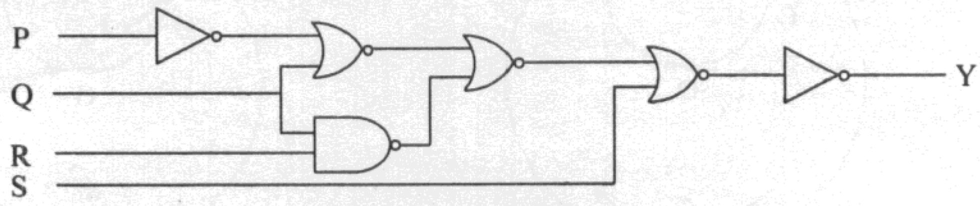
A

Q.11 A photon of wavelength λ is incident on a free electron at rest and is scattered in the backward direction. The fractional shift in its wavelength in terms of the Compton wavelength λ_c of the electron is

(A) $\frac{\lambda_c}{2\lambda}$ (B) $\frac{2\lambda_c}{3\lambda}$ (C) $\frac{3\lambda_c}{2\lambda}$ (D) $\frac{2\lambda_c}{\lambda}$

Q.12 The logic expression for the output Y of the following circuit is



(A) $\overline{\overline{\overline{P}+Q}+\overline{QR}}+S$
(B) $\overline{\overline{\overline{P}+Q}+\overline{QR}+S}$
(C) $\overline{\overline{P}+Q+\overline{QR}}+S$
(D) $\overline{\overline{\overline{P}+Q}+\overline{QR}}+\overline{S}$

Q.13 The activity of a radioactive sample is decreased to 75 % of the initial value after 30 days. The half-life (in days) of the sample is approximately [You may use ln 3 ≈ 1.1, ln 4 ≈ 1.4]

(A) 38 (B) 45 (C) 59 (D) 69

Q.14 The ratio of the second-neighbour distance to the nearest-neighbour distance in an *fcc* lattice is

(A) $2\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ (D) $\sqrt{2}$

Q.15 A thermodynamic system is maintained at constant temperature and pressure. In thermodynamic equilibrium, its

(A) Gibbs free energy is minimum
(B) enthalpy is maximum
(C) Helmholtz free energy is minimum
(D) internal energy is zero

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Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page.

Question No.	Answer	Do not write in this column
01		
02		
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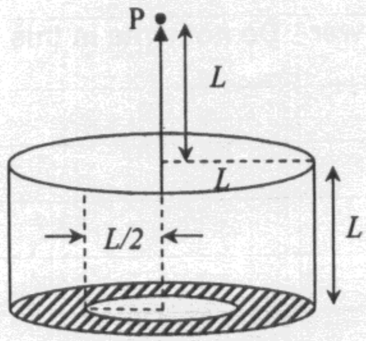
FOR EVALUATION ONLY

No. of Correct Answers		Marks	(+)
No. of Incorrect Answers		Marks	(–)
Total Marks in Question Nos. 1-15			( )

A

Q.16 A thin hollow cylinder of radius and length both equal to L is closed at the bottom. A disc of radius $L/2$ is removed from the bottom as shown in the figure. This object carries a uniform surface-charge density σ. Calculate the electrostatic potential at the point P on the axis of the cylinder as shown in the figure.

[You may use $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2})$.]



(21)

A

Q.17 A particle of mass 1 kg is moving in a central force field given by $\vec{F}_1(\vec{r}) = -\left(\frac{3}{r^2} + \frac{1}{r}\right)\hat{r}$.

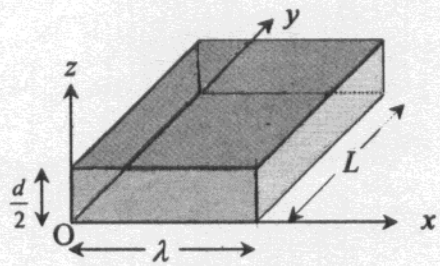
(a) Assuming that the particle is moving in a circular orbit with angular momentum 2 J-s, find out the radius of the orbit. (12)

(b) At $t = 0$, an additional force $\vec{F}_2 = -\lambda \vec{v}$, where $\vec{v}$ is the instantaneous velocity of the particle, is switched on. Show that the magnitude of its angular momentum after a time $\frac{1}{\lambda}$ second is $\frac{2}{e}$ J-s. (9)

A

A

Q.18 An incompressible fluid is enclosed between two horizontal surfaces located at $z = 0$ and $z = d$. The fluid motion is two dimensional, and the velocity field $\vec{V}(x,z,t)$ is given by $\vec{V}(x,z,t) = u(x,z,t)\hat{x} + w(x,z,t)\hat{z}$, where $u(x,z,t)$ and $w(x,z,t)$ are periodic functions of the horizontal coordinate x with wavenumber k.



(a) If the vertical velocity $w(x,z,t) = A(t)\cos(kx)\sin(\frac{\pi z}{d})$, find the horizontal velocity $u(x,z,t)$ using the equation of continuity. What is the vorticity field $\vec{\Omega} = \vec{\nabla} \times \vec{V}$? (15)

(b) Find the net fluid flux from a parallelepiped of size $\lambda \times L \times \frac{d}{2}$ as shown in the figure, where $\lambda = \frac{2\pi}{k}$. (6)

A

A

Q.19 The wave function $\Psi_n(x)$ of a particle confined to a one-dimensional box of length L with rigid walls is given by $\Psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$, $n = 1, 2, 3, \ldots$.

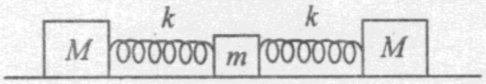
(a) Determine the energy eigenvalues. Also, determine the eigenvalues and the eigenfunctions of the momentum operator. (15)

(b) Show that the energy eigenfunctions are not the eigenfunctions of the momentum operator. (6)

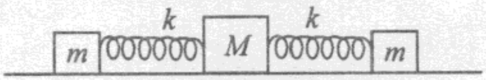
A

A

Q.20 A mass and spring system consists of two blocks of mass M and one block of mass m ($< M$). These blocks are connected with two identical springs of spring constant k as shown in the figure. The system is constrained to move along a straight line on a frictionless horizontal surface. The spring follows Hooke's law. Find the angular frequencies of the independent oscillations (normal modes).



Now, the masses M and m are interchanged and the new arrangement is shown in the following figure:

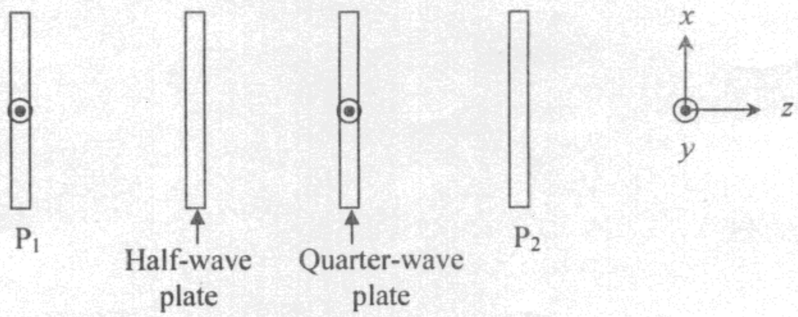


The ratio of the frequency of the new arrangement to that of the old arrangement, when the middle block remains stationary, is $\sqrt{2}$. Find the ratio of the frequencies in the two arrangements when the middle block oscillates. (21)

A

A

Q.21 A half-wave plate and a quarter-wave plate are placed between a polarizer P_1 and an analyzer P_2. All of these are parallel to each other and perpendicular to the direction of propagation of unpolarized incident light (see the figure). The optic-axis of the half-wave plate makes an angle of 30° with respect to the pass-axis of P_1 and that of the quarter-wave plate is parallel to the pass-axis of P_1.



(a) Determine the state of polarization for the light after passing through (i) the half-wave plate and (ii) the quarter-wave plate. (15)

(b) What should be the orientation of the pass-axis of P_2 with respect to that of P_1 such that the intensity of the light emerging from P_2 is maximum? (6)

A

Q.22 Consider a system of N non-interacting distinguishable spin-$\frac{1}{2}$ particles, each of magnetic moment $\vec{\mu}$. The system is at an equilibrium temperature T in a magnetic field $\vec{B}$ such that n particles have their magnetic moments aligned parallel to $\vec{B}$.

(a) Find the energy E and the entropy S of the system. (9)

(b) Using the relationship between E and S, find T. Hence determine the ratio $\frac{n}{N}$ in terms of μ, B and T. [Use $\ln N! = N \ln N - N$] (12)

A

A

Q.23 (a) An ideal gas, kept in contact with a heat reservoir, undergoes a quasistatic process in which its pressure gets doubled. Obtain the Maxwell relation from the differential form $dF = -S\,dT - P\,dV$ and evaluate the expression for the change in entropy of n moles of the gas. (9)

(b) Using $S = S(T, V)$, derive a general expression for $(\partial U/\partial V)_T$, where $U(S, V)$ is the internal energy. Evaluate it for the ideal gas as considered in part (a). Justify that the outcome is consistent with the expression for the average energy known from the kinetic theory. (12)

A

A

Q.24 Consider an ideal Fermi gas consisting of N non-relativistic spin-$\frac{1}{2}$ particles confined to a length L in one dimension at 0 K.

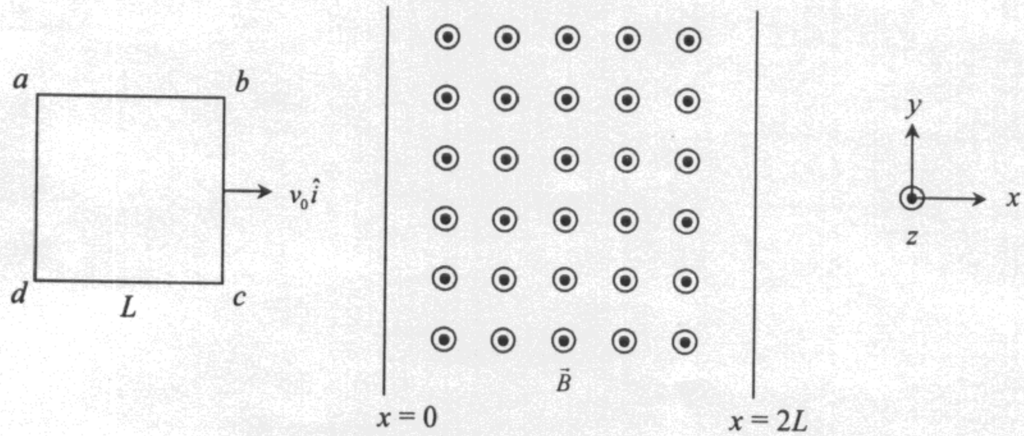
(a) Find an expression for the density of states and hence calculate the Fermi energy of the gas. (15)

(b) Find the mean energy per particle in terms of the Fermi energy. (6)

A

A

Q.25 A square loop of side L and mass M is made of a wire of cross-sectional area A and resistance R. The loop, moving with a constant velocity $v_0\hat{i}$ in the horizontal xy-plane, enters a region $0 \le x \le 2L$ having constant magnetic field $B\hat{k}$.



(a) Find an expression for the x-component of the force $\vec{F}$ acting on the loop in terms of its velocity $\vec{v}(t)$, B, L and R. (12)

(b) Find the speed of the loop as its side ad exits the field region at $x = 2L$ and sketch its variation with x. (9)

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