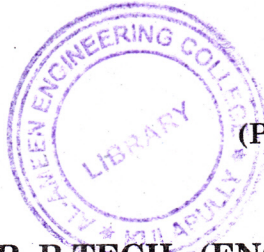


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Name.....

Reg. No.....

**THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION
OCTOBER 2012**

EN 09 301—ENGINEERING MATHEMATICS—III

(2009 Admissions)

[Common to all Branches]

Time : Three Hours

Maximum : 70 Marks

Part A

Answer all questions.

1. Determine constant 'a' such that $u = e^{ax} \cos 3y$ is harmonic.
2. Discuss the nature and location of singularities of the function $f(z) = \frac{\tan z}{z}$.
3. Find the critical points if any of the mapping $W = \sin z$.
4. Show that the set of all 2×2 non-singular matrices is not a vector space.
5. Find the Fourier transform of the function $f(t) = \begin{cases} 5, & -2 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$

(5 × 2 = 10 marks)

Part B

Answer any four questions.

6. Show that $|z|^2$ is not analytic at any point.
7. Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along the line $y = \frac{x}{2}$.
8. Find the Fourier sine transform of $3e^{-2x} + 2e^{-3x}$.
9. If $F(s)$ is the Fourier transform of $f(x)$, then show that $F\{f(ax)\} = \frac{1}{a} F\left(\frac{S}{a}\right)$.
10. Is $U_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $U_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $U_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ a basis of \mathbb{R}^3 .

Turn over

11. If W be a proper sub-space of a finite dimensional vector space V , then show that W is finite dimensional and $\dim W \leq \dim V$.

(4 × 5 = 20 marks)

Part C*Answer all questions as per choice given.*

12. (a) Determine the analytic function $f(z) = u + iV$ where $u + v = (x - y)(x^2 + 4xy + y^2)$.

Or

- (b) Find the image of the first quadrant $x > 0, y > 0$ under $W = \frac{z - i}{z + i}$.

13. (a) (i) Find the Laurent's series expansion of $\frac{e^{2z}}{(z-1)^3}$ about the singularity $z = 1$.

(ii) Evaluate $\int_C \frac{dz}{(z^2 + 4)^2}$; C is $|z - i| = 2$.

Or

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)}$.

14. (a) Find a basis and the dimension of the subspace W of \mathbb{R}^4 generated by

$$(1, -4, 1, 3), (2, -1, 3, -1) \text{ and } (0, 2, 1, -5).$$

Or

- (b) Apply Gram-Schmidt process to the vectors $\beta_1 = (1, 0, 1), \beta_2 = (1, 0, -1), \beta_3 = (0, 3, 4)$ to obtain an orthonormal basis for \mathbb{R}^3 with the standard inner product.

15. (a) Find the Fourier Cosine transform of $f(x) = \frac{1}{1+x^2}$.

Or

- (b) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$$

(4 × 10 = 40 marks)