MATHEMATICS

1. The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer is

(a) 2 (b) 4 (c) 8 (d) 16

2. If one root of the equation $x^2+px+12=0$ is 4 and the equation $x^2+px+q=0$ has equal roots, the value of q is

(a) $\frac{49}{4}$ (b) $\frac{4}{49}$ (c) 4 (d) none of these

3. If $a>0, b>0, c>0$ are in G.P., then $\log a, \log b, \log c$ are in

(a) H.P. (b) A.P. (c) G.P. (d) none of these

4. The maximum number of points of intersections of 8 circles is

(a) 16 (b) 24 (c) 28 (d) 56

5. If $A(\alpha)=\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then $A(\alpha)A(\beta)$ is equal to

(a) $A(\alpha)-A(\beta)$ (b) $A(\alpha)+A(\beta)$ (c) $A(\alpha-\beta)$ (d) $A(\alpha+\beta)$

6. The sum of the series $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+......\infty$ is valid for

(a) $-1<x<1$ (b) all real x (c) $-1\le x\le 1$ (d) $0<x<1$

7. One root of the equation $5x^2+13x+K=0$ is the reciprocal of the other, if

(a) $K=0$ (b) $K=5$ (c) $K=\frac{1}{6}$ (d) $K=6$

8. Which one of the following is $(A-B)\cup(B-A)$?

(a) $(A\cup B)\cup(A-B)$ (b) $(A\cup B)\cup(A\cap B)$

(c) $(A\cup B)-(A\cap B)$ (d) $(A-B)\cap(B-A)$

9. Let $n(U)=700$, $n(A)=200$, $n(B)=300$ and $n(A\cap B)=100$, then $n(A^C\cap B^C)$ is equal to

(a) 400 (b) 600 (c) 300 (d) 200

10. The greatest positive integer, which divides $n(n+1)(n+2)(n+3)$ for all $n\in N$ is

(a) 2 (b) 6 (c) 24 (d) 120

11. The maximum value of $\sin\theta\cos\theta$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

12. If $\cot\theta-\tan\theta=\sec\theta$, then θ is equal to

(a) $n\pi+(-1)^n\frac{\pi}{6}$ (b) $n\pi+\frac{\pi}{2}$

(c) $2n\pi + \frac{3\pi}{2}$ (d) none of these

13. If $\tan A + \tan B + \tan C = 3\sqrt{3}$, then the triangle *ABC*, is

(a) isosceles (b) equilateral (c) right angled (d) none of these

14. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(a) $\frac{5\pi}{3}$ (b) $-\frac{5\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$

15. Three lines $3x - y = 2$, $5x + ay = 3$ and $2x + y = 3$ are concurrent, then *a* is equal to

(a) 2 (b) 3 (c) –10 (d) –2

16. The acute angle between the lines $ax + by + c = 0$ and $(a+b)x = (a-b)y$, $a \neq b$ is

(a) 15° (b) 30° (c) 45° (d) 60°

17. Let $x^2 + y^2 - 2p(x+y) + p^2 = 0$ be the equation of a circle. The circle

(a) passes through the origin (b) touches only *x*-axis

(c) touches only *y*-axis (d) touches both the axes

18. The equation of the directrix of the parabola $x^2 = -4ay$ is

(a) $x + a = 0$ (b) $x - a = 0$ (c) $y + a = 0$ (d) $y - a = 0$

19. If the focal distance of an end of the minor axis of any ellipse (referred to its axes as the axes of *x* and *y* respectively) is *k* and the distance between the foci is 2*h*, then its equation is

(a) $\frac{x^2}{k^2} + \frac{y^2}{h^2} = 1$ (b) $\frac{x^2}{k^2} + \frac{y^2}{k^2 - h^2} = 1$

(c) $\frac{x^2}{k^2} + \frac{y^2}{h^2 - k^2} = 1$ (d) $\frac{x^2}{k^2} + \frac{y^2}{k^2 + h^2} = 1$

20. The product of the perpendiculars, drawn from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is

(a) $\frac{ab}{\sqrt{a} + \sqrt{b}}$ (b) $\frac{ab}{a^2 + b^2}$

(c) $\frac{a^2b^2}{a^2 + b^2}$ (d) $\frac{a^2 + b^2}{a^2b^2}$

21. The lines $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-3}{0}$ and $\frac{x-2}{0} = \frac{y-3}{0} = \frac{z-4}{1}$ are

(a) parallel (b) coincident (c) skew (d) perpendicular

22. If $f(x)$ is a function, then $f(x) + f(-x)$ is

(a) an even function (b) an odd function

(c) not a function (d) none of these

23. Let the function *f* be defined by $f(x) = \frac{2x+1}{1-3x}$, then $f^{-1}(x)$ is

(a) $\frac{x-1}{3x+2}$ (b) $\frac{3x+2}{x-1}$ (c) $\frac{x+1}{3x-2}$ (d) $\frac{2x+1}{1-3x}$

24. If $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)$ exists, then

(a) both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ must exist and $\lim_{x\to a} g(x)\neq 0$

(b) $\lim_{x\to a} f(x)$ need not exist but $\lim_{x\to a} g(x)$ exists

(c) neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists

(d) $\lim_{x\to a} f(x)$ exists but $\lim_{x\to a} g(x)$ does not exist

25. Let $\phi(x)=\frac{1-\cos\lambda x}{x\sin x}$, $x\neq 0$ and $\phi(0)=\frac{1}{2}$. If $\phi(x)$ is continuous at $x = 0$, then λ is

(a) 0 (b) ± 1 (c) 2 (d) none of these

26. Let f be a polynomial. Then the second derivative of $f(e^x)$ is

(a) $f''(e^x)e^{2x}+f'(e^x)e^x$ (b) $f''(e^x)e^x+f'(e^x)$

(c) $f''(e^x)e^{2x}+f''(e^x)e^x$ (d) $f''(e^x)$

27. If $f(x)=\log_a\log_a(x)$, then $f'(x)$ is

(a) $\frac{\log_a e}{x\log_e x}$ (b) $\frac{\log_e a}{x\log_a x}$ (c) $\frac{\log_e a}{x}$ (d) $\frac{x}{\log_e a}$

28. If $f(x)=A\sin\left(\frac{\pi x}{2}\right)+B$, $f'\left(\frac{1}{2}\right)=\sqrt{2}$ and $\int_0^1 f(x)dx=\frac{2A}{\pi}$, then the constants A, B are

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ (c) 0 and $-\frac{4}{\pi}$ (d) $\frac{4}{\pi}$ and 0

29. $\int_0^{\pi/2}\sin^7 x\ dx$ has value

(a) $\frac{37}{184}$ (b) $\frac{17}{45}$ (c) $\frac{16}{35}$ (d) $\frac{16}{45}$

30. $\int\frac{\sqrt{x}}{1+x}dx$ equals

(a) $\log\left(\frac{1+\sqrt{x}}{\sqrt{x}}\right)+c$ (b) $\log\left(\frac{\sqrt{x}}{1+x}\right)+c$

(c) $2\sqrt{x}-2\tan^{-1}\sqrt{x}+c$ (d) $2\sqrt{x}-\tan^{-1}\sqrt{x}+c$

31. The area bounded by the curve $y=\log_e x$ the x-axis and the straight line $x = e$ is

(a) e sq. units (b) 1 sq. unit (c) $1-\frac{1}{e}$ sq. units (d) $1+\frac{1}{e}$ sq. units

32. The solution of $\frac{dy}{dx}+P(x)y=0$ is

(a) $y = Ce^{\int P\,dx}$ (b) $y = Ce^{-\int P\,dx}$ (c) $x = Ce^{-\int P\,dy}$ (d) $x = Ce^{\int P\,dy}$

33. $\tan^{-1} x + \tan^{-1} y = c$ is the general solution of the differential equation

(a) $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$ (b) $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$

(c) $(1+x^2)\,dy + (1+y^2)\,dx = 0$ (d) $(1-x^2)\,dx + (1-y)\,dy = 0$

34. Out of 20 consecutive positive integers, two are chosen at random. The probability that their sum is odd is

(a) $\dfrac{1}{20}$ (b) $\dfrac{10}{19}$ (c) $\dfrac{19}{20}$ (d) $\dfrac{9}{19}$

35. A random variable X is specified by the following distribution law:

X	2	3	4
$p(X = x)$	0.3	0.4	0.3

Then the variance of this distribution is

(a) 0.6 (b) 0.7 (c) 0.77 (d) 1.55

36. If $\vec{a}$ and $\vec{b}$ are any two vectors, then

(a) $|\vec{a}\,.\,\vec{b}| > |\vec{a}|\,|\vec{b}|$ (b) $|\vec{a}\,.\,\vec{b}| < |\vec{a}|\,|\vec{b}|$

(c) $|\vec{a}\,.\,\vec{b}| \geq |\vec{a}|\,|\vec{b}|$ (d) $|\vec{a}\,.\,\vec{b}| \leq |\vec{a}|\,|\vec{b}|$

37. Let $\vec{a}$ and $\vec{b}$ be orthogonal. The value of the parameter m for which $\vec{a} + m\vec{b}$ is orthogonal to the vector $\vec{a} + \vec{b}$ is

(a) –1 (b) 1 (c) $-\left(\dfrac{|\vec{a}|}{|\vec{b}|}\right)^2$ (d) none of these

38. If A and B are the co-efficients of x^r and x^{n-r} in the expansion of $(1+x)^n$, then

(a) $A = B$ (b) $A \neq B$

(c) $A = \lambda B$ for some $\lambda > 1$ (d) none of these

39. Rolle's Theorem holds for the function $x^3 + bx^2 + cx,\ 1 \leq x \leq 2$ at the point $\dfrac{4}{3}$, the value of b and c are

(a) $b = 8,\ c = -5$ (b) $b = -5,\ c = 8$ (c) $b = 5,\ c = -8$ (d) $b = -5,\ c = -8$

40. The equation of the tangent to the curve $(1+x^2)\,y = 2 - x$, where it crosses x-axis is

(a) $x + 5y = 2$ (b) $x - 5y = 2$ (c) $x - 5y = 0$ (d) $5x + y - 2 = 0$

41. The least possible value of K for which the function $f(x) = x^2 + Kx + 1$ may be increasing in [1, 2] is

(a) 2 (b) –2 (c) 0 (d) none of these

42. Let $f(x) = \dfrac{\log(1+x)}{x}$, $x \neq 0$, then $f(x)$ is

(a) monotonically increasing for $x > 0$
(b) monotonically decreasing for $x > 0$
(c) nothing can be said in general
(d) none of these

43. The number of real roots of $(6-x)^4 + (8-x)^4 = 16$ is

(a) 0 (b) 2 (c) 4 (d) none of these

44. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2 + b|x| + c = 0$ is

(a) 2 (b) 4 (c) 0 (d) none of these

45. The solution set of the inequation $2x + y > 5$ is

(a) half plane that contains the origin
(b) open half plane not containing the origin
(c) whole xy-plane except the points lying on the line $2x + y = 5$
(d) none of these