D 20603 (Pages : 3) Name..............................

Reg. No..............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, OCTOBER 2011

EN 09 301—ENGINEERING MATHEMATICS—III

(2009 Admissions)

[Common to all Branches]

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Determine where the Cauchy-Riemann conditions are satisfied for the function $w = xy^2 + iyx^2$.
2. Define isogonal mapping.
3. Express the residue of a function at an isolated singularity as a contour integral.
4. How do you define the linear span of a set of vectors in a vector space.
5. Write down the complex Fourier transform pair.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Find a function w such that $w + u + iv$ is analytic, given that $v = 3x^2y - y^3$.

7. Find the image of the semi-infinite strip $x > 0, 0 < y < 2$ under the transformation $w = iz + 1$. Show the regions graphically.

8. Evaluate $\int \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, around $|z| = 3$.

9. Show that the vectors $u = (1, 2, -1)$, $v = (2, 1, -2)$, and $w = (1, 4, 3)$ generate R^3.

10. Verify that the triangle inequality is satisfied by the vectors $(0, 1, 1)$ and $(1, 1, 0)$ in R^3.

11. Show that $f(x) = e^{-x^2/2}$ is self-reciprocal under Fourier transform.

(4 × 5 = 20 marks)

Turn over

Part C

Answer **all** *questions as per choice given.*

12. (a) Determine the analytic function $f(z) = u + iv$ if $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ given that $f\left(\dfrac{\pi}{2}\right) = 0$.

Or

(b) Find the invariant points of the transformation $w = -\dfrac{2z + 4i}{iz + 1}$, and prove that these two points, together with any point z and its image w, form a set of four points having a constant cross ratio. What is this cross ratio ?

13. (a) Find the Laurent series expansion of the function $f(z) = \dfrac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ valid in the region $3 < |z + 2| < 5$.

Or

(b) Find the value of the integral $\int_C \dfrac{dz}{z^2(z+4)}$ taken counterclockwise around the circle (i) $|z| = 2$' (ii) $|z + 2| = 3$.

14. (a) Find the co-ordinates of the vector $(2, 1, -6)$ of $V_3(R)$ relative to the basis $F_1 = (1, 1, 2)$, $F_2 = (3, -1, 0)$ and $F_3 = (2, 0, -1)$.

Or

(b) Apply the Gram-Schmidt orthogonalisation process to obtain an orthonormal basis for R^3 from the basis given by $v_1 = (3, 0, 4)$, $v_2 = (-1, 0, 7)$, $v_3 = (2, 9, 11)$.

15. (a) Find the Fourier sine and cosine transforms of $f(x) = 2e^{-5x} + 5e^{-2x}$.

Or

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 2 - |x|, & \text{in } |x| \leq 2 \\ 0, & \text{in } |x| \leq 2 . \end{cases}$$

Hence find the value of $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt$.

(4 × 10 = 40 marks)