4}\left(1-\left(\frac{r}{r_o}\right)^2\right)$$

- This equation can be rewritten as:

$$u(r)=2u_m\left(1-\left(\frac{r}{r_o}\right)^2\right)$$

- Shear stress at the wall is given by:

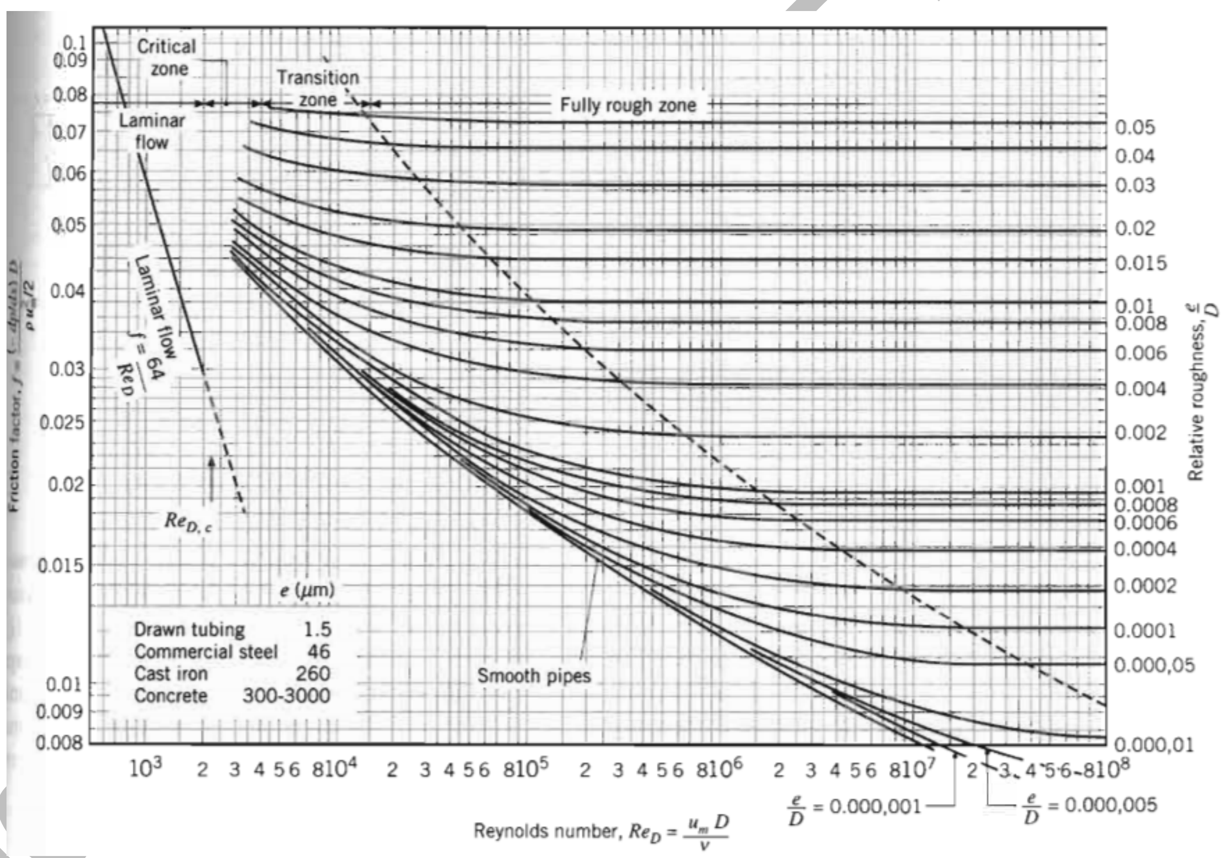
$$\frac{u_m^2}{\quad}$$

- Friction factor f —

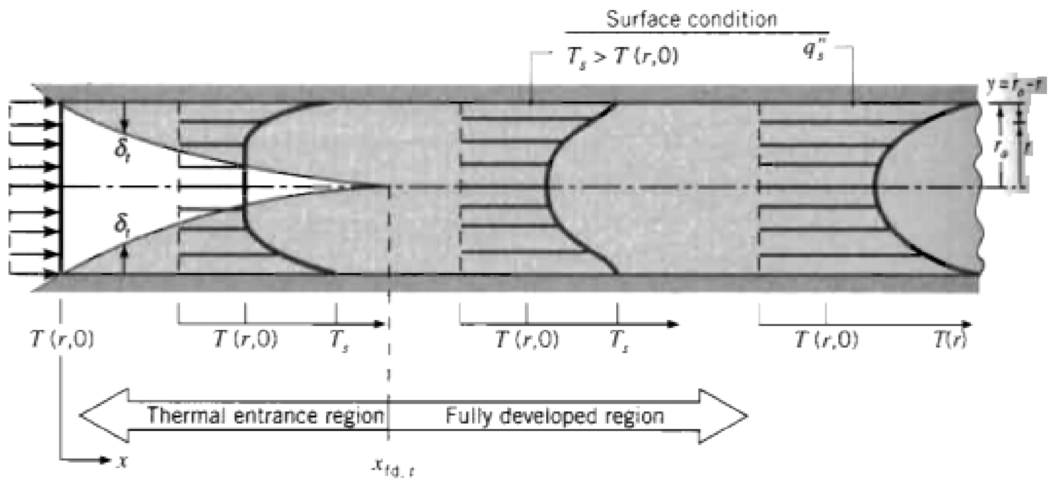
- Friction coefficient = –
- The Reynolds number and Nusselt numbers for flow in these tubes are based on the hydraulic diameter ( ).

—

- The effect of surface roughness on the friction factor and the heat transfer coefficient in laminar flow is negligible.
- The value of friction factor for flow in tubes with smooth as well as rough surface over a wide range of Reynolds number can be find by Moody diagram.

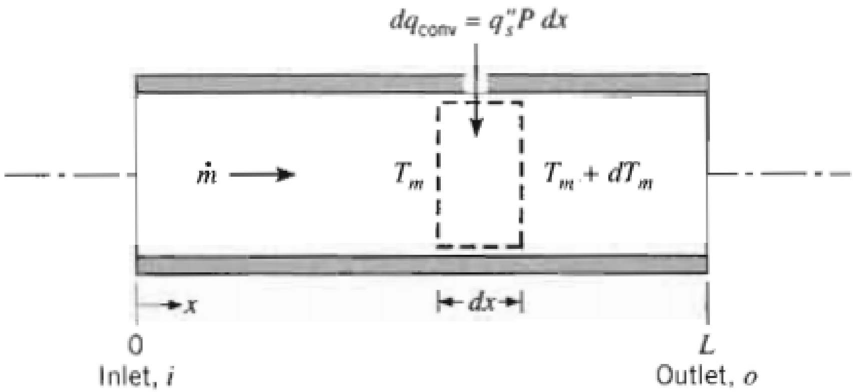


Thermal consideration:



- The shape of the fully developed temperature profile differs according to whether a uniform surface temperature or heat flux is maintained. For both surface conditions, however, the amount by which fluid temperature exceeds the entrance temperature increase with increasing x.
- Although, the temperature profile continue to changes with x, the relative shape of the profile no longer changes and the flow is said to be fully developed flow. The requirement for such a condition is formally stated as:

- In a thermally fully developed flow of a fluid with constant properties, the local convection coefficient is a constant, independent of x.
- Because the thermal boundary layer thickness is zero at the entrance, the convection coefficient is extremely large at x=0. However, h decays rapidly as the thermal boundary layer develops, until the constant value associated with fully developed conditions is reached.
- The mean velocity in actual heating and cooling applications may change somewhat because of the changes in density with temperature.



- Energy equation for this control volume is:
- In case of constant surface heat flux,
- The mean fluid temperature increases linearly in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction.
- The difference ( ) is initially small due to large value of h near the entrance but increases with increasing x due to the decrease in h that occurs as the boundary layer develops. However, in the fully developed region, ( ) is independent of x since h is constant in this region.
- In the case of constant surface temperature,

$$\Rightarrow \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(\frac{-px\bar{h}}{\dot{m}c_p}\right)$$

- The temperature difference decays exponentially with distance along the tube.

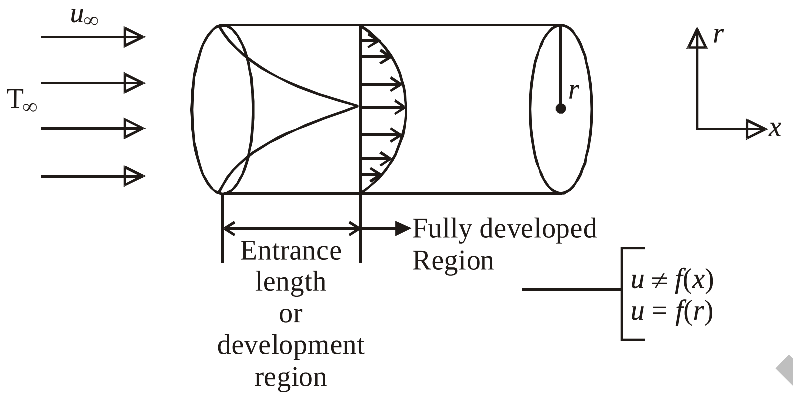
$$q_{conv.} = \dot{m}c_p\left[\left(T_s - T_{m,i}\right) - \left(T_s - T_{m,o}\right)\right] = \dot{m}c_p\left(\Delta T_i - \Delta T_o\right)$$

- Laminar flow:
  1) When temperature of surface is constant Nu = 3.66
  2) When there is uniform heat flux at the surface Nu = 4.36
- Turbulent flow:

1) If the fluid is being heated: $Nu_L = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}$

2) If the fluid is being cooled: $Nu_L = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3}$

Summary: Internal flow:



(*i*) Fully developed Laminar flow inside pipes:

(*a*) When uniform wall heat flux:

$$\boxed{N_{U_D} = 4.364}$$

$$\boxed{q = h \times (\pi DL)(T_s - T_b)}$$

(*b*) At constant wall temperature:

$$\boxed{\bar{N}_U = 3.66}$$

$$\boxed{q = \bar{h} \times (\pi DL)(T_s - T_b)}$$

(*ii*) For development Region:

$$\boxed{\bar{N}_{U_D} = 1.86\left(R_{e_D} . P_r \frac{D}{L}\right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}}$$

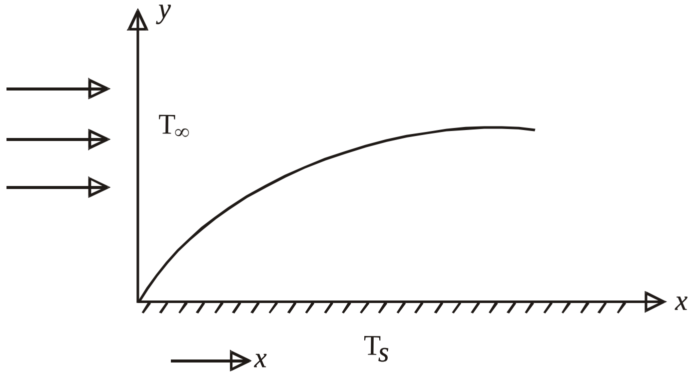
Sieder and Tale (1936)

μ_b – Bulk mean temperature

μ_s – Surface temperature

External Flow : (Plates)

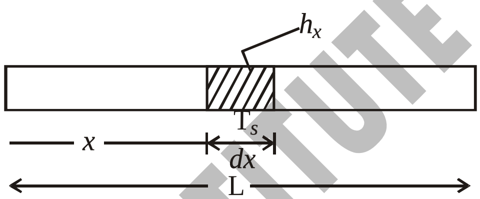
Local and average heat transfer co-efficient.



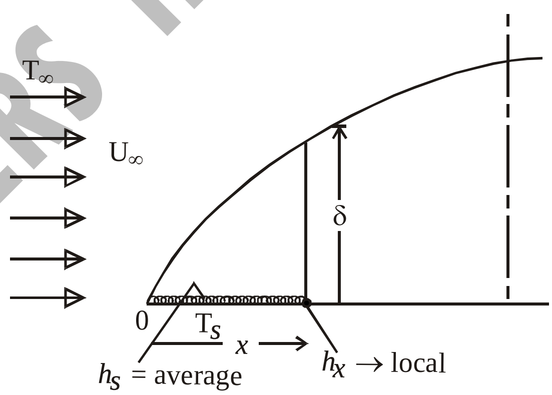
$$q = A_s\, h(T_s - T_\infty) = -K_f A_s \left(\frac{dT}{dy}\right)_{y=0}$$

$$\boxed{h = \frac{-K_f \left(\frac{dT}{dy}\right)_{y=0}}{(T - T_\infty)}}$$

$$\boxed{\bar{h}_L = \frac{1}{L}\int_0^L h_x\, dx} \qquad \boxed{\bar{h}_x = \frac{1}{x}\int_0^x h_x\, dx}$$



Exact solution of Laminar boundary layer over Flat Plate under forced convection:



Note : In all external flow all properties at film temperature (T_f)

$$\boxed{T_f = \frac{T_s + T_\infty}{2}}$$

Local Nusselt number (N_{U_x}) is given by

$$N_{U_x} = 0.332(R_e)^{\frac{1}{2}}(P_r)^{\frac{1}{3}}$$

$$R_{e_x} = \frac{U_\infty x}{\upsilon} < 5 \times 10^5 \qquad C_{f_x} = \frac{0.664}{\sqrt{R_{e_x}}}$$

$$\overline{C}_{f_x} = 2C_{f_x}$$

Reynold Analogy:

$$\frac{N_{U_x}}{P_r \cdot R_{e_x}} = \frac{C_{f_x}}{2} = St_x$$

St_x – Stanton No.

Note : K, P_r, and C_P do not change with pressure but υ (kinematic viscosity) change with pressure.

$K \rightarrow$ Thermal conductivity

$P_r \rightarrow$ Pranditl number

$C_P \rightarrow$ Specific heat