JNUEE M. Sc. PHYSICS
Solved Paper-2012

1. The general solution of the differential equation, $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$ in terms of two arbitrary constants A and B, is

(a) $e^{1/x}\left(A\cos\left(\frac{1}{x}\right)+B\sin\left(\frac{1}{x}\right)\right)$ (b) $Ax+\frac{B}{x}$

(c) $Ax+Bxe^x$ (d) $Ax+Bx^2$

Soln. **(d)** $x^2\dfrac{d^2y}{dx^2}-2x.\dfrac{dy}{dx}+2y=0$

This is Cauchy-Euler homogeneous equation of second order.

Substituting $x=e^z$, we get

$$x^2\frac{d^2y}{dx^2}=\frac{d^2y}{dz^2}-\frac{dy}{dz} \text{ and } x\frac{dy}{dx}=\frac{dy}{dz}$$

So, the given equation can be written in the form; $\dfrac{d^2y}{dz^2}-3\dfrac{dy}{dz}+2y=0$

Taking $y=c\,e^{mz}$, as a trial solution, we get the auxillary equation as $m^2-3m+2=0\cdot$

The roots of the auxillary equation are $m_1=2$ & $m_2=1$

Therefore, the solution is $y=A\,e^z+B\,e^{2z}=Ax+Bx^2$

2. If a, b and c are non-zero real numbers not equal to 1, $\log_a c$ can be expressed as

(a) $\log_b c/\log_b a$ (b) $\log_b a/\log_b c$ (c) $\log_c a/\log_b a$ (d) $\log_c b/\log_a b$

Soln. **(a)**

3. A homogeneous linear transformation takes the point (1, 1) in the xy-plane to the point (3, 3) and keeps the point (1, –1) fixed (i.e., it remains (1, –1) after the transformation). The matrix corresponding to this transformtion is

(a) $\begin{pmatrix}1&2\\2&1\end{pmatrix}$ (b) $\begin{pmatrix}3&0\\0&2\end{pmatrix}$ (c) $\begin{pmatrix}2&1\\1&2\end{pmatrix}$ (d) $\begin{pmatrix}2&-1\\-1&2\end{pmatrix}$

Soln. **(c)** $\begin{pmatrix}2&1\\1&2\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}3\\3\end{pmatrix}$; $\begin{pmatrix}2&1\\1&2\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix}$

4. The function $\dfrac{1}{\cosh x}$ may be expressed around the point x = 0 as a power series as

(a) $1-\frac{1}{2}x^2+\frac{1}{24}x^4-\frac{1}{720}x^6+....$ (b) $1-\frac{1}{2}x^2+\frac{5}{24}x^4-\frac{61}{720}x^6+....$

(c) $1-\frac{1}{2}x^2+\frac{11}{24}x^4-\frac{331}{720}x^6+....$ (d) $1-\frac{1}{2x^2}+\frac{1}{24x^4}-\frac{1}{720x^6}+....$

Soln. **(b)** $\dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}} = \dfrac{2}{2\left[1+\dfrac{x^2}{2}+\dfrac{x^4}{24}+\dfrac{x^6}{720}\right]} = \dfrac{1}{1+\dfrac{x^2}{2}+\dfrac{x^4}{24}+\dfrac{x^6}{720}} = \left[1+\dfrac{x^2}{2}+\dfrac{x^4}{24}+\dfrac{x^6}{720}+...\right]^{-1}$

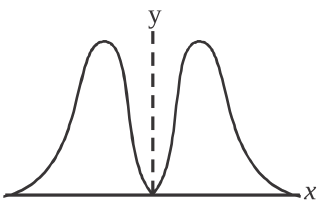
Doing binomial expansion we get,

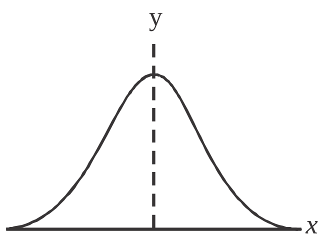
$$\frac{1}{\cos\mathrm{h}\, x} = 1-\frac{x^2}{2}-\frac{x^4}{24}-\frac{x^6}{720}+\left(\frac{x^2}{2}+\frac{x^4}{24}+\frac{x^6}{720}+...\right)^2-\left(\frac{x^2}{2}+\frac{x^4}{24}+\frac{x^6}{720}\right)^3+...$$

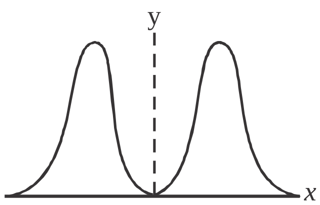
Coefficient of $x^0 = 1$ Coefficient of $x^2 = -\dfrac{1}{2}$ Coefficient of $x^4 = -\dfrac{1}{24}+\dfrac{1}{4} = \dfrac{5}{24}$

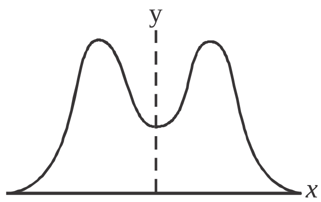
Therefore, $\dfrac{1}{\cos\mathrm{h}\, x} = 1-\dfrac{1}{2}x^2+\dfrac{5}{24}x^4-...$

5. Which of the following graphs gives the best representation of the real-valued function $y = x^2 e^{-x^2}$?

(a)


(b)


(c)


(d)


Soln. (c) $y = x^2 e^{-x^2}$

at $x = 0, y = 0$

at $x = \infty, y = 0$

Since, the given function is a even (symmetric) function of 'x', then option (c) is correct.

6. An observer O uses the coordinate system (x, t) to describe non-relativestic motion in one dimensin. Another observer O′, moving with respect to O with a uniform velocity 'v' (much smaller then the speed of light c) along the positive x-direction, uses (x′, t′), such that at t = 0, t′ = 0 and that instant x and x′ coincide. then

(a) $x' = x - \upsilon t, t' = t, \dfrac{\partial}{\partial x'} = \dfrac{\partial}{\partial x} - \dfrac{1}{\upsilon}\dfrac{\partial}{\partial t}$ and $\dfrac{\partial}{\partial t'} = \dfrac{\partial}{\partial t}$

(b) $x' = x - \upsilon t, t' = t, \dfrac{\partial}{\partial x'} = \dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial t'} = \dfrac{\partial}{\partial t} + \upsilon\dfrac{\partial}{\partial x}$

(c) $x' = x + \upsilon t, t' = t, \dfrac{\partial}{\partial x'} = \dfrac{\partial}{\partial x} + \dfrac{1}{\upsilon}\dfrac{\partial}{\partial t}$ and $= \dfrac{\partial}{\partial t'} = \dfrac{\partial}{\partial t}$

(d) $x' = x + \upsilon t, t' = t, \dfrac{\partial}{\partial x'} = \dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial t'} = \dfrac{\partial}{\partial t} - \upsilon\dfrac{\partial}{\partial x}$

Soln. **(b)** For non-relativistic motion obviously,

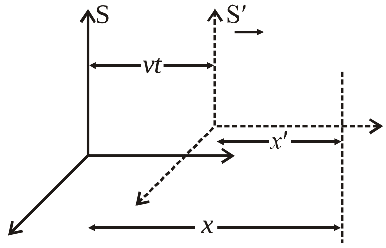
$x' = x - vt; \quad t' = t$

If $y = f(x, t)$

Then, $\dfrac{\partial y}{\partial x'} = \dfrac{\partial y}{\partial x}\cdot\dfrac{\partial x}{\partial x'} + \dfrac{\partial y}{\partial t}\cdot\dfrac{\partial t}{\partial x'}$

$\dfrac{\partial x}{\partial x'} = \dfrac{\partial}{\partial x'}(x' + vt') = 1 \qquad \left\{\because \dfrac{\partial t'}{\partial x'} = 0\right\}$

$\dfrac{\partial t}{\partial x'} = \dfrac{\partial t'}{\partial x'} = 0 \qquad \therefore \quad \dfrac{\partial y}{\partial x'} = \dfrac{\partial y}{\partial x};\ \dfrac{\partial}{\partial x'} = \dfrac{\partial}{\partial x}$



And, $\dfrac{\partial y}{\partial t'} = \dfrac{\partial y}{\partial t}\cdot\dfrac{\partial t}{\partial t'} + \dfrac{\partial y}{\partial x}\cdot\dfrac{\partial x}{\partial t'};\ \dfrac{\partial t}{\partial t'} = \dfrac{\partial t'}{\partial t'} = 1$

$\dfrac{\partial x}{\partial t'} = \dfrac{\partial}{\partial t'}(x' + vt') = \dfrac{\partial x'}{\partial t'} + v\dfrac{\partial t'}{\partial t'} = v \quad \Rightarrow \quad \dfrac{\partial y}{\partial t'} = \dfrac{\partial y}{\partial t} + v\dfrac{\partial y}{\partial x} \quad \Rightarrow \quad \dfrac{\partial}{\partial t'} = \dfrac{\partial}{\partial t} + v\cdot\dfrac{\partial}{\partial x}$

7. A ball dropped from a height h can only attain the height 4h / 5 after bouncing off the floor. If the ball is dropped from a height of 1 m, the time it will take to come to rest is, approximately
[Ignore air resistance and the finite radius of the ball.]

(a) 1.9 s (b) 3.8 s (c) 8.0 s (d) 4.1 s

Soln. (c) Let $e = \dfrac{4}{5}$. Therefore, First height = h, second height = eh, third height = e^2h and so on.

We know that when a ball falls from a height 'h' under gravity then time taken is = $\sqrt{\dfrac{2h}{g}}$

Here, time of striking the floor are $t_1 = \sqrt{\dfrac{2h}{g}}$, $t_2 = t_1 + 2\sqrt{\dfrac{2eh}{g}}$; $t_3 = t_2 + 2\sqrt{\dfrac{2e^2h}{g}}$ and so on.

Therefore, when ball comes to nest, time elapsed is

$$t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2eh}{g}} + 2\sqrt{\frac{2e^2h}{g}} + 2\sqrt{\frac{2e^3h}{g}} + \ldots$$

$$= \sqrt{\frac{2h}{g}}\left\{1 + 2\left[e^{1/2} + e^{2/2} + e^{3/2} + \ldots\right]\right\} = \sqrt{\frac{2h}{g}}\left\{1 + 2e^{1/2}\left[1 + e^{1/2} + e^{2/2} + e^{3/2} + \ldots\right]\right\}$$

$$= \sqrt{\frac{2h}{g}}\left\{1 + \frac{2e^{1/2}}{1 - e^{1/2}}\right\}$$

Taking $h = 1m$, $g = 10$, $e = \dfrac{4}{5}$ we get, $t = 8.02$ sec.

8. A small raindrop of mass m experiences a viscous drag force $F_d = bv$, proportional to its instantaneous speed 'v'. If it starts from rest at a height h, its speed after a time 't' is

(a) $\upsilon(t) = \dfrac{mg}{b}\tanh\left(\dfrac{bt}{m}\right)$ (b) $\upsilon(t) = \dfrac{mg}{b}e^{-bt/m}$

(c) $\upsilon(t) = \dfrac{mg}{2b}(1 - e^{-2bt/m})$ (d) $\upsilon(t) = \dfrac{mg}{b}(1 - e^{-bt/m})$

Soln. (d) Equation of motion of rain drop is $m\dfrac{dv}{dt} = mg - bv$

$$\int_0^v \frac{dv}{mg - bv} = \frac{1}{m}\int_0^t dt \quad -\frac{1}{b}\left[\log(mg - bv)\right]_0^v = \frac{t}{m}$$

$$\Rightarrow \log\left(\frac{mg - bv}{mg}\right) = -\frac{bt}{m} \quad \Rightarrow \quad \frac{mg - bv}{mg} = e^{-bt/m} \quad \Rightarrow v = \frac{mg}{b}(1 - e^{-bt/m})$$

9. The nature of flow in a viscous liquid is characterised by the dimensionless Reynolds' number Re proportional to υ (the flow velocity) : Re $\propto \upsilon$. Given that Re also depends on (i) the density ρ of the fluid, (ii) the dynamical viscosity η and (iii) a characteristic length l, of the flow. By dimensional analysis, we find that

(a) $\text{Re} = \frac{\eta\omega}{\rho}$ (b) $\text{Re} = \frac{\rho\omega}{\eta}$ (c) $\text{Re} = \frac{\rho\eta\upsilon}{l}$ (d) $\text{Re} = \frac{\rho\upsilon}{\eta l}$

Soln. **(b)** Given $Re \propto v$

$\Rightarrow$ $[Re] = [v]\cdot[\rho]^a\,[\eta]^b\,[l]^c$; $[M^0\,L^0\,T^0] = [LT^{-1}]\,[ML^{-3}]^a\,[ML^{-1}\,T^{-1}]^b\,[L]^c$

Equating exponents of M, L and T we get,

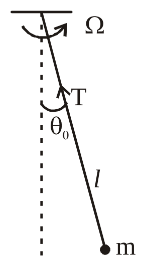
$a + b = 0$

$1 - 3a - b + c = 0$

$-1 - b = 0$

Solving above equations we get, $b = -1$; $a = 1$; $c = 1$ $\Rightarrow Re = \frac{\rho l v}{\eta}$

10. A ball of mass m is hung from a support'by a massless wire of length l. The support is rotated with an angular speed. $\Omega > \sqrt{g/l}$ around a vertical axis through the point of suspension as shown in the figure. The ball rests in equilibrium at an angle θ_0. Which the following statemnts concering θ_0 and the rension T, is true ?



(a) $\theta_0 = 0$ and $T = mg$ (b) $\theta_0 = \text{tin}^{-1}\left(\frac{g}{\Omega^2 l}\right)$ and $T < mg\cos\theta_0$

(c) $\theta_0 = \sin^{-1}\left(\frac{g}{\Omega^2 l}\right)$ and $T > mg\cos\theta_0$ (d) $\theta_0 = \cos^{-1}\left(\frac{g}{\Omega^2 l}\right)$ and $T > mg\cos\theta_0$

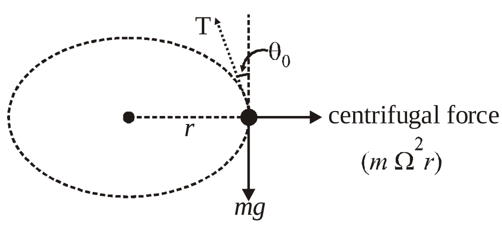
Soln. **(d)** Free body diagram is shown in the figure considering equilibrium of the ball.
(Note that we are considering equilibrium of the ball that is why centrifugal force is being taken, if we consider rotation of the ball then centripetal force is taken).

$\Rightarrow$ $T\cos\theta_0 = mg$ ... (i) and $T\sin\theta_0 = m\Omega^2 r$ ... (ii)

Since $r = l\sin\theta_0$ $\Rightarrow$ $T\sin\theta_0 = m\Omega^2 l\sin\theta_0$ $\Rightarrow$ $T = m\Omega^2 l$ ... (iii)

from (i) $T = \frac{mg}{\cos\theta_0} \Rightarrow T = \frac{mg\cos\theta_0}{\cos^2\theta_0}$ $\Rightarrow \cos^2\theta_0 = \frac{mg\cos\theta_0}{T}$

$\because \theta_0 > 0$ $\cos^2\theta_0 < 1$



$\Rightarrow \frac{mg\cos\theta_0}{T} < 1$ $\Rightarrow T > mg\cos\theta_0$

Using value of T from (iii) into (i) we get,

$m\Omega^2 l\cos\theta_0 = mg$ $\Rightarrow \cos\theta_0 = \frac{g}{\Omega^2 l}$ $\Rightarrow \theta_0 = \cos^{-1}\left(\frac{g}{\Omega^2 l}\right)$

11. In a wire loop of resistance R and inductance L, an e.m.f. ε is switched on at t = 0. The magnetic flux through the loop is biven by

(a) $\frac{L\varepsilon}{R}\left(1-e^{-tR/L}\right)$ (b) $\frac{L\varepsilon}{R}e^{-tR/L}$ (c) $\frac{L\varepsilon}{R}\left(1-\frac{L}{tR}\right)$ (d) $\frac{L\varepsilon}{R}$

Soln. **(a)** Magnetic flux = LI where I = current through the loop.

Since, $I = \frac{\varepsilon}{R}(1-e^{-tR/L}) \Rightarrow$ flux $= \frac{L\varepsilon}{R}(1-e^{-tR/L})$

12. The electric and magnetic fields of an electromagnetic wave in vacuum are given by

$\mathbf{E} = \hat{i}E_0\sin(kz-\omega t)$ and $\mathbf{B} = \hat{j}B_0\sin(kz-\omega t)$ respectively. Which of the following relatins is correct ?

(a) $k^2E_0 = \omega^2B_0$ (b) $\omega E_0 = kB_0$ (c) $kE_0 = \omega B_0$ (d) $E_0B_0 = \omega k$

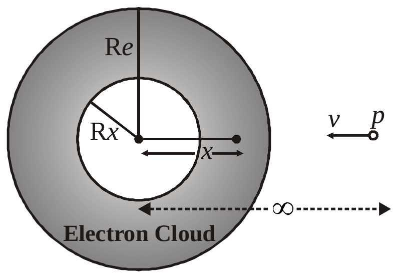
Soln. (c) For electromagnetic waves, $\frac{E_0}{B_0} = c$ and $c = \frac{\omega}{k}$ $\Rightarrow \frac{E_0}{B_0} = \frac{\omega}{k} \Rightarrow kE_0 = \omega B_0$

13. The radius of the nucleus of the Ra atom, which carries an electric charge +88e, is 7.0×10^{-15} m. What should roughly be the speed of a proton, if it has to reach as close as 1.0×10^{-14} m from the centre of the nucleus?[The radius of the cloud of orbital clectrons of the Ra atom is approximately 5.0×10^{-11}m.]

(a) 6.7×10^9 m/s (b) 3.1×10^8 m/s (c) 1.4×10^5 m/s (d) 4.9×10^7 m/s

Soln. **(d)** From conservation of energy we get,

Loss in K.E. = Gain in P.E; $\frac{1}{2}mv^2 = eV$



where V = electric potential at the final point.

Actually, $V = \frac{88e}{4\pi\epsilon_0 x} - V'$

x = distance of the point from center of nucleus where

V′ = potential due to electron cloud

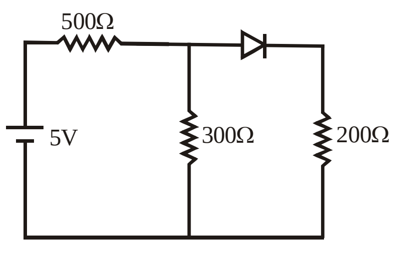
In the expression of V′ radius of electron cloud (R_e) comes

In the denominator, As $R_e >> x$

$\therefore \quad V' << \frac{88e}{4\pi\epsilon_0 x} \Rightarrow V \simeq \frac{88e}{4\pi\epsilon_0 x} \Rightarrow \frac{1}{2}mv^2 = \frac{88e^2}{4\pi\epsilon_0 x}$

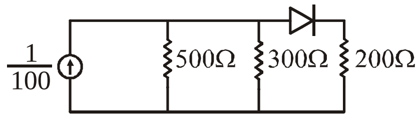
$\frac{1}{2} \times 1.67 \times 10^{-27} v^2 = \frac{9\times10^9\times88\times(1.6\times10^{-19})^2}{1.0\times10^{-14}} \Rightarrow v \simeq 4.5\times10^7$ m/s

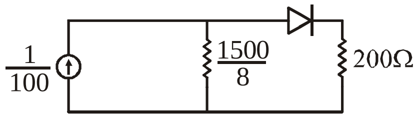
14. In the circuit shown below, the diode is non-ideal and has a voltage drop of 0.7 V What is the value of the diode current ?

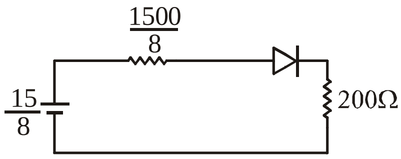


(a) 4.84 mA (b) 8.06 mA (c) 3.03 mA (d) 6.25 mA

Soln. **(c)**







$$I = \frac{1.175 - 0.7}{\left(\frac{1500}{8} - 200\right)} = 3.03\text{mA}$$

15. The Doppler width $\Delta\lambda$ of the orange line (for which λ = 6058 Å) of Kr is 0.0055 Å. What is the spread in frequency of this spectral line?

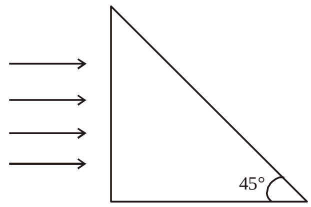
(a) 2.7×10^7 Hz (b) 2.7×10^9 Hz (c) 4.5×10^6 Hz (d) 4.5×10^8 Hz

Soln. **(d)** We know that, $c = \nu\lambda$

$\therefore \quad \Delta c = 0 = \lambda\Delta\nu + \nu\Delta\lambda$

$$\therefore \quad |\Delta\nu| = \frac{\nu}{\lambda}|\Delta\lambda| = \frac{c}{\lambda^2}|\Delta\lambda| = \frac{3\times10^8 \times 5.5\times10^{-3} \times 10^{-10}}{(6.058)^2 \times 10^6 \times 10^{-20}} \simeq 4.5 \times 10^8 \text{ Hz}$$

16. A beam of light, consisting of red (R), green (G) and blue (B) colours, is incident normal to a ace on a face on a right- angled prism (see figure). The refractive indices of the material of the prosm for R, G and B wavelengths are 1.39, 1.44 and 1.47 respectively. Then



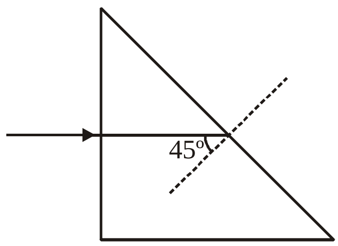
(a) R, G and B get transmitted (without underging total internal reflection)

(b) R and G undergo total internal reflection and B gets transmitted

(c) R gets transmitted, while G and B undergo total internal reflection

(d) All of R, G and B undergo total internal reflection

Soln. **(c)** Critical angle for red. $\theta_R = \sin^{-1}\left(\frac{1}{\mu_R}\right) = \sin^{-1}\left(\frac{1}{1.39}\right) \simeq 46^\circ$



Critical angle for green $\theta_G = \sin^{-1}\left(\frac{1}{1.44}\right) \simeq 44^\circ$

Critical angle for blue $\theta_B = \sin^{-1}\left(\frac{1}{1.47}\right) \simeq 43^\circ$

Here angle of incidence at the interface as shown in the figure is 45°.

$\therefore \quad i > \theta_G, \theta_B$. But $i < \theta_R$.

Therefore, R gets transmitted, while G and B undergo TIR.

17. The two slits in a Young's double-slit experiment are of unequal width, one being four times wider than the other. If I_{max} and I_{min} denote the intensities at a neighbouring maximum and a minimum, then the ratio I_{min}/I_{max} is

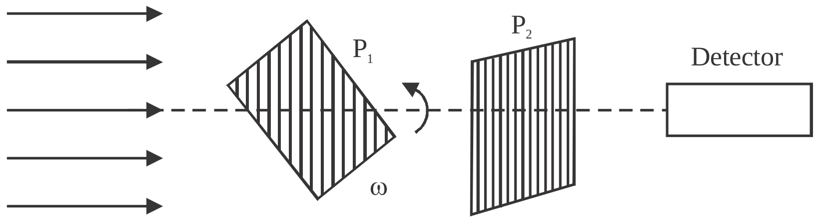
(a) $\frac{1}{9}$ (b) $\frac{1}{4}$ (c) $\frac{3}{5}$ (d) 0

Soln. (a) Given, $\frac{I_1}{I_2} = 4 : 1$

We know that, $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ and $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

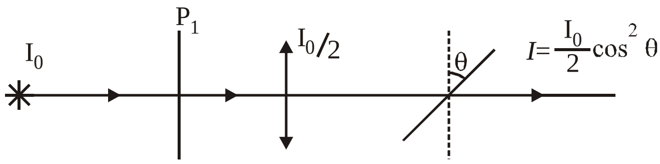
$$\Rightarrow \quad \frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \left(\frac{2-1}{2+1}\right)^2 = \frac{1}{9}$$

18. A linear beam of unpolarised light passes through tow plane polarisers, the planes of which are perpendicular to the direction of propagation of the beam. The first polariser rotates around this direction with an angular velocity of 20π radians per second. If the initial intensity of the light is I_0, then the intensity when it leaves the second polariser



(a) is periodic with frequency of 20 Hz and maximum of $I_0/4$
(b) is periodic with frequency of 20 Hz and maximum of $I_0/2$
(c) is periodic with frequency of 10 Hz and maximum of $I_0/4$
(d) is periodic with frequency of 10 Hz and maximum of $I_0/2$

Soln. **(d)** According to Malus' Law. If pass axis makes an angle θ then transmitted intensity becomes $\cos^2\theta$ times for polarized light.



$\therefore \quad I = \frac{I_0}{2}\cdot\cos^2\theta$ since, $\theta = w\,t = 2\pi\nu t$ $\Rightarrow I = \frac{I_0}{2}\cdot\cos^2 2\pi\nu t$

Given, $2\pi\nu = 20\pi \quad \Rightarrow \quad \nu = 10\text{Hz}$

19. The Boolean expression B.(A + B) + A (A + $\overline{B}$) can be realised using a minimum number of
(a) 1 OR gate (b) 1 AND gate (c) 2 OR gates (d) 2 AND gates

Soln. (a) $B\cdot(A + B) + A\cdot(A + \overline{B}) = BA + B\cdot B + A\cdot A + A\cdot\overline{B}$

$= BA + B + A + A\overline{B}$ [$\because$ $B\cdot B = B,\ A\cdot A = A$]
$= B + A$ [used absorption law]

20. An ideal diatomic gas $(\text{of } \gamma = 5/3)$ is expanded adiabatically so that its volume is doubled. By what ratio is its temperature reduced in this process?

(a) 1/2 (b) $1/2^{1/3}$ (c) $1/2^{2/3}$ (d) $1/2^{5/3}$

Soln. **(c)** In adiabatic process, $TV^{\gamma-1}$ = constant

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \quad \Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = (2)^{\frac{5}{3}-1} = 2^{2/3} \quad \left\{\because \frac{V_2}{V_1} = 2\right\} \quad \Rightarrow \frac{T_2}{T_1} = 1/2^{2/3}$$

21. Two buckets B_1 and B_2, each containing 25 litres of water, are initially at temperatures T_1 and T_2, respectively. Now take 1 litre of water from B_1, put it in B_2 and allow thermal equilibrium to be established. Then take 1 litre of water from B_2, put it back in B_1 and again allow it to come to thermal equilibrium. At the end of this cycle the amount of water in each bucket does not change, but their temperatures will change. When this process is repeated, the difference in temperature reduces by the same factor after each cycle.

If $|T_1 - T_2|$ was 40ºC to begin with, what would be its value after 5 cycles?

(a) 27 ºC (b) 10 ºC (c) 19 ºC (d) 35 ºC

Soln. **(a)** After the first half cycle let equilibrium temperature becomes T.
Therefore, Heat lost by 25 litre = heat gained by 1 litre

$$25(T_1 - T) = 1\times(T - T_2) \quad \Rightarrow \quad T = \frac{25T_1 + T_2}{26}$$

Let T' be the equilibrium temperature after one complete cycle.
Therefore, using principle of calorimetry
Heat lost by 1 litre = Heat gained by 24 litre

$$\text{We get, } T' = \frac{24T_2 + T}{25} \text{ \{similar to previous case\}} = \frac{24T_2 + \frac{25T_1 + T_2}{26}}{25}$$

$$\text{Temperature difference } = T - T' = \frac{25T_1 + T_2}{26} - \frac{24T_2}{25} - \frac{25T_1 + T_2}{25\times 26}\,;\ \Delta T_1 = \frac{24}{26}|T_1 - T_2|$$

After 2nd cycle, temperature difference becomes $\Delta T_2 = \Delta T_1 \cdot \frac{24}{26}$

Therefore, after fifth cycle temperature difference becomes

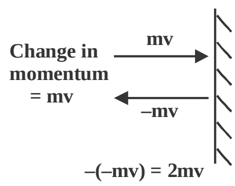
$$\Delta T_5 = \Delta T_4 \frac{24}{26} = \left(\frac{24}{26}\right)^2 \Delta T_3 = \left(\frac{24}{26}\right)^3 \Delta T_2 = \left(\frac{24}{26}\right)^4 \Delta T_1 = \left(\frac{24}{26}\right)^5 \Delta T = \left(\frac{24}{26}\right)^5 \times 40 = 26.8^\circ\text{C} \simeq 27^\circ\text{C}$$

22. A flat plate is constantly being bombarded from one side by particles of mass m. If the number density of the particles is ρ and they strike the plate with speed υ along the normal to the plate, the pressure exerted on the plate is

(a) $m\rho\upsilon^2$ (b) $2m\rho\upsilon^2$ (c) $m\rho\upsilon$ (d) $2m\rho\upsilon$

Soln. **(a)** For the elastic collision, the momentum transfered to the plate per collission = 2mv

In this case the pressure is $2m\rho v^2$



In the case bombarded particles get stuck to the plate after collision, (perfect inelastic collision) in other words. If the particles get absorbed by the plate, the momentum transfered per collission is mv and the pressure is $m\rho v^2$. In the question it should have been made clear whether the collision is elastic or inelastic.

23. Helium atoms at low temperatures make a perfect closed pack structure of hexagonal lattice with parameters a = 0.36 nm and c = 0.59 nm. The density of the crystal is approximately

(a) 2000 kg/m^3 (b) 100 kg/m^3 (c) 123 kg/m^3 (d) 200 kg/m^3

Soln. **(b)** In a conventional cell of hexagonal structure number of atoms present (N) = 6.

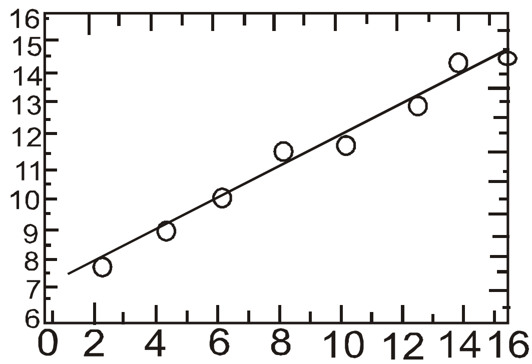
Volume of conventional unit cell (v) = $6\frac{\sqrt{3}}{2}a^2\cdot c$

Number density (n) = $\frac{\text{N}}{\text{V}}=\frac{6}{\text{V}}=\frac{6}{6\times\frac{\sqrt{3}}{2}a^2.c}=\frac{2}{\sqrt{3}\,a^2c}$

Mass density = Mass of one helium atom × number density

$$= 4\times 1.67\times 10^{-27}\times\frac{2\times10^{27}}{\sqrt{3}\times(0.36)^2\times0.59} = 100.88 \text{ kg/m}^3$$

24. The ratio of the specific heat capacity and temperature, C_υ/T, of Cu is plotted as a function of T^2, the square of the absolute temperature, in the graph below:



The values of γ and β (the coefficients corresponding to the electronic and the vibrational components of the specific heat) are, approximately

(a) $\gamma = 7.0\times10^{-4}\,\text{Jmol}^{-1}\text{K}^{-2}\text{and}\,\beta = 5.0\times10^{-5}\,\text{Jmol}^{-1}\text{k}^{-4}$

(b) $\gamma = 5.0\times10^{-5}\,\text{J mol}^{-1}\,\text{k}^{-2}\text{and}\,\beta = 7.0\times10^{-4}\,\text{J mol}^{-1}\text{k}^{-4}$

(c) $\gamma = 1.4\times10^{-3}\,\text{J mol}^{-1}\,\text{k}^{-2}\text{and}\,\beta = 7.0\times10^{-4}\,\text{J mol}^{-1}\text{k}^{-4}$

(d) $\gamma = 5.0\times10^{-4}\,\text{J mol}^{-1}\,\text{k}^{-2}\text{and}\,\beta = 7.0\times10^{-5}\,\text{J mol}^{-1}\text{k}^{-4}$

Soln. **(a)** Specific heat varies as, $\frac{C}{\text{T}}=\gamma+\beta\text{T}^2$

$\Rightarrow$ γ = Intercept = 7×10^{-4} J mol^{-1} k^{-2} & β = slope = $\frac{(9-8)\times10^{-4}}{4-2}$ = 5×10^{-5} J mol^{-1} k^{-4}

25. A paramagnetic gas at room temperature is placed in an external magnetic field of 1.5 T (tesla). Each atom of the gas has a magnetic moment $\mu = 1.0\,\mu_{B,}$ where $\mu_B = 9.3\times10^{-24}$ J/T is the Bohr magneton. The difference in energy when an atom is aligned along the magnetic field and opposite to it, is

(a) 2.8×10^{-23} J (b) 1.4×10^{-23} J (c) 18.6×10^{-24} J (d) 9.3×10^{-24} J

Soln. **(a)** When atom is along the field , $E_1 = -\mu B$

when atom is opposite to the field , $E_2 = +\mu B$

$\therefore$ $\Delta E = E_2 - E_1 = 2\mu B = 2\times1.0\times9.3\times10^{-24}\times1.5 = 2.8\times10^{-23}$ J.

26. The Fermi energy ε_F in metals depends on the number density n_e of mobile electrons, which may be thought of as a free Fermi gas. If n_e of one metal is larger by a factor of 1000 compared to another, then in comparison, its Fermi energy is

(a) 1000 times larger
(b) smaller by a factor of 1/100
(c) 100 times larger
(d) 10 times larger

Soln. **(c)** Fermi energy is proportional to number density as $E_F \propto n^{2/3}$ $\Rightarrow \left(\dfrac{E_1}{E_2}\right) = \left(\dfrac{n_1}{n_2}\right)^{2/3}$

Given $n_1 = 1000 n_2 \quad \Rightarrow \dfrac{E_1}{E_2} = (1000)^{2/3} = 100 \quad \Rightarrow E_1 = 100E_2$

27. The kinetic energy of a proton and an α–particle (not under the influence of any force) are given to be equal. If we denote the de Broglie wavelengths of the proton by λ_p and that of the α–particle by λ_α, then

(a) $\lambda_p \simeq \lambda_\alpha$
(b) $\lambda_p \simeq 4\lambda_\alpha$
(c) $\lambda_p \simeq \dfrac{1}{2}\lambda_\alpha$
(d) $\lambda_p \simeq 2\lambda_\alpha$

Soln. **(d)** For a given kinetic energy K, de-Broglie wavelength λ is given as, $\lambda = \dfrac{h}{\sqrt{2mK}}$

$\Rightarrow \quad \dfrac{\lambda_\alpha}{\lambda_p} = \sqrt{\dfrac{m_p}{m_\alpha}}$ since, $m_\alpha \simeq 4m_p$ then, $\dfrac{\lambda_\alpha}{\lambda_p} \simeq \sqrt{\dfrac{1}{4}} \quad \Rightarrow \lambda_p \simeq 2\lambda_\alpha$

28. When a monochromatic point source of light is placed at a distance of 0.2 m from a photoelectric cell, the stopping potential V_s and the saturation current I_s are found to be 0.6 V and 18.0 mA, respectively. If the same source is now placed 0.6 m away from the photoelectric cell, one finds

(a) $V_S = 0.2$ V and $I_S = 6.0$ mA
(b) $V_S = 0.6$ V and $I_S = 6.0$ mA
(c) $V_S = 0.6$ V and $I_S = 2.0$ mA
(d) $V_S = 0.2$ V and $I_S = 18.0$ mA

Soln. **(c)** Intensity of a point source follows inverse square law i.e., $I \propto \dfrac{1}{r^2}$

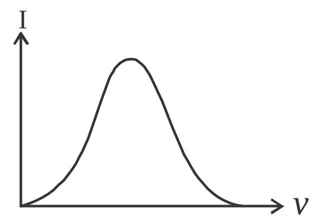
$$\Rightarrow \quad \frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{0.2}{0.6}\right)^2 = \frac{1}{9} \quad \Rightarrow I_2 = \frac{1}{9}I_1$$

Stopping potential depends on energy of photon and saturation current depends on intensity of light. Here, energy of photon is unchanged, therefore, stopping potential will remain the same.

Intensity falls by a factor of $\dfrac{1}{9}$. Therefore, saturation current will also fall by this factor.

$$\Rightarrow \quad I_{s'} = \frac{1}{9} I_s = \frac{1}{9} \times 18.0 \text{ mA} = 2\text{mA} \quad \text{and } V_{s'} = V_s = 0.6\text{v}$$

29. The graph in the fuigure below shows the intensity I as a function of frequency v of a perfect blackbody at a fixed temperature T:



The corresponding graph at teperature 2T can be obtained by which of the following operations ?

For every point of the graph
(a) multiply the v-coordinate by 1 / 2 and the I-coordinate by 8
(b) multiply the v-coordinate by 2 and the I-coordinate by 8
(c) multiply the v-coordinate by 1 / 2 and the I-coordinate by 16
(d) multiply the v-coordinate by 2 and the I-coordinate by 16

Soln. **(d)** Wein's law $\lambda_m T = b \Rightarrow \frac{c}{v_m}T = b \Rightarrow v_m = \frac{cT}{b}$
Therefore, if T is doubled, v_m will be doubled.
Stefan's law, $I \propto T^4$. Therefore, if 'T' is doubled, I will become 16 times.

30. What is the maximum theoretical accuracy ΔE to which an ideal experiment may determine the energy levels of the hydrogen atom ?
[Hint : Use the fact that the age of the universe is estimated to be approximately 1.4×10^{10} years.]
(a) 4.7×10^{-26} eV (b) 9.4×10^{-33} eV (c) 1.2×10^{-63} eV (d) 2.4×10^{-70} eV

Soln. **(b)** $\Delta E \geq \frac{h}{\Delta t} = \frac{6.63\times10^{-34}}{1.4\times10^{10}\times365\times24\times60\times60}$ Joule $= 15.015\times10^{-52}$ Joule $\simeq 9.4\times10^{-33}$ eV

31. A particle in one dimension is in the ground state (lowest energy quantum state) of the potential well given by

$$V(x)\begin{cases}0 & \text{for} |x| < \frac{L}{2} \\ \infty & \text{otherwise}\end{cases}$$

Let P_+ be the probability that the particle is found to move along the positive x-direction and p be the magnitude of the momentum for that state of motion. Then

(a) $P_+ = 0$ and $p = 0$ (b) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{2L}$ (c) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{L}$ (d) $P_+ = 1$ and $p = \frac{\pi}{L}$

Soln. **(c)** Let $\varphi = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$ {we could have taken cosine function as well}

$$P_+ = \int_0^{L/2}\phi^*\phi\,dx = \frac{2}{L}\int_0^{L/2}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/2}\frac{1}{2}\left(1-\cos\frac{2\pi x}{L}\right)dx = \frac{1}{2}$$

Magnitude of momentum $= \frac{n\pi\hbar}{L}$

Therefore, for ground state ($n = 1$), momentum $p = \frac{\pi\hbar}{L}$

32. A particle of mass m is moving in a three-dimensional potential

$$V(x,y,z) = \frac{1}{2}m\omega^2\left(x^2+2y^2+4z^2\right)$$

The energy of the particle in the ground state (lowest energy quantum state) is

(a) $\frac{\sqrt{7}}{2}\hbar\omega$ (b) $\frac{3}{2}\hbar\omega$ (c) $\frac{7}{2}\hbar\omega$ (d) $\frac{\left(3+\sqrt{2}\right)}{2}\hbar\omega$

Soln. **(d)** $V = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m(\sqrt{2}\,\omega)^2y^2 + \frac{1}{2}m(2\omega)^2z^2$, Kinetic energy $= \frac{p^2}{2m} = \frac{1}{2m}\left[\frac{n^2\pi^2\hbar^2}{L^2}\right]$

Energy of the particle, $E = E_1 + E_2 + E_3 = \hbar\omega\left(n_1 + \frac{1}{2}\right) + \hbar\sqrt{2}\,\omega\left(n_2 + \frac{1}{2}\right) + \hbar(2\omega)\left(n_3 + \frac{1}{2}\right)$

For ground state $n_1 = n_2 = n_3 = 0$

$$E = \hbar\omega\left(\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{2}{2}\right) = \frac{\hbar\omega}{2}\left(3 + \sqrt{2}\right)$$

33. A nucleus may be modelled as a drop of liquid consisting of the nucleons (protons and neutrons). In this model, the dominant contribution to the nuclear binding energy is from the volume, which is proportional to A, the total number of nucleons. Then the two important subdominant contributions from the surface tension and the coulomb repulsion of the protons are, proportional to

(a) $A^{2/3}$ and $Z / A^{1/3}$ respectively (b) $A^{2/3}$ and $Z^2 / A^{1/3}$ re spectively

(c) $A^{1/3}$ and $Z^2 / A^{2/3}$ respectively (D) $A^{1/2}$ and $Z^2 / A^{1/3}$ respectively

Soln. **(b)** For nucleus, $R = R_0 A^{1/3}$ $\Rightarrow$ $B.E_{surface} \propto$ Area $\propto R^2$ $\Rightarrow B.E_{surface} \propto A^{2/3}$

and $B.E_{coulombic} \propto \frac{Z^2}{R} \propto \frac{Z^2}{A^{1/3}}$