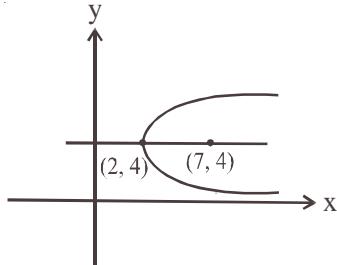


7. The co-ordinates of the focus of the parabola described parametrically by $x = 5t^2 + 2$, $y = 10t + 4$ are
 (A) (7, 4) (B) (3, 4) (C) (3, -4) (D) (-7, 4)

Ans : (A)
Hints : $x = 5t^2 + 2$; $y = 10t + 4$, $\left(\frac{y-4}{10}\right)^2 = \left(\frac{x-2}{5}\right)$
 or, $(y-4)^2 = 20(x-2)$



8. For any two sets A and B, $A - (A - B)$ equals
 (A) B (B) $A - B$ (C) $A \cap B$ (D) $A^c \cap B^c$

Ans : (C)

Hints : $A - (A - B) = A - (A \cap B^c) = A \cap (A \cap B^c)^c = A \cap (A^c \cup B) = (A \cap A^c) \cup (A \cap B) = A \cap B$

9. If $a = 2\sqrt{2}$, $b = 6$, $A = 45^\circ$, then
 (A) no triangle is possible (B) one triangle is possible
 (C) two triangles are possible (D) either no triangle or two triangles are possible

Ans : (A)

Hints : $a = 2\sqrt{2}$; $b = 6$; $A = 45^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin B = \frac{b}{a} \sin A$$

$$\Rightarrow \sin B = \frac{6}{2\sqrt{2}} \sin 45^\circ = \frac{3}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{3}{2} \Rightarrow \text{No triangle is possible since } \sin B > 1$$

10. A Mapping from IN to IN is defined as follows :

$$f: \text{IN} \rightarrow \text{IN}$$

$$f(n) = (n+5)^2, n \in \text{IN}$$

(IN is the set of natural numbers). Then

- (A) f is not one-to-one (B) f is onto
 (C) f is both one-to-one and onto (D) f is one-to-one but not onto

Ans : (D)

Hints : $f: \text{IN} \rightarrow \text{IN}; f(n) = (n+5)^2$

$$(n_1 + 5)^2 = (n_2 + 5)^2$$

$$\Rightarrow (n_1 - n_2)(n_1 + n_2 + 10) = 0$$

$$\Rightarrow n_1 = n_2 \rightarrow \text{one-to-one}$$

There does not exist $n \in \text{IN}$ such that $(n+5)^2 = 1$

Hence f is not onto

11. In a triangle ABC if $\sin A \sin B = \frac{ab}{c^2}$, then the triangle is
 (A) equilateral (B) isosceles (C) right angled (D) obtuse angled
Ans : (C)

Hints : $\sin A \sin B = \frac{ab}{c^2}$

$$\Rightarrow c^2 = \frac{ab}{\sin A \sin B} = \left(\frac{a}{\sin A} \right) \left(\frac{b}{\sin B} \right)$$

$$\Rightarrow c^2 = \left(\frac{c}{\sin C} \right)^2 \Rightarrow \sin^2 C = 1 \Rightarrow \sin C = 1 \Rightarrow C = 90^\circ$$

12. $\int \frac{dx}{\sin x + \sqrt{3} \cos x}$ equals

(A) $\frac{1}{2} \ln \left| \tan \left(\frac{x}{2} - \frac{\pi}{6} \right) \right| + c$ (B) $\frac{1}{2} \ln \left| \tan \left(\frac{x}{4} - \frac{\pi}{6} \right) \right| + c$ (C) $\frac{1}{2} \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{6} \right) \right| + c$ (D) $\frac{1}{2} \ln \left| \tan \left(\frac{x}{4} + \frac{\pi}{3} \right) \right| + c$

where c is an arbitrary constant

Ans : (C)

Hints : $\int \frac{dx}{\sin x + \sqrt{3} \cos x} = \int \frac{dx}{2 \left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right)} = \frac{1}{2} \int \frac{dx}{\sin \left(x + \frac{\pi}{3} \right)}$

$$= \frac{1}{2} \int \operatorname{cosec} \left(x + \frac{\pi}{3} \right) dx = \frac{1}{2} \log \left| \tan \left(\frac{x}{2} + \frac{\pi}{6} \right) \right| + c$$

$$= \frac{1}{2} \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{6} \right) \right| + c$$

13. The value of $(1 + \cos \frac{\pi}{6})(1 + \cos \frac{\pi}{3})(1 + \cos \frac{2\pi}{3})(1 + \cos \frac{7\pi}{6})$ is

(A) $\frac{3}{16}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

Ans : (A)

Hints : $\left(1 + \cos \frac{\pi}{6} \right) \left(1 + \cos \frac{\pi}{3} \right) \left(1 + \cos \frac{2\pi}{3} \right) \left(1 + \cos \frac{7\pi}{6} \right)$

$$= \left(1 + \frac{\sqrt{3}}{2} \right) \left(1 + \frac{1}{2} \right) \left(1 - \frac{1}{2} \right) \left(1 - \frac{\sqrt{3}}{2} \right) = \left(1 - \frac{3}{4} \right) \left(1 - \frac{1}{4} \right) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$$

14. If $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$ then

(A) $\frac{1}{3} \leq P \leq \frac{1}{2}$

(B) $P \geq \frac{1}{2}$

(C) $2 \leq P \leq 3$

(D) $-\frac{\sqrt{13}}{6} \leq P \leq \frac{\sqrt{13}}{6}$

Ans : (A)

Hints : $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta = \frac{1}{2}\sin^2\theta + \frac{1}{3}(1 - \sin^2\theta) = \frac{1}{3} + \frac{1}{6}\sin^2\theta$

$$0 \leq \sin^2\theta \leq 1 \Rightarrow \frac{1}{3} \leq \frac{1}{3} + \frac{1}{6}\sin^2\theta \leq \frac{1}{3} + \frac{1}{6}$$

$$\Rightarrow \frac{1}{3} \leq P \leq \frac{1}{2}$$

15. A positive acute angle is divided into two parts whose tangents are $\frac{1}{2}$ and $\frac{1}{3}$. Then the angle is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{5}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Ans : (A)

Hints : Angle $\theta = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right)$

$$= \tan^{-1}\left(\frac{5/6}{5/6}\right) = \tan^{-1}(1) = \pi/4$$

16. If $f(x) = f(a-x)$ then $\int_0^a xf(x)dx$ is equal to

(A) $\int_0^a f(x)dx$

(B) $\frac{a^2}{2} \int_0^a f(x)dx$

(C) $\frac{a}{2} \int_0^a f(x)dx$

(D) $-\frac{a}{2} \int_0^a f(x)dx$

Ans : (C)

Hints : $f(x) = f(a-x)$, $I = \int_0^a xf(x)dx = \int_0^a (a-x)f(a-x)dx$

$$= \int_0^a (a-x)f(x)dx = a \int_0^a f(x)dx - I$$

$$\therefore 2I = a \int_0^a f(x)dx \Rightarrow I = \frac{a}{2} \int_0^a f(x)dx$$

17. The value of $\int_0^{\infty} \frac{dx}{(x^2 + 4)(x^2 + 9)}$ is
- (A) $\frac{\pi}{60}$ (B) $\frac{\pi}{20}$ (C) $\frac{\pi}{40}$ (D) $\frac{\pi}{80}$

Ans : (A)

Hints : $\int_0^{\infty} \frac{dx}{(x^2 + 4)(x^2 + 9)} = \int_0^{\pi/2} \frac{\sec^2 \theta}{(\tan^2 \theta + 4)(\tan^2 \theta + 9)} d\theta$ (putting $x = \tan \theta$)

$$= \frac{1}{5} \int_0^{\pi/2} \frac{(9 + \tan^2 \theta) - (4 + \tan^2 \theta) \sec^2 \theta}{(\tan^2 \theta + 4)(\tan^2 \theta + 9)} d\theta$$

$$= \frac{1}{5} \left[\int_0^{\pi/2} \frac{\sec^2 \theta}{4 + \tan^2 \theta} d\theta - \int_0^{\pi/2} \frac{\sec^2 \theta}{9 + \tan^2 \theta} d\theta \right]$$

$$= \frac{1}{5} \left[\frac{1}{2} \tan^{-1} \left(\frac{\tan \theta}{2} \right) \Big|_0^{\pi/2} - \frac{1}{3} \tan^{-1} \left(\frac{\tan \theta}{3} \right) \Big|_0^{\pi/2} \right]$$

$$= \frac{1}{5} \left[\frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{3} \cdot \frac{\pi}{2} \right] = \left(\frac{\pi}{2} \right) \left(\frac{1}{5} \right) \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\pi}{2} \cdot \frac{1}{5} \cdot \frac{1}{6} = \frac{\pi}{60}$$

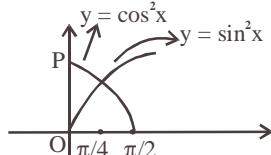
18. If $I_1 = \int_0^{\pi/4} \sin^2 x dx$ and $I_2 = \int_0^{\pi/4} \cos^2 x dx$, then,
- (A) $I_1 = I_2$ (B) $I_1 < I_2$ (C) $I_1 > I_2$ (D) $I_2 = I_1 + \pi/4$

Ans : (B)

Hints : $I_1 = \int_0^{\pi/4} \sin^2 x dx ; I_2 = \int_0^{\pi/4} \cos^2 x dx$

$$\text{In } \left(0, \frac{\pi}{4}\right), \cos^2 x > \sin^2 x \therefore \int_0^{\pi/4} \cos^2 x dx > \int_0^{\pi/4} \sin^2 x dx$$

$$I_2 > I_1 \text{ i.e. } I_1 < I_2$$



19. The second order derivative of $a \sin^3 t$ with respect to $a \cos^3 t$ at $t = \frac{\pi}{4}$ is

(A) 2 (B) $\frac{1}{12a}$ (C) $\frac{4\sqrt{2}}{3a}$ (D) $\frac{3a}{4\sqrt{2}}$

Ans : (C)

Hints : $y = a \sin^3 t ; x = a \cos^3 t$

$$\frac{dy}{dt} = 3a \sin^2 t \cos t ; \frac{dx}{dt} = -3a \cos^2 t \sin t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a \sin^2 t \cos t}{-3a \cos^2 t \sin t} = -\frac{\sin t}{\cos t} = -\tan t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (-\tan t) = \frac{d}{dt} (-\tan t) \cdot \frac{dt}{dx}$$

$$= (-\sec^2 t) \frac{1}{-3\cos^2 t \sin t} = \frac{1}{+3\cos^4 t \sin t}$$

$$\left. \frac{d^2y}{dx^2} \right|_{t=\pi/4} = \frac{1}{3a \left(\frac{1}{\sqrt{2}} \right)^4 \left(\frac{1}{\sqrt{2}} \right)} = \frac{\left(\sqrt{2}\right)^5}{3a} = \frac{4\sqrt{2}}{3a}$$

Ans : (C)

Hints : $5 \cos\theta + 12$, $-1 \leq \cos \theta \leq 1$

$$\Rightarrow -5 \leq 5 \cos \theta \leq 5$$

$$\therefore 5 \cos\theta + 12 \geq -5 + 12 \Rightarrow 5 \cos\theta + 12 \geq 7$$

21. The general solution of the differential equation $\frac{dy}{dx} = e^{y+x} + e^{y-x}$ is

where c is an arbitrary constant

Ans : (B)

Hints : $e^{-y} dy = (e^x + e^{-x}) dx$ Integrate

$$-e^{-y} = e^x - e^{-x} + c, \quad e^{-y} = e^{-x} - e^{+x} + c$$

22. Product of any r consecutive natural numbers is always divisible by

(A) r !

Ans : (A)

$$= \frac{(n+r)!}{r!}$$

$$= \frac{(n+r)!}{r!} r! = r!^{n+r} C_n$$

23. The integrating factor of the differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$ is given by

(A) e^x

Ans. • (B)

$$\text{Hints : } \frac{dy}{dx} + \frac{1}{x \log x} \cdot y = \frac{2}{x}$$

$$\text{If } = \int_{e^0}^{\infty} \frac{1}{x \log x} dx = \int \frac{1/x}{\log x} dx$$

$$= e^{\log(\log x)} = \log x$$

24. If $x^2 + y^2 = 1$ then
 (A) $yy'' - (2y')^2 + 1 = 0$ (B) $yy'' + (y')^2 + 1 = 0$ (C) $yy'' - (y')^2 - 1 = 0$ (D) $yy'' + (2y')^2 + 1 = 0$

Ans : (B)

Hints : $2x + 2yy' = 0$

$$x + yy' = 0$$

$$1 + yy'' + (y')^2 = 0$$

25. If $c_0, c_1, c_2, \dots, c_n$ denote the co-efficients in the expansion of $(1+x)^n$ then the value of $c_1 + 2c_2 + 3c_3 + \dots + nc_n$ is
 (A) $n \cdot 2^{n-1}$ (B) $(n+1)2^{n-1}$ (C) $(n+1)2^n$ (D) $(n+2)2^{n-1}$

Ans. (A)

Hints : $(1+x)^n = c_0 + xc_1 + x^2c_2 + \dots + x^n c_n$

$$n(1+x)^{n-1} = c_1 + 2xc_2 + \dots + nx^{n-1}c_n$$

$$\text{Put } x = 1$$

$$n(2)^{n-1} = c_1 + 2c_2 + 3c_3 + \dots + nc_n$$

26. A polygon has 44 diagonals. The number of its sides is

- (A) 10 (B) 11 (C) 12 (D) 13

Ans : (B)

Hints : ${}^n C_2 - n = 44$

$$\frac{n(n-1)}{2} - n = 44$$

$$n\left[\frac{n-1}{2} - 1\right] = 44$$

$$n(n-3) = 88$$

$$n(n-3) = 11 \times 8$$

$$n = 11$$

27. If α, β be the roots of $x^2 - a(x-1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$

- (A) $\frac{4}{a+b}$ (B) $\frac{1}{a+b}$ (C) 0 (D) -1

Ans : (C)

Hints : $x^2 - ax = a + 3$ $\alpha\beta = a + b$

$$\alpha + \beta = a$$

$$\alpha^2 - a\alpha = -(a+b)$$

$$\beta^2 - a\beta = -(a+b)$$

$$-\frac{1}{a+b} - \frac{1}{a+b} + \frac{2}{a+b} = 0$$

28. The angle between the lines joining the foci of an ellipse to one particular extremity of the minor axis is 90° . The eccentricity of the ellipse is

- (A) $\frac{1}{8}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{1}{2}}$

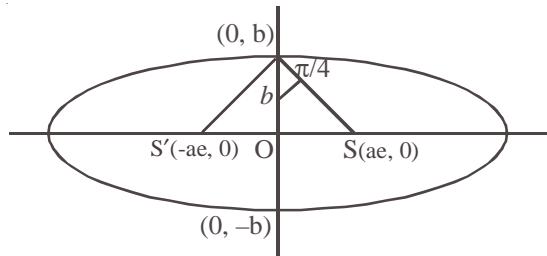
Ans : (D)

$$b = ae \Rightarrow \frac{b}{a} = e$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - e^2$$

$$e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$



29. The order of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 - \left(\frac{dy}{dx}\right)^2}$ is (A) 3 (B) 2 (C) 1 (D) 4
Ans : (B)

30. The sum of all real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ (A) 7 (B) 4 (C) 1 (D) 5

Ans : (B)

Hints : Put $1x - 21 = y$

$$y^2 + y - 2 = 0$$

$$(y-1)(y+2)=0$$

$$y=1$$

$$y = -2$$

$$|x - 2| = 1$$

$$x - 2 = \pm 1$$

$$x = 2 \pm 1$$

$$x=3, 1$$

Sum = 4

$$f(x)dx \geq$$

Ans : (D)

$$\text{Hints : } \int_{-1}^4 f(x) dx = 4$$

$$3(4-2) - \int_2^4 f(x)dx = 7$$

$$\int_2^4 f(x)dx = -1$$

$$\int_{-1}^2 f(x)dx = \int_{-1}^4 f(x)dx + \int_4^2 f(x)dx = 4 - \int_2^4 f(x)dx = 4 - (-1) = 5$$

32. For each $n \in N$, $2^{3n} - 1$ is divisible by

(A) 7 (B) 8 (C) 6 (D) 16

where N is a set of natural numbers

Ans : (A)

Hints : $2^{3n} = (8)^n = (1+7)^n = 1 + {}^nC_1 7 + {}^nC_2 7^2 + \dots + {}^nC_n 7^n$
 $2^{3n} - 1 = 7[{}^nC_1 + {}^nC_2 7 + \dots]$

33. The Rolle's theorem is applicable in the interval $-1 \leq x \leq 1$ for the function

(A) $f(x) = x$ (B) $f(x) = x^2$ (C) $f(x) = 2x^3 + 3$ (D) $f(x) = |x|$

Ans : (B)

Hints : $f(x) = x^2$ and $f(1) = f(-1)$ for $f(x) = |x|$ but at $x = 0$, $f(x) = |x|$ is not differentiable hence (B) is the correct option.

$$f(1) = 1 = f(-1)$$

34. The distance covered by a particle in t seconds is given by $x = 3 + 8t - 4t^2$. After 1 second velocity will be

(A) 0 unit/second (B) 3 units/second (C) 4 units/second (D) 7 units/second

Ans : (A)

Hints : $v = \frac{dx}{dt} = 8 - 8t$

$$t = 1, v = 8 - 8 = 0$$

35. If the co-efficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ be same, then the value of 'a' is

(A) $\frac{3}{7}$ (B) $\frac{7}{3}$ (C) $\frac{7}{9}$ (D) $\frac{9}{7}$

Ans : (D)

Hints : $(3 + ax)^9 = {}^9C_0 3^9 + {}^9C_1 3^8(ax) + {}^9C_2 3^7(ax)^2 + {}^9C_3 3^6(ax)^3$
 ${}^9C_2 3^7 a^2 = {}^9C_3 3^6 a^3$

$$\frac{9}{7} = a$$

36. The value of $\left(\frac{1}{\log_3 12} + \frac{1}{\log_4 12} \right)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

Ans : (C)

Hints : $\log_{12} 3 + \log_{12} 4 = \log_{12} 12 = 1$

37. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then the value of $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ will be

(A) $x + y + z$ (B) 1 (C) $ab + bc + ca$ (D) abc

Ans : (B)

Hints : $1 + x = \log_a a + \log_a bc = \log_a abc$

$$\frac{1}{1+x} = \log_{abc} a, \text{ Similarly } \frac{1}{1+y} = \log_{abc} b$$

$$\frac{1}{1+z} = \log_{abc} c, \text{ Ans. } \log_{(abc)} abc = 1$$

Hints : $A = \pi r^2$ $\frac{dr}{dt} = 5$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi 20(5)$$

$$= 200 \pi$$

40. The quadratic equation whose roots are three times the roots of $3ax^2 + 3bx + c = 0$ is
(A) $ax^2 + 3bx + 3c = 0$ (B) $ax^2 + 3bx + c = 0$ (C) $9ax^2 + 9bx + c = 0$ (D) $ax^2 + bx + 3c = 0$

Ans : (A)

Hints: $3a\alpha^2 + 3b\alpha + c = 0$

$$x = 3\alpha \Rightarrow \alpha = \frac{x}{3}$$

$$3a\frac{x^2}{9} + 3b \cdot \frac{x}{3} + c = 0$$

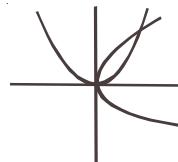
$$ax^2 + bx + c = 0$$

41. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(A) $2\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{4}{3}\right)$ (C) $\frac{\pi}{2}$

Ans : (C)

Hints : Angle between axes (since co-ordinate axes are the tangents for the given curve).



42. In triangle ABC, $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then B is equal to
 (A) 30° (B) 60° (C) 90° (D) 120°

Ans: (C)

$$\text{Hints : } \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{b}{a} \cdot \sin A = \frac{3}{2} \cdot \frac{2}{3} = 1$$

$$B = \frac{\pi}{2}$$

43. $\int_0^{1000} e^{x-[x]}$ is equal to
- (A) $\frac{e^{1000}-1}{e-1}$ (B) $\frac{e^{1000}-1}{1000}$ (C) $\frac{e-1}{1000}$ (D) $1000(e-1)$

Ans : (D)

Hints : $I = 1000 \int_0^1 e^{x-[x]}$

$$= 1000 \int_0^1 e^x dx = 1000(e^x)_0^1 = 100(e-1)$$

Period of function is 1

44. The coefficient of x^n , where n is any positive integer, in the expansion of $(1 + 2x + 3x^2 + \dots + \infty)^{\frac{1}{2}}$ is

- (A) 1 (B) $\frac{n+1}{2}$ (C) $2n+1$ (D) $n+1$

Ans : (A)

$$s = 1 + 2x + 3x^2 + \dots + \infty$$

Hints : $\frac{xs = x + 2x^2 + \dots + \infty}{s(1-x) = 1 + x + x^2 + \dots + \infty}$

$$s = \frac{1}{(1-x)^2}$$

$$f(x) = \frac{1}{1-x}, f(x) = (1-x)^{-1} = 1 + x + x^2 + x^3 + \dots = 1$$

45. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = a^2$ intersect at two distinct points if

- (A) $a < 2$ (B) $2 < a < 8$ (C) $a > 8$ (D) $a = 2$

Ans. (B)

Hints : $C_1(5, 0)$ $r_1 = \sqrt{25-16} = 3$

$$C_2(0, 0) \quad r_2 = a$$

$$r_1 & r_2 < C_1 C_2 < r_1 + r_2$$

$$|a-3| < \sqrt{25} < a+3$$

$$|a-3| < 5 < a+3$$

$$-5 < a-3 < 5 \quad 2 < a$$

$$-2 < a < 8$$

$$2 < a < 8$$

46. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to
 (A) $\log(\sin^{-1} x) + c$ (B) $\frac{1}{2}(\sin^{-1} x)^2 + c$ (C) $\log\left(\sqrt{1-x^2}\right) + c$ (D) $\sin(\cos^{-1} x) + c$

where c is an arbitrary constant

Ans : (B)

Hints : $I = \int t dt$

$$\sin^{-1} x = t$$

$$= \frac{1}{2}t^2 + c$$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$= \frac{1}{2}(\sin^{-1} x)^2 + c$$

47. The number of points on the line $x + y = 4$ which are unit distance apart from the line $2x + 2y = 5$ is

- (A) 0 (B) 1 (C) 2 (D) Infinity

Ans : (A)

Hints : $x + y = 4$

$$x + y = \frac{5}{2}$$

$$PQ = \frac{4 - 5/2}{\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

48. Simplest form of $\frac{2}{\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 4x}}}}$ is

- (A) $\sec \frac{x}{2}$ (B) $\sec x$ (C) $\operatorname{cosec} x$ (D) 1

Ans : (A)

Hints : $\frac{2}{\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos^2 2x}}}} = \frac{2}{\sqrt{2 + \sqrt{2 + 2\cos 2x}}} = \frac{2}{\sqrt{2 + \sqrt{2(2\cos^2 x)}}}$

$$= \frac{2}{\sqrt{2 + 2\cos x}} = \frac{2}{2\cos \frac{x}{2}} = \sec \frac{x}{2}$$

49. If $y = \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

- (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -1

Ans : (A)

$$\text{Hints : } y = \tan^{-1} \left| \frac{1 - \cos\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)} \right|$$

$$= \tan^{-1} \left| \frac{2 \sin^2 \left(\frac{\pi}{4} - \frac{x}{2} \right)}{2 \cos^2 \left(\frac{\pi}{4} - \frac{x}{2} \right)} \right| = \tan^{-1} \left| \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right| = \left(\frac{\pi}{4} - \frac{x}{2} \right)$$

$$\frac{dy}{dx} = -\frac{1}{2}$$

50. If three positive real numbers a, b, c are in A.P. and $abc = 4$ then minimum possible value of b is

(A) $2^{3/2}$

Ans : (B)

s: $(b - d) b (b -$

$$(b^2 - d^2) b =$$

$$b^+ = 4 + d^- b$$

51. If $5\cos 2\theta + 2\cos^2 \frac{\theta}{2} + 1 = 0$, when ($0 < \theta < \pi$), then the values of θ are :

$$(A) \quad -\frac{\pi}{2} + \pi \quad (B)$$

$$s: 5\cos 2\theta + 1 + \cos \theta + 1 = 0$$

$$5(2\cos^2 \theta - 1) + \cos \theta +$$

$$10\cos^2 \theta + \cos \theta - 3 = 0$$

$$(5\cos\theta +$$

$$\theta = \frac{\pi}{3}$$

$$\begin{aligned}\cos \theta &= -\frac{3}{5} \\ \theta &= \cos^{-1}\left(-\frac{3}{5}\right) \\ &= \pi - \cos^{-1}\left(\frac{3}{5}\right)\end{aligned}$$

52. For any complex number z , the minimum value of $|z| + |z - 1|$ is

(A) 0

Ans : (B)

$$s: 1 = |z - (z-1)|$$

Ans : (D)

Hints: C₁(0, 0) r₁ = 4

$$C_2(0, 1) \qquad \qquad r_2 = \sqrt{0+1} = 1$$

$$C_1 C_2 = \sqrt{0+1} = 1$$

$$r_1 - r_2 = 3$$

$$C_1 C_2 < r_1 - r_2$$

54. If C is a point on the line segment joining A (-3, 4) and B (2, 1) such that $AC = 2BC$, then the coordinate of C is

- (A) $\left(\frac{1}{3}, 2\right)$ (B) $\left(2, \frac{1}{3}\right)$ (C) $(2, 7)$ (D) $(7, 2)$

Ans : (A)

Hints :



$$C\left(\frac{4-3}{3}, \frac{2+4}{3}\right)$$

$$C\left(\frac{1}{3}, 2\right)$$

Ans : (C)

Hints : $3x^2 - 2x(a + b + c) + ab + bc + ca = 0$

$$D = 4(a+b+c)^2 - 4 \cdot 3(ab+bc+ca)$$

$$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= 2[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\geq 0$$

56. The sum of the infinite series $1 + \frac{1}{2!} + \frac{1.3}{4!} + \frac{1.3.5}{6!} + \dots$ is

- (A) e (B) e^2 (C) \sqrt{e} (D) $\frac{1}{e}$

Ans : (C)

$$\text{Hints : } T_n = \frac{1.3.5....(2n-1)}{2n}$$

$$= \frac{\lfloor 2n \rfloor}{\lfloor 2n(2.4\dots 2n) \rfloor}$$

$$= \frac{\lfloor 2n \rfloor}{2^n \lfloor n \rfloor \lfloor 2n \rfloor}$$

$$= \frac{x^n}{\lfloor n \rfloor} \quad \frac{1}{2} = x$$

$$\therefore \frac{x}{\lfloor 1 \rfloor} + \frac{x^2}{\lfloor 2 \rfloor} + \dots = e^x - 1$$

$$\exp = 1 + e^x - 1 = e^x = e^{\frac{1}{2}}$$

57. The point $(-4, 5)$ is the vertex of a square and one of its diagonals is $7x - y + 8 = 0$. The equation of the other diagonal is
 (A) $7x - y + 23 = 0$ (B) $7y + x = 30$ (C) $7y + x = 31$ (D) $x - 7y = 30$

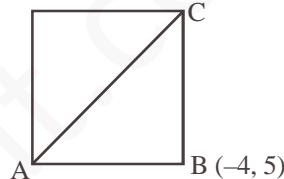
Ans : (C)

Hints : $x + 7y = k$ (1)

$$-4 + 35 = k$$

$$31 = k$$

$$x + 7y - 31 = 0$$



58. The domain of definition of the function $f(x) = \sqrt{1 + \log_e(1-x)}$ is

- (A) $-\infty < x \leq 0$ (B) $-\infty < x \leq \frac{e-1}{e}$ (C) $-\infty < x \leq 1$ (D) $x \geq 1-e$

Ans : (B)

Hints : $1-x > 0 \Rightarrow x < 1$

$$1 + \log_e(1-x) \geq 0$$

$$\log_e(1-x) \geq -1 \Rightarrow 1-x \geq e^{-1}$$

$$x \leq 1 - \frac{1}{e}$$

$$x \leq \frac{e-1}{e}$$

59. For what value of m , $\frac{a^{m+1} + b^{m+1}}{a^m + b^m}$ is the arithmetic mean of 'a' and 'b'?

- (A) 1 (B) 0 (C) 2 (D) None

Ans : (B)

Hints : $\frac{a^{m+1} + b^{m+1}}{a^m + b^m} = \frac{a+b}{2}$

$m = 0$ Satisfy.

60. The value of the limit $\lim_{x \rightarrow 1} \frac{\sin(e^{x-1} - 1)}{\log x}$ is

(A) 0

(B) e

(C) $\frac{1}{e}$

(D) 1

Ans : (D)

$$\text{Hints : } \lim_{h \rightarrow 0} \frac{\sin(e^h - 1)}{\log(1+h)} \quad \text{Put } x = 1 + h$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{\sin(e^h - 1)}{(e^h - 1)} \cdot \frac{(e^h - 1)}{\log(1+h)} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(e^h - 1)}{(e^h - 1)} \cdot \frac{(e^h - 1)}{h} \cdot \frac{h}{\log(1+h)} \\
 &= 1 \cdot 1 \cdot 1 \\
 &\equiv 1
 \end{aligned}$$

61. Let $f(x) = \frac{\sqrt{x+3}}{x+1}$ then the value of $\lim_{x \rightarrow -3-0} f(x)$ is

- Ans : (B)**

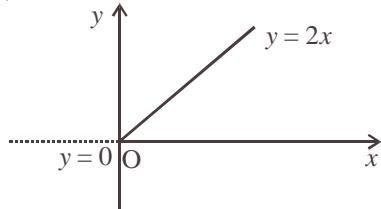
Hints : Because on left hand side of 3 function is not defined.

62. $f(x) = x + |x|$ is continuous for

(A) $x \in (-\infty, \infty)$ (B) $x \in (-\infty, \infty) - \{0\}$ (C) only $x > 0$ (D) no value of x

Ans. • (A)

Hints : *f*



63. $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$ is equal to

$$(A) \quad \frac{2a}{h}$$

$$(B) \quad \frac{2b}{a}$$

(C) $\frac{a}{b}$

(D) $\frac{b}{a}$

Ans : (B)

Hints : Let $\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right) = \theta$, then $\cos 2\theta = \frac{a}{b}$

$$\begin{aligned} & \tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] \\ &= \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 2\left(\frac{1 + \tan^2 \theta}{1 - \tan^2 \theta}\right) = \frac{2}{\cos 2\theta} = \frac{2}{\frac{a^2 - b^2}{ab}} = \frac{2ab}{a^2 - b^2} \end{aligned}$$

Ans : (D)

Hints: $i^n(1+i+i^2+i^3) = i^n(1+i-1-i) = 0$

65. $\int \frac{dx}{x(x+1)}$ equals

- (A) $\ln\left|\frac{x+1}{x}\right| + c$ (B) $\ln\left|\frac{x}{x+1}\right| + c$ (C) $\ln\left|\frac{x-1}{x}\right| + c$ (D) $\ln\left|\frac{x-1}{x+1}\right| + c$

where c is an arbitrary constant.

Ans : (B)

$$\text{Hints : } \int \frac{dx}{x(x+1)} = \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx = \int \frac{dx}{x} - \int \frac{dx}{x+1} = \ln|x| - \ln|x+1| + C = \ln \left| \frac{x}{x+1} \right| + C$$

Ans : (C)

Hints : a, b, c are in GP.

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in A.P.

$\Rightarrow \frac{1}{\log_a a}, \frac{1}{\log_a b}, \frac{1}{\log_a c}$ are in H.P.

$\Rightarrow \log_a x, \log_b x, \log_c x$ are in H.P.

67. A line through the point A (2, 0) which makes an angle of 30° with the positive direction of x -axis is rotated about A in clockwise direction through an angle 15° . Then the equation of the straight line in the new position is

- (A) $(2 - \sqrt{3})x + y - 4 + 2\sqrt{3} = 0$ (B) $(2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$
 (C) $(2 - \sqrt{3})x - y + 4 + 2\sqrt{3} = 0$ (D) $(2 - \sqrt{3})x + y + 4 + 2\sqrt{3} = 0$

Ans : (B)

Hints : Equation of line in new position :

$$y - 0 = \tan 15^\circ (x - 2)$$

$$\Rightarrow y = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) (x-2)$$

$$\Rightarrow y = \frac{(\sqrt{3}-1)^2}{2}(x-2)$$

$$\Rightarrow 2y = (4 - 2\sqrt{3})(x - 2)$$

$$\Rightarrow y = (2 - \sqrt{3})(x - 2)$$

$$\Rightarrow (2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$$

68. The equation $\sqrt{3} \sin x + \cos x = 4$ has

(A) only one solution (B) two solutions (C) infinitely many solutions (D) no solution

Ans : (D)

Hints : $\sqrt{3} \sin x + \cos x = 2 \sin\left(x + \frac{\pi}{6}\right) \leq 2$. Therefore

$\sqrt{3} \sin x + \cos x = 4$ cannot have a solution

69. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through $(-1, 1)$. The equation of the curve is

(A) $y = x^3 + 2$ (B) $y = -x^3 - 2$ (C) $y = 3x^3 + 4$ (D) $y = -x^3 + 2$

Ans : (A)

Hints : $\frac{dy}{dx} = 3x^2 \Rightarrow \int dy = \int 3x^2 dx \Rightarrow y = x^3 + C$

Curve passes through $(-1, 1)$. Hence $1 = -1 + C \Rightarrow C = 2$

$\therefore y = x^3 + 2$

70. The modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$ is

(A) $\sqrt{5}$ unit (B) $\frac{\sqrt{11}}{5}$ unit (C) $\frac{\sqrt{5}}{5}$ unit (D) $\frac{\sqrt{12}}{5}$ unit

Ans : (C)

Hints : $\frac{1-i}{3+i} + \frac{4i}{5} = \frac{5-5i+4i(3+i)}{5(3+i)} = \frac{5-5i+12i-4}{5(3+i)} = \frac{1+7i}{5(3+i)} = \frac{(1+7i)(3-i)}{5(9+1)}$

$$= \frac{3+21i-i+7}{5 \times 10} = \frac{10+20i}{5 \times 10} = \frac{1+2i}{5}$$

$$\therefore \text{Modulus} = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \sqrt{\frac{1}{25} + \frac{4}{25}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5} \text{ unit}$$

71. The equation of the tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at $(2, 1)$ is

(A) $x + 2 = 0$ (B) $2x + 1 = 0$ (C) $x + y + 1 = 0$ (D) $x - 2 = 0$

Ans : (D)

Hints : Equation of tangent at (x_1, y_1) is

$$xx_1 - yy_1 - 4(x + x_1) + (y + y_1) + 11 = 0$$

$$x_1 = 2; y_1 = 1$$

\therefore Equation of tangent is

$$2x - y - 4(x + 2) + (y + 1) + 11 = 0$$

$$\text{or } -2x - 8 + 12 = 0$$

or $-2x + 4 = 0$

or $2x = 4$

or $x = 2$

or $x - 2 = 0$

72. A and B are two independent events such that $P(A \cup B') = 0.8$ and $P(A) = 0.3$. The $P(B)$ is

(A) $\frac{2}{7}$

(B) $\frac{2}{3}$

(C) $\frac{3}{8}$

(D) $\frac{1}{8}$

Ans : (A)

Hints : Let $P(B) = x$

$$P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.3 + (1-x) - 0.3(1-x)$$

or $0.8 = 1 - x + 0.3x$

or $1 - 0.7x = 0.8$

or $0.7x = 0.2$

or $x = \frac{2}{7}$

73. The total number of tangents through the point $(3, 5)$ that can be drawn to the ellipses $3x^2 + 5y^2 = 32$ and $25x^2 + 9y^2 = 450$ is

(A) 0

(B) 2

(C) 3

(D) 4

Ans : (C)

Hints : $(3, 5)$ lies outside the ellipse $3x^2 + 5y^2 = 32$ and on the ellipse $25x^2 + 9y^2 = 450$. Therefore there will be 2 tangents for the first ellipse and one tangent for the second ellipse.

74. The value of $\lim_{n \rightarrow \infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + n^2} \right]$ is

(A) $\frac{\pi}{4}$

(B) $\log 2$

(C) zero

(D) 1

Ans : (A)

Hints : $\lim_{n \rightarrow \infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + n^2} \right]$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{1}{1 + \left(\frac{r}{n}\right)^2} = \int_0^1 \frac{dx}{1 + x^2} = \left[\tan^{-1} x \right]_0^1 = \frac{\pi}{4}$$

75. A particle is moving in a straight line. At time t , the distance between the particle from its starting point is given by $x = t - 6t^2 + t^3$. Its acceleration will be zero at

(A) $t = 1$ unit time

(B) $t = 2$ unit time

(C) $t = 3$ unit time

(D) $t = 4$ unit time

Ans : (B)

Hints : $x = t - 6t^2 + t^3$

$$\frac{dx}{dt} = 1 - 12t + 3t^2$$

$$\frac{d^2x}{dt^2} = -12 + 6t$$

$$\text{Acceleration} = \frac{d^2x}{dt^2}$$

$$\therefore \text{Acceleration} = 0 \Rightarrow 6t - 12 = 0 \Rightarrow t = 2$$

76. Three numbers are chosen at random from 1 to 20. The probability that they are consecutive is

(A) $\frac{1}{190}$

(B) $\frac{1}{120}$

(C) $\frac{3}{190}$

(D) $\frac{5}{190}$

Ans : (C)

Hints : Total number of cases ; ${}^{20}C_3 = \frac{20 \times 19 \times 18}{2 \times 3} = 20 \times 19 \times 3 = 1140$

Total number of favourable cases = 18

$$\therefore \text{Required probability} = \frac{18}{1140} = \frac{3}{190}$$

77. The co-ordinates of the foot of the perpendicular from (0, 0) upon the line $x + y = 2$ are

(A) (2, -1)

(B) (-2, 1)

(C) (1, 1)

(D) (1, 2)

Ans : (C)

Hints : Let P be the foot of the perpendicular. P lies on a line perpendicular to $x + y = 2$.

∴ Equation of the line on which P lies is of the form : $x - y + k = 0$

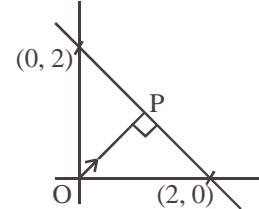
But this line passes through (0, 0).

$$\therefore k = 0$$

Hence, co-ordinates of P may be obtained by solving $x + y = 2$ and $y = x$

$$\therefore x = 1, y = 1$$

Hence, $P \equiv (1, 1)$



78. If A is a square matrix then,

(A) $A + A^T$ is symmetric (B) AA^T is skew - symmetric (C) $A^T + A$ is skew-symmetric (D) A^TA is skew symmetric

Ans : (A)

Hints : $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$

79. The equation of the chord of the circle $x^2 + y^2 - 4x = 0$ whose mid point is (1, 0) is

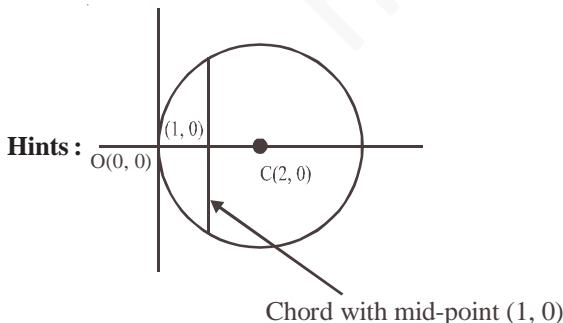
(A) $y = 2$

(B) $y = 1$

(C) $x = 2$

(D) $x = 1$

Ans : (D)



Equation : $x = 1$

80. If $A^2 - A + I = 0$, then the inverse of the matrix A is

(A) $A - I$

(B) $I - A$

(C) $A + I$

(D) A

Ans : (B)

Hints : $A^2 - A + I = 0 \Rightarrow A^2 = A - I \Rightarrow A^2 \cdot A^{-1} = A \cdot A^{-1} - I \Rightarrow A = I - A^{-1} \Rightarrow A^{-1} = I - A$

MATHEMATICS

SECTION-II

1. A train moving with constant acceleration takes t seconds to pass a certain fixed point and the front and back end of the train pass the fixed point with velocities u and v respectively. Show that the length of the train is $\frac{1}{2}(u+v)t$.

A. $v = u + at$ $a = \frac{v-u}{t}$

$$v^2 = u^2 + 2aS$$

$$\frac{v^2 - u^2}{2a} = S \Rightarrow S = \frac{(v+u)(v-u)}{2a} = \frac{at(v+u)}{2a} = \frac{u+v}{2}t$$

2. Show that

$$\frac{\sin \theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2}(\tan 27\theta - \tan \theta)$$

A. $T_1 = \frac{2 \sin \theta}{2 \cos 3\theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{\sin 2\theta}{2 \cdot \cos 3\theta \cdot \cos \theta}$

$$= \frac{1}{2} \cdot \frac{\sin(3\theta - \theta)}{\cos 3\theta \cdot \cos \theta}$$

$$T_1 = \frac{1}{2}(\tan 3\theta - \tan \theta)$$

$$T_2 = \frac{1}{2}(\tan 9\theta - \tan 3\theta)$$

$$T_3 = \frac{1}{2}(\tan 27\theta - \tan 9\theta)$$

$$T_1 + T_2 + T_3 = \frac{1}{2}(\tan 27\theta - \tan \theta)$$

3. If $x = \sin t$, $y = \sin 2t$, prove that

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = 0$$

A. $y = \sin(2 \sin^{-1} x)$

$$\frac{dy}{dx} = \cos(2 \sin^{-1} x) \cdot \frac{2}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} \frac{dy}{dx} = 2 \cos(2 \sin^{-1} x)$$

$$(1-x^2) \left(\frac{dy}{dx} \right)^2 = 4 \cdot \cos^2(2 \sin^{-1} x) = 4[1 - \sin^2(2 \sin^{-1} x)]$$

$$(1-x^2) \left(\frac{dy}{dx} \right)^2 = 4[1 - y^2]$$

Again differentiate

$$(1-x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 (-2x) = -8y \frac{dy}{dx}$$

Divide by $2 \frac{dy}{dx}$

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = 0$$

4. Show that, for a positive integer n, the coefficient of x^k ($0 \leq k \leq n$) in the expansion of

$$1 + (1+x) + (1+x)^2 + \dots + (1+x)^n$$
 is $n+1 C_{n-k}$.

$$\text{A. } S = \frac{1-(1+x)^{n+1}}{1-(1+x)} = \frac{(1+x)^{n+1}-1}{x}$$

$$\text{Coefficient of } x^k \text{ in } \frac{(1+x)^{n+1}}{x} - \frac{1}{x} = \text{Coefficient of } x^{k+1} \text{ in } (1+x)^{n+1} = n+1 C_{k+1} = n+1 C_{n-k}$$

$$5. \text{ If } m, n \text{ be integers, then find the value of } \int_{-\pi}^{\pi} (\cos mx - \sin nx)^2 dx$$

$$\begin{aligned} \text{A. } I &= \int_{-\pi}^{\pi} (\cos^2 mx + \sin^2 nx - 2 \sin nx \cos mx) dx \\ &= \int_{-\pi}^{\pi} \cos^2 mx dx + \int_{-\pi}^{\pi} \sin^2 nx dx - 2 \int_{-\pi}^{\pi} \sin nx \cos mx dx \\ &= 2 \int_0^{\pi} \cos^2 mx dx + 2 \int_0^{\pi} \sin^2 nx dx - 0 && (\text{Odd}) \\ &= 2 \int_0^{\pi} (1 + \cos 2mx) dx + \int_0^{\pi} (1 - \cos 2nx) dx \\ &= \pi + \frac{1}{2m} (\sin 2mx)_0^\pi + \pi - \frac{1}{2n} (\sin 2nx)_0^\pi \\ &= \pi + \pi + \frac{1}{2m} (0-0) - \frac{1}{2n} (0-0) \\ &= 2\pi \end{aligned}$$

6. Find the angle subtended by the double ordinate of length $2a$ of the parabola $y^2 = ax$ at its vertex.

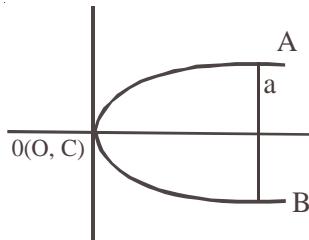
A. $y^2 = ax$, $a^2 = ax$, $a = x$ [put $y = a$]

A (a, a) , B $(a, -a)$

$$\text{Slope } OA = \frac{a}{a} = 1$$

$$\text{Slope of } OB = \frac{-a}{a} = -1$$

$$\text{Ans. } = \frac{\pi}{2}$$



7. If f is differentiable at $x = a$, find the value of

$$\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a}.$$

A. $\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a}, \frac{0}{0}$ form by LH

$$= \lim_{x \rightarrow a} \frac{2x f(a) - a^2 f'(x)}{1}$$

$$= 2af(a) - a^2 f'(a)$$

8. Find the values of 'a' for which the expression $x^2 - (3a - 1)x + 2a^2 + 2a - 11$ is always positive.

A. $x^2 - (3a - 1)x + 2a^2 + 2a - 11 > 0$

$$D < 0$$

$$(3a - 1)^2 - 4(2a^2 + 2a - 11) < 0$$

$$9a^2 - 6a + 1 - 8a^2 - 8a + 44 < 0$$

$$a^2 - 14a + 45 < 0$$

$$(a - 9)(a - 5) < 0$$

$$5 < a < 9$$

9. Find the sum of the first n terms of the series $0.2 + 0.22 + 0.222 + \dots$

A. $S = \frac{2}{9}[0.9 + 0.99 + 0.999 + \dots]$

$$= \frac{2}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots]$$

$$= \frac{2}{9}[n - (0.1 + 0.01 + \dots + n \text{ terms})]$$

$$= \frac{2}{9}n - \frac{2}{9} \frac{(0.1)[1 - (0.1)^n]}{[1 - (0.1)]}$$

$$\frac{2}{9}n - \frac{2}{9} \frac{(0.1)}{(0.9)} [1 - (0.1)^n]$$

$$\frac{2}{9}n - \frac{2}{81} + \frac{2}{81}(0.1)^n$$

10. The equation to the pairs of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. Find the equations of its diagonals.

A. $x = 2$ (i)

$x = 3$ (ii)

$y = 1$ (iii)

$y = 5$ (iv)

A (2, 1), B (3, 1), C (3, 5), D(2, 5)

Equation of AC

$$\frac{x-2}{3-2} = \frac{y-1}{5-1}, \quad x-2 = \frac{y-1}{4}$$

$$4x - 8 = y - 1, \quad 4x - y - 7 = 0$$

$$\text{Equation of BD} \quad \frac{x-3}{2-3} = \frac{y-1}{5-1}$$

$$\frac{x-3}{-1} = \frac{y-1}{4}, \quad -4x + 12 = y - 1$$

$$4x + y - 13 = 0$$