

**Solutions
of
Waves & Thermodynamics**

Lesson 14th to 19th

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14. Wave Motion

Introductory Exercise 14.1

1. A function, f can represent wave equation, if it satisfy

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$

For, $y = a \sin t$,

$$\frac{\partial^2 y}{\partial t^2} = a \sin t \neq v^2 y$$

but, $\frac{\partial^2 y}{\partial x^2} = 0$

So, y do not represent wave equation.

2. $y(x, t) = ae^{(bx - ct)^2} = ae^{(kx - vt)^2}$

$$k = b \text{ and } v = \frac{c}{b}$$

3. $y(x, t) = \frac{1}{1 + (4x - t)^2}$ represent the given pulse, where,

$$y(x, 0) = \frac{1}{1 + k^2 x^2} = \frac{1}{1 + x^2}$$

$$k = 1$$

$$y(x, z) = \frac{1}{1 + (x - 2z)^2} = \frac{1}{1 + (x - 1)^2}$$

$$v = \frac{1/2}{1} = 0.5 \text{ m/s}$$

4. $y = \frac{10}{5 + (x - 2t)^2} = \frac{a}{b + (kx - vt)^2}$

$$\text{Amplitude, } y_{\max} = \frac{a}{b} = \frac{10}{5} = 2 \text{ m}$$

$$\text{and } k = 1; \quad v = 2$$

$v = \frac{2}{k} = 2 \text{ m/s}$ and is travelling in $(-)$ x direction.

5. $y = \frac{10}{(kx - t)^2 + 2}$

$$y(x, 0) = \frac{10}{k^2 x^2 + 2} = \frac{10}{x^2 + 2} \quad k = 1$$

$$vk = 2 \text{ m/s} \quad 1 \text{ m}^{-1} = 2 \text{ rad/s}$$

$$y = \frac{10}{(x - 2t)^2 + 2}$$

Introductory Exercise 14.2

1. $y(x, t) = 0.02 \sin \left(\frac{x}{0.05} - \frac{t}{0.01} \right) \text{ m}$

$$A \sin(kx - \omega t) \text{ m} \quad \cos \frac{0.2}{0.5} - \frac{0.3}{0.01}$$

$$A = 0.02 \text{ m}, k = \frac{1}{0.05} \text{ m}^{-1}, \quad \frac{1}{0.01} \text{ s}^{-1}$$

(a) $v = \frac{0.05}{0.01} \text{ m/s} = 5 \text{ m/s}$

(b) $v_p = \frac{y}{t} = A \cos(kx - \omega t)$

$$v_p(0.2, 0.3) = 0.02 \frac{1}{0.01}$$

$$2 \cos(4 - 30)$$

$$2 \cos 34$$

$$2(-0.85)$$

$$1.7 \text{ m/s}$$

2. Yes, $(v_p)_{\max} = A \frac{A k}{k} = (A k) v$

3. $4 \text{ cm}, v = 40 \text{ cm/s}$ (given)

(a) $\frac{v}{4 \text{ cm}} = \frac{40 \text{ cm/s}}{4 \text{ cm}} = 10 \text{ Hz}$

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$$(b) \quad \frac{2}{4 \text{ cm}} \cdot 2.5 \text{ cm} = \frac{5}{4} \text{ rad}$$

$$(c) \quad t = \frac{T}{2} = \frac{1}{2} \cdot \frac{1}{10} = \frac{1}{20} \text{ s}$$

$$(d) \quad v_p = \frac{(v_p)_{\max}}{A} = \frac{2 \text{ cm}}{2 \text{ cm} \cdot 10 \text{ s}^{-1}} = \frac{40 \text{ cm/s}}{1.26 \text{ cm/s}}$$

$$y = A \sin \left(2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \right)$$

$$0.05 \sin \left(2\pi \left(\frac{t}{0.4} - \frac{x}{0.4} \right) \right)$$

$$0.05 \sin (60\pi t - 5\pi x)$$

$$(b) \quad y(0.25, 0.15)$$

$$0.05 \sin (60\pi \cdot 0.15 - 5\pi \cdot 0.25)$$

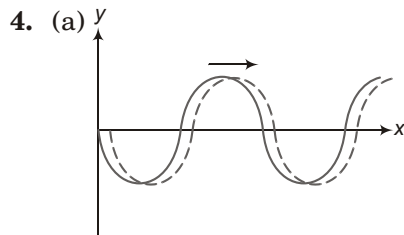
$$0.05 \sin (9\pi - 1.25\pi)$$

$$0.05 \sin (7.75\pi) = 0.05 \sin (1.75\pi)$$

$$0.0354 \text{ m} = 3.54 \text{ cm}$$

$$(c) \quad t = \frac{T}{2} = \frac{0.25}{60}$$

$$\frac{1}{240} \text{ s} = 4.2 \text{ ms}$$



Introductory Exercise 14.3

$$1. \quad v = \sqrt{\frac{T}{m/l}} = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{500 \cdot 2}{0.06}} = \frac{100\sqrt{5}}{\sqrt{3}} = 129.1 \text{ m/s}$$

$$2. \quad v = \sqrt{\frac{T}{A}} = \sqrt{\frac{0.98}{9.8 \cdot 10^3 \cdot 10^{-6}}} = 10 \text{ m/s}$$

Introductory Exercise 14.4

$$1. \quad I = \frac{P}{4r^2} = \frac{1 \text{ W}}{4(1 \text{ m})^2} = \frac{1}{4} \text{ W/m}^2$$

$$I = \frac{1}{r} \text{ and as } I \propto A^2$$

$$2. \quad \text{For line source, } I = \frac{1}{2rl}$$

$$A = \frac{1}{\sqrt{r}}$$

AIEEE Corner

■ Subjective Questions (Level 1)

1. $y(x, t) = 6.50 \text{ mm} \cos 2$

$$\frac{x}{28.0 \text{ cm}} - \frac{t}{0.0360 \text{ s}}$$

$$A \cos 2 \frac{x}{T} - \frac{t}{T}$$

$$A = 6.50 \text{ mm}, \quad \frac{28.0 \text{ cm}}{T} = 27.78 \text{ Hz}$$

$$v = \frac{28.0 \text{ cm}}{27.78 \text{ s}^{-1}} = 778 \text{ cm/s} = 7.78 \text{ m/s}$$

The wave is travelling along ()ve x-axis.

2. $y = 5 \sin 30 \pi t - \frac{x}{240}$

$$5 \sin 30 \pi t - \frac{x}{8} = A \sin (2 \pi t - kx)$$

(a) $y(2, 0) = 5 \sin 3 \pi = 0$

$$5 \sin \frac{\pi}{4} = \frac{5}{\sqrt{2}} = 3.535 \text{ cm}$$

(b) $\frac{2}{k} = \frac{2}{8} = 16 \text{ cm}$

(c) $v = \frac{30}{k} = \frac{30}{8} = 240 \text{ cm/s}$

(d) $\frac{30}{2} = 15 \text{ Hz}$

3. $y = 3 \text{ cm} \sin(3.14 \text{ cm}^{-1}x - 314 \text{ s}^{-1}t)$

$$3 \text{ cm} \sin(\pi x - 100 \pi t)$$

$$A \sin(kx - \omega t)$$

(a) $(v_p)_{\max} = A = 3 \text{ cm} = 0.03 \text{ m}$
 $300 \text{ cm/s} = 3 \text{ m/s} = 9.4 \text{ m/s}$

(b) $a = \omega^2 y = (100 \pi \text{ s}^{-1})^2 \cdot 3 \text{ cm} = 300 \sin(6 \pi t)$

$$300 \sin(105 \pi t) = 0$$

4. (a) $x = \frac{v}{2} t = \frac{350}{500} t = \frac{7}{10} t$

$$\frac{7}{60} - \frac{7}{50} = -0.166 \text{ m}$$

(b) $\frac{2}{T} t = 2 \pi t = 500 \cdot 10^{-3}$

5. $y(x, t) = \frac{180}{(kx - t)^2 + 3}$

$$y(x, 0) = \frac{6}{k^2 x^2 + 3} = \frac{6}{x^2 + 3}$$

$$k = 1 \text{ m}^{-1}$$

$$v k = 4.5 \text{ m/s} = 1 \text{ m}^{-1} \cdot 4.5 \text{ rad/s}$$

$$y(x, t) = \frac{6}{(x - 4.5t)^2 + 3}$$

6. $y = 1.0 \sin \frac{x}{2.0} - \frac{t}{0.01}$

$$1.0 \sin 2 \pi \left(\frac{x}{4.0} - \frac{t}{0.02} \right)$$

$$A \sin 2 \pi \left(\frac{x}{T} - \frac{t}{T} \right)$$

(a) $A = 1.0 \text{ mm}, \quad 4.0 \text{ cm}, \quad T = 0.02 \text{ s}$

(b) $v_p = \frac{y}{t} = A \cos 2 \pi \left(\frac{x}{T} - \frac{t}{T} \right)$

$$\frac{2}{T} A \cos 2 \pi \left(\frac{x}{T} - \frac{t}{T} \right)$$

$$\frac{2}{0.02 \text{ s}} \cdot 1.0 \text{ mm} \cos 2 \pi \left(\frac{x}{4.0} - \frac{t}{0.02 \text{ s}} \right)$$

$$\frac{100}{10} \text{ m/s} \cos \frac{x}{2.0 \text{ cm}} - \frac{t}{0.01 \text{ s}}$$

$$v_p(1.0 \text{ cm}, 0.01 \text{ s})$$

$$\frac{1}{10} \text{ m/s} \cos \frac{1}{2} = \frac{0.01}{2} = 0.01$$

$$\frac{1}{10} \text{ m/s} \cos \frac{0}{2} = 0 \text{ m/s}$$

(c) $v_p(3.0, 0.01)$

$$\frac{3}{10} \cos \frac{3}{2} = 1 = 0 \text{ m/s}$$

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$$v_p(5.0 \text{ cm}, 0.01\text{s}) = \frac{0 \text{ m/s}}{10} \cos \left(\frac{5}{2} x - 3750 t \right)$$

$$v_p(7.0 \text{ cm}, 0.01\text{s}) = \frac{0 \text{ m/s}}{10} \cos \left(\frac{7}{2} x - 3750 t \right)$$

$$(d) v_p(1.0 \text{ cm}, 0.011\text{s}) = \frac{0 \text{ m/s}}{10} \cos \left(\frac{1}{2} x - 3545.5 t \right)$$

$$\cos \left(\frac{1}{2} x - 3545.5 t \right) = \frac{1}{10} \cos \left(\frac{1}{12} x - 1.1 t \right) = \frac{0.6}{10} \cos \left(\frac{3}{5} x - 9.7 \text{ cm/s} t \right)$$

$$v_p(1.0 \text{ cm}, 0.012\text{s}) = \frac{0 \text{ m/s}}{10} \cos \left(\frac{1}{2} x - 4166.7 t \right)$$

$$\cos \left(\frac{1}{2} x - 4166.7 t \right) = \frac{0.5}{10} \cos \left(0.7 x - 18.5 \text{ cm/s} t \right)$$

$$v_p(1.0 \text{ cm}, 0.013 \text{ s}) = \frac{0 \text{ m/s}}{10} \cos \left(\frac{1}{2} x - 4615.4 t \right)$$

7. (a) $k = \frac{2}{40 \text{ cm}} = \frac{1}{20} \text{ cm}^{-1}$

$$T = \frac{1}{8} \text{ s} = 0.125 \text{ s}$$

$$v = \frac{2 \text{ rad/s}}{8 \text{ s}^{-1}} = \frac{16 \text{ rad/s}}{40 \text{ cm}} = \frac{50.26 \text{ rad/s}}{320 \text{ cm/s}}$$

$$(b) y(x, t) = A \cos(kx - \omega t) = 15.0 \text{ cm} \cos(0.157x - 50.3t)$$

8. $A = 0.06 \text{ m}$ and 2.5 cm
 $\frac{20}{2.5} \text{ cm} = 8 \text{ cm}$
 $v = \frac{300 \text{ m/s}}{8 \text{ cm}} = 3750 \text{ Hz}$

$$y = A \sin(kx - \omega t) = 0.06 \text{ m} \sin \left(\frac{2}{0.08} x - 2 \cdot 3750 t \right)$$

$$\sin \left(\frac{2}{0.08} x - 2 \cdot 3750 t \right) = 0.06 \text{ m} \sin(78.5 \text{ m}^{-1} x - 23561.9 \text{ s}^{-1} t)$$

9. (a) $v = \frac{8.00 \text{ m/s}}{0.32 \text{ m}} = 25 \text{ Hz}$

$$T = \frac{1}{15} \text{ s} = 0.043 \text{ Hz}$$

$$k = \frac{2}{0.32 \text{ m}} = 19.63 \text{ rad/m}$$

$$(b) y = A \cos(kx - \omega t) = 0.07 \text{ m} \cos \left(2 \frac{x}{0.32 \text{ m}} - 2 \frac{t}{0.04 \text{ s}} \right)$$

$$(c) y = 0.07 \text{ m} \cos \left(2 \frac{0.36}{0.32} - 2 \frac{0.15}{0.04} \right) = 0.07 \text{ m} \cos \left(2 \frac{9}{8} - \frac{30}{8} \right)$$

$$= 0.07 \text{ m} \cos \frac{39}{4}$$

$$= 0.07 \text{ m} \cos 10 = \frac{0.0495 \text{ m}}{4}$$

$$(d) t = \frac{T}{2} = \frac{1/4}{2} = \frac{1}{8} \text{ s} = 0.125 \text{ s}$$

$$= \frac{3}{200} \text{ s} = 0.015 \text{ s}$$

$$10. v = \sqrt{\frac{T}{A}} = \sqrt{\frac{Mg}{A}} = \sqrt{\frac{2 \cdot 9.8}{8920 \cdot 3.14 \cdot (1.2 \cdot 10^3)^2}} = \sqrt{\frac{2 \cdot 9.8 \cdot 10^4}{89.2 \cdot 3.14 \cdot 1.44}} = 22 \text{ m/s}$$

$$11. \frac{\sqrt{T}}{2} = \frac{\sqrt{M}}{1} \Rightarrow \frac{2}{1} \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{8}{2}} \Rightarrow \sqrt{4} = 2$$

$$2 = 2 \Rightarrow 0.12 \text{ m}$$

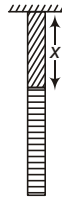
12. $T(x) = (L - x)g, v(x) = \sqrt{\frac{T(x)}{\mu}}$

$$\frac{\sqrt{g(L-x)}}{dx} dt;$$

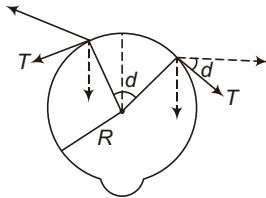
Let, $L - x = y$

$$\frac{dx}{dy} = -1$$

$$t = \frac{1}{\sqrt{g}} \int_0^L \frac{dy}{\sqrt{y}} = \frac{2\sqrt{L}}{\sqrt{g}}$$



13. (a) $dm = \frac{2R}{L} T \sin d$



$$\frac{R \cdot 2d}{R^2} = \frac{2T \sin d}{T}$$

Wave speed, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{2R^2/L}} = R \sqrt{\frac{L}{2R}}$

(b) Kink remains stationary when rope and kink moves in opposite sense i.e., if rope is rotating anticlockwise then kink has to move clockwise.

14. x is being measured from lower end of the string

$$m(x) = \int_0^x \mu dx = \frac{1}{2} \mu x^2$$

$$v(x) = \sqrt{\frac{T(x)}{\mu}} = \sqrt{\frac{\mu x g}{\mu}} = \sqrt{\frac{1}{2} g x}$$

15. $t = \sqrt{\frac{2}{g}} \int_0^L dx = 2\sqrt{l_0}$

$$t = \sqrt{\frac{8l_0}{g}}$$

$$M = \int_0^L dm = \int_0^L kx dx = \frac{1}{2} kL^2$$

$$k = \frac{2M}{L^2}$$

$$v(x) = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{kx}} = \sqrt{\frac{TL^2}{2Mx}} \frac{dx}{dt}$$

$$t = \int_0^L \frac{dx}{v} = \int_0^L \frac{dx}{\sqrt{\frac{TL^2}{2Mx}}} = \sqrt{\frac{2M}{TL^2}} \int_0^L \sqrt{x} dx = \frac{1}{2} \sqrt{\frac{2M}{TL^2}} L^2$$

$$\frac{2}{3} \sqrt{\frac{2ML^3}{TL^2}} = \frac{2}{3} \sqrt{\frac{2ML}{T}}$$

16. (a) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{1.5/0.055}} = 16.3 \text{ m/s}$

(b) $\frac{v}{120/\text{s}} = 0.136 \text{ m}$

(c) $v \propto \sqrt{T} \propto \sqrt{M}$ i.e., if M is doubled both speed and wavelength increases by a factor of $\sqrt{2}$.

17. $E = I A t = \frac{1}{2} \mu a^2 v A t$

$$E = \frac{1}{2} \mu a^2 (A) (v) t$$

$$E = \frac{1}{2} \mu a^2 \cdot l$$

$$E = \frac{1}{2} \mu a^2 m$$

$$E = \frac{1}{2} (3.14)^2 (120)^2 (0.16 \cdot 10^{-3})^2 \cdot 80 \cdot 10^3$$

$$E = 582 \cdot 10^6 \text{ J} = 582 \text{ J} = 0.58 \text{ mJ}$$

18. $P = \frac{E}{t} = I A v = \frac{1}{2} \mu a^2 v A = \frac{1}{2} \mu a^2 v$

$$P = \frac{1}{2} \mu a^2 \sqrt{T}$$

$$P = \frac{1}{2} (3.14)^2 (60)^2$$

$$P = \frac{1}{2} (6 \cdot 10^2)^2 \sqrt{80 \cdot 5 \cdot 10^2}$$

$$P = 4(3.14 \cdot 60 \cdot 0.06)^2 = 511.6 \text{ W}$$

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$$19. P = IA = 2^2 \cdot 2^2 \cdot a^2 \cdot \sqrt{T}$$

$$= 2 \cdot (3.14)^2 \cdot (200)^2 \cdot 10^6 \cdot \sqrt{60 \cdot 6 \cdot 10^3}$$

$$= 8 \cdot (3.14)^2 \cdot 10^2 \cdot 6 \cdot 10^1 \text{ W}$$

$$= 0.474 \text{ W}$$

$$E = Pt = \frac{P \cdot l}{v}$$

$$= \frac{0.474 \cdot 2}{\sqrt{\frac{60}{6 \cdot 10^3}}} = \frac{0.474 \cdot 2}{100} \text{ J} = 9.48 \text{ mJ}$$

$$20. P = 2^2 v^2 a^2 \cdot vA = 2^2 v^2 a^2 \cdot v; v = \sqrt{\frac{T}{\mu}}$$

$$= 2^2 \cdot 2^2 \cdot a^2 \cdot \frac{T}{v^2} \cdot v = 2^2 \cdot 2^2 \cdot a^2 \cdot \frac{T}{v}$$

$$= \frac{2 \cdot (3.14)^2 \cdot (100)^2 \cdot (0.5 \cdot 10^{-3})^2 \cdot 100}{2 \cdot (3.14)^2 \cdot 10^4 \cdot 0.25 \cdot 10^6}$$

$$= 4.93 \cdot 10^{-2} \text{ W} = 49 \text{ mW}$$

■ Objective Questions (Level 1)

1. $\frac{150}{60} = 2.5 \text{ rad/s}$, $A = 0.04 \text{ m}$ and $\frac{4}{4}$

$$y = A \sin(\omega t) = 0.04 \sin 5 t \quad \frac{4}{4}$$

2. 600 , $v = 300$, $k = \frac{2}{v}$

$$y = A \sin(\omega t - kx)$$

$$= 0.04 \sin(600 t - 2 x)$$

$$y(0.75, 0.01) = 0.04 \sin 600 \cdot 0.01 - 2 \cdot \frac{3}{4}$$

$$= 0.04 \sin 6 - \frac{3}{2}$$

$$= 0.04 \sin 4 - \frac{3}{2} = 0.04 \text{ m}$$

3. $y(x, t) = \frac{1}{2} \cdot \frac{1}{3(kx - t)^2}$

$$y(x, 0) = \frac{1}{2} \cdot \frac{1}{3k^2 x^2} = \frac{1}{2} \cdot \frac{1}{3x^2}$$

$$y(x, 2) = \frac{1}{2} \cdot \frac{1}{3(x - 2)^2} = \frac{1}{2} \cdot \frac{1}{3(x - 2)^2}$$

$$1 = v \cdot \frac{1}{k} = 1 \text{ m/s}$$

4. $y = A \sin(\omega t - kx) = \frac{A}{2}$

$$\omega t - kx = \frac{\pi}{6}$$

$$\frac{2}{T} \cdot \frac{T}{6} - \frac{1}{6} = kx$$

5. $\frac{12 \cdot 0.04}{6} = 0.04 \text{ m}$

$$v = \frac{300 \text{ m/s}}{25 \text{ Hz}} = 12 \text{ m}$$

$$\frac{2}{12} x = \frac{2}{12 \text{ m}} (16 - 10) \text{ m}$$

6. $y = 0.02 \sin(x - 30t) = A \sin(kx - \omega t)$

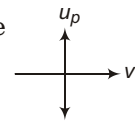
$$k = 1, \omega = 30$$

$$v = \frac{\omega}{k} = 30 \text{ m/s} = \sqrt{\frac{T}{\mu}}$$

$$T = v^2 \cdot \mu = 1.3 \cdot 10^4 = 900 = 0.117 \text{ N}$$

7. $v_p = \frac{y}{t} = \frac{y}{x} \cdot \frac{x}{t} = v \cdot \frac{y}{x}$ slope v

In transverse wave u_p they are



perpendicular i.e., $\frac{\pi}{2}$. In longitudinal

wave u_p , they are either at 0 or π so, $0, \frac{\pi}{2}$ and π are the possible angles

between v_p and v .

8. $2 = 200 \text{ rad/s}$,

$$k = \frac{1}{v} \cdot \sqrt{\frac{3.5 \cdot 10^3}{35}}$$

$$y = A \cos(200t - 2x) \quad \text{2 rad/m}$$

$$\frac{y}{x} = 2A \sin(200t - 2x)$$

When, $y = 0$

$$\sin(200t - 2x) = 0$$

$$\sin(200t - 2x) = 1$$

$$2A = \frac{1}{20} \quad A = \frac{1}{40} = 0.025 \text{ m}$$

$$y = 0.025 \cos(200t - 2x)$$

9. $\frac{2}{T} = \frac{2}{0.25} = 8 \text{ rad/s;}$

$$k = \frac{8}{v} = \frac{8}{48} = \frac{1}{6} \text{ rad/cm}$$

$$y = A \sin(8t - \frac{1}{6}x)$$

$$A \sin 8t - \frac{1}{6}x = \frac{67}{6}$$

$$A \sin \frac{\pi}{6} = A \sin 30^\circ = \frac{A}{2} = 3 \text{ cm}$$

$$A = 6 \text{ cm}$$

10. $\frac{v_A}{v_B} = \sqrt{\frac{T_A}{A_A} \frac{A_B}{T_B}} = \sqrt{\frac{T_A d_B^2}{T_B d_A^2}}$

$$\frac{d_B}{d_A} \sqrt{\frac{T_A}{T_B}} = \frac{d_B}{d_B/2} \sqrt{\frac{T_B/2}{T_B}}$$

$$2 \frac{1}{\sqrt{2}} = \sqrt{2}$$

11. $E = A^2 v^2$ for E to constant, A constant

$$\frac{A_A}{A_B} = \frac{A_B}{4A_A} \quad \frac{A_A^4}{A_B} = \frac{A_B}{A_B}$$

12. $k = 1 \text{ rad/m; } v = 4 \text{ m/s}$

$$y = \frac{vk}{6} \frac{4 \text{ rad/s}}{3} \frac{6}{(kx - t)^2} - \frac{6}{(x - 4t)^2} \frac{6}{3}$$

13. $v_l = \sqrt{\frac{Y}{\rho}}$ and $v_t = \sqrt{\frac{Y}{\rho} \frac{l}{l}} = v_l \sqrt{\frac{l}{l}}$

$$\sqrt{\frac{l}{l}} = \frac{v_l}{v_t} = 10 \quad \frac{l}{l} = \frac{1}{100}$$

$$\text{Stress} = Y \frac{l}{l} = E \frac{l}{l} = \frac{E}{100}$$

14. $A = 4 \text{ m, } \frac{1}{5}, k = \frac{1}{9}, \frac{1}{6}$

$$v = \frac{1/5}{k} = \frac{9}{5} \text{ m/s}$$

$$\frac{2}{k} = \frac{2}{9} = 18 \text{ m}$$

$$\frac{1}{2} = \frac{1/5}{2} = \frac{1}{10} \text{ Hz}$$

15. 10 and $k = 0.1$

$$\frac{2}{k} = \frac{2}{0.1} = 20 \text{ m}$$

$$\frac{2}{x} = \frac{2}{20} = 10$$

16. $y = \frac{2}{(2x - 6.2t)^2} \frac{2}{20}$

$$A = \frac{2}{20} = 0.1 \text{ m, } k = 2 \text{ rad/m}$$

and

$$v = \frac{6.2 \text{ rad/s}}{k} = \frac{6.2}{2} = 3.1 \text{ m/s}$$

$$\frac{6.2}{2} = \frac{6.2}{2} = 3.1 \text{ Hz}$$

$$\frac{2}{k} = \frac{2}{2} = \text{m}$$

17. $I = 2^2 \frac{1}{2} A^2 v = \frac{1}{2} A^2 v$

$$u = \frac{E}{V} = \frac{IST}{V} = \frac{2^2 A^2 v St}{V}$$

$$2^2 A^2 \frac{1}{2} A^2$$

$$P = \frac{E}{t} = I.S = 2^2 A^2 v S$$

$$\frac{1}{2} A^2 v S$$

$$E = Pt = P \frac{E}{t} = IS = I \frac{P}{S}$$

18. $y = A \sin(x - t)$

$$y(x, 0) = A \sin(x) \quad y = 0 \text{ for } x = 0 \text{ and } 1$$

$$a = \frac{2}{2} y = A \sin(x)$$

$$a = \frac{2}{2} A \text{ at } x = \frac{1}{2} \text{ and } \frac{3}{2}$$

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$$v_p = A \cos(x) \quad v_p = 0 \text{ for } x = \frac{1}{2} \text{ and } \frac{3}{2}$$

So all the above options are correct.

$$19. \quad y = A \sin \frac{2}{a} (x - bt) = A \sin (kx - \omega t)$$

$$k = \frac{2}{a}, \quad \omega = \frac{2b}{a}$$

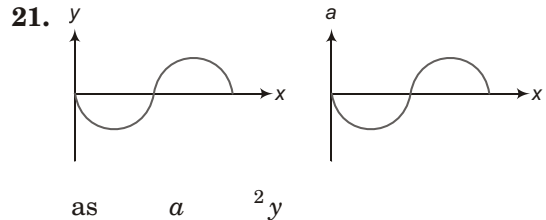
$$v = \frac{\omega}{k} = \frac{2b/a}{2/a} = b$$

$$\frac{2}{k} = \frac{2}{2/a} = a$$

$$20. \quad y = A \sin 2\pi \left(\frac{x}{a} - \frac{t}{T} \right) = A \sin 2\pi \left(\frac{x}{a} - \frac{t}{T} \right)$$

$$a, T, b$$

$$v = \frac{a}{T} = \frac{a}{b}$$



JEE Corner

■ Assertion and Reason

- For propagation of transverse waves medium require tension which is possible due to modulus of rigidity. And in gases there is no such Young's modulus or surface tension. So the reason given is correct explanation.
- Surface tension of water plays the role of modulus of rigidity and that is why transverse waves can travel on liquid surface.
- Both the waves are travelling in same direction with a phase difference of π . So reason is false.
- v/f is constant for a particular medium so if frequency is doubled wavelength becomes half, and speed remains constant. Thus assertion is false.
- Sound is mechanical wave which requires material medium for propagation and as on moon there is no atmosphere, sound cannot travel.
- Angular wave number, $k = \frac{2\pi}{\lambda}$ while wave number, $n = \frac{1}{\lambda}$ which is defined as the number of waves per unit length.
- Electromagnetic wave are non-mechanical, they travel depending upon electric and magnetic properties of medium. They can travel in medium as well as an vacuum. So reason is false.
- As speed, $v = \sqrt{\frac{T}{\mu}}$ $v \propto \frac{1}{\sqrt{\mu}}$ in second string μ is more (by looking) so v will be less. Thus reason is true explanation of assertion.
- At point A both v_p and l is zero i.e., K.E. and P.E. are minimum while at B both v_p and l are maximum i.e., both K.E. and P.E. are maximum. Thus both assertion and reason are true but not correct explanation.
- If P is moving downward then it shows that the wave is travelling in () ve x direction. So assertion is false.

11. $A = 2a \cos \frac{\omega}{2}$, for $A = a$
 $\cos \frac{\omega}{2} = \frac{1}{2} \Rightarrow \cos \frac{\omega}{3}$

$\frac{2}{3} = \frac{360}{3} = 120$

Assertion is true but the reason is false.

■ Match the Columns

1. $y = a \sin(bt - cx) = A \sin(t - kx)$

(a)	$v = \frac{b}{k} = \frac{b}{c}$	r
(b)	$(v_p)_{\max} = A = ab$	s
(c)	$\frac{b}{2} = \frac{b}{2}$	p
(d)	$\frac{2}{k} = \frac{2}{c}$	s

2. $y = 4 \text{ cm} \sin(t - 2x)$

$v_p = 4 \text{ cm/s} \cos(t - 2x)$
 $a = 4^2 \text{ cm/s}^2 \sin(t - 2x)$

(a)	$v_p(0, t) = 4 \text{ cm/s} \cos t$ 4 for $\cos t = 1$ or $t = n\pi, n = 0, 1, 2, 3,$	q, r
(b)	$a(0, t) = 4^2 \text{ cm/s}^2 \sin t$ 4 ² for $\sin t = 1$ or $t = (2n - 1)\frac{\pi}{2}$ $t = n\pi, n = \frac{1}{2}, 0.5, 2.5$	p, s
(c)	$v_p(0.5, t) = 4 \text{ cm/s} \cos(t - 0.5)$ 4 for $t = n\pi, n = 0, 1, 2, 3,$	q, r
(d)	$a(0.5, t) = 4^2 \text{ cm/s}^2 \sin(t - 0.5)$ 4 ² for $\sin(t - 0.5) = 1$ or $t = n\pi + \frac{1}{2}, n = 0.5, 1.5, 2.5$	p

3. $y = A \sin(t - kx)$ at $t = 0$

$y = A \sin kx$
 $v_p = A \cos kx$ and $a = -A^2 \sin kx$

(a)	$v_p = A \cos kx$	s
(b)	$a_A = (-A^2) \sin kx$ as y_A is negative	p

(c)	$v_B = A$	s
(d)	$a_B = 0$ ve $y_0 = 0$	r

4. (a) $u = \frac{E}{V} = \frac{IST}{V} = \frac{2^2 \cdot 2^2 \cdot A^2 \cdot vst}{2^2 \cdot 2^2 \cdot A^2 \cdot \frac{1}{2} \cdot 2^2 \cdot A^2}$

$[u] = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}] = s$

(b) $P = \frac{E}{t} = \frac{ISt}{t} = IS = 2^2 \cdot 2^2 \cdot A^2 \cdot vs$
 $\frac{1}{2} \cdot 2^2 \cdot A^2 \cdot vs = \frac{1}{2} \cdot 2^2 \cdot A^2 \cdot s \cdot v = q$

$[P] = \frac{E}{t} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}] = p$

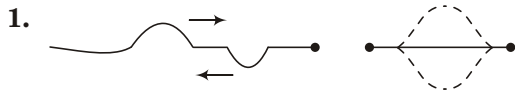
(c) $I = \frac{E}{St} = \frac{[ML^2T^{-2}]}{[L^2T]} = [MT^{-3}] = [ML^0T^{-3}] = s$

(d) $\frac{1}{L} = \frac{1}{[L^1]} = [M^0L^{-1}T^0] = s$

5.	(a)	$y = A \sin(t - kx)$	p
		$v_p = A \cos(t - kx)$	r
		$a = -A^2 \sin(t - kx)$	
	(b)	$y = A \sin(kx - t)$	p
		$v_p = A \cos(kx - t)$	
		$a = -A^2 \sin(kx - t)$	
	(c)	$y = A \cos(t - kx)$	q
		$v_p = A \sin(t - kx)$	
		$a = -A^2 \cos(t - kx)$	s
	(d)	$y = A \cos(kx - t)$	p
		$v_p = A \sin(kx - t)$	
		$a = -A^2 \cos(kx - t)$	d, s

15. Superposition of Waves

Introductory Exercise 15.1



When displacement of all the particles is momentarily zero, then there is no elastic potential energy stored in the string and as the speed is maximum at mean position, so entire energy is purely kinetic.

2. (a) $v = \sqrt{\frac{T}{\mu}}$

$$\frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{1}{0.25}} = \frac{1}{\sqrt{0.25}} = \frac{1}{0.5} = 2$$

(b) $a_t = \frac{2v_2}{v_1 + v_2} a_i = \frac{2 \cdot 20}{10 + 20} a_i = \frac{4}{3} a_i$

and $a_r = \frac{v_2 - v_1}{v_2 + v_1} a_i = \frac{20 - 10}{20 + 10} a_i = \frac{1}{3} a_i$

3. (a) For fixed end, a phase change of π takes place in reflected wave and direction becomes opposite.

as $Y_i = 0.3 \cos(2x - 40t)$
 $Y_r = 0.3 \cos(2x + 40t)$

- (b) For free end, there is no change in phase for reflected wave and direction becomes opposite.

as $Y_i = 0.3 \cos(2x - 40t)$
 $Y_r = 0.3 \cos(2x - 40t)$

4. $v_1 = \frac{50}{k_1} = 25 \text{ m/s}$ and $v_2 = 50 \text{ m/s}$

$$a_t = \frac{2v_2}{v_1 + v_2} a_i = \frac{2 \cdot 50}{25 + 50} a_i = \frac{4}{3} a_i = \frac{4}{3} \cdot 2 \cdot 10^3 \text{ m} = \frac{8}{3} \text{ mm}$$

$$a_r = \frac{v_2 - v_1}{v_2 + v_1} a_i = \frac{50 - 25}{50 + 25} a_i = \frac{1}{3} a_i = \frac{1}{3} \cdot 2 \cdot 10^3 \text{ m} = \frac{2}{3} \text{ mm}$$

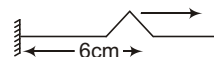
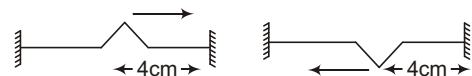
- as $v_2 > v_1$ the boundary is rarer and there is no phase change.

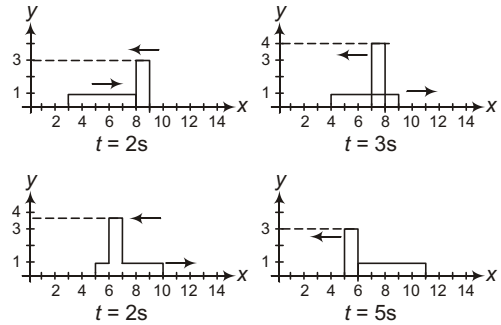
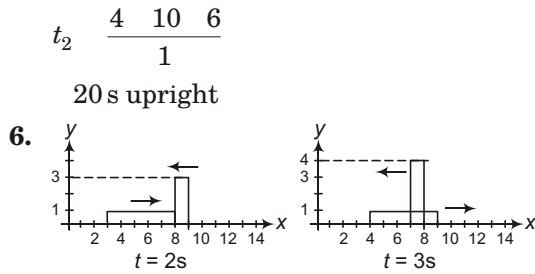
$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{50}$$

$$y_r = \frac{2}{3} \cdot 10^3 \cos(0.2x - 50t)$$

and $y_t = \frac{8}{3} \cdot 10^3 \cos(1.0x - 50t)$

5. $t_1 = \frac{2 \cdot 40 \text{ cm}}{1 \text{ cm/s}} = 8 \text{ s}$, inverted





Introductory Exercise 15.2

1. $y = 5 \sin \frac{x}{3} \cos 40 t - 2a \sin kx \cos t$

$a = \frac{5}{2} = 2.5 \text{ cm}, k = \frac{1}{3} \text{ cm}^{-1}, \omega = 40 \text{ s}^{-1}$

$v = \frac{\omega}{k} = \frac{40}{\frac{1}{3}} = 120 \text{ cm/s}$

$x = \frac{1}{2} \cdot \frac{2}{k} = \frac{1}{k} = \frac{1}{\frac{1}{3}} = 3 \text{ cm}$

$v_P = \frac{dy}{dt} = 200 \sin \frac{x}{3} \sin 40 t$

$v_P = 1.5 \cdot \frac{9}{8} = 200 \sin \frac{3}{3} \sin 40 t = \frac{9}{8} \sin 40 t$

$200 \sin \frac{1}{2} \sin (45^\circ)$

$200 \cdot 1 \cdot 0 = 0 \text{ cm/s}$

2. Two waves with different amplitudes can produce partial stationary waves with amplitude of antinodes being $a_1 + a_2$ and amplitude of nodes being $a_1 - a_2$. As here node is not stationary that is why energy is also transported through nodes.

3. (a) $\frac{0}{2} = 2 \text{ m}$ 4 m ,

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{4 \cdot 10^{-2}}} = \frac{10^2}{2} = 50 \text{ m/s}$
 $\frac{v}{\lambda} = \frac{50}{4} = 12.5 \text{ Hz}$

and is fundamental tone or first harmonic.

$y = 0.1 \sin \frac{2}{x} \sin 2 t$

$0.1 \sin \frac{2}{4} x \sin 2 t = 12.5 t$

$0.1 \sin \frac{2}{2} x \sin 25 t$

(b) $3 \cdot \frac{2}{2} = 2 \text{ m}$ $\frac{4}{3} \text{ m}$ and $v = 50 \text{ m/s}$

$\frac{v}{\lambda} = \frac{50}{\frac{4}{3}} = 37.5 \text{ Hz}$ and is 2nd

overtone or 3rd harmonic.

$y = 0.04 \sin \frac{2}{4/3} x \sin 2 t = 37.5 t$

$0.04 \sin \frac{3}{2} x \sin 75 t$

4. $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Fl}{m}} = \sqrt{\frac{400 \cdot 4}{160 \cdot 10^{-3}}}$
 $\sqrt{\frac{1600}{16 \cdot 10^{-2}}} = 10^2 = 100 \text{ m/s}$

(a) $\frac{0}{4} = l = 0$ $4l = 16 \text{ m}$

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- $\frac{3}{4} l = \frac{4l}{3} = \frac{16}{3} \text{ m} = 5.33 \text{ m}$
 and $\frac{5}{4} l = \frac{4l}{5} = \frac{16}{5} \text{ m} = 3.2 \text{ m}$
- (b) $\frac{v}{16} = 6.25 \text{ Hz}$
 $\frac{v}{16/3} = 18.75 \text{ Hz}$
 $\frac{v}{16/5} = 31.25 \text{ Hz}$
5. $l = \frac{0.54}{2} n = 0.27 n$
 and $l = \frac{0.48}{2} (n - 1)$
 $0.24 (n - 1)$
 $0.27 n - 0.24 n = 0.24$
 $0.03 n = 0.24 \Rightarrow n = 8$
- (a) These are 8th and 9th harmonic
 (b) $l = 0.27 n = 0.27 \times 8 = 2.16 \text{ m}$
 (c) $\frac{l}{2} = 1.08 \text{ m}$
6. $5 \times 2 = 10 \text{ Hz}$
 $3 \times 6 = 18 \text{ Hz}$

7. $\frac{1}{2l} \sqrt{\frac{F}{M}}$
 $\frac{1}{2l} \sqrt{\frac{F_2}{F_1}}$
 $\sqrt{\frac{M}{2.2}} = \frac{260}{220} = \frac{13}{11}$
 $\frac{M}{2.2} = \frac{169}{121} \Rightarrow M = \frac{48}{121} \times 2.2 = 0.873 \text{ kg}$
8. $n = 5$ and $n = 6$ So these are 5th and 6th harmonics.
 $\frac{1}{2l} \sqrt{\frac{F}{M}}$
 $F = 4l^2 v_0^2 = 4 \times 50^2 \times \frac{36 \times 10^3}{1} = 360 \text{ N}$

AIEEE Corner

■ Subjective Question (Level 1)

1. $A = \sqrt{A_1^2 + A_1^2 - 2A_1 A_1 \cos 90}$
 $A = \sqrt{2} A_1 = 4\sqrt{2} \text{ cm} = 5.66 \text{ cm}$
2. $v_2 = 2v_1$
 $A_r = \frac{v_2 - v_1}{v_2 + v_1} A = \frac{v_1 - v_1}{3v_1} A = \frac{1}{3} A$
 $A_t = \frac{2v_2}{v_2 + v_1} A = \frac{4v_1}{3v_1} A = \frac{4}{3} A$
 $\frac{I_r}{I_i} = \left(\frac{A_r}{A}\right)^2 = \frac{1}{9}$
 and $\frac{I_t}{I_i} = 1 + \frac{1}{9} = \frac{8}{9}$

$$A = \sqrt{10^2 + 20^2 - 2 \times 10 \times 20 \cos \frac{\pi}{3}}$$

$$= \sqrt{100 + 400 - 200} = \sqrt{300} = 10\sqrt{3}$$

26.46 units

$$\tan^{-1} \frac{20 \sin \frac{\pi}{3}}{10 - 20 \cos \frac{\pi}{3}}$$

$$= \tan^{-1} \frac{\sqrt{3}}{2} = 0.714 \text{ rad}$$

Phase = $5x - 25t + 0.714 \text{ rad}$.

4. $y_1 = 1 \text{ cm} \sin (\pi x - 50 \pi t)$

$$y_2 = 1.5 \text{ cm} \sin \left(\frac{9}{2} \text{ cm}^{-1} x - 100 \text{ s}^{-1} t \right)$$

$$y_1 = (4.5, 5 \cdot 10^{-3}) \cdot 1 \text{ cm} \sin \left(4.5 \frac{250}{1000} \right)$$

$$1 \sin \frac{9}{2} \frac{-}{4}$$

$$1 \sin \frac{17}{4}$$

$$1 \text{ cm} \sin \left(4 \frac{-}{4} \right)$$

$$1 \sin \frac{1}{4} \frac{1}{\sqrt{2}} \text{ cm and}$$

$$y_2 = (4.5, 5 \cdot 10^{-3}) \cdot 1.5 \text{ cm} \sin \left(\frac{9}{4} \frac{500}{1000} \right)$$

$$1.5 \text{ cm} \sin \left(\frac{9}{4} \frac{-}{2} \right)$$

$$= 1.5 \sin \frac{5}{4}$$

$$1.5 \sin \frac{-}{4}$$

$$1.5 \sin \frac{-}{4}$$

$$\frac{1.5}{\sqrt{2}} \text{ cm}$$

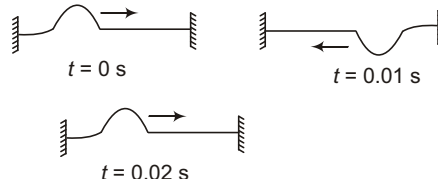
$$y = y_1 + y_2 = \frac{1}{\sqrt{2}} \frac{1.5}{\sqrt{2}} + \frac{0.5}{\sqrt{2}} \frac{1}{2\sqrt{2}} \text{ cm}$$

$$5. v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{16 \text{ N}}{0.4 \cdot 10^{-3} \cdot 10^2 \text{ kg/N}}} = \sqrt{\frac{16 \cdot 10^2}{4}} = 20 \text{ m/s}$$

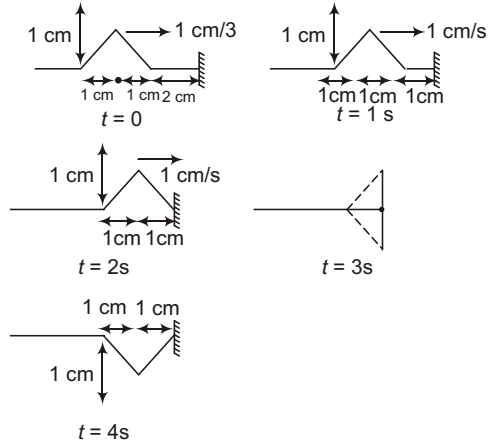
(a) For same shape, time,

$$t = \frac{2l}{v} = \frac{2 \cdot 0.2}{20} \text{ s} = 0.02 \text{ s}$$

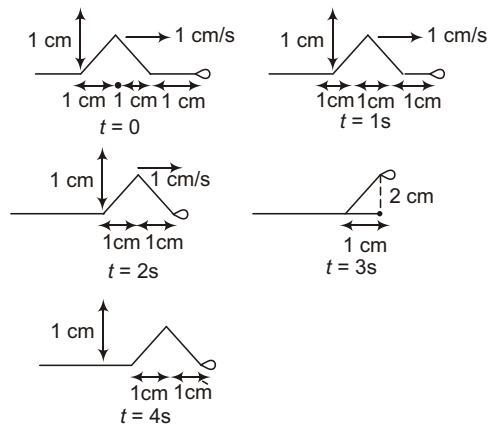
(b)



6. (a)



(b)



$$7. y = 1.5 \sin(0.4x) \cos(200t)$$

$$\frac{2A}{k} = \frac{2}{0.4} = 5 \text{ m} = 15.7 \text{ m}$$

$$\frac{2}{2} \frac{200}{2} = 100 \text{ Hz} = 31.8 \text{ Hz}$$

$$v = \frac{200}{0.4} = 500 \text{ m/s}$$

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8. $y = y_1 + y_2 = 3 \text{ cm} \sin(x - 0.6t) + 3 \text{ cm} \sin(x + 0.6t)$

$6 \text{ cm} \sin x \cos 0.6t = R \cos 0.6t$
 where, $R = 6 \text{ cm} \sin x$.

(a) $R(0.25) = 6 \text{ cm} \sin \frac{1}{4}$

$\frac{6}{\sqrt{2}} = 3\sqrt{2} \text{ cm} = 4.24 \text{ cm}$

(b) $R(0.50) = 6 \text{ cm} \sin \frac{1}{2} = 6 \text{ cm}$

(c) $R(1.50) = 6 \text{ cm} \sin \frac{3}{2} = 6 \text{ cm}$

$|R| = 6 \text{ cm}$

(d) For antinodes, $R = 6 \text{ cm}$

$\sin x = 1 \Rightarrow x = (2n - 1)\frac{\pi}{2}$

or $x = n \frac{\pi}{2} = 0.5 \text{ cm}, 1.5 \text{ cm}, 2.5 \text{ cm}$

9. $\frac{2}{4} = \frac{2}{\lambda/2} = 4 \text{ cm}$

(a) Distance between successive antinodes $= \frac{\lambda}{2} = 2 \text{ cm}$

(b) $R(x) = 2A \sin kx = 2 \text{ cm} \sin \frac{x}{2} = 0.5$

$2 \sin \frac{x}{4}$

$\frac{2}{\sqrt{2}} = \sqrt{2} \text{ cm}$

10. $n \frac{n-1}{2l} \sqrt{\frac{T}{\mu}} = \frac{n-1}{2 \cdot 20} \sqrt{\frac{20}{9 \cdot 10^{-3}}}$

$\frac{n-1}{60} = \frac{100\sqrt{2}}{3} = \frac{5\sqrt{2}}{9} = \frac{5\sqrt{2}}{9} (n-1)$

$0.786(n-1)$

0.786 Hz

$1.57 \text{ Hz}, 2.36 \text{ Hz}, 3.14 \text{ Hz}$

11. (a) $T = v^2 \cdot \frac{2}{\lambda^2}$

$\frac{1.2 \cdot 10^3}{0.7} = (220)^2 = (1.4)^2$

162.6 N

(b) $\frac{2}{\lambda} = \frac{3}{\lambda_0} = \frac{3}{220 \text{ Hz}} = 660 \text{ Hz}$

12. $n \frac{n-1}{2l} \sqrt{\frac{T}{\mu}} = \frac{n-1}{2 \cdot 0.6} \sqrt{\frac{50}{0.01}}$
 $\frac{50\sqrt{2}}{1.2} (n-1) = 58.93 (n-1) \text{ Hz}$

$n = 20,000 \text{ Hz} \Rightarrow n = 338$

$338 = 339 - 58.93 = 19975.8 \text{ Hz}$

19.976 kHz

13. $n_0 = 420 \text{ Hz}$ and $(n-1)_0 = 490 \text{ Hz}$

$\lambda_0 = 70 \text{ Hz}$ and $n = 6$

$\frac{1}{2l} \sqrt{\frac{T}{\mu}} = \frac{1}{2 \cdot 0} \sqrt{\frac{T}{\mu}} = \sqrt{\frac{450}{0.005 \cdot 70}}$

$\frac{300}{140} = 2.143 \text{ m}$

14. $\frac{v}{\lambda} = \frac{400 \text{ m/s}}{800 \text{ Hz}} = \frac{1}{2} \text{ m}$,

$l = 4 \cdot \frac{\lambda}{2} = 2 \cdot 1 \text{ m}$

(a) $4_0 = 400 \text{ Hz}$ and $0 = 100 \text{ Hz}$

(b) $7_0 = 700 \text{ Hz}$

16. $\frac{1}{2l} \sqrt{\frac{T}{\mu}}$

$\frac{1}{l}$

$1 : 2 : 3 \Rightarrow \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$

$l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$

$6 : 3 : 2 \Rightarrow 6x : 3x : 2x$

$6x = 3x = 2x = 1 \text{ m}$

$x = \frac{1}{11} \text{ m}$

position of first bridge $6x = \frac{6}{11} \text{ m}$

and position of second bridge

$6x = 3x = 9x = \frac{9}{11} \text{ m}$

From the same end or $1 \frac{9}{11} = \frac{2}{11} \text{ m}$

from other end .

$$17. \quad \frac{v}{2l} = \frac{v}{2 \times 124} = \frac{v}{248} = \frac{90}{186} = \frac{60}{124}$$

Thus length of the vibrating string has to be 60 cm.

$$18. \quad \frac{v}{2} = 15 \text{ cm} \quad 30 \text{ cm},$$

$$R_{\max} = 2A = 0.85 \text{ cm},$$

$$T = 0.075 \text{ s}$$

$$(a) \quad y = 2A \sin kx \sin \omega t$$

$$0.85 \text{ cm} \sin \frac{2\pi}{0.3 \text{ m}} x \sin \frac{2\pi}{0.075 \text{ s}} t$$

$$(b) \quad v = \frac{\omega}{k} = \frac{2\pi/0.075}{2\pi/0.3} = \frac{0.3}{0.075} = 4 \text{ m/s}$$

$$(c) \quad \frac{30}{4} = 7.5 \text{ cm}$$

$$R(7.5 \text{ cm}) = 2A \sin kx$$

$$0.85 \sin \frac{2\pi}{30} \times 7.5 = 10.5$$

$$R(115 \text{ cm}) = 0.85 \sin \frac{2\pi}{30} \times 115$$

$$= 0.85 \sin (0.7\pi) = 0.85 (126) = 0.688 \text{ cm}$$

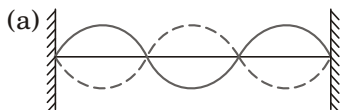
$$19. \quad \frac{v}{2l} = \frac{48}{2 \times 1.5} = 16 \text{ Hz}$$

and $\frac{v}{2l} = \frac{3}{2} = 48 \text{ Hz}$ and $\frac{v}{2} = \frac{48}{2} = 1 \text{ m}$

and $\frac{v}{3} = \frac{4}{3} = \frac{64 \text{ Hz}}{48} = \frac{3}{4} = 0.75 \text{ m}$

$$20. \quad y = 5.60 \text{ cm} \sin (0.340 \text{ rad/cm } x) \sin (50.0 \text{ rad/s } t)$$

$$2A \sin (kx) \sin (\omega t)$$



$$(b) \quad 2A = 5.60 \text{ cm} \quad A = 2.80 \text{ cm}$$

$$(c) \quad l = 3 \frac{\frac{3}{2}}{\frac{2}{k}} = \frac{3}{2} \frac{2}{k} = \frac{3}{k}$$

$$0.0340 \text{ cm} = \frac{3}{k} \Rightarrow k = \frac{3}{0.0340} = 88.2 \text{ rad/cm}$$

$$(d) \quad \frac{2}{k} = \frac{2}{88.2} = 0.0227 \text{ cm} = 184.8 \text{ cm}$$

$$\frac{50}{2} = 25 \text{ Hz}$$

$$T = \frac{1}{25} = 0.04 \text{ s}$$

$$v = 25 \text{ Hz} \times 184.8 \text{ cm} = 4620 \text{ cm/s}$$

$$(e) \quad (v_p)_{\max} = R_{\max} \omega = 5.60 \text{ cm} \times 50 \text{ rad/s} = 280 \text{ cm/s}$$

$$(f) \text{ for eight harmonic, } \frac{8}{2} \frac{l}{4} = \frac{277.2}{4} = 69.3 \text{ cm}$$

$$k = \frac{2\pi}{69.3} = 0.0907 \text{ rad/cm}$$

$$v = 8 v_0 = \frac{8}{3} \times 7.96 \text{ Hz} = 21.22 \text{ Hz}$$

$$\omega = 2\pi \times 21.22 = 133.4 \text{ rad/s}$$

$$y = 5.60 \text{ cm} \sin (0.0907 \text{ rad/s } x) \sin (133 \text{ rad/s } t)$$

$$21. \quad (a) \quad v = \frac{96 \text{ m/s}}{2} = 48 \text{ m/s}$$

$$(b) \quad T = \frac{40}{80} \frac{10^3}{10^2} = \frac{40}{80} \times 10 = 5 \text{ s}$$

$$\frac{96^2}{20} = 460.8 \text{ N}$$

$$(c) \quad (v_p)_{\max} = R_{\max} \omega = 0.3 \text{ cm} \times 2 \times 60 \text{ rad/s} = 36 \text{ cm/s}$$

$$a_{\max} = \omega^2 R_{\max} = (120)^2 \times 0.3 \text{ cm/s}^2 = 426.4 \text{ m/s}^2$$

Objective Questions (Level 1)

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$$1. \frac{2}{1} \sqrt{\frac{T_2}{T_1}} \quad \frac{3}{2} \sqrt{\frac{T}{T}}$$

$$\frac{9T}{5T} \quad \frac{4(T)}{10T} \quad \frac{2.5}{2N}$$

$$2. \frac{n}{2l} \sqrt{\frac{T}{r^2}} \quad \frac{n}{2l} \sqrt{\frac{T}{r^2}} \quad \text{constant.}$$

$$\frac{n}{2lr} \sqrt{\frac{T}{ld}} \quad \frac{n}{ld} \sqrt{\frac{T}{ld}}$$

$$n \frac{ld}{\sqrt{T}}$$

$$\frac{n_1}{n_2} \frac{l_1}{l_2} \frac{d_1}{d_2} \sqrt{\frac{T_2}{T_1}}$$

$$\frac{1}{2} \frac{1}{3} \sqrt{2}$$

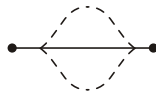
$$\frac{1}{3\sqrt{2}} \quad 1:3\sqrt{2}$$

$$\text{or } \frac{n_2}{n_1} \frac{1}{1} \quad 3\sqrt{2}$$

$$3. f \frac{1}{l}; l \quad l_1 \quad l_2 \quad l_3$$

$$\frac{1}{f_0} \quad \frac{1}{f_1} \quad \frac{1}{f_2} \quad \frac{1}{f_3}$$

4. During overlapping the displacement of particles is zero while velocity is maximum. So the entire energy is purely kinetic.



$$5. y(x, y) = y_1 + y_2 = a \cos(kx - t) + y_2$$

$2a \sin kx \sin t$ is necessary for a node at $x = 0$. Thus,

$$y_2 = 2a \sin kx \sin t = a \cos(kx - t)$$

$$2a \sin kx \sin t = a \cos kx \cos t$$

$$a \sin kx \sin t$$

$$a[\cos kx \cos t - \sin kx \sin t]$$

$$a \cos(kx - t)$$

6. In transverse stationary wave, longitudinal strain is maximum at node. While in longitudinal stationary wave at displacement node pressure and density are maximum. So all are correct.

7. In stationary wave all particles errors the mean position simultaneously and are at their maximum displacement simultaneously at different instant at this time all of them are at rest. So all are correct.

8. Maximum displacement

$$y_{\max} = 3A \quad \frac{A}{2A} \quad \frac{2A}{4A}$$

$$\frac{v_t}{v_l} = \frac{v_t}{v_l} \sqrt{\frac{Y}{l}}$$

$$\frac{v_t}{v_l} = \frac{v_t}{v_l} \sqrt{\frac{Y}{l}}$$

$$\sqrt{\frac{1}{l}} \quad \frac{1}{\sqrt{l}}$$

$$10. f_n = \frac{n}{2l} \sqrt{\frac{100}{0.01}} = 50(n-1)$$

50 Hz, 100 Hz, 150 Hz

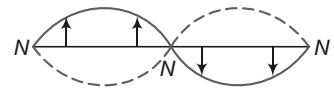
$$n_n = \frac{2n}{4l} \sqrt{\frac{100}{0.01}} = 25(2m-1)$$

25 Hz, 75 Hz, 125 Hz

$$n_2 = 75 \text{ Hz} \quad \frac{f_1}{2} \quad \frac{f_2}{2}$$

$$\frac{50 \text{ Hz}}{2} \quad \frac{100 \text{ Hz}}{2} \quad 75 \text{ Hz}$$

11. In stationary waves all particles perform SHM such that they are at their positive and negative extremes



one time

each in a

time period, where they come to rest.

Particles between two successive nodes are in phase while beside node are in opposite phase. So all the particles cannot be at positive extreme simultaneously.

12. The question is wrong, string has to be fix at one end and free at other. Then

$$(2n-1) \cdot 0 = 90 \text{ Hz}, (2n-3) \cdot 0 = 50 \text{ Hz}$$

$$\text{and } (2n-5) \cdot 0 = 210 \text{ Hz}$$

$2 \times 60 \text{ Hz}$ or 30 Hz and $n = 1$
 i.e., vibrations are 3rd, 5th and 7th harmonic.

$$v = \frac{2l}{\lambda} \times 30 \text{ Hz} = \frac{1.6 \text{ m}}{48 \text{ m/s}}$$

13. $y = y_1 + y_2 + y_3 = 12 \sin \frac{x}{2}$

$$6 \sin \left(\frac{x}{2} \right) + 4 \sin \frac{x}{2}$$

$$R = \sqrt{6^2 + 4^2} = 10 \text{ mm}$$

14. $(2n + 1) \times 105 \text{ Hz}$

and $(2n + 3) \times 175 \text{ Hz}$
 $2 \times 70 \text{ Hz}$
 35 Hz

15. $l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 1 : \frac{1}{2} : \frac{1}{3}$
 $12 : 4 : 3$

$$12x = 4x = 3x = 114 \text{ cm}$$

$$x = \frac{114}{19} \text{ cm} = 6 \text{ cm}$$

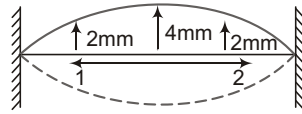
$$l_1 = 12x = 72 \text{ cm}, l_2 = 4x = 24 \text{ cm},$$

$$l_3 = 3x = 18 \text{ cm}$$

16. $f = \frac{1}{2l} \sqrt{\frac{Vg}{V_1g}}$
 $\frac{f/2}{f} = \sqrt{\frac{Vg}{V_1g}} = \sqrt{1 - \frac{\rho_2}{\rho_1}} = \frac{1}{2}$
 $1 - \frac{\rho_2}{\rho_1} = \frac{1}{4} \Rightarrow \frac{\rho_2}{\rho_1} = \frac{3}{4}$
 $\frac{f/3}{f} = \sqrt{\frac{Vg}{V_2g}} = \sqrt{1 - \frac{\rho_2}{\rho_1}} = \frac{1}{3}$
 $1 - \frac{\rho_2}{\rho_1} = \frac{1}{9} \Rightarrow \frac{\rho_2}{\rho_1} = \frac{8}{9}$
 $\frac{1}{2} = \frac{3/4}{8/9} = \frac{27}{32} = \frac{\rho_2}{\rho_1} = 1.18$

where ρ_1 is density of water and ρ_2 is density of the other liquid.

17. $R = 2A \sin Kx = 4 \text{ mm} \sin \frac{2\pi}{3} x$



$$4 \text{ mm} \sin \frac{2\pi}{3} x$$

$$2 \text{ mm} + 4 \text{ mm} \sin \frac{2\pi}{3} x$$

$$\frac{2\pi}{3} x = \frac{\pi}{3} \Rightarrow x = 0.5 \text{ m}$$

Thus points 1 and 2 are at 0.5 m from their nearest boundary. So separation between them is $1.5 \text{ m} - 2 \times 0.5 \text{ m} = 0.5 \text{ m} = 50 \text{ cm}$

18. $y = A \sin \left(t - kx \right)$
 $A \sin 2\pi t - \frac{2\pi}{v} x$
 $A \sin (6\pi t - 2\pi x)$
 $y(3, t) = A + A \sin (6\pi t - 6\pi)$
 $A \sin (6\pi t - 6\pi) = 6\pi t - \frac{11\pi}{2} = 6\pi t - 5.5\pi$

19. $\frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda} \times \frac{3}{2} = \frac{3\pi}{\lambda}$
 and $\frac{2\pi}{\lambda} = \frac{3\pi}{\lambda} - \frac{5\pi}{\lambda}$

20. $n = 0, 400 \text{ Hz}, (n + 1) = 450 \text{ Hz}$
 50 Hz and $n = 8$
 $\frac{1}{2l} \sqrt{\frac{T}{\rho}} = l \frac{1}{2} \sqrt{\frac{T}{\rho}}$
 $\frac{1}{2 \times 50} \sqrt{\frac{490}{0.1}} = \frac{70}{100} = 0.7 \text{ m}$

21. $3 \times \frac{2}{2} = 1 \text{ m}, \frac{2}{3} \text{ m}$
 $v = 300 \text{ Hz} \times \frac{2}{3} \text{ m} = 200 \text{ m/s}$

22. $l = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}$

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$$1 \quad 2l, \quad 2 \quad \frac{2l}{2}, \quad 3 \quad \frac{2l}{3}$$

$$1 : 2 : 3 \quad 2l : \frac{2l}{2} : \frac{2l}{3}$$

$$1 : \frac{1}{2} : \frac{1}{3}$$

23. $\frac{2}{vT} x \quad \frac{2}{300 \cdot 0.04} (16 \quad 10)$

$$\frac{2}{12} \quad 6$$

24. $v = \frac{30}{k} = 30 \text{ m/s}$

$$\sqrt{\frac{T}{A}} \quad \sqrt{\frac{T}{A}}$$

$$T = Av^2 = 8000 \cdot 10^6 = 900 \cdot 7.2 \text{ N}$$

25. $5_0 \quad 480 \text{ Hz}, \quad 2_0 \quad \frac{2}{5} \quad 480 \text{ Hz}$

192 Hz

26. $\frac{I_r}{I_i} = 0.64 = \left(\frac{A_r}{A_i}\right)^2$

$$\frac{A_r}{A_i} = 0.8 \quad A_r = 0.8 A_i$$

$$A_r \frac{v_2}{v_2} \frac{v_1}{v_1} A_i = 0.8 A_i \quad \frac{4}{5} A_i$$

$$5v_2 = 5v_1 \quad (4v_2 = 4v_1)$$

$$v_2 = 9v_1, \frac{1}{9} v_1$$

For, $v_2 > v_1$ the boundary is rarer and there will not be any change in phase of reflected wave and for $v_2 < v_1$ a phase change of 180° takes place.

27. $Y_r = 0.8 A \sin(kx - t - 30 \quad 180)$

$$A \quad \frac{1}{2l} \sqrt{\frac{T}{d^2}} \quad \frac{1}{ld} \sqrt{\frac{T}{d^2}}$$

$$B \quad \frac{1}{2} \frac{1}{2l} \sqrt{\frac{2T}{4d^2}}$$

$$\frac{1}{4ld} \sqrt{\frac{T}{d^2}} \quad \frac{1}{4} A$$

Third overtone of $B = 4 B = A$

■ Passage (Q 28 to 30)

$$I_r = (100\% - 36\%) I_i = 64\% I_i = 0.64 I_i$$

$$\frac{A_r}{A_i} = \sqrt{\frac{I_r}{I_i}} = \sqrt{0.64} = 0.8 \quad \frac{v_2}{v_2} \frac{v_1}{v_1}$$

$$0.8 v_2 = 0.8 v_1 \quad (v_2 = v_1)$$

$$0.2 v_2 = 1.8 v_1$$

$$v_2 = 9v_1$$

for rarer boundary

or $1.8 v_2 = 0.8 v_1 \quad v_2 = \frac{1}{9} v_1$

for denser boundary

28. $A_r = 0.8 A$

29. $Y = A \sin(ax - bt) - \frac{0.8}{2}$

$$A \sin(ax - bt) - \frac{0.8}{2}$$

$$A \sin(ax - bt) - \frac{0.8}{2}$$

$$A \sin(ax - bt) - \frac{0.8}{2}$$

$$A \cos(ax - bt) = 0.8 A \cos(ax - bt)$$

$$A \cos ax \cos bt = A \sin ax \sin bt$$

$$0.8 A \cos ax \cos bt$$

$$0.8 A \sin ax \sin bt$$

$$0.2 A \cos ax \cos bt = 1.8 A \sin ax \sin bt$$

$$0.2 A \cos ax \cos bt = 0.2 A \sin ax \sin bt$$

$$1.6 A \sin ax \sin bt$$

$$0.2 A \cos(ax - bt)$$

$$1.6 A \sin(ax) \sin(bt)$$

$$cA \cos(ax - bt) = 1.6 A \sin ax \sin bt$$

$$e = 0.2$$

30. For antinodes, $\sin ax = 1$

$$ax = (2n - 1) \frac{\pi}{2}$$

$$x = (2n - 1) \frac{\pi}{2a}, \frac{3\pi}{2a}, \frac{5\pi}{2a}$$

So for second antinode, $x = \frac{3}{2a}$

$$31. \frac{15}{0} \sqrt{\frac{1.021}{1}} \quad 1.1$$

$$\frac{15}{2} \sqrt{\frac{0.1}{T_2}} \quad 150 \text{ Hz}$$

$$\frac{1}{1} \sqrt{\frac{1.21}{T_1}} \quad 1.1$$

$$2 \quad 110\% \text{ of } v_1$$

So, (a), (c) and (d) are correct.

32. For interference, sources must be coherent their frequency has to be equal and phase difference has to be constant. So, (a) and (d) are correct.

33. Stationary waves are formed due to superposition (**here use of the term 'interference' is literary and not scientific because interference is a different phenomenon than stationary waves**) of waves having same amplitude, same frequency and travelling opposite direction. Here nodes are the points who always remain at rest. Total energy is always conserved.

34. A medium is said to be rarer if speed of wave in it is higher. And as frequency is

constant, wavelength increases while frequency is constant, wavelength increases while phase does not change during change in medium.

$$35. Y = A \sin kx \cos t - 2a \sin kx \cos t$$

$$a = \frac{A}{2}, \text{ third overtone means fourth}$$

harmonic and wire oscillate with four loops.

$$l = 4 \frac{\lambda}{2} = 2 \frac{2}{k} = \frac{4}{k}$$

and stationary wave does not propagate.

36. For stationary waves, frequency and amplitude has to be same and direction has to be opposite with constant phase difference.

It is satisfied in (b) and (d) only.

$$37. y = y_1 + y_2 = 2A \cos kx \sin t$$

$$R \sin t$$

$R = 2A \cos kx$ so at $x = 0$ there is antinode.

$$\cos kx = 1$$

$$kx = n\pi, x = \frac{n\pi}{k} = 0, \frac{\pi}{4}, \frac{2\pi}{k}, \dots$$

are antinodes.

JEE Corner

■ Assertion and Reason

1. $y_1 = y_2 = A \sin(t - kx)$

$$A \cos(t - kx)$$

$$A \sin(t - kx) + A \sin(t - kx)$$

$$2A \sin \frac{t - kx}{2}$$

$$\cos \frac{t - kx}{2} \cot kx$$

$$2A \sin kx = \frac{4}{4} \cos t = \frac{4}{4}$$

$$R \cos t = \frac{4}{4}$$

where, $R = 2A \sin kx = \frac{4}{4}$;

$$R(0) = 2A \sin \frac{\pi}{4} = A\sqrt{2}$$

So, at $x = 0$, node is not present, i.e., Assertion is false.

2. In stationary waves only nodes are at rest and not other particles. It is so

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called as energy is not transmitted, thus assertion is false.

3. In rarer medium speed of wave is higher and as

$$A_t = \frac{2v_2}{v_1 + v_2} A_i$$

$$A_r = A_i$$

so reason is correct explanation to assertion.

4. In second overtone or third harmonic there are three loops or three antinodes or four nodes. And length of the string, $l = 3 \frac{\lambda}{2}$ so, assertion and reason are both true.



5. As speed of wave is constant in stretched wire, and $v \propto f$, so with increase in frequency, wavelength decreases. So reason is correct explanation of assertion.
6. In stationary waves, amplitude of nodes is zero and it is possible only when superposing waves has same amplitude. But it is not the only condition, there has to be same frequency, opposite direction of propagation and constant phase difference. So assertion is not completely true.
7. Energy lying between conservative node and antinode is constant where it moves to and fro between node and antinode.
8. $\frac{I_{\max}}{I_{\min}} = \frac{25}{1} = \frac{5^2}{1^2} = \frac{A_1^2 + A_2^2}{A_1^2 - A_2^2}$
- $$\frac{5(A_1^2 + A_2^2)}{4A_1^2 - 6A_2^2} = \frac{A_1^2 + A_2^2}{A_1^2 - A_2^2} = 3:2$$

Thus reason is the correct explanation of assertion.

9. $y = A \sin \frac{\pi}{2} = A \sin$

$$A \sin \frac{\pi}{2}$$

$$A \cos \frac{\pi}{2} = A \sin \frac{\pi}{2} = A \cos$$

$$A \sin \frac{\pi}{2} = R A \frac{I_r}{I_i}$$

Assertion and reason are both true but reason do not explain assertion.

10. For two coherent sources phase difference has to be constant and that constant be same at all points as (t). Different light sources can never be coherent. So phase difference must be same, thus assertion is false.

■ Match the Columns

1. $v_1 = \sqrt{\frac{T}{\mu}}$ and $v_2 = \sqrt{\frac{T}{9\mu}} = \frac{v_1}{3}$

$$\frac{v_1}{v_2} = 3$$

$$A_r = \frac{v_1 - v_2}{v_1 + v_2} A_i = \frac{2/3}{4/3} A_i = \frac{1}{2} A_i$$

$$\text{and } A_t = \frac{2v_2}{v_1 + v_2} A_i$$

$$= \frac{2 \cdot \frac{v_1}{3}}{v_1 + \frac{v_1}{3}} A_i = \frac{1}{2} A_i$$

(a) $\frac{A_1}{A_2} = \frac{A_r}{A_t} = \frac{1/2 A_i}{1/2 A_i} = 1$ q

(b) $\frac{v_1}{v_2} = 3$ r

(c) $\frac{I_r}{I_i} = \frac{A_r^2}{A_i^2} = \frac{1}{4} = \frac{1}{4}$ and

$$\frac{I_t}{I_i} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\frac{I_1}{I_2} = \frac{I_r}{I_t} = \frac{I_r/I_i}{I_t/I_i} = \frac{1/4}{3/4} = \frac{1}{3}$$
 s

(d) $P = IS = 2^2 \cdot 2^2 \cdot A^2 v$

$$= \frac{1}{2} \cdot 2^2 \cdot A^2 \sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2} \cdot 2^2 \cdot A^2 \sqrt{T}$$

$$\frac{P_1}{P_2} = \frac{\frac{1}{2} A_1^2 \sqrt{T_1}}{\frac{1}{2} A_2^2 \sqrt{T_2}} = \frac{A_1^2 \sqrt{\frac{1}{2}}}{A_2^2 \sqrt{\frac{1}{9}}}$$

2. (a) $\frac{2}{4} \frac{\frac{3}{5}v}{\frac{2l}{2l}} = \frac{3}{5} r$

(b) Number of nodes in 3rd harmonic is 4 and in Fifth harmonic 6, so, $\frac{4}{6} = \frac{2}{3} p$

(c) Number of antnodes in 3rd harmonic is 3 and in fifth harmonic 5, $\frac{3}{5} = \frac{3}{5} r$

(d) $\frac{2}{4} = \frac{4}{2} = \frac{5}{3} s$

3. In denser medium speed of wave is lesser and in rarer medium it is greater.

(a) When wave goes from denser to rarer medium its speed increases p

(b) As frequency do not change with change in medium then with

increase in speed wavelength increases p

(c) As $v_t = v_i$ then $A_t = A_i = p$

(d) Frequency remains unchanged r

4. $R = \frac{\sqrt{A^2 + A^2}}{2} = \frac{A\sqrt{2}}{2} = 2A \cos \frac{90}{2}$

(a) $R(60) = 2A \cos \frac{60}{2}$

$2A \cos 30 = 2A \frac{\sqrt{3}}{2} = A\sqrt{3} s$

(b) $R(120) = 2A \cos 120/2 = 2A \cos 60 = 2A \frac{1}{2} = A s$

(c) $R(90) = 2A \cos 90/2 = 2A \cos 45 = A\sqrt{2}$

(d) $R(0) = 2A \cos (0/2) = 2A$
 $I_R = 2A^2 = 4I_i = r$

5. $\frac{2}{3} = \frac{0}{210 \text{ Hz}} = \frac{0}{70 \text{ Hz}}$

(a) $\frac{0}{70 \text{ Hz}} = s$

(b) $\frac{2}{3} = \frac{0}{210 \text{ Hz}} = p$

(c) $\frac{3}{4} = \frac{0}{4 \cdot 70 \text{ Hz}} = \frac{280 \text{ Hz}}{r}$

(d) $\frac{1}{2} = \frac{0}{140 \text{ Hz}} = s$

16. Sound Waves

Introductory Exercise 16.1

1. $P_0 = S_0 k B$

$$B = \frac{P_0}{S_0 k} = \frac{P_0}{2 S_0} = \frac{14 \times 0.35}{2 \times 3.14 \times 5.5 \times 10^6} = 1.4 \times 10^5 \text{ N/m}^2$$

2. $v_{\max} = \frac{1450 \text{ m/s}}{20 \text{ Hz}} = 72.5 \text{ m}$
 $v_{\min} = \frac{1450 \text{ m/s}}{20000 \text{ Hz}} = 7.25 \text{ cm}$

3. Pressure wave and displacement wave has a phase difference of $\frac{\pi}{2}$, so,

(a) When pressure is maximum, displacement is minimum *i.e.*, zero.

(b) $S_0 = \frac{P_0}{k B} = \frac{P_0}{2 \frac{P_0}{v^2}} = \frac{P_0 v^2}{2 P_0} = \frac{v^2}{2}$
 $= \frac{10}{2 \times 3.14 \times 10^3 \times 1.29 \times 340} = 3.63 \times 10^{-6} \text{ m}$

4. $S_0 = \frac{P_0}{k B} = \frac{P_0}{2 \frac{P_0}{v}} = \frac{P_0 v}{2 P_0} = \frac{v}{2}$
 $= \frac{12 \times 8.18}{2 \times 129 \times (2700)^2} = 1.04 \times 10^{-5} \text{ m}$

Introductory Exercise 16.2

1. $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{2 \times T_2}{4 T_1}} = \sqrt{\frac{2 \times 273 \text{ K}}{4 \times 273 \text{ K}}} = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = 0.707$

2. $v_t = v_0 \left(1 + \frac{t}{273}\right)^{1/2} = v_0 \left(1 + \frac{t}{546}\right)$
 $v_{30} = v_3 = v_0 \left(1 + \frac{30}{546}\right) = v_0 \left(1 + \frac{3}{54.6}\right) = v_0 \frac{33}{54.6}$
 $332 \times \frac{33}{54.6} = 20.06 \text{ m/s}$

3. $v = 250 \times 8 = 2000 \text{ m/s}$
 $B = v^2 = 900 \times (2000)^2 = 36 \times 10^8 \text{ N/m} = 3.6 \times 10^9 \text{ Pa}$

4. $v = \sqrt{\frac{R t}{M}} = \sqrt{\frac{7 \times 8.314 \times 273}{32 \times 10^3}} = 315 \text{ m/s}$

Introductory Exercise 16.3

1. $P_0 = S_0 k B = 2 v S_0$
 $2 \times 3.14 \times 300 \times 1.2 \times 344 \times 6 \times 10^{-6}$
 4.67 Pa
 $I = \frac{P_0^2}{2 v} = \frac{(4.67)^2}{2 \times 1.2 \times 344}$
 $2.64 \times 10^{-2} \text{ W/m}^2$
 $L = 10 \log \frac{I}{I_0} = 10 \log \frac{2.64 \times 10^{-2}}{10^{-12}}$
 104 dB

2. $2L = L + 10 \log \frac{I}{I_0} + 10 \log \frac{I}{I_0}$
 $10 \log () = 9 \text{ dB}$
 $\log () = 0.9, \quad 10^{0.9} = 7.9$

3. $I = \frac{1}{r^2} = I = \frac{k}{r^2}$
 $L_F - L_M = 10 \log \frac{I_F}{I_M}$
 $10 \log \frac{r_M^2}{r_F^2}$

$20 \log \frac{3}{0.3} = 20 \text{ dB}$

4. (a) $I = \frac{P_0^2}{2 v}; I_{\max} = \frac{(28)^2}{2 \times 1.29 \times 345}$
 0.881 W/m^2
 $L_{\max} = 10 \log \frac{0.881}{10^{-12}} = 119.45 \text{ dB}$
 $I_{\min} = \frac{(2 \times 10^{-5})^2}{2 \times 1.29 \times 345}$
 $4.49 \times 10^{-13} \text{ W/m}^2$
 $L_{\min} = 10 \log \frac{4.49 \times 10^{-13}}{10^{-12}} \text{ dB}$

(b) $S_0 = \frac{P_0}{k B} = \frac{P_0}{2 v}$
 3.48 dB
 $(S_0)_{\max} = \frac{28}{2 \times 3.14 \times 500 \times 1.29 \times 345}$
 $2 \times 10^{-5} \text{ m}$
 $(S_0)_{\min} = \frac{2 \times 10^{-5}}{2 \times 3.14 \times 500 \times 1.29 \times 345}$
 $1.43 \times 10^{-11} \text{ m}$

Introductory Exercise 16.4

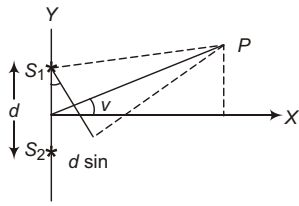
1. $(2n - 1) \frac{\lambda}{2} = 12 \text{ cm}$
 and $(2n - 1) \frac{\lambda}{2} = 36 \text{ cm}$
 $\frac{36}{12} = \frac{2n - 1}{2n - 1} \times \frac{24 \text{ cm}}{12 \text{ cm}}$
 $\frac{v}{0.24 \text{ m}} = 1375 \text{ Hz}$

2. $x = \frac{v}{6} = \frac{350}{6 \times 500} = 0.117 \text{ m} = 11.7 \text{ cm}$
 $\frac{2}{T} = t \times 2 = t \times 2 = 500 \times 10^3$
 rad 180

3. $x_1 = 2 \sqrt{H^2 - \frac{d^2}{4}} = d \quad n$

and
 $x_2 = 2 \sqrt{(H - h)^2 - \frac{d^2}{4}} = d \quad n \quad \frac{1}{2}$
 $\frac{d}{2} = 2 \sqrt{(H - h)^2 - \frac{d^2}{4}} = 2 \sqrt{H^2 - \frac{d^2}{4}}$
 or $4 \sqrt{(H - h)^2 - \frac{d^2}{4}} = 4 \sqrt{H^2 - \frac{d^2}{4}}$
 $2 \sqrt{4(H - h)^2 - d^2} = 2 \sqrt{4H^2 - d^2}$

4. $x_p = d \sin \left(n \frac{1}{2} \right)$ for minima



(a) $d \sin \frac{\theta}{2}$ for first minima

$$\sin^{-1} \frac{\lambda}{2d} = \sin^{-1} \frac{v}{2d}$$

$$\sin^{-1} \frac{340}{2 \cdot 600} = \sin^{-1} \frac{340}{1200}$$

$$\sin^{-1}(0.142) = 0.142 \text{ rad}$$

8.14

(b) For, first maxima $d \sin \theta = \lambda$

$$\sin^{-1} \frac{\lambda}{d} = \sin^{-1} \frac{340}{1200}$$

$$16.46^\circ$$

(c) $x_{\text{max}} = \frac{d}{n} = \frac{d}{3}$

$$\frac{2 \cdot 600}{340} = 3.53$$

$n = 3$ maxima.

5. (a) For coherent speakers in phase,

$$I_R = 4I_0 \cos^2 \frac{\theta}{2}$$

$$I_R = 4I_0 \cos^2 \frac{\theta}{2} = 0$$

(b) For incoherent sources,

$$I_R = I_1 + I_2 = I_0 + I_0 = 2I_0$$

(c) For coherent speakers with a phase difference 180° .

$$I_R = 4I_0 \cos^2 \frac{180^\circ}{2} = 4I_0 \cos^2 90^\circ = 0$$

6. $60 \text{ dB} = 10 \log \frac{I_0}{10^{-12}}$

$$\frac{10^6}{10^{-12}} = I_0$$

$$I_0 = 10^6 \text{ W/m}^2$$

$$\frac{2}{340} x = \frac{2}{v} x$$

$$\frac{2}{340} (11.8) = \frac{2}{v} (8) \cdot 3$$

(a) $I_R = 4I_0 \cos^2 \frac{\theta}{2} = 4I_0 \cos^2 \frac{3}{2} = 0$

(b) $I_R = 4I_0 \cos^2 \frac{4}{2} = 4I_0$

$$L_R = 10 \log \frac{4 \cdot 10^6 \text{ W/m}^2}{10^{-12}}$$

$$10 \log 10^6 \text{ dB} = 10 \log 4 + 60 \text{ dB} = 2 \log 2 \text{ dB} + 60 \text{ dB} = 66 \text{ dB}$$

(e) $\frac{2}{v} x = \frac{2}{340} (11.8)$

$$I_R = 4I_0 \cos^2 \frac{3}{4} = 4I_0 \cos^2 \frac{3}{4} = 2I_0$$

$$L_R = 10 \log \frac{2 \cdot 10^6}{10^{-12}} = 63 \text{ dB}$$

7. (a) $I_1 = \frac{10^3}{4 \cdot 2^2} = \frac{10^3}{16}$

$$19.9 \cdot 10^6 \text{ W/m}^2$$

$$199 \text{ W/m}^2$$

$$I_2 = \frac{10^3}{4 \cdot 3^2} = \frac{10^3}{36}$$

$$8.84 \cdot 10^6 \text{ W/m}^2$$

$$884 \text{ W/m}^2$$

(b) $(I_P)_{\text{max}} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$= (4.46 + 2.97)^2 = 55.27 \text{ W/m}^2$$

(c) $(I_P)_{\text{min}} = (\sqrt{I_1} - \sqrt{I_2})^2$

$$= (4.46 - 2.97)^2 = 2.22 \text{ W/m}^2$$

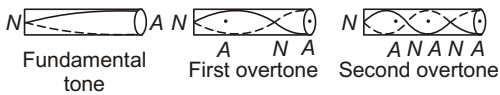
(d) $I_P = I_1 + I_2 = 28.7 \text{ W/m}^2$

Introductory Exercise 16.5

1. (a) $l_c = \frac{v}{4f} = \frac{345 \text{ m/s}}{4 \times 220 \text{ Hz}} = 0.392 \text{ m}$

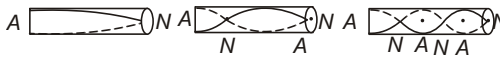
(b) $l_0 = \frac{3v}{10f} = \frac{3 \times 345}{10 \times 220} = 0.470 \text{ m}$

2. (a)



$d_A = l = 0.8 \text{ m}$, $d_A = \frac{l}{2} = 0.4 \text{ m}$, $d_A = \frac{l}{3} = 0.267 \text{ m}$, $d_A = \frac{l}{4} = 0.2 \text{ m}$, $d_A = \frac{l}{5} = 0.16 \text{ m}$, $d_A = \frac{l}{6} = 0.133 \text{ m}$

(b)



$d_A = 0, \frac{2l}{3}, \frac{4l}{3}$, $d_A = 0, \frac{2l}{5}, \frac{4l}{5}, \frac{6l}{5}$

3.

2	400, 560
0	
4	20, 28

HCF of the two shows, 80 and the values, 400 Hz and 560 Hz are odd multiples of 80. These conservative

harmonics are odd, which can be seen in closed organ pipe only.

(b) These are 5th and 7th harmonic.

(c) $l_c = \frac{v}{4f} = \frac{344}{4 \times 80} = 1.075 \text{ m}$

4. $v = 1000 \times 2 \times 6.77 \times 10^2 \text{ m/s}$

$v = \sqrt{\frac{RT}{M}} = r \sqrt{\frac{Mv^2}{RT}}$
 $n = \frac{127 \times 10^3 \times (135.4)^2}{8.314 \times 400} = 0.7 n$

As 1 r 2

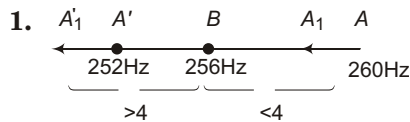
$n = 2, r = 0.7, 2 \times 1.4 = \frac{7}{5}$ diatonic

5.

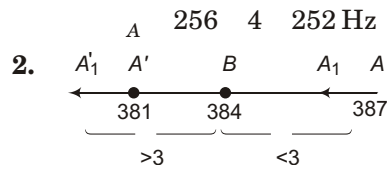
$\frac{(2n-1)v}{4l_1} = \frac{(2n-3)v}{4l_2}$
 $\frac{2n-3}{2n-1} = \frac{l_2}{l_1} = \frac{100}{60} = \frac{5}{3}$

$v = \frac{4l_1 n}{2n-1} = \frac{4 \times 0.6 \times 440}{3} = 352 \text{ m/s}$

Introductory Exercise 16.6



$A = 252 \text{ Hz}$
 $A = (256/4) \text{ Hz}$
 and $A = n(256/6) \text{ Hz}$
 $256/4 = n(256/6)$
 $4 \mp 6 = n \quad n = 4 \quad 6 \quad 2$



$A = 387 \text{ Hz}$
 $A = (384/3) \text{ Hz}$
 and $A = n(384/3) \text{ Hz}$
 $384/3 = n(384/3)$
 $384/3 = n \quad n = 384 \text{ m, m} \quad 3$

$$\frac{3}{2} \frac{v}{v_s} = \frac{1}{1 \mp \frac{v_s}{v}}$$

$$6 \text{ Hz} = 600 \text{ Hz} \frac{1}{2l} \sqrt{\frac{T_A}{T_B}}$$

and

$$\frac{606}{600} = \sqrt{\frac{T_A}{T_B}} \quad 1.01$$

$$\frac{T_A}{T_B} = 1.02$$

$$4. \quad 256 \text{ Hz} = \frac{v}{2(0.25 \text{ m})}$$

and

$$256 \text{ Hz} = \frac{v}{2(0.25 \text{ m})} \frac{1}{4x}$$

$$256 \text{ Hz} = \frac{v}{2(0.25 \text{ m})} \frac{1}{4x}$$

$$x = \frac{1}{256} \text{ m} = \frac{100}{256} \text{ cm} = 0.4 \text{ cm}$$

Introductory Exercise 16.7

1. When source is moving,

$$\frac{v}{v - v_s} = \frac{1}{1 \mp \frac{v_s}{v}}$$

$$1 \mp \frac{v_s}{v}$$

$$1 \mp \frac{v_s}{v} = 1 \mp \frac{u}{v}$$

When observer is moving,

$$1 \mp \frac{v_0}{v} = 1 \mp \frac{v}{v}$$

So, it can be seen that, v_0 and v_s are equal if $u = v$.

2. $\frac{340}{200} = 1.7 \text{ m}$

(a) $uT = 1.7 \text{ m} \frac{80}{200} = 1.7 \text{ m}$

(b) $\frac{0.4 \text{ m}}{340 \text{ m/s}} = \frac{1.3 \text{ m}}{262 \text{ Hz}}$

3. For doppler effect there has to be relative motion between source and receiver, but as they are at rest relative to each other that's why there is no shift in wavelength and frequency.

4. $\frac{v}{500} = 0.688 \text{ m}$

(a) front $uT = 0.688 \frac{30}{500}$

$0.688 \times 0.060 = 0.628 \text{ m}$

(b) behind $uT = 0.688 \times 0.060$

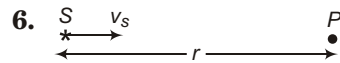
0.748 m

(c) front $\frac{344}{0.628} = 547.8 \text{ Hz}$

(d) behind $\frac{344}{0.748} = 459.9 \text{ Hz}$

5. $\frac{v}{v} = \frac{w}{w} \frac{v_0}{v_s} = \frac{340}{340} \frac{5}{5} \frac{20}{10} = 300 \text{ Hz}$

$\frac{315}{345} = 300 \text{ Hz} \Rightarrow 273.9 \text{ Hz}$



AIEEE Corner

■ Subjective Questions (Level 1)

$$1. \quad d \quad d_1 \quad d_2 \quad v \quad \frac{t_1}{2} \quad v \quad \frac{t_2}{2}$$

$$\frac{v}{2} (t_1 \quad t_2) \quad \frac{332}{2} \quad \frac{3}{2} \quad \frac{5}{2}$$

$$332 \quad 2 \quad 664 \text{ m}$$

The time for third eco is,

$$t \quad t_1 \quad t_2 \quad \frac{3}{2} \quad \frac{5}{2} \quad 4 \text{ s}$$

$$2. \quad v \quad \sqrt{\frac{RT}{M}} \quad \sqrt{\frac{7}{5} \frac{8.314}{2} \frac{300}{10^3}}$$

$$\sqrt{21} \quad 8.314 \quad 10^4 \quad 1321 \text{ m/s}$$

$$3. \quad v \quad \sqrt{\frac{P}{\frac{5}{3} \frac{76}{10^2} \frac{13.6}{10^3} \frac{9.8}{0.179}}}$$

$$\sqrt{\frac{5}{3} \frac{76}{10^2} \frac{136}{0.179} \frac{9.8}{0.179}} \quad 971 \text{ m/s}$$

$$4. \quad (a) \quad B \quad v^2 \quad 2^2$$

$$1300 \quad 16 \quad 10^4 \quad 64$$

$$1.33 \quad 10^{10} \text{ N/m}^2$$

$$(b) \quad Y \quad v^2 \quad \frac{l^2}{t^2} \quad \frac{6400}{(3.9 \cdot 10^4)^2} \quad \frac{(15)^2}{(3.9 \cdot 10^4)^2}$$

$$5. \quad v_t \quad \sqrt{\frac{l}{l} v_l} \quad \frac{l}{l} \quad \frac{v_t}{v_l} \quad \frac{F}{A} \quad Y \quad \frac{l}{l}$$

$$Y \quad \frac{v_t}{t_l} \quad \frac{Y}{30} \quad \frac{Y}{900}$$

$$6. \quad M_{\text{mix}} \quad \frac{2 \cdot 2 \cdot 1 \cdot 14}{2 \cdot 1} \quad 6 \text{ m/mole}$$

$$\frac{v_{\text{mix}}}{v_{\text{H}_2}} \quad \sqrt{\frac{M_{\text{H}_2}}{M_{\text{mix}}}} \quad \sqrt{\frac{2}{6}} \quad \frac{1}{\sqrt{3}}$$

$$v_{\text{mix}} \quad \frac{1}{\sqrt{3}} v_{\text{H}_2} \quad \frac{1}{\sqrt{3}} v_0 \quad \sqrt{\frac{T_2}{T_1}} \quad \frac{v_0}{\sqrt{3}} \quad \sqrt{\frac{300}{273}}$$

$$\frac{v_0}{\sqrt{2.73}} \quad \frac{1300}{\sqrt{2.73}} \quad 787 \text{ m/s}$$

$$7. \quad L_1 \quad 10 \log \frac{10^6}{10^{12}} \quad 60 \log 10 \quad 60 \text{ dB}$$

$$L_2 \quad 10 \log \frac{10^9}{10^{12}} \quad 30 \log 10 \quad 30 \text{ dB}$$

$$L_1 \quad 2L_2$$

$$8. \quad 100 \text{ dB} \quad 10 \log \frac{I}{I_0} \text{ dB}$$

$$I \quad 10^{10} I_0 \quad 10^2 \text{ W/m}^2$$

$$P \quad 4 r^2 I \quad 4 \quad (40)^2 \quad 10^2$$

$$64 \text{ W} \quad 201 \text{ W}$$

$$9. \quad (a) \quad 60 \text{ dB} \quad 10 \log \frac{I}{I_0} \text{ dB}$$

$$I \quad 10^6 I_0 \quad 10^6 \text{ W/m}^2$$

$$(b) \quad P \quad AI \quad 120 \quad 10^4 \quad 10^6 \text{ W}$$

$$1.2 \quad 10^8 \text{ watt}$$

$$10. \quad (a) \quad L \quad 13 \text{ dB} \quad 10 \log \frac{I_2}{I_1} \text{ dB}$$

$$I_2 \quad 10^{1.3} I_1 \quad 20 I_1$$

(b) As with doubling the intensity, loudness increases by 3 dB irrespective of the initial intensity.

$$11. \quad I \quad \frac{P}{4 r^2} \quad \frac{5}{4 (20)^2} \quad \frac{5}{4 \cdot 400}$$

$$\frac{1}{320} \text{ W/m}^2 \quad 9.95 \quad 10^4 \text{ W/m}^2$$

$$(b) \quad I \quad 2^2 \cdot 2 a^2 \quad v \quad a \quad \frac{1}{\sqrt{2}} \frac{I}{v}$$

$$\frac{1}{300} \sqrt{\frac{1}{320 \cdot 2 \cdot 129 \cdot 330}}$$

$$\frac{1}{300} \frac{1}{10^{12}} \sqrt{\frac{1}{85.5}}$$

$$1.15 \cdot 10^6 \text{ m}$$

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12. $60 \text{ dB} = 10 \log \frac{I}{10^{-12}} \text{ dB}$

$$I = 10^6 \text{ W/m}^2 \text{ and } a = \frac{1}{\sqrt{2}} \sqrt{\frac{T}{v}}$$

$$\frac{1}{800} \sqrt{\frac{10^6}{2 \cdot 1.29 \cdot 330}} = 13.6 \cdot 10^{-9} \text{ m}$$

13. $102 \text{ dB} = 10 \log \frac{I}{I_0} \text{ dB}$

$$I = 10^{10.2} I_0 = 10^{10.2} \cdot 10^{-12} = 10^{1.8} \text{ W/m}^2$$

$$P = 4 \pi r^2 I = 4 \cdot 3.14 \cdot (20)^2 \cdot 10^{1.8} = 80 \text{ W}$$

14. $I = 2^2 v^2 a^2 = 2 \cdot (3.14)^2 \cdot (300)^2 \cdot (0.2 \cdot 10^{-3})^2 = 1.29 \cdot 330 \text{ W/m}^2 = 30.25 \text{ W/m}^2$

$$L = 10 \log \frac{I}{I_0} \text{ dB} = 10 \log \frac{30.25}{10^{-12}} \text{ dB} = 134.8 \text{ dB}$$

15. (a) $v_w = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \cdot 10^9}{10^3}} = 1.48 \cdot 10^3 \text{ m/s}$

$$A_w = \sqrt{\frac{I}{2 \pi^2 v}} = \frac{1}{\sqrt{2}} \sqrt{\frac{I}{v}}$$

$$\frac{1}{3400} \sqrt{\frac{3 \cdot 10^6}{2 \cdot 10^3 \cdot 1.48 \cdot 10^3}} = 9.44 \cdot 10^{-11} \text{ m}$$

$$v_N = \frac{1.48 \cdot 10^3}{3400} = 0.43 \text{ m}$$

(b) $v_a = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.4 \cdot 10^5}{1.2}} = 341.6 \text{ m/s}$

$$A_a = \frac{1}{3400} \sqrt{\frac{3 \cdot 10^6}{2 \cdot 1.2 \cdot 341.6}} = 5.66 \cdot 10^{-9} \text{ m}$$

$$a = \frac{341.6}{3400} = 0.1 \text{ m}$$

(c) $A_a = A_w; \frac{A_a}{A_w} = \frac{5.66 \cdot 10^{-9}}{9.44 \cdot 10^{-11}} = 60$

As bulk modulus of water is much larger than air, such that displacement of particles of medium becomes less.

16. $I = \frac{p_0^2}{2 v} = \frac{(6 \cdot 10^5)^2}{2 \cdot 1.29 \cdot 343} \text{ W/m}^2 = 4 \cdot 10^{12} \text{ W}$

$$L = 10 \log \frac{I}{I_0} = 10 \log \frac{4 \cdot 10^{12}}{10^{-12}} = 20 \log 2 = 6 \text{ dB}$$

17. $v = 2l f = 594 \text{ Hz}$

$c = \frac{v}{4l} = \frac{594}{2} = 297 \text{ Hz}$

18. $f_n = \frac{(n-1)v}{2l} = (n-1) \frac{344}{2 \cdot 0.45}$

$f_1 = 382.2 \text{ Hz}$
 $f_2 = 382.2 \text{ Hz}, 764.4 \text{ Hz}, 1146.7 \text{ Hz},$
 $f_3 = (2n-1)v$

$f_4 = \frac{344}{2 \cdot 0.45}$

$f_5 = (2n-1) 191.1 \text{ Hz}$
 $f_6 = 191.1 \text{ Hz}, 573.3 \text{ Hz}, 955.6 \text{ Hz}$

19. $c = \frac{v}{4l}$

$$v = 4l c = 4 \cdot 0.15 \cdot 500 = 300 \text{ m/s}$$

$f = \frac{v}{2l_0} = \frac{300}{2 \cdot 0.6} = 250 \text{ Hz}$

20. $y = A \cos kx \cos \omega t$

$$A \cos \frac{2}{1.6} x \cos 2 \frac{330}{1.6} t$$

21. $A \cos 3.93x \cos 1296 t$

$$\frac{2n-1}{4} v = \frac{2n-1}{4} \frac{3}{0.84} v$$

$$\frac{2n-1}{2n} = \frac{3}{1} \frac{84}{50} = 1.68$$

$$\begin{aligned}
 & \frac{3 \cdot 1.68 \cdot 2n \cdot 0.68}{n \cdot 0.97 \cdot 1} \text{ as } n \text{ is an integer} \\
 & v = \frac{4lv}{2n-1} = \frac{4 \cdot 0.5 \cdot 512}{3} \text{ m/s} \\
 & \frac{341.3 \text{ m/s}}{2n-5} \\
 & l = \frac{4l}{2n-5} v = \frac{7}{4 \cdot 512} \cdot 341.3 \\
 & \frac{1.167 \text{ m}}{2n-5} = \frac{116.7 \text{ cm}}{2n-5} \\
 \text{22. } & c = \frac{v}{4l} = \frac{340}{4 \cdot 1} = 85 \text{ Hz} \\
 & s = \frac{v}{0.4} \sqrt{\frac{F}{\dots}} = 85 \\
 & F = (85 \cdot 0.4)^2 \cdot (34)^2 \cdot \frac{4 \cdot 10^3}{0.4} \\
 & = 11.65 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \text{23. } & c = \frac{v}{4(l-e)} = v \cdot 4 \cdot (l-l) \\
 & 4 \cdot (l-0.3d) \\
 & 4 \cdot 480(0.16-0.3 \cdot 0.05) = 336 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 \text{24. (a) } & e = \frac{(2n-1)}{4l} \\
 & \frac{440}{4l} = \frac{5 \cdot 330}{4l} \\
 & l = \frac{5 \cdot 330}{4 \cdot 440} = \frac{15}{16} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } & \text{N} \left(\text{Diagram of a standing wave with 5 loops} \right) A = \frac{5}{4} l = \frac{15}{16} \\
 & \frac{15}{16} \cdot \frac{4}{5} = \frac{3}{4} \text{ m} \\
 & p = p_0 \cos kx = p_0 \cos \frac{2}{3/4} = \frac{15}{32} \\
 & p_0 \cos \frac{15}{12} = p_0 \cos \frac{5}{4} = \frac{p_0}{\sqrt{2}}
 \end{aligned}$$

(c) At open end there is pressure node, so, $p_{\max} = p_{\min} = p_0$
 (d) At closed end there is pressure antinode, such that, $p_{\max} = p_0$ and $p_{\min} = p_0$

$$\begin{aligned}
 \text{25. (a) } & c = \frac{v}{4l_c} \\
 & l_c = \frac{v}{4 \cdot c} = \frac{345}{4 \cdot 220} = 0.392 \text{ m} \\
 \text{(b) } & l_0 = \frac{5}{4} l_c, l_0 = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{4}{5} l_c \\
 & \frac{6}{5} l_c = \frac{6}{5} \cdot 0.392 \text{ m} = 0.47 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{26. } & s = c = \frac{v_s}{2 \cdot 0.8 l_c} = \frac{v_s}{4 l_c} \\
 & \frac{v_s}{v_a} = \frac{1.6}{4} = 0.4
 \end{aligned}$$

$$\begin{aligned}
 \text{27. (a) } & s = \frac{v}{300} = \frac{17}{15} \text{ m} = 1.13 \text{ m} \\
 \text{(b) } & a = v_s T = \frac{v \cdot v_s}{\dots}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{340 \cdot 30}{300} = \frac{31}{30} = 1.03 \text{ m} \\
 & b = v_s T = \frac{v \cdot v_s}{\dots} \\
 & \frac{340 \cdot 30}{300} = \frac{37}{30} = 1.23 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{28. } & \frac{1}{2l} \sqrt{\frac{F}{\dots}} = \frac{1}{2} \frac{F}{\dots} \\
 & \frac{F}{F} = 2 - 2 = \frac{15}{440} - \frac{3}{440} = 0.68\% \\
 & 440 \cdot 1.5 = 438.5 \text{ Hz or } 441.5 \text{ Hz}
 \end{aligned}$$

$$\begin{aligned}
 \text{29. } & v = 0.32 \text{ m/s;} \\
 & vT = 0.32 \cdot 1.6 \text{ m} = 0.512 \text{ m.} \\
 & a = v_s T = \frac{v_s}{T} = v \cdot \frac{\dots}{\dots} \\
 & 0.32 \cdot \frac{0.12}{1.6} = 0.245 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 & b = v_s T = \frac{0.512 \text{ m}}{0.392} = 1.6 \\
 & 0.512 \cdot 0.392 = 0.904 \text{ m.}
 \end{aligned}$$

$$\begin{aligned}
 \text{30. (a) } & a = \frac{v \cdot v_0}{v \cdot v_s} = \frac{340 \cdot 18}{240 \cdot 30} = 262 \text{ Hz} \\
 & \frac{358}{310} = 262 \text{ Hz} \quad 302.5 \text{ Hz} \\
 \text{(b) } & r = \frac{v \cdot v_0}{v \cdot v_s} = \frac{340 \cdot 18}{340 \cdot 30} = 262
 \end{aligned}$$

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$$\frac{322}{370} \quad 262 \text{ Hz} \quad 228 \text{ Hz}$$

31. $\frac{v}{v} \frac{v}{v_s} \sim \frac{2v_s}{v} \quad v \frac{v}{2v_s}$

$$\frac{340}{2} \frac{4}{1} \quad 680 \text{ Hz}$$

32. $l_c \frac{v}{4} \frac{330}{4} \quad 110 \quad 2.2; \quad c \frac{v}{4l_c}$

$$l_c \frac{3}{4} \text{ m}; \quad 0 \frac{2v}{2l_0}$$

$$l_0 \frac{2v}{2} \frac{2}{2(330 \quad 2.2)}$$

0.993 m or 1.007 m

33. $\frac{7}{2}$ and $\frac{P}{Q}$ as beat
frequency increases waxing of P.

$$Q \frac{5}{v} \frac{v}{v_s} \quad Q \frac{332}{327} \frac{5}{Q}$$

$$Q \frac{327 \text{ Hz}}{327} \text{ and } \frac{7}{2} \quad 323.5 \text{ Hz}$$

When Q gives 5 beats with its own echo.

OR

$$P \quad Q \quad \frac{7}{2} \quad q \quad 5 \quad \frac{332}{327} \quad Q \quad 5$$

$$5 \quad \frac{7}{2} \quad \frac{5}{327} \quad Q$$

$$Q \quad \frac{327 \quad 1.5}{5} \quad 98.1 \text{ Hz}$$

$$P \quad 98.1 \quad 2.5 \quad 94.6 \text{ Hz}$$

When P gives 5 beats with the echo of Q.

34. $\frac{v}{v} \frac{v}{v_s} \sim \frac{2v_s}{v} \quad \frac{2v_s}{v}$

$$v_s \frac{v}{2} \frac{340}{2} \quad 680$$

$$v_s \quad \frac{1}{2} \text{ m/s}$$

35. $(2n - 1) \frac{1}{2} \quad 11.5 \text{ cm}$

$$(2n - 3) \frac{1}{2} \quad 34.5 \text{ cm}$$

$$\frac{2n}{2n} \frac{3}{1} \frac{34.5}{11.5} \quad 3 \quad 4n \quad 0 \quad n \quad 0$$

$$\frac{11.5 \text{ cm}}{2} \quad 23 \text{ cm}$$

$$\frac{v}{0.23 \text{ m}} \quad \frac{331.2 \text{ m/s}}{0.23 \text{ m}} \quad 1440 \text{ Hz}$$

36. $\frac{v}{220} \quad 1.5 \text{ m}$

$$x \quad S_2 P \quad S_1 P \quad 3 \quad \frac{3}{4} \quad \frac{9}{4} \text{ m}$$

$$\frac{3}{2} \quad \frac{3}{2} \quad \frac{3}{2} \quad (2n - 1) \frac{1}{2}$$

Here, $S_1 P \quad \frac{3}{4} \quad \frac{1}{2}$

$$1 \quad \frac{2}{2} S_1 P \quad \frac{2}{2} \quad \frac{1}{2}$$

and $S_2 P \quad 3 \quad 2 \quad \frac{3}{2} \quad 2$

$$2 \quad \frac{2}{2} \quad 2 \quad 4$$

Destructive interference will take place at P.

$$P_P \quad P_{\min} \quad (\sqrt{P_1} \quad \sqrt{P_2})^2$$

$$(\sqrt{1.8 \quad 10^3} \quad \sqrt{1.2 \quad 10^3})^2$$

$$0.6 \quad 10^3 (\sqrt{3} \quad \sqrt{2})^2$$

$$0.6 \quad 10^3 \quad 0.1 \quad 6 \quad 10^5 \text{ W}$$

37. $x \quad 2 \sqrt{2^2 \quad \frac{x^2}{2}} \quad x \quad n \quad 1$

$$\frac{360 \text{ m/s}}{360 \text{ Hz}} \quad 1 \text{ m}$$

$$2 \sqrt{4 \quad \frac{x^2}{4}} \quad 1 \quad x$$

or $4 \quad 4 \quad \frac{x^2}{4} \quad 1 \quad 2x \quad x^2$

$$16 \quad 1 \quad 2x$$

$$x \quad 7.5 \text{ m}$$

■ Objective Questions (Level 1)

1. Sound cannot travel in vacuum, as it is mechanical wave.
2. Longitudinal waves can travel through all mechanical mediums.

3. $\sqrt{\frac{RT}{32}} \sqrt{\frac{R}{28}}$
 $T \frac{32}{28} 288 \text{ K } \frac{8}{7} 288 \text{ K } 56 \text{ C}$

4. Third overtone is 7th harmonic *ie*, there 4 nodes and 4 antinodes.



5. $\frac{v}{l} \propto \sqrt{T}$ so with increase in temperature, frequency increases.

6. For sound water is rarer medium and air is denser medium so, it bends towards normal while going from water to air.

7. $c \frac{v}{4l_c} o \frac{v}{2l_o} \frac{l_c}{l_o} \frac{2}{4} 1:2$

8. $\frac{2}{1} \sqrt{\frac{F_2}{F_1}}$
 $F_2 \frac{2}{1} F_1$
 $M_2 \frac{2}{1} M_1 \frac{256}{320} 10 \text{ kg}$
 6.4 kg

$OM \ M_2 \ M_1 \ 6.4 \ 10 \ 3.6 \text{ kg}$
ie., Mass has to be decreased by 3.6 kg

9. direct $\frac{v}{v - v_s}$ and reflected $\frac{v}{u + v_s}$
 as $D = R$ so there will be no beats *ie.*, beat frequency will be zero.

10. $\frac{2}{v} \frac{1}{v} \frac{v}{2} \frac{v}{1} v \frac{(1 \ 2)}{1 \ 2}$
 $v \frac{1 \ 2}{v}$

$v \frac{1 \ 1.01 \ \frac{10}{3}}{0.01} 337 \text{ m/s}$

11. $\frac{v}{v} \frac{1}{2} n \ n \ \frac{1}{2} \ 0.5$

12. $I_{\max} (\sqrt{I} \ \sqrt{I})^2 \ 4I \ NI \ N \ 4$

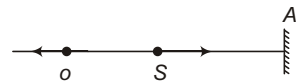
13. $\frac{v}{4(l_1 \ e)} \frac{3v}{4(l_2 \ e)}$
 $\frac{l_2 \ e \ 3l_1 \ 3e}{e \ l_2 \ 3l_1}$
 $e \frac{42 \ 3 \ 17}{2} \text{ cm } 0.5 \text{ cm}$

$v \ 4 \ (l_1 \ e) \ 4 \ 500(17 \ 8.5) \ 10^2$
 $20 \ 17.5 \ 350 \text{ m/s}$

14. At the moment when velocity of source is perpendicular to the line joining source and observer then there is no Doppler effect *ie.*, $n = n_1 = n_2 = 0$

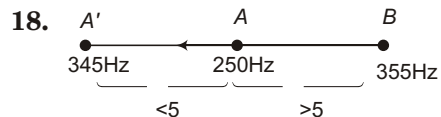
15. $\frac{(n \ 1)v}{4l} (2n \ 1) \frac{340}{4 \ 1} 85(2n \ 1)$
 85, 255, 425, 595, 765, 935
 6 frequencies below 1 kHz.

16. $\frac{v \ v_0}{v \ v_s} \frac{v \ v_0}{v \ v_s} \ 1 \ \frac{v \ v_0}{v \ v_s}$



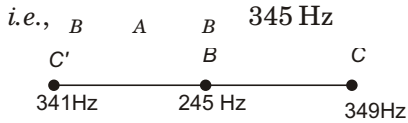
$\frac{v_s \ v_0}{v \ v_s} \frac{10}{360} 180 \ 5 \text{ Hz}$

17. $n \ 1 \ 2 \ \frac{v}{1} \ \frac{v}{2}$
 $\frac{v(2 \ 1)}{1 \ 2} \frac{v}{v} \frac{n \ 1 \ 2}{1 \ 2}$



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As beat frequency between A and B decreases on loading A.



After loading A, $f_A = 345 - 2 = 247$ Hz and $f_C = 345 + 6 = 351$ or 353 Hz.

As possible frequency of C are 341 Hz and 249 Hz then only 341 Hz is justified.

19. $e \frac{l_2}{2} = \frac{3l_1}{2} = \frac{122}{2} = 61$ cm

So, $\frac{l_2}{v} = \frac{3l_1}{5v}$

20. $\frac{l_3}{5} = \frac{5l_1}{40} = \frac{4l}{4} = l = 204$ cm

$\frac{1}{2} \frac{F}{F} = \frac{1}{2} \frac{F}{F} = 200$ Hz

21. $\frac{2n-1}{4l} v = \frac{2n-1}{4} \frac{v}{l}$

$\frac{1}{4} m, \frac{3}{4} m, \frac{5}{4} m.$

As, $l_{\max} = 120$ cm $l = 25$ cm 75 cm.

Height of water column

22. $7 \frac{1}{4} = 105$ cm $\frac{105}{7} = 15$ cm



$\frac{60}{4} = 15$ cm

So, nodes are at, $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}$ and $7 \frac{1}{4}$ from closed end i.e., they are at, 15 cm, 45 cm, 75 cm and 105 cm.

23. $c \frac{v}{4l} = 512$ Hz, $o \frac{v}{2l} = 2 \frac{v}{4l}$

$2 \frac{v}{4l} = 1024$ Hz

24. $M_{\min} = \frac{1}{1} \frac{32}{1} = 17$

$\frac{v_{\min}}{v_2} = \sqrt{\frac{M_{H_2}}{M_{\min}}} = \sqrt{\frac{2}{17}}$

25. f_a and f_r but f_a constant and f_r constant.

So, curve in (b) represents correctly.

26. $u = \frac{(2n-1)v}{4l} = \frac{(m-1)v}{2l}$

How, $\frac{(2n-1)v}{4 \cdot 2l} = \frac{(m-1)v}{4 \cdot 2l}$

$\frac{4}{2} = 2$ beat/s

27. $a_r \frac{v}{v_1} = \frac{v}{v_1}$

$\frac{2vv_1}{(v_1)(v_1)} = \frac{2v}{v_1}$

$\frac{2}{320} = \frac{2}{4 \cdot 243} = 6$ Hz

28. $c \frac{(2n-1)v}{4l_c} = \frac{320}{4} (2n-1)$

$(2n-1) = 80$ Hz 80 Hz, 240 Hz, 400 Hz, ...

$o \frac{(n-1)v}{2l_0} = \frac{320}{2 \cdot 1.6} (n-1)$

$(n-1) = 100$ Hz 100 Hz, 200 Hz, 300 Hz, 400 Hz, ...

$c \quad o \quad 400$ Hz

29. $I_{\max} = 4I_0$

and $I_{\max} = \frac{4I_{\max}}{L} = \frac{16I_0}{10 \log 16}$

$10 \log 16 = 40 \log 2 = 22$ dB

30. $\frac{2}{k} = \frac{2}{2} = 4$ m

$l = 5 \frac{1}{4} = 5 \frac{1}{4} m = 5$ m

31. $d = (2n - 1) \frac{v}{4} = \frac{(2n - 1)}{4} \frac{330}{660} \text{m}$
 $\frac{330}{24} (2n - 1) \text{cm} = (2n - 1) \cdot 13.75 \text{cm}$
 13.75 cm, 41.25 cm, 68.75 cm, 96.25 cm
 etc.

32. $\frac{v_1}{332} - \frac{v_2}{10^2} = \frac{v(2 - 1)}{2}$
 $\frac{0.49}{332} - \frac{0.5}{10^2} = 13.15 \text{ Hz}$

33. $\frac{300}{300} - \frac{300}{30} = 300 - 300$
 33.33 Hz and $\frac{A}{B}$
 So both (a) and (b) options are wrong.

34. $f_a = \frac{v - v_0}{v} f = 1 - \frac{v_0}{v} f$ and
 $f_r = 1 + \frac{v_0}{v} f$
 $\frac{f_a}{f_r} = \frac{v - v_0}{v + v_0}$
 $(f_a - f_r)v = (f_a - f_r)v_0$
 $\frac{v}{v_0} = \frac{f_a - f_r}{f_a + f_r}$

and
 $f_a - f_r = \frac{2v_0}{v} f = 2 \frac{f_a - f_r}{f_a + f_r} f$
 $f = \frac{f_a + f_r}{2}$

JEE Corner

■ Assertion and Reason

1. $c = (2n - 1) \frac{v}{4l} = \frac{v}{4l}, \frac{3v}{4l}, \frac{5v}{4l}, \dots$

while, $c_0 = \frac{(n - 1)v}{2l} = \frac{v}{2l}, \frac{2v}{2l}, \frac{3v}{2l}, \frac{4v}{2l}, \dots$

it can be seen that $c = c_0$ at all situation and $c = \frac{1}{2} c_0$ so assertion is true but reason is false.

2. Apparent frequency is constant for constant relative velocity so assertion is false.

3. At a point of minimum displacement pressure amplitude is maximum i.e., pressure difference is maximum not pressure. So assertion is false.

4. The driver receiver two sounds one direct, c_0 and other $c_R = \frac{v + u}{v - u}$ such

that he detects beats. So reason is true explanation of assertion.

5. With increase in intensity sound level increases in logarithmic order so assertion is false.

6. Speed of sound $v = \sqrt{\frac{p}{\rho}}$, with increase in only pressure density increases such that $\frac{p}{\rho}$ remains constant. Again $v = \sqrt{\frac{RT}{M}}$ so both assertion and reason are true but reason is not correct explanation of assertion.

7. $f_A - f_B = 4$ when A is loaded with little wax then f_A slightly decreases and then beat frequency decreases, but if it is heavily loaded with wax then its frequency goes much below f_B such that beat frequency increases. So, assertion and reason are both true but reason is not correct explanation of assertion.

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8. $\frac{150}{3}, \frac{450}{5}, \frac{750}{5}$

The frequencies are odd harmonics then the pipe is closed and fundamental frequency is also 150 Hz. So assertion and reason are both true but reason is not correct explanation of assertion.

9. $\frac{1}{l e}$ with increase in diameter end

correction, e , increases and decreases. So reason is correct explanation of assertion.

10. With increasing length of air column, number of overtone increases and not the wavelength so assertion is false.

■ Objective Questions (Level 2)

1. At the boundary between two mediums, one part of incident wave gets reflected and other part gets transmitted or refracted.

2. $k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.9} = \frac{2\pi}{1.5} \times \frac{1.5}{3.9}$
 $S_0 = k B = \frac{2\pi}{3.9} \times \frac{3}{1.5} \times 10^2 = \frac{2\pi}{1.5} \times 10^5$

$S_0 = \frac{0.3}{k B} = \frac{10^2}{3.9} \times \frac{10^5}{1.3} = \frac{3.9 \times 10^7}{12 \times 1.3} = 0.025 \text{ m} \times 2.5 \text{ cm}$

3. $\frac{A_t}{A_i} = \frac{2v_2}{v_1 + v_2} = \frac{2 \times 100}{200 + 100} = \frac{2}{3}$

4. $\frac{v}{v_1 + v_2} \sim \frac{v}{v_1 + v_2}$
 $v_1 + v_2 = \frac{340 \times 10}{1700} = 2 \text{ m/s}$

5. $v_s = \frac{gt}{v} = \frac{10 \times 10}{300} = \frac{2}{30} = \frac{2}{10} = \frac{1}{5} = 0.2 \text{ m/s}$
 $\frac{302}{290} = \frac{298}{310} = 150 \text{ Hz}$
 $\frac{302}{290} = \frac{298}{310} = 150 \text{ Hz}$

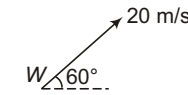
6. $\frac{7L}{2} = \frac{2L}{7}$
 $A = a \cos kx = a \cos \frac{2\pi}{7} \times \frac{L}{7} = a \cos \frac{2\pi L}{7}$



7. For maxima, $n = 3$
 $\frac{3}{n} = \frac{v}{nv} = \frac{110}{n}$

110, 220, 330 Hz, ..etc. maxima will be formed so maximum will not be formed at 120 Hz and 100 Hz.

8. $W = 20 \text{ m/s}$
 $S = 20 \text{ m/s}$
 $\frac{v}{v - w \cos 60} = \frac{300}{300 - 10} = 500 \text{ Hz}$
 $\frac{310}{290} = 500 \text{ Hz}$
 534 Hz



9. $R = \frac{v}{v - \frac{20}{10}} = \frac{v}{v - 2} = 500$
 $\frac{360}{300} = \frac{360}{350} = 500 \text{ Hz} \times 31 \text{ Hz}$

10. $\frac{404}{2} = \frac{400}{2} = 202 \text{ Hz}$
 $\frac{I_{\max}}{I_{\min}} = \frac{2}{1} = 9:1$

$$11. \frac{3}{4} \cdot 34 \text{ cm} = \frac{4}{3} \cdot 34 \text{ cm}$$

$$\frac{v}{v_{51}} = \frac{136}{3}$$

$$\frac{v_{16}}{v_{51}} = \frac{\sqrt{273}}{\sqrt{273}} \cdot \frac{16}{51} = \frac{\sqrt{289}}{\sqrt{324}} \cdot \frac{1}{\sqrt{1.121}} = \frac{1}{1.1}$$

$$16 \cdot \frac{51}{1.1} = \frac{136}{3 \cdot 1.1} = 41.21 \text{ cm}$$

$$12. 176 \frac{v}{v} \cdot \frac{v}{22} = 165 \frac{v}{v}$$

$$176v(v-v) = 165(v-v)(v-22)$$

$$176 = 330(330-v) = 165(330-v)(330-22)$$

or $1.143(330-v) = 330-v$
 or $0.143 \cdot 330 = 2.143v - v = 22 \text{ m/s}$

$$13. M_{\min} = \frac{2 \cdot 32 \cdot 3 \cdot 48}{2 \cdot 3} = 41.6$$

$$\frac{2}{1} \frac{v_2}{v_1} = \sqrt{\frac{1}{2}} \sqrt{\frac{32}{41.6}} = \sqrt{0.77}$$

$$= 0.875 \cdot 175 \text{ Hz}$$

$$14. v_0 = gt = 30 \text{ m/s}$$

$$1100 \frac{v}{v} = \frac{30}{v} = 1000, 1.1v - v = 30$$

$$0.1v = 30 \Rightarrow v = 300 \text{ m/s}$$

Passage (Q 5 to 17)

$$v_m = v_p = 8 \text{ m/s}, 50v_m = 150v_p$$

$$v_m = 3v_p, 4v_p = 8 \text{ m/s}$$

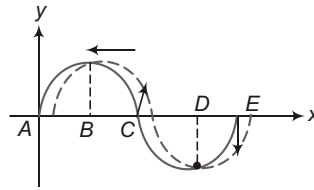
$$v_p = 2 \text{ m/s and } v_m = 6 \text{ m/s}$$

$$15. \frac{v}{v} \cdot \frac{2}{6} f_0 = \frac{332}{324} f_0 = \text{constant}$$

$$16. \frac{v}{v} \cdot \frac{2}{6} f_0 = \frac{328}{336} f_0 = \text{constant}$$

$$17. f_0 = v \text{ and graph is (a)}$$

$$18.$$



Both (a) and b are correct.

More Than One

$$19. \frac{(2n-1)v}{4l}$$

$$l \frac{v}{4} (2n-1) = \frac{330}{4} \frac{330}{264} (2n-1) \text{ m}$$

$$(2n-1) = 31.25 \text{ cm}$$

$$31.25 \text{ cm}, 93.75 \text{ cm}, 156.25 \text{ cm}$$

$$20. (a) v = p^0, (b) v = \sqrt{T} = v^2 T,$$

where T is absolute temperature.

$$(c) v = \sqrt{F} \quad (d) \frac{1}{l}$$

(c) and (d) are correct.

$$21. P_0 = BA k; B = \frac{P}{V} = \frac{P}{p}$$

$$\frac{P}{B} = BA k = \frac{P}{B} = A k$$

Pressure and density equations are in opposite phase i.e., $\frac{P}{2}$ and not $\frac{P}{2}$.

So, (a), (b) and (c) are correct.

$$22. \frac{5v}{4l_c} = \frac{3v}{2l_o} = \frac{125}{l_c} = \frac{2}{l_o} = \frac{l_o}{l_c} = \frac{2}{1.25} = \frac{8}{5}$$

$$(a) c = \frac{v}{4l_c} = \frac{v}{4 \cdot \frac{5}{8} l_o} = \frac{2v}{5l_o}$$

$$\frac{4}{5} \frac{v}{2l_o} = \frac{4}{5} \frac{v}{l_o} = c$$

$$(b) c = \frac{3v}{4l_c} = \frac{3v}{4 \cdot \frac{5}{8} l_o} = \frac{12}{5} \frac{v}{2l_o}$$

$$\frac{6}{5} \frac{2v}{2l_o} = \frac{6}{5} \frac{v}{l_o} = c$$

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(c) $c \frac{15v}{4l_c} \frac{15v}{4 \frac{5}{8}l_o} \frac{6v}{l_o} 12 \frac{v}{2l_o}$

$12v_o$ twelfth harmonic.

(d) Closed organ pipe cannot have tenth harmonic it only has odd harmonics.

23. $f \frac{v}{4(l-e)} \frac{1}{4(l-e)} \sqrt{\frac{RT}{M}}$

(a) increase in r increase in e
decrease in f

(b) increase in T increase in v
increase in f

(c) increase in M decrease in v
decrease in f

(d) increase in P increase in no
change in v no change in f

24. $f_a \frac{v}{v-v_s} f$ and $f_r \frac{v}{v+v_s} f$ are constants during approach and received.

■ Match the Columns

1. $\frac{v}{2l} f$

(a) $c \frac{v}{4 \cdot 2l} \frac{f}{4} 0.25f \quad s$

(b) $c_2 \frac{5v}{4 \cdot 2l} \frac{5}{4} f 1.25f \quad p$

(c) $c_1 \frac{3v}{4 \cdot 2l} \frac{3}{4} f 0.75f \quad r$

(d) $c_1 \frac{3v}{4 \cdot 2l} 0.75f \quad r$

2. $\frac{v}{v-v_s} \frac{v}{v+v_s} f \frac{2vv_f}{v^2-v_s^2} f$

$\frac{2v \frac{v}{4}}{v^2 \frac{16}{v^2}} f \frac{16}{15} \frac{1}{2} f \frac{8}{15} f$

2 $\frac{v}{v-v_s} \frac{v}{v+v_s} 1 f \frac{2v_s}{v-v_s} f$

$\frac{2v/4}{v-v/4} f \frac{2}{3} f$

3 $\frac{v}{v-v_5} \frac{v}{v+v_5} f 0$

(a) $1 \frac{8}{15} f \quad q$

(b) $2 \frac{2}{3} f \quad p$

(c) $3 0 \quad s$

(d) $3 0 \quad s$

3. $f \quad f_T \quad f_S$

(a) If tuning fork is loaded f_T decreases such that beat frequency may increase or decrease depending upon amount of wax r, s

(b) If prongs are filed, beat frequency must increase p

(c) If tension is increased beat frequency may increase or decrease depending upon the amount of change in tension. r, s

(d) If tension is decreased, beat frequency must increase p

4. (a) For point source, $I \frac{1}{r}$, and $A \frac{1}{r} \quad r$

(b) q

(c) For line source, $I \frac{1}{r}$ and $A \frac{1}{\sqrt{r}} \quad q$

(d) $\frac{2}{k} \frac{2}{2} 2m$

$5 - \frac{5}{4} \frac{5}{2} m 2.5m$

(a) $l 2.5m \quad s$

(b) $\frac{2}{2} m \quad r$

(c) $\frac{2}{2}, \frac{2}{2} 1m, 2m \quad p, r$

(d) $\frac{3}{4}, 3 - \frac{3}{4} 0.5m, 1.5m \quad q$

17. Thermometry, Thermal Expansion & Kinetic Theory of Gases

Introductory Exercise 17.1

1. (a) $\frac{C}{5} = \frac{F - 32}{9}$ for $F = 0, C = \frac{5}{9} \times 32$

(b) $\frac{K - 273.15}{5} = \frac{F - 32}{9}$ for $K = 0,$
 $F = \frac{9}{5} \times 273.15 + 32 = 459.67 \text{ F}$

2. (a) $\frac{x}{5} = \frac{2x - 32}{9}$ $x = \frac{10x}{9} = 17.8$
 $17.8 = \frac{10}{9} \times 1 \times x$

(b) $\frac{x}{5} = \frac{x/2 - 32}{9}$ $x = \frac{5}{18}x = 17.8$
 $17.8 = \frac{13}{18}x$ $x = 24.65 \text{ C}$

3. $\frac{C - 5}{99} = \frac{F - 32}{52}$ $\frac{C - 5}{180} = \frac{F - 32}{94}$
 $\frac{C - 5}{52} = \frac{F - 32}{94}$ $47 = 122 \text{ F}$

4. $\frac{K - 273.15}{5} = \frac{F - 32}{9}$
 $x = 273.15 + \frac{5}{9}x = 17.8$
 $\frac{4}{9}x = 255.35$ $x = 574.54$

5. $\frac{C}{5} = \frac{F - 32}{9}$ $\frac{9}{5}x = x - 32$
 $\frac{4}{5}x = 32$
 $x = \frac{5}{4} \times 32 = 40 \text{ C}$

6. $t = \frac{1}{2}t$
 $\frac{1}{2} = 1.2 \times 10^5 = 86400 \times 30$
 $1.5 = 1.2 \times 8.64 \text{ s} = 15.55 \text{ s given.}$

7. As from 0°C to 4°C, density of water increases so the volume of wooden block above water level increases and as from 4°C to 10°C density of water decreases so the volume of block above water decreases.

8. $V_1 = 1g$ $V_2 = 1g$ and $V_2 = 2g$ $V_2 = 2g$

$$\frac{V_1}{V_1} = 1 = \frac{V_1}{V_1} = 1 = \frac{1}{1}$$

and $\frac{V_2}{V_2} = 1 = \frac{2}{2}$

$$\frac{V_2}{V_2} = \frac{V_1}{V_1}$$

$$1 = \frac{2}{2} = 1 = \frac{1}{1} = \frac{1}{1} = \frac{2}{2}$$

$$\frac{1}{1} = \frac{1}{1} (1 = \frac{2}{1} T)$$

$$\frac{1}{1} = \frac{1}{1} (1 = \frac{1}{1} T)$$

$$\frac{1}{1} \frac{2}{1} \frac{1}{1} T$$

$$\frac{1}{1} \frac{1}{1} \frac{1}{1} T$$

$$\frac{1}{1} \frac{1}{1} \frac{2}{1} T$$

$$\frac{1}{1} \frac{2}{1} \frac{1}{1} T$$

9. On cooling brass contracts more than iron ($\alpha_{Br} > \alpha_{Fe}$) such that brass disk gets loosen from hole of iron.

10. $V \propto T$ $V \propto kT$ $\ln V \propto \ln k \propto \ln T$

$$\frac{V}{V} \frac{T}{T} \frac{V}{V} \frac{1}{T}$$

Introductory Exercise 17.2

1. For ideal gases, $pV = nRT$

$$\text{Slope} = \frac{VM}{mR}$$

As slope $\frac{1}{m} = \frac{m_2}{m_1}$

2. $pV = nRT$ $\frac{p_2}{p_1} = \frac{T_2}{T_1}$

$$\frac{360}{300} = \frac{6}{5}$$

$$p_2 = \frac{6}{5} p_1 = \frac{6}{5} \times 10 \text{ atm} = 12 \text{ atm}$$

3. $M_{\text{mix}} = \frac{1}{4} \times 28 + \frac{1}{4} \times 44 + \frac{7}{4} \times \frac{11}{2} = 36$

$$pV = nRT = \frac{m}{M} RT$$

$$pM = \frac{m}{V} RT = \rho RT$$

$$\frac{101}{8.31} = \frac{10^5}{290} \times 36 \times 10^3$$

$$\frac{101}{8.31} = \frac{36}{290} \times 15 \text{ kg/m}^3$$

4. $pV = nRT = \frac{N}{N_A} RT$

$$N = \frac{pVN_A}{RT}$$

$$10^6 = \frac{13.6 \times 10^3 \times 10 \times 250 \times 10^6}{8.31 \times 300}$$

$$N = \frac{13.6 \times 5 \times 6.02}{8.31 \times 6} \times 10^{15} = 8.21 \times 10^{15}$$

5. $pV = nRT$

$$V = \frac{nR}{p} T$$

Slope $\frac{1}{p}$

6. $pV = nRT$ $\frac{p_1}{p} = \frac{p_2}{(nRT) \frac{1}{V}}$

$y = mx$ is a straight line passing through origin.

Introductory Exercise 17.3

1. Average velocity depends on the direction of motion of gas molecules and as container do not move such that their net effect becomes zero, due to the reason that some molecules are moving

in one direction while other are moving in opposite direction. But in case of average speed only magnitudes are in use which do not cancel each other.

$$2. \text{ KE } \frac{3}{2} kT = \frac{3}{2} \frac{8.31}{6} \times 10^{23} \times 300 \text{ J}$$

$$= \frac{3}{4} \times 8.31 \times 10^{21} \text{ J}$$

$$= 6.21 \times 10^{21} \text{ J}$$

$$3. v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$v_{\text{He}} = \sqrt{\frac{3 \times 8.31 \times 300}{4 \times 10^{-3}}}$$

$$= \frac{1.37 \times 10^3 \text{ m/s}}{\sqrt{\frac{3 \times 8.31 \times 300}{20.2 \times 10^{-3}}}} = 608.5 \text{ m/s}$$

$$v_{\text{Ne}} = \sqrt{\frac{3 \times 8.31 \times 300}{20.2 \times 10^{-3}}} = 608.5 \text{ m/s}$$

$$\text{KE} = \frac{3}{2} kT = 6.21 \times 10^{21} \text{ J}$$

$$4. v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$T = \frac{Mv_{\text{rms}}^2}{3R} = \frac{4 \times 10^{-3} \times 10^6}{3 \times 8.31}$$

$$= 160.45 \text{ K}$$

$$5. \frac{n_1}{n_1} = \frac{n_2}{n_2} = \frac{(1 - n_2)}{1 - n_2} = \frac{n_2}{n_2}$$

$$= \frac{1 - n_2}{1 - n_2} = \frac{1.293}{1.251} = \frac{1.429}{1.429}$$

$$n_2 = \frac{1}{2} = \frac{1.293}{1.251} = \frac{1.429}{1.429}$$

$$\frac{136}{178} = 0.764 = 76.4\% \text{ by mass}$$

$$6. \frac{V_2}{V_1} = \frac{p_1 T_2}{p_2 T_1} = \frac{(p_0 - h) g}{p_0} = \frac{277}{277}$$

$$= \frac{(1.01 \times 10^5 - 40 \times 10^3) \times 10}{1.01 \times 10^5 \times 277}$$

$$= \frac{5.01 \times 293}{1.01 \times 277} = 5.25$$

$$V_2 = 5.25 V_1 = 105 \text{ cm}^3$$

$$7. N = nN_A = \frac{1}{18} \times 6 \times 10^{23}$$

$$\frac{1}{3} \times 10^{23};$$

$$S = 4R^2 = 4 \times 3.14 \times (6400 \times 10^3 \times 10^2)^2$$

$$= \frac{5.14 \times 10^{18} \text{ cm}^2}{\frac{N}{S} = \frac{10^{23}}{3 \times 5.14 \times 10^{18}}}$$

$$(a) nC_V = \frac{3}{2} nR = 35 \text{ J/K}$$

$$n = \frac{70}{3R} = 2.8 \text{ mole}$$

$$(b) U = \frac{3}{2} nRT = 35 \text{ J/K} \times 273 \text{ K} = 9555 \text{ J}$$

$$(c) C_p = C_V + R = \frac{5}{2} R = 20.8 \text{ J/K mole}$$

$$8. (a) n(C_p - C_V) = nR = 29.1 \text{ J/K}$$

$$n = \frac{29.1}{8.314} \text{ mole} = 3.5 \text{ mole}$$

$$(b) C_V = nc_V = n \frac{3}{2} R = 3.5 \times 1.5 \times 8.314$$

$$= 43.65 \text{ J/K}$$

$$C_p = nc_p = n \frac{5}{2} R = C_V + nR$$

$$= 43.65 + 3.5 \times 8.314$$

$$= 72.75 \text{ J/K}$$

$$(c) C_V = nc_V = n \frac{5}{3} R = 72.75 \text{ J/K}$$

$$C_p = nc_p = n \frac{7}{2} R$$

$$= 72.75 + 3.5 \times 8.314$$

$$= 101.85 \text{ J/K}$$

$$10. v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ and } v_{\text{av}} = \sqrt{\frac{8RT}{M}}$$

$$\text{Here } 3 \frac{8}{3} v_{\text{rms}} = v_{\text{av}},$$

i.e., the statement is true.

AIEEE Corner

■ Subjective Questions (Level 1)

$$1. \frac{C}{5} \frac{68}{9} \frac{32}{9} \frac{36}{9} \frac{4}{9} \\ C \quad 20 \text{ C}; \frac{K}{5} \frac{273}{9} \frac{68}{9} \frac{32}{9} \quad 4$$

$$\frac{K}{C} \frac{293 \text{ K}}{5} \frac{32}{9} \frac{27}{9} \quad 3 \\ C \quad 15 \text{ C}; \frac{K}{5} \frac{273}{9} \frac{5}{9} \frac{32}{9} \quad 3$$

$$\frac{K}{C} \frac{258 \text{ K}}{5} \frac{176}{9} \frac{32}{9} \frac{144}{9} \quad 16 \\ C \quad 80 \text{ C}; \frac{K}{5} \frac{273}{9} \quad 16$$

$$2. \frac{30}{5} \frac{F}{9} \frac{32}{9} \quad F \quad 54 \quad 32 \quad 86 \text{ F} \\ \frac{546 \text{ R}}{5} \frac{F}{9} \frac{32}{9} \quad F \quad 9 \quad 32 \quad 41 \text{ F} \quad 501 \text{ R} \\ \frac{20}{5} \frac{F}{9} \frac{32}{9} \\ F \quad 36 \quad 32 \quad 41 \text{ F}$$

$$3. \frac{x}{5} \frac{x}{9} \frac{32}{9} \quad 32 \quad x \quad \frac{9}{5}x \quad \frac{4}{5}x \\ x \quad \frac{5}{4} \quad 32 \quad 40$$

$$4. \frac{C}{5} \frac{F}{9} \quad F \quad \frac{9}{5} \quad C \quad \frac{9}{5} \quad 40 \quad 72 \\ 40 \text{ C} \quad 40 \text{ F}$$

$$5. \frac{F_2}{80} \frac{F_1}{20} \frac{72}{100} \frac{140.2 \text{ F}}{0} \\ \frac{12}{60} \frac{C}{100} \quad C \quad \frac{12}{60} \frac{100}{60} \quad 20 \text{ C}$$

$$6. \frac{T_2}{T_1} \frac{p_2}{p_1} \frac{160}{80} \quad 2 \quad T_2 \quad 2T_1$$

$$T_2 \quad 2 \quad 273.15 \text{ K} \quad 546.30 \text{ K}$$

$$7. R_t \quad R_0(1 \quad) \\ 3.50 \quad 250(1 \quad 100 \quad) \quad 1 \quad 250 \text{ K} \\ \text{or} \quad \frac{10}{250} \quad 4 \quad 10^3 / ^\circ\text{C}$$

$$650 \quad 250(1 \quad 4 \quad 10^3 \quad) \\ 4 \quad 10^2 \\ 400 \quad 2 \quad 400^\circ\text{C} \\ 400 \quad 2 \quad 400 \text{ C}$$

i.e., boiling point of sulphur is 400°C .

$$8. \frac{T_2}{T_1} \frac{p_2}{p_1} \frac{75}{75} \frac{45}{5} \frac{120}{80} \frac{3}{2}$$

$$T_2 \quad \frac{3}{2} T_1 \quad \frac{3}{2} \quad 300.15 \text{ K} \\ 450.225 \text{ K} \quad 177.08 \text{ C}$$

$$9. \quad (\text{ Br} \quad \text{ Fe}) \\ \frac{1}{\text{Br} \quad \text{Fe}} \\ 0.01 \quad 10^3 \quad 1 \\ \frac{6 \quad 10^2}{\text{Br} \quad \text{Fe}} \\ 10^3$$

$$6(\text{ Br} \quad \text{ Fe}) \\ 2 \quad 1 \quad \frac{10^3}{6(\text{ Br} \quad \text{ Fe})} \\ 30 \text{ C} \quad \frac{10^3}{6(\text{ Br} \quad \text{ Fe})} \quad 30 \text{ C} \quad \frac{100}{6 \quad 0.63} \\ 57.78 \text{ C}$$

$$10. (a) \quad l \quad l \quad \sim 88.42 \quad 2.4 \quad 10^5 \quad 30 \\ 0.064 \text{ cm}$$

$$(b) \quad l \quad l(\text{ Al} \quad \text{ St}) \\ 88.42(2.4 \quad 1.2) \quad 10^5 \quad 30 \\ 0.032 \text{ cm}$$

$$l_S \quad l \quad l \quad 88.42 \quad 0.032 \text{ cm} \\ 88.45 \text{ cm}$$

11. $\frac{l}{l}$ 100% 100%
 $\frac{1.2 \times 10^5}{0.042\%} = 35 \times 100\%$

12. $F = YA \frac{l}{l}$
 $\frac{2 \times 10^{11} \times 2 \times 10^6}{4 \times 1.2 \times 40 \text{ N}} = \frac{1.2 \times 10^5}{160} = 1.2 \text{ N}$
 $1.2 \text{ N} \times 192 \text{ N} = 192 \text{ N}$

13. $V = g (50 \text{ 45}) \times 10^3 \text{ kg}$
 $5 \times 10^3 \text{ kg}$
 $V = g (50 \text{ 45.1}) \times 10^3 \text{ kg}$
 $4.9 \times 10^3 \text{ kg}$
 $V(1 \text{ s}) = \frac{4.9}{1 \text{ l}} \times 4.9 \times 10^3$
 $\frac{1 \text{ s}}{1 \text{ l}} = \frac{4.9}{5}$
 $5 \times \frac{5 \text{ s}}{0.1 \text{ 5 s}} = \frac{4.9}{49} = \frac{4.9 \text{ e}}{1} = \frac{5}{4.9 \text{ s}}$
 $\frac{1}{49} = \frac{5}{75} = \frac{5}{49} \times 12 \times 10^6$
 $272.1 \times 10^6 = 12.2 \times 10^6$
 $2.84 \times 10^4 \text{ C}$

14. $M = 14 \times 3 = 17 \text{ g/mole}$
 $17 \times 10^3 \text{ kg/mole}$
 $M = \frac{17 \times 10^3}{6033 \times 10^{23}} \text{ kg/molecule}$
 $282 \times 10^{26} \text{ kg/molecule}$

15. $n = \frac{pV}{RT} = \frac{1.52 \times 10^6 \times 10^2}{8.314 \times 298.15} = 6.13$
 $\frac{m}{V} = \frac{nM}{V} = \frac{6.13 \times 2 \times 10^3}{10^2} = 1.23 \text{ kg/m}^3$
 $\frac{m}{V} = \frac{nM}{V} = \frac{16 nM}{V} = 16$
 19.62 kg/m^3

16. $p_2 = p_1 \frac{V_1}{V_2} = 1 \text{ atm} \times \frac{76}{6} = 12.7 \text{ atm}$

17. $V_2 = \frac{p_1 V_1}{T_1} \frac{T_2}{p_2} = \frac{p_1}{p_2} \frac{T_2}{T_1} V_1$
 $\frac{1}{0.5} \times \frac{270}{300} \times 500 \text{ m}^3 = 900 \text{ m}^3$

18. $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$
 $\frac{mg}{A} = p_0 A h_i = \frac{mg}{A} = p_0 A h_f$
 $\frac{293}{373} h_i = \frac{373}{293} h_f$
 $h_f = \frac{293}{373} h_i = 4 \text{ cm} \times \frac{293}{373} = 50.9 \text{ cm}$

19. $p_1 = p_2 \frac{n_1}{V_1} = \frac{n_2}{V_2} \frac{25/28}{L_1 A} = \frac{40/4}{L_2 A}$
 $\frac{L_1}{L_2} = \frac{25}{28} \times \frac{1}{10} = \frac{5}{56} = 0.089$
 $\frac{n_1}{n_2} = \frac{25/28}{40/4} = \frac{25}{280} = \frac{5}{56} = 0.089$

20. $n = n_1 = n_2$
 $p(V_1 + V_2) = p_1 V_1 + p_2 V_2$
 $p = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2}$
 $p = \frac{1.38 \times 0.11 + 0.69 \times 0.16}{0.11 + 0.16} \text{ MP}_a$
 $\frac{0.1518 + 0.1104}{0.27} = \frac{0.2622}{0.27} = 0.97 \text{ MP}_a$

21. $\frac{pV_1}{T} = \frac{pV_2}{T} = \frac{p_1 V_1}{T_1} = \frac{p_1 V_2}{T_2}$
 $\frac{1 \text{ atm}}{293 \text{ K}} = 600 \text{ cm}^3$
 $p_1 = \frac{400 \text{ cm}^3}{373 \text{ K}} = \frac{200 \text{ cm}^3}{273 \text{ K}}$
 $p_1 = \frac{600/293}{\frac{400}{373} + \frac{200}{273}} \text{ atm}$
 $p_1 = \frac{600/293}{\frac{2}{373} + \frac{1}{273}} \text{ atm}$
 $\frac{600/293}{\frac{3}{373} + \frac{1}{273}} = \frac{1.57 + 1.07}{2.64} \text{ atm} = 1.136 \text{ atm}$

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22. $V = \frac{nRT}{p} = \frac{1 \times 8.314 \times 273.15}{1.013 \times 10^5} \text{ m}^3$

$0.02242 \text{ m}^3 = 22.42 \text{ litre}$

23. $p_2 = \frac{p_1 V_1}{T_1} \frac{T_2}{V_2} = p_1 \frac{V_1}{V_2} \frac{T_2}{T_1}$
 $1.5 \times 10^5 \times \frac{0.75}{0.48} \times \frac{430}{300}$
 $3.36 \times 10^5 \text{ Pa}$

24. $\frac{p_1}{n_1 RT} = \frac{p_2}{n_2 RT} = \frac{p_1 V_1}{n_1 RT} = \frac{p_2 V_2}{n_2 RT}$
 $\frac{0.7}{20} \times \frac{1.4}{10} = \frac{0.3}{10} \times \frac{1.4}{5} \times \frac{0.4}{10} \times \frac{RT}{1500}$
 $\frac{0.7}{20} \times \frac{0.3}{10} = \frac{1}{10} \times \frac{8.314}{5} \times \frac{1500}{10^3}$
 $\frac{3.3}{20} \times 8.314 = 3 \times 10^5 \text{ Pa}$
 $4.11 \times 10^5 \text{ Pa}$

25. RKE $= 2 \times \frac{1}{2} kT = \frac{1}{2} I \omega^2$
 $\sqrt{\frac{2kT}{I}} = \sqrt{\frac{2 \times 1.38 \times 10^{23} \times 300}{8.28 \times 10^{38} \times 10^{-7}}}$
 $10^{12} \sqrt{\frac{6 \times 1.38}{8.28}} = 10^{12} \text{ rad/s}$

26. $v = \sqrt{\frac{p}{\rho}}$
 $\frac{v^2}{p} = \frac{1.3 \times (330)^2}{1.013 \times 10^5} = 1.398$
 $\frac{f}{f} = \frac{2}{f} \sim 5$
 $f = \frac{2}{0.398} \sim 5$

27. $\frac{n_1 C_{p1}}{n_1 C_{V1}} = \frac{n_2 C_{p2}}{n_2 C_{V2}} = \frac{3 \times \frac{5}{2}}{3 \times \frac{3}{2}} = \frac{2 \times \frac{7}{2}}{2 \times \frac{5}{2}}$
 $\frac{15}{9} = \frac{14}{10} = \frac{29}{19} = 1.53$

28. $K = \frac{3}{2} pV$
 $\frac{K_2}{K_1} = \frac{\frac{3}{2} p_2 V_2}{\frac{3}{2} p_1 V_1} = \frac{3}{2} \times \frac{15}{5} = 4.5$

29. $C_p = \frac{f}{2} R = \frac{K_2}{29} = \frac{4.5 \text{ K}}{29} \times \frac{58}{R} = 2 \times 5$

$pT = p \frac{pV}{nR}$

$p^2 V = \text{constant}$

$pV^{1/2} = \text{constant} \Rightarrow a = \frac{1}{2}$

$c = \frac{f}{2} R = \frac{R}{1} \times \frac{1}{2} = \frac{f}{2} \times 4 R = 29$

$f = \frac{58}{R} = 4 \times 3$

30. TKE $= \frac{3}{5}$ of total energy and RKE $= \frac{2}{5}$ of total energy, so the gas is diatomic.

TKE $= \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{23} \times 300 \text{ J}$

$6.21 \times 10^{21} \text{ J/molecule}$

$Q = n C_V T = 1 \times \frac{5}{2} \times 8.314 \times 1 \times 20.8 \text{ J}$

31. $C_p = \frac{n_1 C_{p1}}{n_1} = \frac{n_2 C_{p2}}{n_2}$
 $\frac{2.5 R}{1} = \frac{3.5 R}{1} = 3R$

$C_V = \frac{n_1 C_{V1}}{n_1} = \frac{n_2 C_{V2}}{n_2}$
 $\frac{1.5 R}{1} = \frac{2.5 R}{1} = 2R$

$\frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$

32. $\frac{n_1 C_{p1}}{n_1 C_{V1}} = \frac{n_2 C_{p2}}{n_2 C_{V2}} = \frac{(n_1 + n_2) C_{p1}}{(n_1 + n_2) C_{V1}}$

$\frac{C_{p1}}{C_{V1}}$

33. $p = aV^b \Rightarrow pV^b = \text{constant}$

$C = \frac{Q}{n T} = 0$ for adiabatic process for

which $pV^b = \text{constant}$ comparing, we get, b

34. $p \propto kV \propto pV^1$ constant
 $pV \propto a$ constant $\frac{a}{1} \propto \frac{1}{1}$
 $C \propto C_V \frac{R}{1} \propto C_V \frac{R}{1} \propto C_V \frac{R}{2}$

35. $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \cdot 8.314 \cdot 373.15}{2 \cdot 10^3}}$
 $2.16 \cdot 10^3$ m/s
 2.16 km/s

36. $v_{rms} = \sqrt{\frac{(500)^2 + (600)^2 + (700)^2 + (800)^2 + (900)^2}{5}}$
 $\frac{100}{\sqrt{5}} \sqrt{25 + 36 + 49 + 64 + 81}$
 714 m/s
 $v_{av} = \frac{500 + 600 + 700 + 800 + 900}{5}$
 $20(5 + 6 + 7 + 8 + 9) = 700$ m/s

37. KE $\frac{3}{2} pV$
 $N = 6 \cdot 10^{26} \cdot 1.5 \cdot 2 \cdot 10^5 \cdot 100$
 $10^3 \cdot 10^3$
 $N = \frac{3 \cdot 10}{6 \cdot 10^{26}} \cdot 5 \cdot 10^{26}$
 $\frac{5000}{6.023} \cdot 6.023 \cdot 10^{23} = 830.15$ Na
 $n = 830.15$ moles

38. Frequency of collision, $\frac{v}{2\sqrt{3}l} \cdot \frac{v}{2\sqrt{3}V}$
 $\frac{1}{2\sqrt{3}V} \sqrt{\frac{3RT}{M}}$
 $\sqrt{\frac{RT}{4VM}} \cdot \sqrt{\frac{RT}{4 \frac{nRT}{p} M}} \cdot \sqrt{\frac{p}{4nM}}$
 $\sqrt{\frac{2 \cdot 10^5}{4 \cdot 1 \cdot 46 \cdot 10^3}}$
 $41.04 \cdot 10^3$ /s

39. KE $\frac{3}{2} pV = \frac{3}{2} \cdot 10^5 \cdot 2 \cdot 10^6 = 0.3$ J

$N = \frac{m}{m_1} \frac{50 \cdot 10^6}{8 \cdot 10^{26}} = 6.25 \cdot 10^{20}$

$K_1 = \frac{K}{N} = \frac{0.3}{6.25 \cdot 10^{20}} = \frac{30}{6.25} \cdot 10^{22}$ J

$4.8 \cdot 10^{22}$ J

40. $v_0 = \sqrt{\frac{3RT_0}{M_0}}$

(a) $\frac{v}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{573}{293}} = 1.4$ $v = 1.4 v_0$

(b) $v \propto v_0$ as RMS speed changes with temperature and not with pressure.

(c) $\frac{v}{v_0} = \sqrt{\frac{M_0}{M}} = \sqrt{\frac{M_0}{3M_0}} = \frac{1}{\sqrt{3}}$

$v = \frac{v_0}{\sqrt{3}} = 0.58 v_0$

41. $\sqrt{\frac{RT}{M_{H_2}}} = \sqrt{\frac{RT}{M_{O_2}}}$ $T \frac{M_{H_2}}{M_{O_2}} = T$
 $\frac{2}{32} \cdot 320 = 20$ K 253 C

42. $\frac{1}{2} m v_e^2 = \frac{GMm}{R_e} = g_e R_e m$
 $v_e = \sqrt{2g_e R_e} = v_{H_2} \sqrt{\frac{3RT}{M}}$
 $T_e = \frac{2g_e R_e M}{3R}$
 $\frac{2 \cdot 9.8 \cdot 6367 \cdot 10^6}{3 \cdot 8.314} = \frac{2 \cdot 10^3}{3}$

and $T_m = \frac{10007 \text{ K} \cdot 2g_m R_m M}{3R}$
 $\frac{2 \cdot 1.6 \cdot 1.75 \cdot 10^6}{3 \cdot 8.314} = \frac{2 \cdot 10^3}{3}$

43. (a) KE $\frac{3}{2} kT = \frac{3}{2} \cdot 1.38 \cdot 10^{23} \cdot 300$ J
 $6.21 \cdot 10^{21}$ J

(b) KE $\frac{3}{2} kT N_a = 6.023 \cdot 10^{23}$
 $6.21 \cdot 10^{21}$ J
 3740 J

$$(c) v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{32 \times 10^3}}$$

483.6 m/s

■ Objective Questions (Level 1)

1. $v_{\text{av}} = \sqrt{\frac{8RT}{M}}$

2. $v_{\text{rms}} = \sqrt{\frac{1^2 + 0^2 + 2^2 + 3^2}{4}}$
 $\sqrt{\frac{14}{4}} = \sqrt{3.5} \text{ m/s}$

3. $\frac{l}{l} = \frac{12 \times 10^6 \times 50}{600 \times 10^6 \times 6 \times 10^4}$

4. $V \propto T \frac{V_2}{V_1} \frac{T_2}{T_1} = 2$;
 $\frac{V}{V} = \frac{V_2}{V_1} \frac{V_1}{V_2} = 2 \times 1 \times 1 = 100\%$

5. $\text{KE} \propto T \frac{K_2}{K_1} \frac{T_2}{T_1} = \frac{2E}{E} = 2$
 $\frac{T_2}{T_1} = \frac{2T_1}{T_1} = 2$ 283 K
 $\frac{T_2}{T_1} = \frac{566 \text{ K}}{293 \text{ C}}$

6. $\text{TE} = \frac{f}{2} kT = \frac{n}{2} kT$

7. $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1200}{300}} = 2$ $v_2 = 2v_1$

8. (a) $p_{\text{av}} = m_1 v$ is different for different m_1
 (b) $(\text{KE})_{\text{molecule}} = \frac{3}{2} kT$ is same for any gas.
 (c) $(\text{KE})/V = \frac{3}{2} \frac{pV}{V} = \frac{3}{2} p$ is different as p is different for different.
 (d) $(\text{KE})_m = \frac{3}{2} \frac{pV}{m} = \frac{3}{2} \frac{p}{m}$ is different as $\frac{p}{m}$ is different.

9. $\frac{p_1 V_1}{RT_1} = \frac{p_2 V_2}{RT_2} = \frac{p V_1}{RT} = \frac{p V_2}{RT} = \frac{p(V_1 + V_2)}{RT}$

$$T = \frac{p(V_1 + V_2)}{p_1 V_1} = \frac{T_1 T_2 p(V_1 + V_2)}{p_1 V_1 T_2} = \frac{p_2 V_2 T_1}{p_2 V_2 T_1}$$

10. $\frac{l}{l} = \frac{0.08 \times 10^3}{10 \times 10^2 \times 100}$
 $\frac{V}{V} = \frac{8 \times 10^6 / \text{C}}{3V \times 3 \times 100 \text{ cc} \times 8 \times 10^6 \times 100}$
 $\frac{0.24 \text{ cc}}{V \times 100 \text{ cc} \times 0.24 \text{ cc} \times 100.24 \text{ cc}}$

11. $T = T_0 \tan 45^\circ = V = T_0 = V$
 $pV = nRT = \frac{nR(T_0 + V)}{V} = \frac{nRT_0 + nRV}{V}$
 or $p = \frac{nR}{V} \left(\frac{nRT_0}{p} + a \frac{b}{V} \right)$
ie, p versus V graph will be hyperbola.

12. $p^2 V = \text{constant}$
 $\frac{nRT}{V} = \frac{\text{constant}}{V}$

$$\frac{T_2}{T_1} = \frac{\sqrt{V_2}}{\sqrt{V_1}} = \frac{\sqrt{3V_0}}{\sqrt{V_0}} = \sqrt{3}$$

$$\frac{T_2}{T_1} = \frac{\sqrt{3} T_1}{\sqrt{3} T_0}$$

13. $p = \frac{1}{3} v_{\text{rms}}^2 = \frac{1}{3} \frac{m}{V} v_{\text{rms}}^2$
 $\frac{mT}{m_1} = \frac{\text{constant}}{T_2} = \frac{310}{280} = 1.1$

14. As temperature of vessels A and B are same so is average velocity of O_2 , i.e., u .

15. $N = nN_a = \frac{pV}{RT} N_a$
 $\frac{10^{13}}{8.314} = \frac{10^6}{300} \times 6.023 \times 10^{23}$

$$16. \frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h} = mgh$$

$$h \sim \frac{v^2}{2g} = \frac{3RT}{2gM}$$

$$h = \frac{3 \times 8.314 \times 273}{2 \times 10 \times 28 \times 10^3} \text{ m}$$

$$12.16 \times 10^3 \text{ m} = 12 \text{ km}$$

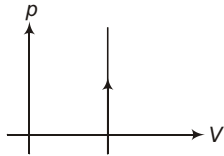
$$17. \frac{l_1}{l_2} = \frac{2}{1} \times \frac{19}{11} \text{ and } l_1 = l_2 = 30 \text{ cm}$$

$$l_1 = \frac{11}{19} l_2 = 30 \text{ cm}$$

$$l_1 = \frac{19}{8} \times 30 \text{ cm} = 71.25 \text{ cm}$$

$$\text{and } l_2 = \frac{11}{19} l_1 = 41.25 \text{ cm}$$

$$18. p \propto \frac{V}{T} \text{ constant}$$



$$19. V = V_0 \tan T, pV = nRT = \frac{m}{M} RT$$

$$p(V_0 \tan T) = \frac{m}{M} RT$$

$$\tan T = \frac{1}{pT} \times \frac{m}{M} RT = \frac{pV_0}{pT}$$

$$\text{or } \tan T = \frac{mR}{pM} \times \frac{V_0}{T}$$

$\tan T$ remains same when $m = 2m$ and $p = 2p$

$$20. n_1 = \frac{p_1 V}{RT_1} \text{ and } n_2 = \frac{p_2 V}{RT_2}$$

$$\frac{n_1}{n_2} = \frac{p_1}{T_1} \times \frac{T_2}{p_2} = \frac{10}{5} \times \frac{300}{330} = \frac{600}{330} = \frac{20}{11}$$

$$n_2 = \frac{11}{20} n_1$$

$$m = m_1 = m_2 = m_1 \times \frac{11}{20}$$

$$21. \frac{m}{V} = \frac{m}{(n_1 + n_2) \frac{RT}{p}} = \frac{mp}{(n_1 + n_2)RT}$$

$$\frac{12}{(2+2)} \times \frac{1.01 \times 10^5}{8.314 \times 300} = \frac{10^3}{300} \times 0.12 \text{ kg/m}^3$$

$$22. p \propto k \frac{KM}{V}$$

$pV = \text{constant}$ is for isothermal process, i.e., $T = \text{constant}$

$$23. \frac{p^2}{T} = \text{constant}$$

$$\frac{p^2 V}{T} = \text{constant} \implies \frac{p_2^2}{p_1^2} \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{p_2 T_2}{p_1 T_1}} = \frac{1}{\sqrt{2}}$$

$$\frac{p_2}{p_1} = \frac{1}{\sqrt{2}}$$

$$\frac{T_2}{T_1} = \frac{p_1}{p_2} = \sqrt{2}$$

$$T_2 = \sqrt{2} T_1 = \sqrt{2} T$$

as $pT = \text{constant} \implies p = \frac{1}{T}$

i.e., $p - T$ graph is hyperbola.

$$24. p^2 V = \text{constant}$$

$PT = \text{constant}$ and $T^2 V = \text{constant}$.

$$\frac{p_2}{p_1} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V}{4V}} = \frac{1}{2}$$

$$p_2 = \frac{p_1}{2} = \frac{p}{2}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{4V}{V}} = 2$$

$$T_2 = 2T_1 = 2T$$

as $p = \frac{1}{T}$ $p - T$ graph is hyperbola.

$$25. \begin{matrix} C & 0 & F & 32 & F & MP \\ 100 & 0 & 212 & 32 & BP & MP \end{matrix}$$

ice point 32 F and steam point 212 F

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26. $p = p_0$ and $V = V_0$ at 'a',
 $p = 2p_0$ and $V = 2V_0$ at 'b',

$$\frac{p_b}{p_a} = \frac{V_a}{V_b} = \frac{V_0}{2V_0} = \frac{1}{2}$$

$$p_b = \frac{1}{2} \text{ Pa}$$

$$\frac{T_b}{T_a} = \frac{P_b V_b}{P_a V_a} = \frac{2p_0 \cdot 2V_0}{p_0 V_0} = 4$$

$$T_b = 4T_a$$

$$\text{as } \frac{U}{P} = \frac{T}{V} \Rightarrow \frac{U_b}{V_b} = \frac{4U_a}{V^2}$$

Parabola passing through origin

27. (a) $\frac{3}{2} nRT$ is independent of type of gas true.

(b) In one degree of freedom for one mole of gas, $V = \frac{1}{2} RT$

(c) false

(d) false

28. $V = T \tan T$

$$pV = nRT = \frac{m}{M} RT$$

$$p = T \tan \frac{mRT}{M} \Rightarrow \tan \frac{mR}{MP}$$

$$\tan \theta_1 = \tan \theta_2 \Rightarrow \frac{m_1}{p_1} = \frac{m_2}{p_2}$$

all a, b, c and d are possible.

29. $pV = nRT$

$$p = \frac{n}{V} RT = \frac{N_a m/V}{M} RT = \frac{m}{M} RT$$

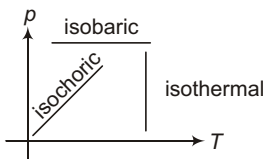
$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kN_a T}{M}} = \sqrt{\frac{3kT}{M}}$$

(a) and (d) are correct.

JEE Corner

■ Assertion & Reason

1. Assertion is false.



2. Assertion and reason are both true but reason is not correct explanation of assertion. As at low temperature atoms in molecules are tightly bound such that they cannot oscillate.

3. $pV = nRT = \frac{2}{3} \text{ KE}$

$$p = \frac{2 \text{ KE}}{3 V} \Rightarrow p = \frac{2}{3} E.$$

Assertion and reason are both true but reason cannot explain assertion.

4. Internal energy remains same in train frame of reference, so temperature do not change, but KE of gas molecules in ground frame increases.

5. According to equipartition theory, energy is equally distributed for each degree of freedom, so assertion is false.

6. At high temperature and low pressure intermolecular distance is much larger than size of the molecules and intermolecular forces can be neglected. So, assertion and reason are both true but not correct explanation.

7. At 4°C, volume is minimum or density is maximum i.e., liquid will overflow on increasing or decreasing temperature. This reason is false.

8. Temperature remains constant as pressure is double and volume is halved, so internal energy remains constant. So reason partially explains assertion.
9. Assertion and reason are both true but not correct explanation.

10. $V \propto \frac{nR}{Mp} T$ slope m ; reason is correct explanation.

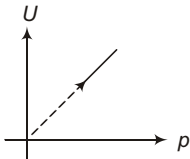
■ Match The Columns

1.

- (a) TKE $\frac{3}{2}nRT$ $\frac{3}{2} 2RT$ $3RT$ r
- (b) RKE $\frac{2}{2}nRT$ $\frac{2}{2} 2RT$ $2RT$ p
- (c) PE s
- (d) TKE $\frac{5}{2}nRT$ $5RT$ s

- (d) $T \propto \frac{1}{2} T$ increases with p increasing temperature

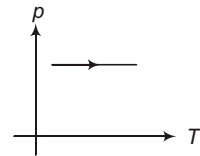
2.



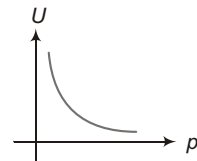
$U \propto p \propto T \propto \frac{1}{V}$ VT constant
 pT^2 constant and pV^2 constant

- (a) U increases T increases q
 P decreases r
- (b) p increase V decreases r
- (c) U increases T increases q
- (d) $\frac{T}{V} \propto \frac{TV}{V^2} \propto \frac{\text{constant}}{V^2}$ increase as V decreases q

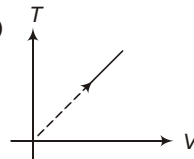
5. (a) p constant q



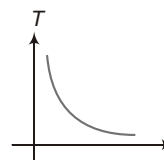
- (b) $V \propto \frac{1}{U} T$



- (c) $T \propto V$



- (d) $V \propto \frac{1}{T} T$



3. $x_1 = 3, x_2 = \frac{8}{-}, x_3 = 2$ and $x_4 = r$

- (a) r , (b) s , (c) q , (d) s

4.

- (a) density of water is maximum of 4 °C s
- (b) depends of change in density of solid and liquid s
- (c) depends of change in density of solid and liquid s

R

p

r

18. First Law of Thermodynamics

Introductory Exercise 18.1

1. (a) $W = p \Delta V = 1.7 \times 10^5 (1.2 - 0.8) \text{ J}$
 $6.8 \times 10^4 \text{ J}$
 (b) $V = 1.1 \times 10^5 \text{ J}$
 $Q = U - W = 17.8 \times 10^4 \text{ J}$
i.e., $1.78 \times 10^5 \text{ J}$ of heat has flown out of the gas.
 (c) No, it is independent of the type of the gas.
2. (a) In p - V graph of cyclic process, clockwise rotation gives positive work and anticlockwise gives negative work. And as loop 1 has greater area than loop 2, that is why total work done by the system is positive.
 (b) As in cyclic process change in internal energy is zero, that's why for positive work done by the system, heat flows into the system.
 (c) In loop '1' work done is positive so, heat flows into the system and in loop '2' work done is negative so heat flows out of the system.
3. As the box is insulated *i.e.*, no heat exchange takes place with surrounding and as the gas expands against vacuum *i.e.*, zero pressure that's why no work has been done and there is no change in internal energy. Thus, temperature do not change, internal energy and gas does not do any work.
4. $U = \frac{f}{2} nRT = \frac{3}{2} nRT$
 $n = \frac{2U}{3RT} = \frac{2 \times 100}{3 \times 8.314 \times 300}$
 0.0267 mole.
5. $Q = ms = 1 \times 387 \times 30 \text{ J} = 11610 \text{ J}$
 $V = \frac{m}{\rho} = \frac{3}{8.92 \times 10^3} = 3.7 \times 10^{-6} \text{ m}^3$
 $W = p \Delta V = 1.01 \times 10^5 \times 7.06 \times 10^{-8}$
 $7.13 \times 10^{-3} \text{ J}$
 $U = Q - W = 11609.99 \text{ J}$

Introductory Exercise 18.2

1. (a) At constant volume,
 $U = 0$ $W = 0$
 $Q = nC_V \Delta T = \frac{Q}{nC_V} = \frac{200}{1 \times \frac{3}{2} \times 8.314} = 16.04 \text{ K}$
 $T_f = T_i + \Delta T = 300 + 16.04 = 316.04 \text{ K}$
- (b) At constant pressure,
 $T = \frac{Q}{nC_p} = \frac{200}{1 \times \frac{5}{2} \times 8.314} = 9.62 \text{ K}$
 $T_f = 300 + 9.62 \text{ K} = 309.62 \text{ K}$
2. For adiabatic process,
 $pV^c = \text{constant}$ c (say)
 $\int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \frac{c}{V^{c+1}} dV = \frac{c}{-c} \left[\frac{1}{V^c} \right]_{V_i}^{V_f} = \frac{c}{c} \left[\frac{1}{V_i^c} - \frac{1}{V_f^c} \right]$

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$$c \frac{V_f - V_i}{1} \left| \frac{V_f}{V_i} \right. \quad c \frac{V_f - V_i}{1} \quad \frac{V_f - V_i}{1}$$

$$\frac{p_f V_f - p_i V_i}{1} \quad \frac{p_i V_i - p_f V_f}{1} \quad \text{(Proved)}$$

3. $W_{AB} = 500 \text{ J}$, $Q_{AB} = 250 \text{ J}$
 $U_{AB} = 250 \text{ J}$
 $W_{AC} = 700 \text{ J}$, $Q_{AC} = 300 \text{ J}$
 $U_{AC} = 400 \text{ J}$

(a) Path BC is isochoric process, i.e.,

$$W_{BC} = 0$$

$$Q_{BC} = U_{BC} = U_{AC} - U_{AB}$$

$$400 \text{ J} - (250 \text{ J}) = 150 \text{ J}$$

- (b) $W_{CDA} = W_{CD} + W_{DA}$
 $800 \text{ J} = 0 + 800 \text{ J}$

(Work is negative as volume is decreasing)

$$U_{CDA} = U_{AC} - U_{AD} = 400 \text{ J}$$

$$Q_{CDA} = W_{CDA} + U_{CDA} = 800 \text{ J} + 400 \text{ J}$$

4. (a) $T = \frac{pV}{nR} = \frac{1 \times 10^2 \times 2 \times 10^5}{1 \times 8.314}$

(b) $W = \frac{p}{1} \frac{V}{2} = \frac{10^5 \times 5 \times 10^3}{\frac{5}{3}} = 10^3 \text{ J}$
 $\frac{2}{3} \text{ J}$

5. (a)

$$K = \frac{p^2}{2m_i} - \frac{p^2}{2m_f} = \frac{p^2}{2} \left(\frac{1}{m_i} - \frac{1}{m_f} \right)$$

$$= \frac{(10 \times 10^3 \times 200)^2}{2} \left(\frac{1}{10 \times 10^3} - \frac{1}{2.01} \right)$$

$$= 2 \times 100 \times \frac{1}{2} \times 199 \text{ J}$$

(b) $Q = nC_V T_f - nC_V T_i = nC_V (T_f - T_i)$
 $\frac{Q}{nC_V} = T_f - T_i = \frac{Q}{m} C_V$

$$\frac{M}{m} = \frac{Q}{3R} = \frac{200 \times 199}{2010 \times 3 \times 8.314}$$

$$= 0.8 \text{ C}$$

6. $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$$nRT_1 m \frac{p_1}{p_2} - p_2 (V_C - V_{BC})$$

$$nRT m \frac{p_2}{p_1} - p_1 (V_1 - V_2)$$

$$nR(T_2 - T_1) \ln \frac{p_2}{p_1} - p_1 V_2 + p_1 V_1$$

$$p_1 V_1 - p_1 V_2$$

$$(p_2 V_2 - p_1 V_1) \ln \frac{p_2}{p_1}$$

7. W_{ABCA} (+)ve, W_{AB} (+)ve,

$$W_{BC} = 0, W_{CA} \text{ (-)ve}$$

For BC, Q (-)ve, U_{BC} (-)ve and

$$W_{BC} = 0$$

For CA, U (-)ve, Q_{CA} (-)ve as W_{CA} (-)ve.

	U	W	Q
AB	+	+	+
BC		0	
CA			
Total	0	+	+

For AB, as $U_{ABCA} = 0$ and

$$U_{BC} \text{ (-)ve,}$$

$$U_{CA} \text{ (-)ve}$$

$$U_{AB} \text{ (-)ve}$$

As $Q_{ABCA} = W_{ABCA}$ (-)ve and

$$Q_{BC} \text{ (-)ve}$$

$$Q_{CA} \text{ (-)ve, } Q_{AB} \text{ (-)ve}$$

In isobaric process, $W = p \Delta V = nR \Delta T$

$$= 0.2 \times 8.314 \times (300 - 200) = 166.3 \text{ J}$$

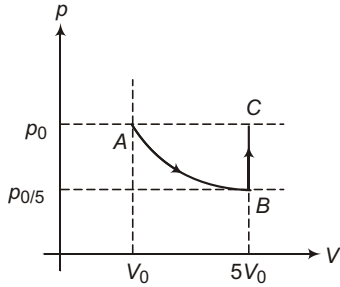
9. $W = \int p dV = \int V^2 dV = \frac{1}{3} V^3$

$$= \frac{1}{3} (5 \times 1.01 \times 10^5)^3 - (2^3 \times 1^3)$$

$$= 1.18 \times 10^6 \text{ J}$$

Introductory Exercise 18.3

1.



$$W_{BB} = nRT \ln \frac{V_B}{V_A} = 3R \cdot 273 \ln 5$$

$$10959 \text{ J}$$

$$W_{BC} = 0$$

$$Q = U = W$$

$$U = Q = W$$

$$80000 \quad 10959$$

$$69041$$

$$T_f = 5T_i = 5 \cdot 273 \text{ K} = 1365 \text{ K}$$

$$Q_{ABC} = Q_{AB} + Q_{BC}$$

$$W_{BC} = 0 \quad 0$$

$$U_{BC}$$

$$Q_{BC} = nC_V T + C_V \frac{Q_{BC}}{n T}$$

$$\frac{69041}{3 \cdot 4 \cdot 273} = 21.07$$

$$C_p \quad C_V \quad R \quad 29.39$$

$$\frac{C_p}{C_V} = \frac{29.39}{21.07} = 1.4$$

2. $Q = U = W; Q = nC_p T$

$$1600 = 1 \cdot C_p \cdot 72$$

$$C_p = 22.22$$

$$C_V = C_p - R = 13.9 \quad \frac{C_p}{C_V} = 1.6$$

$$W = Q = U = 1600 = nC_V T$$

$$1600 = 1 \cdot 13.9 \cdot 72$$

$$1600 = 1000.8 \text{ J}$$

$$599.2 \text{ J}$$

and $U = nC_V T = 1 \cdot 13.9 \cdot 72$

$$1001$$

$$1 \text{ kJ}$$

3. $W = \frac{1}{2} p \Delta V$

$$\frac{1}{2} \cdot 20 \cdot 1.01 \cdot 10^5 = 1 \cdot 10^3$$

$$10 = 101 = 1010 \text{ J}$$

$$p = \frac{n W}{t} = \frac{100 \cdot 1010 \text{ J}}{60 \text{ s}}$$

$$1.68 \text{ kW}$$

AIEEE Corner

■ Subjective Questions (Level 1)

1. $U = Q = W = 254 \text{ J} = 73 \text{ J}$

$$327 \text{ J}$$

2. (a) $T = \frac{Q}{nC_V} = \frac{2 Q}{3nR} = \frac{2 \cdot 200}{2 \cdot 1 \cdot 8.314}$

$$16 \text{ K}$$

$$T_f = T_i = T = 316 \text{ K}$$

(b) $T = \frac{Q}{nC_p} = \frac{2 Q}{5nR} = \frac{2 \cdot 200}{5 \cdot 1 \cdot 8.314}$

$$9.6 \text{ K}$$

$$T_f = T_i = T = 309.6 \text{ K}$$

3. $U = nC_V T$, in adiabatic process,

$$Q = 0 \text{ and } U = W$$

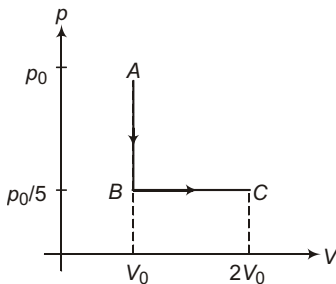
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where, $W = \frac{nR \Delta T}{1}$
 $U = \frac{nR \Delta T}{1}$ for all process.

4. $V = 0$ $W = 0$
 $Q = U = nC_V \Delta T = n \frac{5}{2} R \Delta T$
 $\frac{5}{2} (p_f V_f - p_i V_i) = \frac{5}{2} (p_f - p_i) V$
 $\frac{5}{2} (5 \times 10^5 - 10^5) = 10 \times 10^3$
 $\frac{5}{2} \times 4 \times 10^5 = 10^2 \times 10^4 \text{ J}$

5. $Q_1 = U_1 = nC_V \Delta T = n \frac{5}{2} R (3T_i - T_i)$
 $5 nRT_i$
 $Q_2 = nC_p \Delta T = n \frac{5}{2} R (6T_i - 3T_i)$
 $7.5 nRT_i$
 $c = \frac{Q}{n \Delta T} = \frac{12.5 nRT_i}{n (6T_i - 3T_i)} = \frac{12.5R}{3} = 2.5R$

6. $W_{AB} = 0$, $W_{BC} = \frac{p_0}{2} V_0 - \frac{1}{2} p_0 V_0$
 $\frac{1}{2} nRT_0 = 300 R$



$Q = (U_f - U_i) = (U_{BC} - U_{AB}) = (U_C - U_B) - (U_B - U_A) = U_C - U_A$
 $0 - 300 R$
 (As $T_A = T_C$)
 $2.49 \times 10^3 \text{ J} = 2.49 \text{ kJ}$

7. $U = Q = W = 1200 \text{ J} = 2100 \text{ J}$
 900 J

$T = \frac{U}{nC_V} = \frac{900}{5 \times \frac{3}{2} \times 8.314} = 14.43$

$T_f - T_i = T = 127 \text{ C} + 14.43 \text{ C} = 141.6 \text{ C}$

8. When gas expands it does positive work on the surrounding and for this purpose heat has to be supplied into the system.

9. $W = -p \Delta V = -p(V_f - V_i)$
 $m \frac{1}{f} - \frac{1}{i} = m \frac{1}{1000} - \frac{1}{999.9}$
 $\frac{10^5}{1000} - \frac{0.1}{999.9} = 0.02 \text{ J}$

(work done is negative as volume decreases)

$Q = ms = 2 \times 4200 = 8400 \text{ J}$
 33600 J
 $U = Q = W = 33600.02 \text{ J}$

10. $W = p \Delta V = p(V_f - V_i) = p \frac{m}{\rho}$
 $\frac{10^5}{0.6} (10 - 10^3) = 1666.67 \text{ J}$
 $Q = ms = mL = 10^2 \times 4200 = 420000 \text{ J}$
 $100 \times 10^2 = 10^4 \text{ J}$
 $25 \times 10^6 = 250000 \text{ J}$
 29200 J
 $U = Q = W = 29200 \text{ J} = 1666.67 \text{ J}$
 $27533.33 \text{ J} = 2.75 \times 10^4 \text{ J}$

11. $W = p \Delta V = 1.013 \times 10^5 (1670 - 10^6)$
 $1.013 \times 167 \text{ J} = 169.2 \text{ J}$
 $Q = mL = 10^3 \times 2.256 = 2256 \text{ J}$
 10^6 J
 $U = Q = W = (2256 - 169.2) \text{ J} = 2086.8 \text{ J} = 2087 \text{ J}$

12. $W = p \Delta V = 2.3 \times 10^5 (0.5 - 1.15) = -1.15 \times 10^5 \text{ J}$
 $U = 1.4 \times 10^5 \text{ J}$
 $Q = U - W = 1.4 \times 10^5 \text{ J} + 1.15 \times 10^5 \text{ J} = 2.55 \times 10^5 \text{ J}$

$$(1.4 + 1.15) \cdot 10^5 \text{ J}$$

$$2.55 \cdot 10^5 \text{ J}$$

Thus, $2.55 \cdot 10^5 \text{ J}$ of heat flows out of the system and it is independent of the type of the gas.

13. In a cyclic process, $U = 0$, $Q = W$

$$(a) \quad W = (Q_1, Q_2, Q_3, Q_4)$$

$$(W_1, W_2, W_3)$$

$$(5960, 5585, 2980, 3645)$$

$$(2200, 825, 1100)$$

$$1040, 275, 765 \text{ J}$$

$$(b) \quad \frac{\text{work done}}{\text{heat supplied}} = \frac{1040}{9605} = 10.83\%$$

14. (a) $W = \frac{1}{2} AB + AC = \frac{1}{2} (2p_0 V_0 + p_0 V_0)$

$$(b) \quad T_C = \frac{2 p_0 V_0}{nR} \text{ and } T_A = \frac{p_0 V_0}{nR}$$

$$Q_{CA} = nC_p T = nC_p \frac{p_0 V_0}{nR}$$

$$= \frac{5}{2} R \frac{p_0 V_0}{R} = \frac{5}{2} p_0 V_0$$

$$T_B = \frac{3 p_0 V_0}{nR}, \quad Q_{AB} = nC_V T$$

$$= n \frac{3}{2} R \frac{3 p_0 V_0}{nR} = \frac{9}{2} p_0 V_0$$

$$\frac{3}{2} (2 p_0 V_0 + 3 p_0 V_0)$$

$$(c) \quad Q_{AB} = Q_{BC} = Q_{CA} = W$$

$$3 p_0 V_0, \quad Q_{BC} = \frac{5}{2} p_0 V_0, \quad p_0 V_0$$

$$Q_{BC} = \frac{p_0 V_0}{2}$$

(d) Temperature is maximum at a point D lying somewhere between B and C where the product pV is maximum.

$$p = \frac{2p_0}{V_0} = 5p_0$$

$$pV = \frac{2p_0}{V_0} V = 5p_0 V$$

$$\frac{2p_0}{V_0} V^2 = 5p_0 V$$

For pV maximum $\frac{d}{dV}(pV) = 0$

$$2V \frac{2p_0}{V_0} = 5p_0 \Rightarrow 0$$

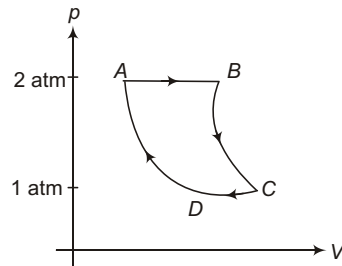
$$V = \frac{5V_0}{4}$$

$$p = \frac{2p_0}{V_0} \frac{5V_0}{4} = 5p_0 = \frac{5}{2} p_0$$

$$T_{\max} = \frac{(pV)_{\max}}{nR}$$

$$= \frac{\frac{5}{2} p_0 \frac{5}{4} V_0}{1 R} = \frac{25 p_0 V_0}{8R}$$

15. $V_A = \frac{nRT_A}{p_A} = \frac{2R \cdot 300}{2 \cdot 10^5} = 3 \cdot 10^{-3} R$



$$V_B = \frac{2R \cdot 400}{2 \cdot 10^5} = 4 \cdot 10^{-3} R,$$

$$V_C = \frac{2R \cdot 400}{10^5} = 8 \cdot 10^{-3} R$$

$$V_0 = \frac{2R \cdot 300}{10} = 6 \cdot 10^{-3} R$$

$$W = 2 \cdot 10^5 (4 - 3) = 10^3 R$$

$$2R \cdot 400 \ln \frac{8}{4} = 1 \cdot 10^5 (6 - 8) = 10^3 R$$

$$K = 2R \cdot 300 \ln \frac{3}{6}$$

$$W = 200R - 800R \ln 2 = 200R - 600R \ln 2$$

$$= 2000R \ln 2 - 1153 J$$

As $Q = W = 1153 \text{ J}$ and $U = 0$ cyclic process.

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$$16. \quad W = \frac{1}{2} \frac{3V_0}{2} \frac{V_0}{2} (p_B - p_0) - \frac{1}{2} \frac{3V_0}{2} \frac{V_0}{2} (p_0 - p_D) + \frac{1}{2} V_0 (p_B - p_0) - \frac{1}{2} V_0 (p_0 - p_D) - \frac{1}{2} V_0 (p_B - p_D)$$

where, $p_B = p_0 \frac{p_0}{V_0} \frac{V_0}{2} = \frac{3p_0}{2}$

and $p_D = p_0 \frac{p_0}{V_0} \frac{V_0}{2} = \frac{p_0}{2}$

$$W = \frac{1}{2} V_0 \left(\frac{3}{2} p_0 - \frac{1}{2} p_0 \right) - \frac{1}{2} p_0 V_0$$

$$W_{ABC} = p_0 \left(\frac{3V_0}{2} - \frac{V_0}{2} \right) - \frac{1}{2} \frac{3V_0}{2} \frac{V_0}{2} + \frac{V_0}{2}$$

$$(p_B - p_0) p_0 V_0 - \frac{1}{2} V_0 \left(\frac{3}{2} p_0 - p_0 \right)$$

$$= \frac{5}{4} p_0 V_0$$

$$U_{ABC} = nC_V \frac{(T_C - T_A)}{\frac{3V_0}{2}} = n \frac{3}{2} R \frac{p_0 \frac{V_0}{2}}{nR} - \frac{p_0 \frac{V_0}{2}}{nR} = \frac{3}{2} p_0 V_0$$

$$Q_{\text{supplied}} = \frac{5}{4} p_0 V_0 - \frac{11}{4} p_0 V_0 = \frac{1}{2} p_0 V_0 = \frac{2}{11} \cdot \frac{11}{4} p_0 V_0 = 0.1818 \cdot 11 p_0 V_0 = 18.18\%$$

17. (a) As the cyclic process is clockwise *i.e.*, work done is positive, so heat is absorbed by the system.

(b) In cyclic process work done is equal to the net heat absorbed (as change in internal energy is zero) so, work done in one cycle is 7200 J.

(c) In anticlockwise rotation, work done is negative and heat is liberated by the system, and its magnitude is 7200 J.

18. (a) As area under clockwise loop is more than that at anticlockwise loop, so network done is positive.

(b) In loop I work done is positive and in loop II work done is negative.

(c) As network done in one cycle is positive so heat flows into the system.

(d) In loop I heat flows into the system and in loop II heat flows out of the system.

$$19. \quad T_A = \frac{p_A V_A}{nR} = \frac{1.01 \cdot 10^5 \cdot 22.4}{10^3 \cdot 8.314}$$

$$= 273 \text{ K}$$

$$T_B = \frac{p_B V_A}{nR} = \frac{2p_A V_A}{nR} = 2T_A$$

$$= 546 \text{ K}$$

$$V_c = \frac{nRT_c}{p_c} = \frac{T_c}{p_A} \frac{nRT_B}{p_A}$$

$$= \frac{2nRT_A}{p_A} = 2V_A = 44.8 \text{ m}^3$$

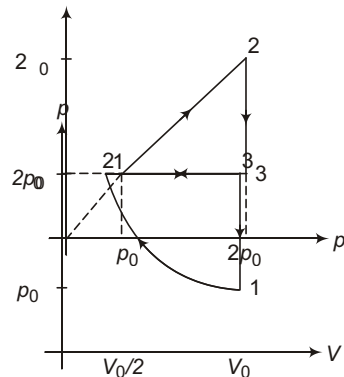
20. (a) $W_{AB} = BC$

$$(4 \cdot 1.5) \cdot 10^6 - (4 \cdot 2) \cdot 10^5 = 2.5 \cdot 0.2 \cdot 0.5 \text{ J}$$

(b) $Q = W$ as $U = 0$ in a cycle

$$Q = 0.5 \text{ J}$$

21. As $\frac{1}{V}$



$$(a) W_{12} = nRT_0 \ln \frac{V_0/2}{V_0} = p_0 V_0 \ln 2 = p_0 \frac{M}{\rho} \ln 2$$

$$W_{23} = 2p_0 V_0 \left[\frac{V_0}{2} - p_0 V_0 \right] = p_0 \frac{M}{\rho}; W_{31} = 0$$

$$(b) Q_{231} = Q_{23} + Q_{31} = nC_V T_{23} + W_{23} + nC_V T_{31} = n \left[\frac{3}{2}R \frac{2p_0 V_0}{nR} + \frac{2p_0 V_0}{nR} + p_0 V_0 \right]$$

$$= n \left[\frac{3}{2}R \frac{p_0 V_0}{nR} + \frac{2p_0 V_0}{nR} + \frac{p_0 V_0}{nR} \right]$$

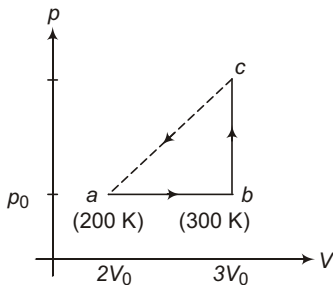
$$p_0 V \left[\frac{3}{2} p_0 V_0 + p_0 V_0 + \frac{5}{2} p_0 V_0 \right]$$

Heat rejected $Q_{231} = W = \frac{5}{2} p_0 V_0 + p_0 V_0 + p_0 V_0 \ln 2 = \frac{3}{2} p_0 V_0 + p_0 V_0 \ln 2$

$$p_0 V_0 \left[\frac{5}{2} \ln 2 + \frac{p_0 M}{p_0} \frac{3}{2} \ln 2 \right]$$

$$(c) \frac{\text{work done}}{\text{heat supplied}} = \frac{W}{Q_{231}} = \frac{2}{3} (1 - \ln 2)$$

22. $W_{AB} = p_0(3V_0 - 2V_0) = p_0 V_0;$



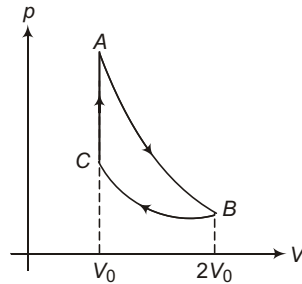
$$W_{BC} = 0, W_{CA} = ?$$

$$Q = W_{AB} + W_{BC} + W_{CA} = 800 \text{ J} + p_0 V_0 + 0 = W_{CA}$$

$$W_{CA} = 800 \text{ J} + p_0 V_0 = 800 \text{ J} + \frac{1}{2} nRT_A$$

$$W_{CA} = 800 \text{ J} + 200R = 2463 \text{ J}$$

23. $W_{AB} = \frac{p_B V_B - p_A V_A}{1}$



$$\frac{3}{2} (p_A V_A - p_B V_B)$$

$$\frac{3}{2} nR(T_A - T_B) = \frac{3}{2} nRT_B \left(\frac{T_A}{T_B} - 1 \right); TV^{-1}$$

$$\frac{3}{2} nRT_B \left(\frac{2}{T} \right)^{\frac{5}{3}-1} - 1 = \frac{3}{2} nRT_B (2^{2/3} - 1)$$

$$W_{BC} = nRT_B \ln \frac{V_0}{2V_0}$$

$$= nRT_B \ln 2 \text{ and } W_{CA} = 0$$

Heat Supplied

$$Q_{CA} = U_{CA} = \frac{3}{2} nR(T_A - T_C)$$

$$= \frac{3}{2} nR(T_A - T_B)$$

$$\frac{3}{2} nRT_B \left(\frac{T_A}{T_D} - 1 \right) = \frac{3}{2} nRT_B (2^{2/3} - 1)$$

$$\frac{W}{Q_{CA}} = \frac{nRT_B \ln 2}{\frac{3}{2} nRT_B (2^{2/3} - 1)} = 0$$

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$$1 \frac{2}{3} \frac{\ln 2}{2^{2/3}} \frac{1}{1} \quad 1 \quad 0.7867 \quad 0.213$$

21.3%

■ Objective Questions (Level 1)

1. $U = nC_V T = \frac{3}{2} RT = \frac{3}{2} RT$

$$T = \frac{2U}{3R}$$

$$T_D = \frac{2V_0}{3R} = 300 \text{ K} \quad U_0 = 450R,$$

$$T_A = \frac{4V_0}{3R} = 600 \text{ K}$$

$$W = W_{AB} = W_{CD} = nRT_A \ln \frac{2V_0}{V_0}$$

$$nRT_D \ln \frac{V_0}{2V_0} = nR(T_A - T_D) \ln 2$$

$$1 \quad R \quad (600 - 300) \ln 2$$

$$300R \ln 2 = Q$$

2. $W_{12} = p \Delta V = nR \Delta T$

$$2R \cdot 300 = 600R$$

$W_{23} = ?; W_{31} = 0.$

As, $Q = W_{12} + W_{23} + W_{31}$

$$300 \text{ J} = 600R + W_{23} + 0$$

$$W_{23} = 300 \text{ J} - 600R = 5288 \text{ J}$$

3. $nC_p T_1 = nC_V T_2$

$$\frac{7}{2} T_1 = 30 \frac{5}{2} T_2$$

$$T_2 = 42 \text{ K}$$

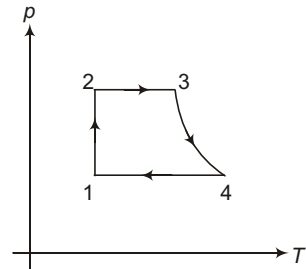
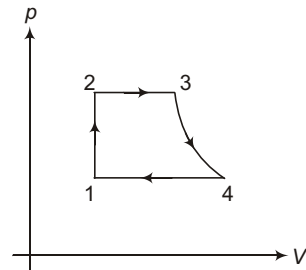
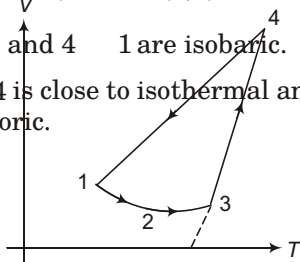
4. $TV^{n-1} = \text{constant}$

$$pV = V^n \cdot \frac{1}{V} = pV^n = \text{constant}$$

$$\ln p + n \ln V = \ln c$$

$$\frac{p}{p} = n \frac{V}{V} = \frac{p}{V/V} = np = B$$

5. 2, 3 and 4, 1 are isobaric.
3, 4 is close to isothermal and 1, 2 is isochoric.



6. $W = \int p dV = \int kV dV = \frac{k}{2} V^2$

$$\frac{1}{2} pV = \frac{1}{2} nR(T_2 - T_1) = \frac{R}{2}(T_2 - T_1)$$

7. $p = V^{-2}, W = \int p dV = \int kV^{-2} dV$

$$= \frac{1}{3} kV^{-1} = \frac{1}{3} pV$$

$$= \frac{1}{3} nR(T_f - T_i) \quad () \text{ ve}$$

8. $W = nRT \ln \frac{V_f}{V_i}$

$$nRT \ln \frac{1}{2} = nRT \ln 2$$

9. $U = 600 \text{ J} - 150 \text{ J} = 450 \text{ J}$

$$nC_V T = \frac{3}{2} R n T$$

$$C = \frac{Q}{nT} = \frac{600 \text{ J}}{450 \text{ J}} = \frac{3}{2}R = \frac{600}{450} \cdot \frac{3}{2}R = \frac{4}{3} \cdot 2R$$

10. W_1 () ve, $W_2 = 0$, W_3 () ve
and $U_1 = U_2 = U_3$
as $Q = U - W$ $Q_1 = Q_2 = Q_3$

11. $U = 2p_0 \cdot 2V_0 = 2p_0 V_0 = 2p_0 V_0$
and $W = p_0(2V_0 - V_0) = p_0 V_0$
 $Q = U - W = 3p_0 V_0$

12. In adiabatic compression, temperature of the gas increases and as $pV = T$ so, pV increases.

13. As $W_1 = W_2$ while $U_1 = U_2$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{C_1}{C_2} = 1$$

14. $W = nR(4T - T) = \frac{nR(5T - 4T)}{1}$
 $nR(3T - 5T) = \frac{nR(T - 3T)}{1}$
 $3nRT - 2nRT = \frac{nRT}{1} - \frac{2nRT}{1}$
 $nRT = \frac{nRT}{1}$
 $\frac{nRT}{1} (1 - 1) = -\frac{nRT}{1}$
 $\frac{5/3}{5/3 - 1} 1RT = 2.5RT$

15. Up constant
 $\frac{3}{2}nRT = \frac{nM}{V} = \frac{3}{2}n^2MR \frac{T}{V}$
 $T = V$ i.e., isobaric process.
 $\frac{U}{W} = \frac{U}{Q - U} = \frac{3/2}{5/3 - 3/2} = \frac{3}{2}$

$$\frac{C_V}{C_p} = \frac{C_V}{R}$$

16. $W = 50(0.4 - 0.1) = \frac{1}{2} \cdot 50(0.2 - 0.1)$
 $15 = 2.5 = 27.5 \text{ J}$
 $U = 2.5 \text{ J}$

17. $W_1 = \int_{V_0}^{2V_0} p dV = p(2V_0 - V_0) = pV_0$
 $W_2 = \int_{V_0}^{2V_0} kV dV = \frac{1}{2}kV^2$
 $\frac{1}{2}k(4V_0^2 - V_0^2) = \frac{3}{2}kV_0^2 = \frac{3}{2}PV_0$

18. $W = \frac{W_1}{r_1 r_2} = \frac{W_2}{ab}$
 $\frac{r_2}{r_1} = \frac{(p_2 - p_1)}{2}$
 $-\frac{(p_2 - p_1)(V_2 - V_1)}{4}$

19. $W = \int_{V_0}^{2V_0} PdV = \frac{nRT}{V} \frac{dx}{x}$
 $x = V - b$
 $dx = dV$
 $nRT \ln x = nRT \ln (V - b) \Big|_V^{2V}$
 $nRT [\ln (2V - b) - \ln (V - b)]$
 $nRT \ln \frac{2V - b}{V - b} = RT \ln \frac{2V - b}{V - b}$

as $n = 1$ mole

20. AB is isochoric process, so, $W_{AB} = 0$
 BC is isothermal process, so,
 $W_{BC} = nRT_2 \ln \frac{V_2}{V_1} = RT_2 \ln \frac{V_2}{V_1}$

CA is close to isobaric process, so,

$W_{CA} = nRT = nR(T_1 - T_2) = R(T_1 - T_2)$
21. $Q = U - W = Q - W$
 $W = 2Q$
 $U = nC_V T = n \frac{f}{2} R T = \frac{n}{1} R T$

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$$f = \frac{2}{1} = 2$$

$$W = \int p dV = \frac{2n}{1} R T$$

$$\frac{2nR}{1} \frac{T}{1} = \frac{nR}{1} \frac{T}{a}$$

for polytropic process with pV^a constant

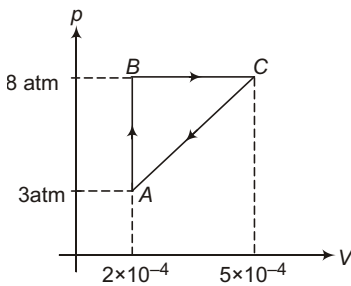
$$\frac{2}{1} = \frac{1}{1} \frac{1}{a} = 1 - a \Rightarrow \frac{1}{2} = -a$$

or $a = -\frac{1}{2}$

$$pV^{-1/2} = \text{constant}$$

$$TV^{1/2} = \text{constant}$$

22. $W_{AB} = 0, U_{AB} = 600 \text{ J}$



$$W_{BC} = 8 \times 10^5 (5 - 2) \times 10^{-4} = 240 \text{ J}$$

$$U_{BC} = Q_{BC} - W_{BC} = 200 - 240 = -40 \text{ J}$$

$U_{CA} = U_{AB} - U_{BC} - U_{CA} = 0$ in cyclic process.

$$U_{CA} = U_{AB} - U_{BC} = 600 \text{ J} - 40 \text{ J} = 560 \text{ J}$$

23. Starting and ending points along x-axis in graph are not clear, so nothing can be said about the magnitude of work.

It can only be said that work done in ABC is negative and that in DEF is

positive. Looking at the graph, area can be assumed to be equal so,

$$W_{DEF} = W_{ABC}$$

24. $W_{\text{isobaric}} = p(V_2 - V_1) = pV \ln 2$

$$W_{\text{isothermal}} = nRT \ln \frac{2V}{V} = pV \ln 2$$

$$W_{\text{adiabatic}} = \frac{0.693 pV}{1} = 0.693 pV$$

$$\frac{p_i V_i}{2V_i} = \frac{p_i V_i}{V_i} (2^{1-r})$$

$$pV \frac{1}{r} \frac{2^{1-r}}{1} = pV \frac{1}{2/3}$$

$$0.55 pV$$

So, work done is minimum in adiabatic process.

25. $Q = U - W$

$$\frac{7}{2} RT_0 = 10 \frac{5}{2} R T - 10R T - 35R T$$

$$T_0 = 100T - 10(T - T_0)$$

$$11T_0 = 10T$$

$$\frac{pV_0}{RT_0} = \frac{1.1T_0}{pV}$$

$$V = \frac{11}{10} V_0 = 1.1V_0$$

26. $W = (3p_0 - p_0)(2V_0 - V_0) = 2p_0 V_0$

$$Q_{\text{supplied}} = n \frac{3}{2} R \frac{3p_0 V_0}{nR} - \frac{p_0 V_0}{nR}$$

$$= n \frac{5}{2} R \frac{3p_0 \cdot 2V_0}{nR} - \frac{3p_0 V_0}{nR}$$

$$\frac{3}{2} nR \frac{2p_0 V_0}{nR} - \frac{5}{2} nR \frac{3p_0 V_0}{nR}$$

$$3p_0 V_0 - \frac{15}{2} p_0 V_0 = -\frac{9}{2} p_0 V_0$$

$$\frac{W}{r} = \frac{2p_0 V_0}{2} = p_0 V_0$$

27. W_{12} W_{13} can be seen from area under the curve, while V_1 V_2

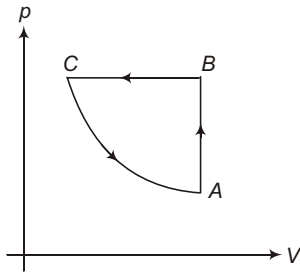
$$Q_{12} = Q_{13} = Q_2 = Q_1 \text{ or } Q_1 = Q_2$$

28. $W_{CA} = p_0(V_0 - 2V_0) = -p_0V_0$

and $U_{CA} = \frac{3}{2}p_0V_0$

$$Q_{CA} = \frac{5}{2}p_0V_0$$

29. $Q_{AB} = 200 \text{ kJ} = nC_V T$;

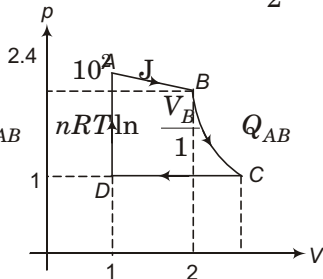


$U_{BC} = 100 \text{ kJ}$ and $W_{BC} = 50 \text{ kJ}$
 $W_{AB} = 0$ $U_{AB} = 200 \text{ kJ}$, $Q_{CA} = 0$
 $U_{ABC} = U_{AB} + U_{BC} = U_{CA} = 0$
 or $200 \text{ kJ} = 100 \text{ kJ} + U_{CA} = 0$

$U_{CA} = 100 \text{ kJ}$
 $Q_{AB} = Q_{BC} = Q_{CA} = 0$
 $200 \text{ kJ} = (100 \text{ kJ} + 50 \text{ kJ}) + 0$
 50 kJ

$W_{AB} = W_{BC} = W_{CA} = W_{CA}$
 $0 = 200 \text{ kJ} + W_{CA}$
 $Q_{ABC} = 50 \text{ kJ}$
 $W_{CA} = 150 \text{ kJ}$

30. $Q = W = ab = \frac{20 \cdot 10^3}{2} = \frac{20 \cdot 10^3}{2}$



31. $W_{AB} = nRT \ln \frac{V_B}{V_A} = Q_{AB} = 9 \cdot 10^4 \text{ J}$

$$W_{ABCD} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= nRT \ln \frac{V_B}{V_A} + \frac{p_C p_C}{p_B p_B} \frac{V_D - V_C}{1} + 0$$

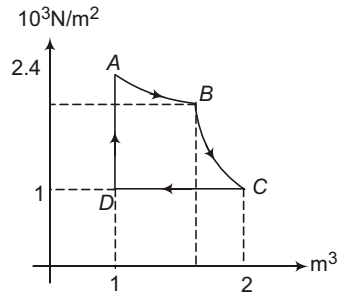
$$= 9 \cdot 10^4 + \frac{10^5 \cdot nRT_B}{1 \cdot \frac{5}{3}} = 10^5 (2 + 1)$$

$$= 19 \cdot 10^4 = \frac{3}{2} (10^5 + 800 T_B)$$

$$= 4 \cdot 10^4 = 1200 T_B$$

$$4 \cdot 10^4 = 1200 \cdot \frac{2.4 \cdot 10^5}{100 \cdot 8} = 4 \cdot 10^5 \text{ J}$$

31. $W = W_{AB} = \frac{p_C V_C - p_B V_B}{1} = W_{CD}$



$$9 \cdot 10^4 = \frac{2 \cdot 10^5 - 9 \cdot 10^4}{1 \cdot \frac{5}{3}} = 1 \cdot 10^5$$

$$9 \cdot 10^4 = \frac{3}{2} \cdot 11 \cdot 10^4 = 10 \cdot 10^4$$

$$\frac{33}{2} = 1 \cdot 10^4 = 15.5 \cdot 10^4$$

32. $W = \int p dV = \int kV dV = \frac{1}{2} kV^2$

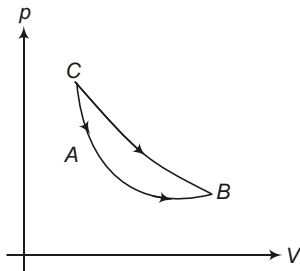
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$$\frac{1}{2} pV = \frac{1}{2} nRT_0 = \frac{1}{2} RT_0$$

$$U = nC_V T = \frac{3}{2} RT_0$$

$$Q = \frac{3}{2} \frac{1}{2} RT_0 = 2RT_0$$

33. $pT = \text{constant}$ $p \frac{pV}{nR} = \frac{p^2 V}{nR}$
 $p^2 V = \text{constant}$



$$p_0^2 V_0 = \frac{p_0^2}{2} V = V = 4V_0$$

$$T = \frac{p_0}{2} \frac{4V_0}{nR} = 2 \frac{p_0 V_0}{nR} = 2T_0$$

$$U = nC_V T = 2 \frac{3}{2} R (2T_0 - T_0)$$

$$= 3R \frac{p_0 V_0}{2R} = \frac{3}{2} p_0 V_0$$

35. $W_{BC} = nRT_0 \ln \frac{V_C}{V_B}$

$$= nRT_0 \ln \frac{p_B}{p_C} = 2 nRT_0 \ln \frac{V_B}{V_A}$$

$$= 2nRT_0 \ln \frac{p_A}{p_B}$$

$$\ln \frac{p_B}{p_C} = \ln \frac{p_0}{p_0/2} = \ln 4$$

$$\frac{p_B}{p_C} = \frac{4p_C}{p_C} = \frac{p_0}{8}$$

36. As, $W_a = W_b = W_1 = W_2$
 while, $U_1 = U_2 = Q_1 = Q_2$

37. $1 \frac{T_{\text{sink}}}{T_{\text{source}}} = 1 \frac{300}{600}$
 $1 \frac{1}{2} = 0.5 = 50\%$

38. As the volume is adiabatically decreased, temperature of the gas increases and as the time elapsed, temperature normalizes *i.e.*, decreases and so pressure also decreases.

39. As the compression is quick, the process is adiabatic while leads to heating of the gas.

40. $pV = \text{constant}$
 $\frac{nRT}{V} V = nRTV^{-1}$
 $TV^{-1} = \text{constant}$
 $\frac{T_1}{T_2} = \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{L_2}{L_1} = \frac{2}{3}$

41. $pV = \text{constant}$ $p \frac{nT}{p}$
 $p^1 T = \text{constant}$
 $p = T^{-1}$

As $\frac{7/5}{1} = \frac{7}{2}$ for diatom gases.

$$p = T^{3.5} = 3.5$$

42. $pV^x = \text{constant}$, $W = \frac{nR T}{1-x}$,

$$U = n \frac{5}{2} R T$$

$$C = \frac{Q}{n T} = \frac{\frac{nR T}{1-x} \frac{5}{2} nR T}{n T}$$

$$= \frac{5}{2} R \frac{R}{1-x} = 0$$

$$\frac{5}{2} R \frac{R}{x} = 1 \frac{2}{5}$$

$$x = \frac{7}{5} \quad x = 1.4 \text{ but } x = 1 \text{ as for } x = 1,$$

C will become positive.

$$43. C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{13}{6} R$$

$$(a) \frac{2 \cdot \frac{5}{2} R + 4 \cdot \frac{5}{2} R}{2 + 4} = \frac{15}{6} R$$

$$(b) \frac{2 \cdot \frac{5}{2} R + 4 \cdot \frac{3}{2} R}{2 + 4} = \frac{11}{6} R$$

$$(c) \frac{2 \cdot \frac{3}{2} R + 4 \cdot \frac{5}{2} R}{2 + 4} = \frac{13}{6} R \text{ and}$$

$$(d) \frac{2 \cdot \frac{6}{2} R + 4 \cdot \frac{3}{2} R}{2 + 4} = \frac{12}{6} R$$

Passage 44 & 45

$$44. W_{ABCA} = \frac{1}{2} p V = \frac{pV}{2} = Q_{\text{net}}$$

$$45. CA \text{ isobaric and } BC \text{ isochoric,}$$

$$\frac{C_p}{C_v} = \frac{5}{3}$$

$$46. pV = \text{constant} \quad p = \frac{nRT}{V}$$

$$p^1 T = \text{constant}$$

$$T = \frac{\text{constant}}{p}$$

$$T = p^{\frac{5/3 - 1}{5/3}} = T p^{2/5}$$

$$\frac{T_B}{T_A} = \frac{p_B}{p_A} = \frac{2p_c}{3p_c} = 0.85$$

$$47. W_{AB} = \frac{nRT}{1} \left[\frac{1}{\frac{25}{3}} - \frac{1}{\frac{5}{3}} \right] = 150 - 75 = 75 \text{ J}$$

$$1875 \text{ J}$$

$$48. W_{BC} = 0, \quad Q_{BC} = U_{BC}$$

$$n \cdot \frac{3}{2} R (T_C - T_B)$$

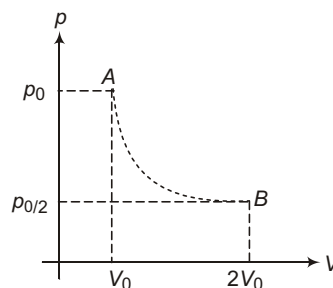
$$n \cdot \frac{3}{2} R \frac{p_C V}{nR} = \frac{p_B V}{nR}$$

$$\frac{3}{2} \cdot \frac{1}{3} p_A = \frac{2}{3} p_A = V$$

$$\frac{1}{2} p_A V = \frac{1}{2} \cdot \frac{3}{2} p_B V = \frac{3}{4} nRT_B$$

$$\frac{3}{4} \cdot 1 \cdot \frac{25}{3} = 850 \quad 5312.5 \text{ J}$$

$$49. W_{AB} \text{ () ve, } T_A = T_B$$



$$p = \frac{p_0}{2V_0} V = \frac{3}{2} p_0$$

$$\frac{nRT}{V} = \frac{p_0}{2V_0} V = \frac{3}{2} p_0$$

$$\text{or } T = \frac{p_0}{2nRV_0} V^2 = \frac{3p_0}{2nR} V_0$$

$$y = ax^2 + bx \text{ is parabola.}$$

$$\text{Again, } p = \frac{p}{2V_0} = \frac{nRT}{p} = \frac{3}{2} p_0$$

is also equation of parabola.

While going from A to B temperature first increases and then decreases.

$$50. pV^2 = \text{constant}$$

$$W = \int p dV = \int \frac{k}{V^2} dV = k \left[\frac{1}{V} \right]$$

$$pV \Big|_i^f = p_i V_i = p_f V_f$$

$$nR(T_i - T_f) = nR(T_f - T_i) \text{ () ve}$$

$$\text{as } T_f > T_i$$

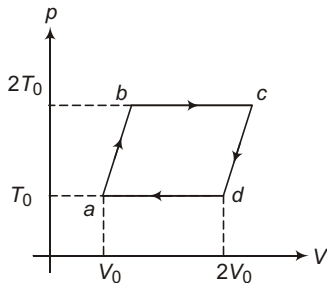
$$\text{as } T_i > T_f \quad U_i > U_f$$

$$U \text{ () ve}$$

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$Q = nC_V \Delta T = nR \Delta T = n(C_V + R) \Delta T$
 ()ve as $C_V + R$
 i.e., heat is given to the system.

51. In cyclic process, $U = 0$



$$W = 0 = nR2T_0 \ln \frac{2V_0}{V_0} + nRT_0 \ln \frac{V_0}{2V_0}$$

$$= 2nRT_0 \ln 2 - nRT_0 \ln 2 = 0$$

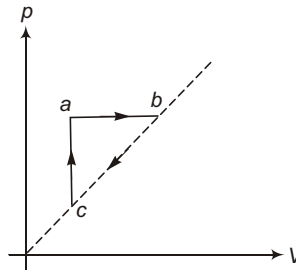
i.e., $W = 0$

$$Q_{\text{supplied}} = U_{ab} + W_{bc} = nC_V(2T_0 - T_0) + nR2T_0 \ln \frac{2V_0}{V_0}$$

$$= 2 \left(\frac{3}{2} RT_0 + 4RT_0 \ln 2 \right)$$

$$= 3RT_0 + 4RT_0 \ln 2$$

52. ab isochoric, bc isobaric and ca isothermal.



$$W_{ab} = 0, U_{ca} = 0$$

as in ca density is increasing, so volume is decreasing i.e.,

$$W_{ca} \text{ ()ve, i.e., } W_{ca} = 0$$

in isochoric process Q_{ab} is positive for increase in temperature.

53. In isochoric process $W = 0$.

and in adiabatic process

$$Q = 0 \quad Q_3 \text{ to be minimum}$$

$$Q_2 < Q_1 < Q_3$$

JEE Corner

■ Assertion & Reasons

1. In adiabatic expression, W ()ve while $Q = 0$ and as according to first law of thermodynamics,

$$Q = \Delta U = W = \Delta U = W$$

i.e., U ()ve this implies decrease in temperature. So, Assertion and reason are both true but not correct explanation.

2. Assertion is false, as work done is a path function and not a state function i.e., it

depends on the path through which the gas was taken from initial to final state.

3. Assertion is false, as first law can be applied for both real and ideal gases.

4. During melting of ice its volume decreases, so work done by it is negative and that by atmosphere is positive. So, reason is true explanation of assertion.

5. As $Q = \Delta U = W = \Delta U = Q = W$, where U is state function while Q and W are path function as for definite

initial and final state U is constant and so is $Q = W$. Thus assertion and reason are both true but not correct explanation.

6. Carnot's engine is ideal heat engine with maximum efficiency but it is not also 100%. So assertion and reason are both true but not correct explanation.

7. $pT = \text{constant}$ $p = \frac{pV}{R}$ $\frac{p^2V}{nR}$

$$p^2V = \text{constant}$$

$$W = \int p dV = \int \sqrt{k} \frac{dV}{\sqrt{V}} = \sqrt{k} \frac{V^{1/2}}{1/2}$$

$$= 2\sqrt{k}\sqrt{V} = 2\sqrt{kV} = 2\sqrt{p^2/V}$$

$$= \frac{2pV}{2nR(T_f - T_i)} = \frac{2nRT}{2nRT} T$$

W () ve for T () ve
and $\frac{nRT}{V} T = \text{constant}$.

$$T^2 \propto V$$

or, $V \propto T^2$

Thus assertion is true but reason is false.

8. In adiabatic changes for free expansion, $Q = 0, W = 0$ and $U = 0$ as in free expansion no work is done against any force. For ideal gases $pV = \text{constant}$ as $U = 0$ $T = \text{constant}$ So, assertion and reason are both true but not correct explanation.
9. Assertion and reason are both true and correct explanation.
10. Assertion and reason are both true and correct explanation.

■ Match the Columns

1. (a) $W = \int p dV = \int \frac{pV}{nR} nR(T_f - T_i)$

(b) $U = nC_V T = 2 \cdot \frac{3}{2} R(2T - T)$

(c) $W = \frac{3RT}{1 - 5/3} = \frac{3}{2} \cdot 2RT$

(d) $U = nC_V T = 3RT = pV$

2. (a) In ab slope is more so, pressure is less as $V \propto \frac{nR}{p} T$, but is constant and in

isobaric process. $W = p \Delta V = nR \Delta T$ and as T is same in both process so, W is same for both r

(b) As $U = nC_V T$ is same for both process r

(c) As $Q = U - W$, it is also same for both process s

(d) Nothing can be said about molar heat capacity s

3. (a) $W = \int p dV$

$$= \int \frac{k}{V} dV = \sqrt{k} \frac{dV}{\sqrt{V}}$$

$$= 2\sqrt{kV} = \frac{2pV}{2nR} T = pV$$

(b) $U = nC_V T = \frac{3}{2} nR T = \frac{3}{2} pV$ s

(c) $Q = 2nR T = \frac{3}{2} nR T$

$$= \frac{7}{2} nR T = \frac{7}{2} pV$$

(d) s

4. (a) $W = \int p dV = nR \Delta T$ and $U = nC_V \Delta T$

$$W = U = q$$

(b) $W = 0$ $Q = U$, U () ve p, r

(c) W () ve, U () ve, $Q = 0$ p

(d) W () ve, $U = 0$, Q () ve p

5. (a) $W_{AB} = p_0 V_0 - \frac{1}{2} p_0 V_0$

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$$\frac{3}{2} p_0 V_0 \quad s$$

$$(b) \quad U_{AB} \quad Q \quad W$$

$$6 p_0 V_0 \quad \frac{3}{2} p_0 V_0 \quad \frac{9}{2} p_0 V_0 \quad s$$

$$(c) \quad Q \quad 6 p_0 V_0$$

$$nC \quad \frac{4 p_0 V_0}{nR} \quad \frac{p_0 V_0}{nR}$$

$$\frac{3 p_0 V_0}{R} C$$

$$(d) \quad U \quad C \quad 2R \quad p$$

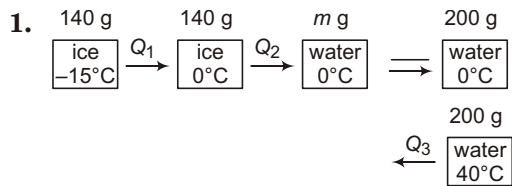
$$nC_V \quad \frac{4 p_0 V_0}{nR} \quad \frac{p_0 V_0}{nR}$$

$$3C_V \quad \frac{p_0 V_0}{R} \quad \frac{9}{2} p_0 V_0$$

$$C_V \quad \frac{3}{2} R \quad s$$

19. Calorimetry and Heat Transfer

Introductory Exercise 19.1.



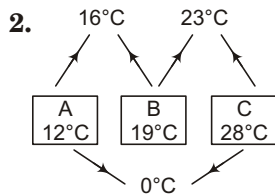
As Heat gain Heat loss

$$\begin{matrix} Q_1 & Q_2 & Q_3 \\ 140 & 0.53 & 15 & m & 80 \end{matrix}$$

$m = \frac{8000 - 1113}{80} = 86 \text{ g}$ is the mass of ice melt

Mass of water
 $200 \text{ g} + 86 \text{ g} = 286 \text{ g}$
 and mass of ice
 $140 \text{ g} - 86 \text{ g} = 54 \text{ g}$

while final temperature of mixture is 0°C .



$$\begin{matrix} ms_A (16 & 12) & ms_B (19 & 16) \\ & 4s_A & & 3s_B \end{matrix}$$

$$ms_B (23 \quad 19) \quad ms_C (28 \quad 23)$$

$$ms_A \left(\begin{matrix} 4s_B & 5s_C \\ & \frac{4}{5}ms(28 \quad) \end{matrix} \right)$$

$$\text{or} \quad \frac{3}{4}s_B \left(\begin{matrix} & 4 \\ & 5 \end{matrix} s_B (28 \quad) \right)$$

$$\text{or} \quad 15 \left(\begin{matrix} & 12 \\ & 16 \end{matrix} (28 \quad) \right)$$

$$\text{or} \quad \begin{matrix} 31 & 448 & 180 \\ & 20.26 \text{ C} & \end{matrix}$$

3. $mL \quad ms$

$$\begin{matrix} 80 \text{ cal} & 1 \text{ cal}/^\circ\text{C} & (\quad 0 \text{ C}) \\ & 80 \text{ C} & \end{matrix}$$

4. As Heat gain Heat loss

$$\begin{matrix} (100 & m) & 529 & m & 80 \\ & 100 & 529 & 609 \text{ m} \end{matrix}$$

$$m = \frac{100 \cdot 529}{609} \text{ g} = 86.86 \text{ g of ice will}$$

be formed.

5. $P \frac{d}{dt} \frac{d}{dt} (ms \quad) \quad \frac{dm}{dt} s$

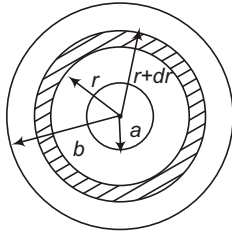
$$\frac{dm}{dt} = \frac{P}{s} = \frac{500 \cdot 10^6 \text{ J/s}}{4200 \text{ J/kg C} \cdot 10 \text{ C}}$$

$$\frac{5}{42} \cdot 10^4 \text{ kg/s} = 12 \cdot 10^4 \text{ kg/s}$$

Introductory Exercise 19.2

1. Rest of the liquid will be heated due to conduction and not convection.

$$2. \frac{dQ}{dt} = \frac{k \cdot 4 \cdot r^2 \cdot (d)}{dr}$$



$$\frac{dQ}{dt} = \frac{dr}{r^2} \cdot 4 \cdot k \cdot d$$

$$\text{or } \frac{dQ}{dt} = \frac{b \cdot dr}{a \cdot r^2} \cdot 4 \cdot k \cdot \frac{T_2}{T_1} \cdot d$$

$$\text{or } \frac{dQ}{dt} = \frac{1}{a} \cdot \frac{1}{b} \cdot 4 \cdot k \cdot (T_2 - T_1)$$

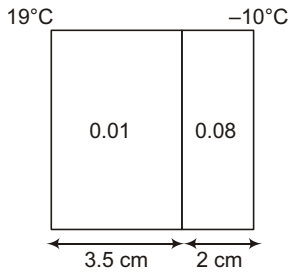
$$\frac{dQ}{dt} = \frac{4 \cdot k \cdot (T_1 - T_2)}{\frac{1}{a} - \frac{1}{b}} = 4 \cdot kab \cdot \frac{T_1 - T_2}{b - a}$$

$$3. \frac{dQ}{dt} = \frac{kA}{t}$$

$$k = \frac{dQ}{dt} \cdot \frac{t}{A}$$

Unit of k = watt $\frac{m}{m^2 \cdot K}$ = W/m-K

$$4. \frac{K_1 A_1}{l_1} = \frac{K_2 A_2}{l_2}$$



$$\frac{0.01(19 - (-10))}{3.5} = \frac{0.08(19 - (-10))}{2}$$

$$\text{or } 2(19 - (-10)) = 28(19 - (-10))$$

$$\text{or } 38 = 280 - 30$$

$$\text{or } \frac{242}{30} = 8.07 \text{ C}$$

$$\frac{dQ}{dt} = \frac{0.01 \cdot 1 \cdot (19 - 8.1)}{3.5 \cdot 10^{-2}}$$

$$7.74 \text{ W/m}^2$$

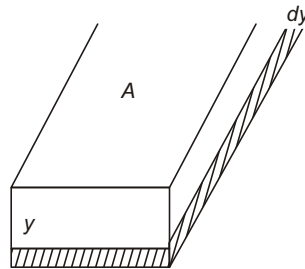
$$5. \frac{dQ}{dt} = \frac{dm}{dt} \cdot L = \frac{0.44 \text{ kg}}{300 \text{ s}} \cdot 2.256 \cdot 10^6 \text{ J/kg}$$

$$\frac{3308.8 \text{ J/s}}{kA} = \frac{50.2 \cdot 0.15 \cdot (100)}{t \cdot 1.2 \cdot 10^{-2}}$$

$$627.5 \cdot (100) = 100 \cdot \frac{3308.8}{627.5} \cdot 5.27$$

$$105.27 \text{ C}$$

$$6. \frac{dQ}{dt} = \frac{kA [0 - (-)]}{y} = \frac{dm}{dt} \cdot L$$



$$\frac{dV}{dt} = L \cdot A \cdot \frac{dy}{dt}$$

$$\frac{kA}{y} = AL \cdot \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{k}{L \cdot y} \quad (\text{Proved})$$

$$7. \frac{dQ}{dt} = e \cdot AT^4$$

$$4 \cdot 5.67 \cdot 10^{-8} \cdot 4 \cdot (4 \cdot 10^{-2})^2 = (3000)^4$$

$$0.4 \cdot 4 \cdot 5.67 \cdot 4^2 \cdot 3^4 \text{ J/s} = 3.7 \cdot 10^4 \text{ watt}$$

$$8. \frac{dQ}{dt} = \frac{1}{R_{th}} \cdot R_{th} \cdot \frac{K}{W} \cdot \text{KW}^{-1}$$

AIEEE Corner

■ Subjective Questions (Level-1)

$$\begin{array}{ccccccc}
 1. & \text{ice} & Q_1 & \text{Water} & Q_2 & \text{Water} & Q_3 & \text{steam} \\
 & 0^\circ\text{C} & & 0^\circ\text{C} & & 100^\circ\text{C} & & 100^\circ\text{C} \\
 & Q & Q_1 & Q_2 & Q_3 & mL_f & ms & mL_v \\
 & 10 & [80 & 1 & 100 & 540] & & \\
 & 10 & 720 \text{ cal} & 7200 \text{ cal} & & & &
 \end{array}$$

2. 10 g of water at 40°C do not have sufficient heat energy to melt 15 g of ice at 0°C, so there will be a mixture of ice-water at 0°C. Let the mass of ice left is *mg*.

$$\begin{array}{ccccccc}
 (15 & m) & 80 & 10 & 1 & 40 \\
 & 15 & m & 5 & m & 10 & g
 \end{array}$$

Mass of ice 10 g

and mass of water (10 + 5) g = 15 g

3. $4 s_P (60 \ 55) \ 1 \ s_R (55 \ 50)$

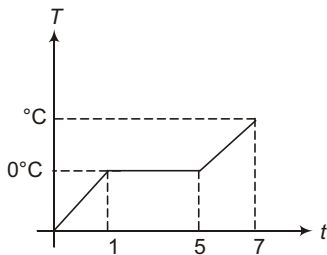
$$\begin{array}{ccccccc}
 & 4s_P & s_R & & & & \\
 1 & s_P (60 \ 55) & 1 & s_Q (55 \ 50) & & &
 \end{array}$$

$$\begin{array}{ccccccc}
 & s_P & s_Q & & & & \\
 1 & s_Q (60 \) & 1 & s_R (\ 50) & & &
 \end{array}$$

or $s_P (60 \) \ 4s_P (\ 50)$

$$\begin{array}{ccccccc}
 260 & 5 & & \frac{260}{5} & 52 & \text{C} &
 \end{array}$$

4. $\frac{dQ}{dt} = \frac{m}{4} \frac{336 \ 10^3 \text{ J/kg}}{60 \text{ s}}$



$$\begin{array}{ccccccc}
 & 1400 \text{ J/kg} & & & & & \\
 & 1400 \text{ mW/kg} & & & & & \\
 \frac{m \ s}{t} & \frac{m}{2} & \frac{4200 (\ 0) \ c}{60 \ s} & & & & \\
 & 1400 & 2 & 60 & & & \\
 & 4200 & & & & &
 \end{array}$$

$$\begin{array}{ccccccc}
 5. & Q & \frac{1}{2} & \frac{1}{2} & mv^2 & ms & mL \\
 & v & \sqrt{4(s \ L)} & & & & \\
 & v & \sqrt{4(125 \ 300 \ 25 \ 10^4)} & & & & \\
 & & \sqrt{4(3.75 \ 2.5) \ 10^4} & & & & \\
 & & \sqrt{4 \ 6.25 \ 10^4} & & 500 \text{ m/s} & &
 \end{array}$$

6. $mg \ h \ ms$

$$\begin{array}{ccccccc}
 \frac{g \ h}{s} & \frac{0.4 \ 10 \ 0.5}{800} & \frac{1}{400} & \text{C} & & &
 \end{array}$$

7. $\frac{K_1 A (\ 0) \ 2.5 \ 10^3 \ \text{C}}{l} \ \frac{K_2 A (100 \)}{l}$

$$\begin{array}{ccccccc}
 (K_1 \ K_2) & 100 \ K_2 & & & & & \\
 \frac{100 \ K_2}{K_1 \ K_2} & \frac{100 \ 46}{390 \ 46} & 1055 \ \text{C} & & & &
 \end{array}$$

8. $i_{CD} \ i_{AC} \ i_{CB}$

$$\begin{array}{ccccccc}
 \frac{KA(\ 25) \ i_{CB}}{l} & \frac{KA(100 \)}{l/2} & \frac{KA(\ 0) \ i_{AC}}{l/2} & & & &
 \end{array}$$

or $25 \ 2(100 \) \ 2$

or $5 \ 225 \ \frac{45 \ \text{C}}{5} \ 4 \ \text{W}$

9. $i_A \ i_C \ i_D$

$$\begin{array}{ccccccc}
 \frac{KA(T_1 \)}{l} & \frac{KA(\ T_3) \ i_D}{3l/2} & \frac{KA(\ T_2) \ i_C}{3l/2} & & & &
 \end{array}$$

$$\begin{array}{ccccccc}
 T_1 & \frac{2}{3} (\ T_3) & \frac{2}{3} (\ T_2) & & & &
 \end{array}$$

or $T_1 \ \frac{2}{3} (T_2 \ T_3) \ 1 \ \frac{4}{3}$

$$\begin{array}{ccccccc}
 T_1 & \frac{2}{3} (T_2 \ T_3) & & & & & \\
 & 7/3 & & & & &
 \end{array}$$

$$\begin{array}{ccccccc}
 3T_1 & 2(T_2 \ T_3) & & & & &
 \end{array}$$

10. $\frac{KA(200 \ 1) \ 7}{l} \ \frac{2KA(\ 1 \ 2) \ 7}{l}$

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$$\frac{3KA(200 - 100)}{l}$$

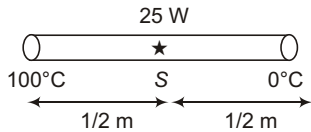
$$200 \times \frac{1}{3} [200 - 100] + 3 \times \frac{1}{3} [200 - 100]$$

$$\frac{3 \times 1 \times 200}{11} = 118.2 \text{ C}$$

$$1 \times \frac{1}{3} [200 - 100] = 145.45 \text{ C}$$

11. 25 $\frac{400 \times 10^4 (100)}{1/2}$

$$\frac{400 \times 10^4 (100)}{1/2}$$



or $25 = 8 \times 10^2 [100 - 0]$

$$\frac{25}{8 \times 10^2} = 100$$

$$\frac{1}{l} \times \frac{106.25 \text{ C}}{1/2 \text{ m}} = 212.5 \text{ C/m}$$

and $\frac{2}{l} \times \frac{206.25 \text{ C}}{1/2 \text{ m}} = 412.5 \text{ C/m}$

12. $\frac{dQ}{dt} = e AT^4$ $0.6 \times 5.67 \times 10^8$

$$0.6 \times 5.67 \times (0.1)^2 \times (1073)^4$$

$$902 \text{ W}$$

13. $\frac{dQ}{dt} = e AT^4$ and $\frac{dQ}{dt} = AT^4$

$$e \frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{210}{700} = 0.3$$

14. $\frac{(80 - 50)_c}{5} = \frac{80 - 50}{2} = 20 \text{ c}$

$$K = \frac{6}{45}$$

$$\frac{(60 - 30)}{t} = \frac{6}{45}$$

$$\frac{60 - 30}{2} = 20 \quad t = 9 \text{ min}$$

■ Objective Questions (Level-1)

1. $\frac{3KA(35 - 0)}{10} = \frac{KA(0 - 0)}{20}$

$$\frac{6(35 - 0)}{10} = \frac{KA(0 - 0)}{20}$$

$$\frac{6 \times 35}{7} = 30 \text{ C}$$

2. $\frac{T_S}{T_N} = \frac{N}{S} = \frac{350}{510} = 0.69$

According to Wien's law

3. $\frac{dQ}{dt} \propto \frac{1}{l^2} \frac{K}{l} = \frac{4}{K} \frac{1}{l^2}$

4. $\frac{dm}{dt} \propto \frac{dQ}{dt} \propto \frac{1}{l^2} \frac{K}{l} = \frac{4}{K} \frac{1}{l^2}$

$$\frac{dm}{dt} \propto \frac{1}{l^2} \frac{K}{l} = \frac{4}{K} \frac{1}{l^2}$$

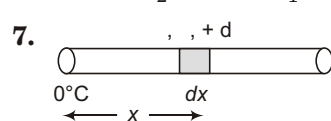
5. $\frac{K_1 A (T_2 - T_1)}{d} = \frac{K_2 A (T_3 - T_2)}{3d}$

$$K_1 (T_2 - T_1) = \frac{1}{3} K_2 (T_3 - T_2)$$

$$K_1 = \frac{1}{3} K_2 \quad K_1 : K_2 = 1 : 3$$

6. $\frac{dQ}{dt} \propto \frac{2K}{2l} \frac{2A}{l} = \frac{KA}{l}$

$$\frac{dQ}{dt} \propto \frac{KA}{l}$$



$$P \frac{dQ}{dt} = \frac{K Ad}{dx} \frac{K_0(1 - \alpha x) A d}{dx}$$

$$l \frac{dx}{dt} = \frac{K_0 A}{P} \frac{10^2}{1} \frac{10^4}{1} \frac{10^3}{1} \frac{1}{1}$$

$$\frac{1}{a} \ln(1 - \alpha x) \Big|_0^l = \frac{10^2}{1} \frac{10^4}{1} \frac{10^3}{1} \frac{1}{1}$$

$$\ln(1 - \alpha l) = \ln 1 - 1$$

$$\ln(1 - \alpha l) = \ln 1 - 1$$

$$\ln(1 - \alpha l) = 1$$

or $1 - \alpha l = e^{-1}$
 or $l = \frac{1}{\alpha} (e - 1) = e - 1 = 1.7 \text{ m}$

8. $\frac{2}{1} \frac{T_1}{T_2} = \frac{2}{3} = 2 \frac{2}{3} \text{ m}$

9. Heat required to boil 1 g of ice is 180 cal while 1 g of steam can release 540 cal during condensation. So, temperature of the mixture will be 100°C with 2/3 g steam and 4.3 g water.

10. T_1, T_2, T_3 as temperature of a body decreases in rate of cooling also decreases such that time increases for equal temperature difference.

11. Conduction is maximum for which thermal resistance is minimum, as $R_{th} = \frac{l}{r^2}$ then for

(a) 50 (b) 25 (c) 100 (d) 33.33,

So option 'b' has minimum resistance.

12. Slope of temperature *versus* heat graph gives increase of specific heat or heat capacity and the portion DE is the gaseous state.

13. $dQ = m s dt = maT^3 dT$
 $\frac{Q}{m} = \frac{a}{4} T^4 \Big|_1^2 = \frac{a}{4} (16 - 1) = \frac{15a}{4}$

14. Resistance becomes 1/4th in parallel of that in series, so times taken will also become 1/4th i.e., 12/4 = 3 min.

15. $ms_1 = 12, ms_2 = 8, s_1 : s_2 = 2 : 3$

16. $\frac{KA(T - T_c)}{\sqrt{2}l} = \frac{KA(T_c - \sqrt{2}T)}{l}$
 $\frac{T}{\sqrt{2}} = \sqrt{2}T - T_c = \frac{T_c}{\sqrt{2}}$
 $\frac{3}{\sqrt{2}} T = \frac{1}{\sqrt{2}} T_c$
 $T_c = \frac{3}{1} \frac{1}{\sqrt{2}} T$

17. $P = \frac{(1000 - 160) W}{2} = \frac{840 W}{2} = 4200 \text{ W}$

$t = \frac{42 \times 10^4}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$

18. $\frac{dQ}{dt} = \frac{KA(T_2 - T)}{x} = \frac{2KA(T - T_1)}{4x}$
 $T_2 - T = \frac{1}{2} T = \frac{1}{2} T_1$
 $T_2 = \frac{1}{2} T_1 + \frac{3}{2} T$
 $T = \frac{2}{3} T_2 = \frac{1}{2} T_1 + \frac{1}{3} (2T_2 - T_1)$
 $\frac{dQ}{dt} = \frac{KA}{x} T_2 = \frac{1}{3} (2T_2 - T_1)$
 $\frac{KA}{x} [3T_2 - 2T_2 - T_1] = \frac{1}{3}$
 $\frac{KA}{x} (T_2 - T_1) = \frac{1}{3}$
 $f = \frac{1}{3}$

19. $\frac{1}{K} = \frac{A}{B} \frac{K_B}{K_A} = \frac{1}{2}$
 $A = \frac{1}{2}, B = 18 \text{ C}$

■ More than One Correct Options

20. Amount of heat radiated or absorbed depends upon. Surface type, surface area, surface temperature and temperature of surrounding, so (a) and (b) are correct.

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21. $\frac{KA(40)}{l} = \frac{KA(30)}{l} = \frac{KA(20)}{l}$
 $\frac{40}{3} = \frac{30}{2} = \frac{20}{1}$

or $30 = 90$
 So, (b) and (d) are correct.

22. $m_1 s_1 (2 - 0) = m_2 s_2 (0 - 4)$
 $4 m_1 s_1 = 4 m_2 s_2$
 $m_1 : m_2 = s_2 : s_1 = 1 : 2$

So, (b) and (c) are correct.

23. In series rate of $R = R_1 + R_2$
 $\frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2}$
 In parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
 $q = q_1 + q_2$ as $q = \frac{1}{R}$

So, (b) and (c) are correct.

24. (a), (c) and (d) are correct.

JEE Corner

■ Assertion and Reason

- Assertion is false.
- According to Wien's law assertion and reason are correct.
- Assertion and reason are true but not correct explanation.
- Assertion is true but reason is false as resistance becomes 1/4th.
- Assertion and reason are both false.
- Assertion is false as this statement was not given by Newton.
- Assertion and reason are both true with correct explanation.
- Both are true but not correct explanation.
- Assertion is false as temperature at different points become different.
- As mass of follow sphere is less so cooling will be faster. So, both are true with correct explanation.

■ Match the Columns

1.

(a)	$\frac{(dQ/dt)}{AT^4}$ [MT ⁻³ L ⁴]	$\frac{ML^2 T^{-2} T^{-1}}{L^2 T^{-4}}$	s
(b)	b	T	L

(c)	$\frac{E}{At}$ [ML ² T ⁻²] [L ² T]	[MT ⁻³]	r
(d)	Rth $\frac{d}{dQ/dt}$ [M ⁻¹ L ² T ³]	[ML ² T ⁻² T ⁻¹]	s

2.

(a)	Slope of line ab	s
(b)	Length of line bc	m
(c)	Solid liquid bc	s
(d)	Only liquid cd	q

3. $\frac{KA(100 - b)}{l} = \frac{KA(b - d)}{l}$
 $\frac{KA(d - 80)}{l}$

100 - b = b - d and
 100 - b = d - 80
 $d = 2b - 100$
 $d = b - 20$
 $b = 40$ C
 $d = 20$ C
 $\frac{40 - 20}{2} = 10$ C

(a) q, (b) p, (c) p, (d) r

4. (a) $ms(1 - 5)$
 $3 \times 1 \times 5 = 1 \times \frac{5}{3} \times q$

$$(b) \frac{ms}{4} \left(\frac{2}{10} \right) + 3ms \left(3 \frac{5}{2} \right) \quad p$$

$$(c) \frac{2ms}{5} \left(\frac{3}{13} \right) + 3ms \left(3 \frac{13}{2} \right) \quad s$$

$$(d) \frac{ms}{6} \left(\frac{4}{14} \right) + 2ms \left(\frac{4}{3} \right) \quad r$$

5.

(a)	s	$\frac{1}{m} \frac{dQ}{d}$	$\frac{J}{kg \cdot C}$	q
(b)	c	ms	$m \frac{dQ}{md}$	J/C
(c)	i	$\frac{dQ}{dt}$	J/s	r
(d)	L	$\frac{E}{m}$	J/kg	s