Risk Management and Financial Institutions

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Chapter 3 How Traders manage Their Exposures

1. **Linear products**: a product whose value is linearly dependent on the value of the underlying asset price. Forward, futures, and swaps are linear products; options are not.

E.g. Goldman Sachs have entered into a forward with a gold mining firm. Goldman Sachs borrows gold from a central bank and sell it at the current market price. At the end of the life of the forward, Goldman Sachs buys gold from the gold mining firm and uses it to repay the central bank.

- 2. **Delta neutrality** is more feasible for a large portfolio of derivatives dependent on a single asset. Only one trade in the underlying asset is necessary to zero out delta for the whole portfolio.
- 3. **Gamma**: if it is small, delta changes slowly and adjustments to keep a portfolio delta neutral only need to be made relatively infrequently.

$$Gamma = \frac{\partial^2 \Pi}{\partial S^2}$$

- Gamma is positive for a long position in an option (call or put).
- A linear product has zero Gamma and cannot be used to change the gamma of a portfolio.
- 4. Vega
- Spot positions, forwards, and swaps **do not** depend on the volatility of the underlying market variable, but options and most exotics do.

$$- v = \frac{\partial \Pi}{\partial \sigma}$$

- Vega is positive for long call and put;
- The volatilities of short-dated options tend to be more variable than the volatilities of long-dated options.
- 5. Theta: time decay of the portfolio.
- **Theta is usually negative for an option**. An exception could be an in-the-money European put option on a non-dividend-paying stock or an in-the-money European call option on a currency with a very high interest rate.
- It makes sense to hedge against changes in the price of the underlying asset, but it does not make sense to hedge against the effect of the passage of time on an option portfolio. In spite of this, many traders regard theta as a useful statistic. In a delta neutral portfolio, when theta is large and positive, gamma tends to be large and negative, and vice versa.
- Rho: the rate of change of a portfolio with respect to the level of interest rates.
 Currency options have two rhos, one for the domestic interest rate and one for the foreign interest rate.
- 7. Taylor expansions:
- if ignoring terms of higher order dt and assuming delta neutral and volatility and interest rates are constant, then

$$\Delta \Pi = \Theta \Delta t + \frac{1}{2} \Gamma \Delta S^2$$

- Options traders make themselves delta neutral or close to delta neutral at the end of each day. Gamma and vega are monitored, but are not usually managed on a daily basis.
- There is one aspect of an options portfolio that mitigates problems of managing gamma and vega somewhat. Options are often close to the money when they are first sold so that they have relatively high gammas and vegas. However, as time elapses, the underlying asset price has often changed sufficiently for them to become deep out of the money or deep in the money. Their gammas and vegas are then very small and of little consequence. The nightmare scenario for a trader is where written options remain very close to the money as the maturity date is approached.

Chapter 4 Interest Rate Risk

- 1. LIBID, the London Interbank Bid Rate. This is the rate at which a bank is prepared to accept deposits from another bank. The LIBOR quote is slightly higher than the LIBID quote.
- Large banks quote 1, 3, 6 and 12 month LIBOR in all major currencies. A bank must have an AA rating to qualify for receiving LIBOR deposits.
- How LIBOR yield curve be extended beyond one year? Usually, create a yield curve to represent the future short-term borrowing rates for AA-rated companies.
- The LIBOR yield curve is also called the swap yield curve or the LIBOR/swap yield curve.
- Practitioners usually assume that the LIBOR/swap yield curve provides the risk-free rate. **T-rates are regarded as too low to be used as risk-free rates because**:
- a. T-bills and T-bonds must be purchased by financial institutions to fulfill a variety of regulatory requirements. This increase demand for these Ts driving their prices up and yields down.
- b. The amount of capital a bank is required to hold to support an investment in Ts is substantially smaller than the capital required to support a similar investment in other very low-risk instruments.
- c. In USA, Ts are given a favorable tax treatment because they are not taxed at the state level.
- 2. Duration

- Continuous compounding.
$$D = -\frac{1}{B}\frac{dB}{dy} = \sum_{i=1}^{n} t_i (\frac{c_i e^{-yt_i}}{B}), \Delta B = -BD\Delta y$$

This is the weighted average of the times when payments are made, with the

weight applied to time t being equal to the proportion of the bond's total present value provided by the cash flow at time t.

- Compounding m times per year, then modified duration. $D^* = \frac{D}{1 + v/m}$
- 3. Convexity.

- $C = \frac{1}{B} \frac{d^2 B}{dy^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-yt_i}}{B}$. This is the weighted average of the square of the time

to the receipt of cash flows.

$$\Delta B = -BD\Delta y + \frac{1}{2}BC(\Delta y)^2$$

- The duration (convexity) of a portfolio is the weighted average of the durations (convexity) of the individual assets comprising the portfolio with the weight assigned to an asset being proportional to the value of the asset.
- The convexity of a bond portfolio tends to be greatest when the portfolio provides payments evenly over a long period of time. It is least when the payments are concentrated around one particular point in time.
- Duration zero \rightarrow protect small parallel shit in the yield curve.
- Both duration and convexity zero \rightarrow protect large **parallel shifts** in the yield curve.
- Duration and convexity are analogous to the delta and gamma.

4. Nonparallel yield curve shifts

- Definition: $D_p = -\frac{1}{P} \frac{dP_i}{dy_i}$, where dyi is the size of the small change made to the

ith point on the yield curve and dpi is the resultant change in the portfolio value.

- The sum of all the partial duration measures equals the usual duration measure.

5. Interest rate deltas

- **DV01**: define the delta of a portfolio as the change in value for a one-basis-point parallel shift in the zero curve. It is the same as the duration of the portfolio multiplied by the value of the portfolio multiplied by 0.01%.
- In practice, traders like to calculate several deltas to reflect their exposures to all the different ways in which the yield curve can move. One approach corresponds to the partial duration approach. → the sum of the deltas for all the points in the yield curve equals the DV01.

6. Principal components analysis

- The interest rate move for a particular factor is *factor loading*. The sum of the squares of factor loadings is 1.
- Factors are chosen so that the factor scores are uncorrelated.
- 10 rates and 10 factors \rightarrow solve the simultaneous equations: the quantity of a particular factor in the interest rate changes on a particular day is known as the

factor score for that day.

- The importance of a factor is measured by the standard deviation of its factor score.
- The sum of the variances of the factor scores equal the total variance of the data. \rightarrow the importance of 1st factor = the variance of 1st factor's factor score/total variance of the factor scores.
- Use principal component analysis to calculate delta exposures to factors: dP/dF = dP/dy * dy/dF.

Chapter 5 Volatility

- 1. What causes volatility?
- It is natural to assume that the volatility of a stock price is caused by new information reaching the market. Fama (1965), French (1980), and French and Roll (1986) show that the variance of stock price returns between Friday and Monday is only 22%, 19% and 10.7% higher than the variance of stock price return between two consecutive trading days (not 3 times).
- Roll (1984) looked at the prices of orange juice futures. By far the most important news for orange juice futures prices is news about the weather and news about the weather is equally likely to arrive at any time. He found that the Friday-to-Monday variance is only 1.54 times the first variance.
- The only reasonable conclusion from all this is that volatility is to a large extent caused by trading itself.
- 2. **Variance rate**: risk managers often focus on the variance rate rather than the volatility. It is defined as the square of the volatility.
- 3. **Implied volatilities** are used extensively by traders. However, risk management is largely based on historical volatilities.
- 4. Suppose that most market investors think that exchange rates are log normally distributed. They will be comfortable using the same volatility to value all options on a particular exchange rate. But you know that the lognormal assumption is not a good one for exchange rates. What should you do? You should buy deep-out-of-the-money call and put options on a variety of different currencies and wait. These options will be relatively inexpensive and more of them will close in the money than the lognormal model predicts. The present value of your payoffs will on average be much greater than the cost of the options. In the mid-1980s, the few traders who were well informed followed the strategy and made lots of money. By the late 1980s everyone realized that out-of-the-money options should have a higher implied volatility than at the money options and the trading opportunities disappeared.
- 5. An alternative to normal distributions: the power law- has been found to be a good descriptions of the tails of many distributions in practice.
- The power law: for many variables, it is approximately true that the value of v of

the variable has the property that, when **x** is large, $\Pr ob(v > x) = Kx^{-\alpha}$, where K and alpha are constants.

6. Monitoring volatility

- The exponentially weighted moving average model (EWMA).

$$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$
, RiskMetrics use $\lambda = 0.94$.

- The GARCH(1,1) MODEL

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$
, Where $\gamma + \alpha + \beta = 1$, if $\gamma = 0$, then GARCH

model is EWMA

- ML method to estimate GARCH (1,1)

7. How good is the model:

- The assumption underlying a GARCH model is that volatility changes with the passage of time. If a GARCH model is working well, it should remove the autocorrelation of u_i^2 . We can consider the autocorrelation of the variables u_i^2 / σ_i^2 . If these show very little autocorrelation, the model for volatility has succeed in explaining autocorrelations in the u_i^2 . We can use Ljung-Box statistic. If this statistic is greater than 25, zero autocorrelation can be rejected with 95% confidence. 8. Using GARCH to forecast future volatility.

$$\sigma_{n+t}^2 - V_L = \alpha (u_{n+t-1}^2 - V_L) + \beta (\sigma_{n+t-1}^2 - V_L), \text{ since the expected value of } u_{n+t-1}^2$$

is σ_{n+t-1}^2 , hence:

$$E[\sigma_{n+t}^2 - V_L] = (\alpha + \beta)E[\sigma_{n+t-1}^2 - V_L] = (\alpha + \beta)^t (\sigma_n^2 - V_L)$$

9. <u>Volatility term structures</u> – the relationship between the implied volatilities of the options and their maturities.

- Define
$$V(t) = E(\sigma_{n+t}^2)$$
 and $a = \ln \frac{1}{\alpha + \beta} \rightarrow$

$$-\frac{1}{T}\int_0^T V(t)dt = V_L + \frac{1 - e^{-at}}{aT} [V(0) - V_L], \text{ or in per year} \rightarrow$$

$$-\sigma(T)^2 = 252 \left\{ V_L + \frac{1 - e^{-at}}{aT} [V(0) - V_L] \right\} \Rightarrow \text{ can be used to estimate a}$$

volatility term structure based on the GARCH (1,1) model. 10. Impact of volatility changes.

$$\sigma(T)^{2} = 252 \left\{ V_{L} + \frac{1 - e^{-at}}{aT} \left[\frac{\sigma(0)^{2}}{252} - V_{L} \right] \right\}.$$

When $\sigma(0)$ change by $d\sigma(0)$, $\sigma(T)$ changes by $\frac{1-e^{-at}}{aT}\frac{\sigma(0)}{\sigma(T)}\Delta\sigma(0)$

 Many banks use analyses such as this when determining the exposure of their books to volatility changes.

Chapter 6 Correlations and Copulas

- 1. **Correlation measures linear dependence**. There are many other ways in which two variables can be related. E.g. for normal values, two variables may be unrelated. However, their extreme values may be related. "During a crisis the correlations all go to one."
- 2. Monitoring correlation
- Using EWMA:

 $\operatorname{cov}_n = \lambda \operatorname{cov}_{n-1} + (1 - \lambda) x_{n-1} y_{n-1}$

- Using GARCH

 $\operatorname{cov}_n = \omega + \alpha \operatorname{cov}_{n-1} + \beta x_{n-1} y_{n-1}$

- 3. Consistency condition for covariances
- Variance-covariance matrix should be positive-semidefinite. That is, $\omega^T \Omega \omega \ge 0$ for any vector omega.
- To ensure that a positive-semidefinite matrix is produced, <u>variances and</u> <u>convariances should be calculated consistently</u>. For example, <u>if variance rates are</u> <u>updated using an EWMA model with $\lambda = 0.94$, the same should be done for covariance rates.</u>
- 4. Multivariate normal distribution

-
$$E[Y|x] = \mu_Y + \rho(\sigma_Y/\sigma_X)(x - \mu_X)$$
, and $\sigma_{V|X}^2 = \sigma_V^2(1 - \rho^2)$

- The conditional mean of Y is linearly dependent on X and the conditional standard deviation of Y is constant.
- 5. Factor models

$$U_i = a_{i1}F_1 + a_{i2}F_2 + \dots + a_{iM}F_M + \sqrt{1 - a_{i1}^2 - a_{i2}^2 - \dots - a_{iM}^2}Z_i \quad . \quad \text{The}$$

factors have uncorrelated standard normal distributions and the Zi are uncorrelated both with each other and with the factors. In this case the correlation between Ui and

Uj is
$$\sum_{m=1}^{M} a_{im} a_{jm}$$

6. <u>Copulas</u>

- The marginal distribution of X (unconditional distribution) is its distribution assuming we know nothing about Y. But often there is no natural way of defining a correlation structure between two marginal distributions.
- **Gaussian copula approach**. Suppose that F1 and F2 are the cumulative marginal probability distributions of V1 and V2. We map V1 = v1 to U1 = u1 and V2 = v2 to U2 = u2, where

$$F_1(v_1) = N(u_1)$$
 and $F_2(v_2) = N(u_2)$, and N is the cumulative normal distribution function

distribution function.

- The key property of a copula model is that it preserves the marginal distributions of V1 and V2 while defining correlation structure between them.
- Student t-copula \rightarrow U1 and U2 are assumed to have a bivariate Student t-distribution. Tail correlation is higher in a bivariate t-distribution than in a bivariate normal distribution.
- A factor copula model: analysts often assume a factor model for the correlation structure between the Ui.
- 7. Application to loan portfolios
- Define Ti (1<= i <= N) as the time when company i default and cumulative probability distribution of Ti by Qi. To define the correlation structure between the Ti using the one-factor Gaussian copula model, we map, for each i, Ti to a variable Ui that has a standard normal on a percentile-to-percentile basis.

- We assume:
$$U_i = a_i F + \sqrt{1 - a_i^2} Z_i \rightarrow \operatorname{Pr} ob(U_i < U \mid F) = N[\frac{U - a_i F}{\sqrt{1 - a_i^2}}]$$

- The mappings between the Ui and Ti imply Prob(Ui<U) = Prob(Ti<T) when

$$U = N^{-1}[Q(T)] \rightarrow \Pr{ob(T_i < T \mid F)} = N[\frac{N^{-1}[Q_i(T)] - a_i F}{\sqrt{1 - a_i^2}}]$$

- Assuming Qi of time to default is the same for all i and equal to Q and the copula correlation between any two names is the same and equals rho $\Rightarrow a_i = \sqrt{\rho}$ for all i. \Rightarrow

-
$$\Pr{ob(T_i < T \mid F)} = N[\frac{N^{-1}[Q(T)] - \sqrt{\rho F}}{\sqrt{1 - \rho}}]$$
. For a large portfolio of loans, this

equation provides a good estimate of the proportion of loans in the portfolio that default by time T. \rightarrow We refer to this as the default rate.

- As F decreases, the default rate increases. Since F is standard normal, the probability that F will be less than $N^{-1}(Y)$ is Y. There is therefore a probability of
 - Y that the default rate will be greater than $N[\frac{N^{-1}[Q(T)] \sqrt{\rho}N^{-1}(Y)}{\sqrt{1-\rho}}]$
- Define V(T,X) as the default rate that will not be exceeded with probability X, so

that we are X% certain that the default rate will not exceed V(T,X). The value of V(T,X) is determined by substituting Y = 1 - X into the above expression:

V(T, X) =
$$N[\frac{N^{-1}[Q(T)] + \sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}}]$$

Chapter 7 Bank Regulation and Basel II

- 1. The capital a financial institution requires should cover the **difference between expected losses over some time horizon and 'worst-case losses'** over the same time horizon. The idea is that <u>expected losses are usually</u> <u>covered by the way a financial institution prices its products</u>. For example, the interest charged by a bank is designed to recover expected loan losses. Capital is a cushion to protect the bank from an extremely unfavorable outcome.
- 2. The 1988 BIS Accord (Basel I)
- Assets-to-capital <+20
- The Cooke Ratio \rightarrow calculate risk-weighted assets.
 - 3. Netting: If a counterparty defaults on one contract if has with a financial institution then it must default on all outstanding contracts with that financial institution.

Without netting the financial institution's exposure in the event of a default today

is
$$\sum_{i=1}^{N} \max(V_i, 0)$$
 (N contracts with the defaulted party). With netting, it is

$$\max(\sum_{i=1}^N V_i, 0)$$

4. Basel II is based on three pillars:

- Minimum capital requirements

- <u>Supervisory review</u>: allow regulators in different countries some discretion in how rules are applied but seeks to achieve overall consistency in the application of the rules. It places more emphasis on early intervention when problems arise. Part of their role is to encourage banks to develop and use better risk management techniques and to evaluate these techniques.
- <u>Market discipline</u>: require banks to disclose more information about the way they allocate capital and the risks they take.

5. Minimum capital requirements:

<u>Total capital = 0.08*(credit risk RWA + Market risk RWA + Operational risk</u> <u>RWA).</u>

- 6. Market risk (1996 amendment to Basel I, continue to be used under Basel II):
- It requires financial institutions to hold capital to cover their exposures to market

risks as well as credit risks. It distinguishes between a bank's trading book (normally marked to market daily) and its banking book.

- <u>The market risk capital requirement: k * VaR + SRC</u>, where SRC is a specific risk charge. The VaR is the greater of the pervious day's VaR and the average VaR over the last 60 days. The minimum value for k is 3.
- SRC is a capital charge for the idiosyncratic risks related to individual companies. E.g. a corporate bond has two risks: interest rate risk and credit risk. The interest rate risk is captured by the bank's market VaR measure; the credit risk is specific risk.

7. Credit risk capital (NEW FOR BASEL II)

- For an on-balance-sheet item a risk weight is applied to the principal to calculate risk-weighted assets reflecting the creditworthiness of the counterparty. For off-balance-sheet items the risk weight is applied to a credit equivalent amount. This is calculated using either credit conversion factors or add-on amount.
- Standardized approach (for small banks. In USA, Basel II will apply only to the largest banks and these banks must use the foundation internal ratings based (IRB) approach). → risk weights for exposures to country, banks, and corporations as a function of their ratings.
- IRB approach one-factor Gaussian copula model of time to default.

WCDR: the worst-case default rate during the next year that we are 99.9% certain will not be exceeded

PD: the probability of default for each loan in one year

EAD: The exposure at default on each loan (in dollars)

LGD: the loss given default. This is the proportion of the exposure that is lost in the event of a default.

Suppose that the copula correlation between each pair of obligors is rho. Then

WCDR =
$$N[\frac{N^{-1}[PD] + \sqrt{\rho N^{-1}(0.999)}}{\sqrt{1-\rho}}]$$

It follows that there is a 99.9% chance that the loss on the portfolio will be less than N times EAD*LGD*WCDR.

- For corporate, sovereign, and bank exposures, Basel II assumes the relationship

between rho and PD is: $\rho \approx 0.12(1 + e^{-50 \times PD})$. As PD increases, rho decreases.

 \rightarrow As a firm becomes less creditworthy, its PD increases and its probability of default becomes more idiosyncratic and less affected by overall market conditions.

- The formula for the capital required is: EAD*LGD*(WCDR PD)*MA. We use WCDR PD instead of WCDR because we are interested in providing capital for the excess of the 99% worst-case loss over the expected loss. The MA is the maturity adjustment.
- The risk-weighted assets (RWA) are calculated as 12.5 times the capital required: RWA = 12.5 * EAD*LGD*(WCDR – PD)*MA

- 8. **Operational risk**: Banks have to keep capital for operational risk. Three approaches.
- Basic indicator approach: operational risk capital = the bank's average annual gross income (=net interest income + noninterest income) over the last three years multiplied by 0.15.
- Standardized approach: similar to basic approach, except that a different factor is applied to the gross income from different business lines.
- Advanced measurement approach: the bank uses its own internal models to calculate the operational risk loss that it is 99.9% certain will not be exceeded in one year.

9. The limitation of Basel II:

- The total required capital under Basel II is the sum of the capital for three different risks (credit, market and operational). <u>This implicitly assumes that the risks are perfectly correlated</u>. It will be desirable if banks can assume less than perfect correlation between losses from different types of risk.
- <u>Basel II does not allow a bank to use its own credit risk diversification</u> <u>calculations when setting capital requirements for credit risk within the banking</u> <u>book</u>. The prescribed rho must be used. In theory a bank with \$1 billion of lending to BBB-rated firms in a single industry is liable to be asked to keep the same capital as a bank that has \$1 billion of lending to a much more diverse group of BBB-rated corporations.

Chapter 8 The VaR Measure

1. Choice of parameters for VaR

- $VaR = \sigma N^{-1}(X)$, Assuming normal distribution and the mean change in the

portfolio value is zero, where X is the confidence level, sigma is the standard deviation (in dollars) of the portfolio change over the time horizon.

- The time horizon: N-day VaR = 1-day VaR * square root N.

- autocorrelation \rightarrow the above formula a rough one.

- The confidence level: if the daily portfolio changes are assumed to be normally distributed with zero mean \rightarrow convert a VaR to another VaR with different confidence level.

$$VaR(X^*) = VaR(X) \frac{N^{-1}(X^*)}{N^{-1}(X)}$$

2. Marginal VaR: $\frac{\partial (VaR)}{\partial x_i}$, where xi is the ith component of a portfolio. For an

investment portfolio, <u>marginal VaR is closely related to the CAPM's beta. If an</u> asset's beta is high, its marginal VaR will tend to be high.

3. **Incremental VaR** is the incremental effect on VaR of a new trade or the incremental effect of closing out an existing trade. It asks the question: "what is the difference between VaR with and without the trade." If a component is small relative to the size of a portfolio, it may be reasonable to assume that the marginal

VaR remains constant as xi is reduced to zero $\Rightarrow \frac{\partial (VaR)}{\partial x_i} x_i$

- 4. Component VaR is the part of the VaR of the portfolio that can be attributed to this component:
- The ith component VaR for a large portfolio should be approximately equal to the incremental VaR.
- The sum of all the component VaRs should equal to the portfolio VaR.

$$VaR = \sum_{i=1}^{N} \frac{\partial (VaR)}{\partial x_i} x_i$$

5. Back testing:

- The percentage of times the actual loss exceeds VaR.

Let p = 1-X, where x is confidence level. m = the number of times that the VaR limits is exceeded, n the total number of days.

Two hypotheses:

a. The probability of an exception on any given day is p.

b. The probability of an exception on any given day is greater than p.

The probability (binominal distribution) of the VaR limit being exceeded on m or

more days is:
$$\sum_{k=m}^{n} \frac{n!}{k!(n-k)!} p^{k} (1-p)^{n-k}$$
. We usually use confidence level as 5%.

If the probability of the VaR limit being exceeded on m or more days is less than 5%, we reject the first hypothesis that the probability of an exception is p.

The above test is one-tailed test. Kupiec has proposed a two-tailed test (Frequency-of-tail-losses or Kupiec test). If the probability of an exception under the VaR model is p and m exceptions are observed in n trials, then

$$-2\ln[(1-p)^{n-m}p^m] + 2\ln[(1-m/n)^{n-m}(m/n)^m]$$
 should have a chi-square

distribution with one degree of freedom.

- The Kupiec test is a large sample test
- Kupiec test focuses solely on the frequency of tail losses. It throws away potentially valuable information about the sizes of tail losses. This suggests that the Kupiec test should be relatively inefficient, compared to a suitable test that took account of the sizes as well as the frequency of tail losses.

- Sizes-of-tail-losses test: compare the distribution of empirical tail losses against the tail-loss distribution predicted by model – Kolmogorov-Smirnov test (it is the maximum value of the absolute difference between the two distribution functions).

Another backtest is Crnkovic and Drachman (CD) test. The test is to evaluate a market model by testing the difference between the empirical P/L distribution and the predicted P/L distribution, across their whole range of values.

- <u>The extent to which exceptions are bunched</u>: in practice, exceptions are often bunched together, suggesting that losses on successive days are not independent.
- 6. <u>Stress testing</u>: involves estimating how the portfolio would have performed under extreme market moves.

Two Main approaches:

- a. Scenario analyses, in which we evaluate the impact of specified scenarios (e.g., such as a particular fall in the stock market) on our portfolio. The emphasis is on specifying the scenario and working out its ramifications.
- b. Mechanical stress tests, in which we evaluate a number (often a large number) of mathematically or statistically defined possibilities (e.g., such as increases or decreases of market risk factors by a certain number of standard deviations) to determine the most damaging combination of events and the loss it would produce.
- 7. stress testing scenario analyses:
- Stylized scenarios: Some stress tests focus on particular market variables.
 - a. Shifting a yield curve by 100 basis points
 - b. Changing implied volatilities for an asset by 20% of current values.
 - c. Changing an equity index by 10%.
 - d. Changing an exchange rate for a major currency by 6% or changing the exchange rate for a minor currency by 20%.
- Actual historical events: stress tests more often involve making changes to several market variables use historical scenarios. E.g. set the percentage changes in all market variables equal to those on October 19, 1987.
- If movements in only a few variables are specified in a stress test, one approach is to set changes in all other variables to zero. Another approach is to regress the nonstressed variables on the variables that are being stressed to obtain forecasts for them, conditional on the changes being made to the stressed variables (conditional stress testing)
- 8. Stress testing mechanical stress testing
- Factor push analysis: we push the price of each individual security or (preferably) the relevant underlying risk factor in the most disadvantageous direction and work out the combined effect of all such changes on the value of the portfolio.
 - a. We start by specify a level of confidence, which gives us a confidence level parameter alpha.
 - b. We then consider each risk factor on its own, 'push' it by alpha times its standard deviation, and revalue the portfolio at the new risk factor value;
 - c. We do the same for all risk factors, and select that set of risk factor

movements that has the worst effect on the portfolio value.

- d. Collecting these worst price movements for each instrument in out portfolio gives us our worst-case scenario, and the maximum loss (ML) is equal to the current value of our portfolio minus the portfolio value under this worst-case scenario.
- e. Factor push test is only appropriate for certain relatively simple types of portfolio in which the position value is a monotonic function of a risk factor.
- Maximum loss optimization: search over the losses that occur for intermediate as well as extreme values of the risk variables

Chapter 9 Market Risk VaR: Historical Simulation

Approach

1. Methodology

Suppose that we calculate VaR for a portfolio using a one-day time horizon, a 99% confidence level, and 500 days of data.

- Identify the market variables affecting the portfolio. They typically are exchange rates, equity prices, interest rates, and so on.
- Collect data on the movements in these variables over the most recent 500 days.
- The value of the market variable tomorrow under ith scenario is $v_n \frac{v_i}{v_{i-1}}$ (n –

today, say n=500, i = 0, 1, ... 499)

- Get change in portfolio value under each scenario according to the calculated market variables.
- $VaR = 5^{th}$ worst number of the change in portfolio value.
- 2. The confidence interval:
- Kendall and Stuart calculate a confidence interval for the quantile of a distribution when it is estimated from sample data.
- The standard errors of the estimate is: $\frac{1}{f(x)}\sqrt{\frac{q(1-q)}{n}}$, where f(x) is the

probability density function of the loss evaluated at x.

3. Weighting of observations: the basic historical simulation approach assumes that each day in the past is given equal weight. Boudoukh et alpha. Suggest that more recent observations should be given more weight – the weight given to the change

between day n-i and n-i+1 is $\frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$. VaR is calculated by ranking the observations from the worst outcome to the best. Starting at the worst outcome,

weights are summed until the required quantile of the distribution is reached.

- 4. Incorporating volatility updating (Hull and White (1998)).
- Get daily volatility for each market variable.
- Suppose σ_{n+1} (current estimate of the volatility of the market variable between

today and tomorrow) is twice σ_i . \rightarrow we expect to see changes between today and tomorrow that are twice as big as changes between day i-1 and day i. \rightarrow the value

of a market variable under the ith scenario becomes: $v_n \frac{v_{i-1} + (v_i - v_{i-1})\sigma_{n+1} / \sigma_i}{v_{i-1}}$

- 5. Extreme value theory is a way of smoothing the tails of the probability distribution of portfolio daily changes calculated using historical simulation. It leads to estimates of VaR that reflect the whole shape of the tail of the distribution, not just the positions of a few losses in the tails. It can also be used to estimate VaR when the VaR confidence level is very high. E.g., even if we have only 500 days of data, it could be used to come up an estimate of VaR for a VaR confidence level of 99.9%:
- F(x) is CDF, $F_u(y)$ as the probability that x lies between u and u+y conditional on

$$x>u. \rightarrow F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)}$$

- Gnedenko states that for a wide class of distributions F(x), the distribution of $F_u(y)$ converges to a generalized **<u>Pareto distribution</u>** as the threshold u is increased.

 $G_{\xi,\beta}(y) = 1 - (1 + \xi \frac{y}{\beta})^{-1/\xi}$, where ξ is the shape parameter and determines

the heaviness of the tail of the distribution. Beta is a scale parameter.

- Estimating ξ and beta – MLE. The PDF of G(y) is

$$g_{\xi,\beta}(y) = \frac{1}{\beta} (1 + \xi \frac{y}{\beta})^{-1/\xi-1}$$

First, choose a value of u, say the 95 percentile. Then focus attention on those observations for which x > u. Suppose there are n_u such observations and they are x_i (i<=n_u). →

$$\ln g(\mathbf{x}) = \sum_{i=1}^{n_u} \ln \left[\frac{1}{\beta} (1 + \xi \frac{x_i - u}{\beta})^{-1/\xi - 1} \right]$$

- Prob(x>u+y| x>u) = 1 -G(y), prob(x>u) = 1 - F(u) ~ n_u/n \rightarrow prob(x>u+y) = [1-F(u)][1-G(y)] $\rightarrow F(x) = 1 - \frac{n_u}{n} (1 + \xi \frac{x-u}{\beta})^{-1/\xi}$, this is the estimator of the tail of the CDF of x when x is large. This is reduced to the Power Law if we set

$$u=\beta/\xi$$

- Calculation of VaR with a confidence level of $q \rightarrow F(VaR) = q \rightarrow$

$$VaR = u + \frac{\beta}{\xi} \left[\left(\frac{n}{n_u} (1-q) \right)^{-\xi} - 1 \right]$$

Chapter 10 Market Risk VaR: Model-Building Approach

1. <u>Basic methodology</u> – two-asset case \rightarrow diversification. Note: <u>VaR does not always</u> reflect the benefits of diversification.

- The linear model: $\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$, where dx is the return on asset i in one day.

Alphai is the dollar value being invested in asset i. \rightarrow get mean and standard deviation of dp then we are done under multivariate normal distribution.

- 2. Handling interest rates- cash flow mapping. The cash flows from instruments in the portfolio are mapped into cash flows occurring on the standard maturity dates. Since it is easy to calculate zero T-bills or T-bonds' volatilities and correlations, after mapping, it is easy to calculate the portfolio's VaR in terms of cash flows of zero T-bills.
- 3. Principal components analysis: A PCA can be used (in conjunction with cash flow mapping) to handle interest rates when VaR is calculated using the model-building approach.
- 4. Applications of Linear model:
- A portfolio with no derivatives.
- Forward contracts on foreign exchange (to be treated as a long position in the foreign bond combined with a short position in the domestic bond).
- Interest rate swaps
- 5. The linear model and options:

$$\Delta P = \delta \Delta S = S \delta \Delta x = \sum_{i=1}^{n} S_i \delta_i \Delta x_i = \sum_{i=1}^{n} \alpha_i \Delta x_i$$
, where dx = ds/s is the return on a

stock in one day. Ds is the dollar change in the stock price in one day.

- <u>The weakness of the model</u>: when gamma is positive (negative), the pdf of the value of the portfolio tends to be positively (negatively) skewed \rightarrow have a less heavy (heavier) left tail than the normal distribution. \rightarrow if we assume the distribution is normal, we will tend to calculate a VaR that is too high (low).

6. The Quadratic model:

-
$$\Delta P = \delta \Delta S + \frac{1}{2} \gamma (\Delta S)^2 = S \delta \Delta x + \frac{1}{2} S^2 \gamma (\Delta x)^2$$
 (ignoring theta term).

- For a portfolio with n underlying market variables, with each instrument in the portfolio being dependent on only one of the market variables, \rightarrow

$$\Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i + \frac{1}{2} \sum_{i=1}^{n} S_i^2 \gamma_i (\Delta x_i)^2$$
, where Si is the value of the ith market

variable.

- When some of the individual instruments are dependent on more than one market variable, this equation takes the more general form:

$$\Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} S_i S_j \gamma_{ij} \Delta x_i \Delta x_j \text{ , where gamma}_{ij} \text{ is a 'cross gamma'.}$$

 <u>Cornish – Fisher expansion</u>: → estimate percentiles of a probability distribution from its moments that can take account of the skewness of the probability distribution.

Using the first three moments of dP, the Cornish-Fisher expansion estimates the

q-percentile of the distribution of dP as $\mu_P + \omega_q \sigma_P$, where $\omega_q = z_q + \frac{1}{6}(z_q^2 - 1)\xi_P$ and Z is q-percentile of the standard normal distribution

and ξ_P is the skewness of dp.

7. The model-building approach is frequently used for investment portfolios. It is less popular for the trading portfolios of financial institutions because it does not work well when deltas are low and portfolios are nonlinear.

Chapter 11 Credit Risk: Estimating Default Probabilities

- 1. <u>Credit-risk arises from the possibility that borrowers, bond issuers, and counter-parties in derivatives transactions may default</u>. In theory, a credit rating is an attribute of a bond issue, not a company. However, in most cases all bonds issued by a company have the same rating. A rating is therefore often referred to as an attribute of a company.
- <u>Ratings changes relatively infrequently</u>. One of rating agencies' objectives is ratings stability. They want to avoid ratings reversals where a firm is downgraded and then upgraded a few weeks later. Ratings therefore change only when there is reason to believe that a long-term change in the firm's creditworthiness has taken place. The reason for this is that bond traders are major users of ratings. Often they are subject to rules governing what the credit ratings of the bonds they hold

must be. If these ratings changed frequently they might have to do a large amount of trading just to satisfy the rules.

- <u>Rating agencies try to 'rate through the cycle'</u>. Suppose that an economic downturn increases the probability of a firm defaulting in the next 6 months, but makes very little difference to the firm's cumulative probability of defaulting over the next three to five years. A rating agency would not change the firm's rating.
- 2. Internal credit ratings: most banks have procedures for rating the creditworthiness of their corporate and retail clients. The internal ratings based (IRB) approach in Basel II allows bank to use their internal ratings in determining the probability of default, PD. Under the advanced IRB approach, they are also allowed to estimate the loss given default, LGD, the exposure at default, EAD, and the maturity, M.

3. <u>Altman's Z-score</u>

Using discriminant analysis, Altman attempted to predict defaults from five accounting ratios.

Z = 1.2*working capital + 1.4*Retained earnings + 3.3* EBIT + 0.6*Market value of equity + 0.999*sales, all variables are scaled by assets, except for market equity, which is scaled by book value of total liabilities.

<u>If Z-score > 3</u>, the firm is unlikely to default. If it is between 2.7 and 3.0, we should be 'on alert'. If it is between 1.8 and 2.7, there is a good chance of default. If it is less than 1.8, the probability of a financial embarrassment is very high.

4. <u>Historical default probabilities</u>

- For investment-grade bonds, the probability of default in a year tends to be an increasing function of time.
- For bonds with a poor credit rating, the probability of default is often a decreasing function of time. The reason is that, for a bond like this, the next year or two may be critical. If the issuer survives this period, its financial health is likely to have improved.

- **Default intensities (hazard rate)**: λ (t) at t is defined so that λ (t) Δ t is the probability of default between time t and t + Δ t conditional on no default between time 0 and time t.

Let V(t) is the cumulative probability of the firm surviving to time t (no default by time t), then V(t + Δ t) – V(t) = - λ (t) V(t) Δ t \rightarrow

$$\frac{dV(t)}{dt} = -\lambda(t)V(t) \rightarrow V(t) = e^{-\int_0^t \lambda(\tau)d\tau} = e^{-\overline{\lambda}t}, \text{ where } \overline{\lambda} \text{ is the}$$

average default intensity between time zero and t. Define Q(t) as the probability of default by time t \rightarrow

$$Q(t) = 1 - V(t) = 1 - e^{-\int_0^t \lambda(\tau) d\tau} = 1 - e^{-\overline{\lambda}t} \rightarrow \overline{\lambda}(t) = -\frac{1}{t} \ln[1 - Q(t)], \text{ where } Q(t)$$

comes from historical data.

- 5. <u>**Recovery rates**</u>: the bond's market value immediately after a default as a percent of its face value = 1 LGD.
- <u>Recovery rates are significantly negatively correlated with default rates</u>.

- Average recovery rate = 0.52 - 6.9*average default rate. \rightarrow <u>a bad year for the default rate is usually doubly bad because it is accompanied by a low recovery rate</u>.

6. Estimating default probabilities from bond prices

- An approximate calculation: $[1 - PD]100e^{-RfT} + PD100 \operatorname{Re}^{-RfT} = 100e^{-yT}$

$$\Rightarrow PD(T) = \frac{1 - e^{-[y - Rf]T}}{1 - R} \sim \text{spread}/(1 - R)$$

- **Risk-free rate: Traders usually use LIBOR/swap rates** as proxies for risk-free rates when calculating default probabilities.
- Credit default swaps can be used to imply the risk-free rate assumed by traders. The rate used appears to be approximately equal to the LIBOR/swap rate minus 10 basis points on average. (Credit risk in a swap rate is the credit risk from making a series of 6-month loans to AA-rated counterparties and 10 basis points is a reasonable default risk premium for an AA-rated 6-month instrument.
- 7. Asset swap: traders often use asset swap spreads as a way of extracting default probabilities from bond prices. This is because asset swap spreads provide a direct estimate of the spread of bond yields over the LIBOR/swap curve. → The present value of the asset swap spread is the amount by which the price of the corporate bond is exceeded by the price of a similar risk-free bond.
- 8. <u>Real-world vs. Risk-neutral probabilities</u>
- The default probabilities implied from bond yields are risk-neutral default probabilities.
- The default probabilities implied from historical data are real-world default probabilities. <u>If there was no expected excess return, the real-world and risk-neutral default probabilities would be the same, and vice versa.</u>
- Reasons for the difference
 - a. Corporate bonds are relatively illiquid and bond traders demand an extra return to compensate for this.
 - b. The subjective default probabilities of traders may be much higher than those given in table 11.1. Traders may be allowing for depression scenarios much worse than anything seen during the 1970 to 2003 period.
 - c. <u>Bonds do not default independently of each other.</u> This gives rise to systematic risk and traders should require an expected excess return for bearing the risk. The variation in default rates from year to year may be because of overall economic conditions or because a default by one company has a ripple effect resulting in defaults by other companies (the latter is referred as credit contagion).
 - d. Bond returns are highly skewed with limited upside. As a result it is much more difficult to diversify risks in a bond portfolio than in an equity portfolio.
- When valuing credit derivatives or estimating the impact of default risk on the pricing of instruments, we should use risk-neutral default probabilities.
- When carrying out scenario analyses to calculate potential future losses from

defaults, we should use real-world default probabilities. The PD used to calculate regulatory capital is a real-world default probabilities.

- 9. Estimating default probabilities from equity prices:
- Equity is an option on the assets of the company. \rightarrow the value of equity today is:

-
$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$
, where

$$- d_1 = \frac{\ln(V_0 / D) + (r + \sigma_V^2 / 2)T}{\sigma_V \sqrt{T}} \text{ and } d_2 = d_1 - \sigma_V \sqrt{T}$$

- The risk-neutral probability that a firm will default on the debt is $N(-d_2)$. We need

V0 and σ_v , which are not directly observable.

- From Ito's lemma
$$\rightarrow \sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0$$

- The above two equations \rightarrow the values of V0 and $\sigma_V \rightarrow N(-d_2)$.
- 10. <u>The default probability can be estimated from historical data (real-world</u> probabilities), bond prices (risk-neutral probabilities), or equity prices (in theory risk-neutral probabilities. But, the output from the model can be calibrated so that either risk-neutral or real-world default probabilities are produced).

Chapter 12 Credit Risk Losses and Credit VaR

1. Estimating credit losses:

- <u>Credit losses on a loan depend primarily on the probability of default and the recovery rate</u>.
- The credit risk in a derivative transaction is more complicated because the exposure at the time of default is uncertain. Some derivatives transactions (e.g., written options) are always liabilities and give rise to no credit risk. Some (e.g., long positions in options) are always assets and entail significant credit risks. Some may become either assets or liabilities during their life (e.g., forward and swaps).
- 2. Adjusting derivatives valuations for counterparty default risk
- Assume the value of the derivative to the financial institution at time t_i is f_i , i=0,1,...n, and q_i is the risk-neutral probability of default at time t_i , the expected recovery rate is R. \rightarrow the risk-neutral expected loss from default at time t_i is:

$$q_i(1-R)\widehat{E}[\max(f_i,0)]$$
, taking present values and sum them up over $i \rightarrow$

costs of defaults. $\sum_{i=1}^{n} q_i (1-R) v_i$, where v is the value today of an instrument that

pays off the exposure on the derivative under consideration at time t_i.

- <u>The impact of default risk on interest rate swaps is considerably less than that on</u> <u>currency swaps</u>, largely because principals are exchanged at the end of the life of a currency swap and there is uncertainty about the exchange rate at that time.
- <u>Two sided default risk</u> (when contracts can become either assets or liabilities). Human nature being what it is, most firms consider that there is no chance that they themselves will default but want to make an adjustment to contract terms for a possible default by their counterparty. This can make it very difficult for the two firms to agree on terms and explains why it is difficult for financial institutions that are not highly creditworthy to be active in the derivatives market.

3. Credit risk mitigation

- <u>Netting</u>. Because of netting, the incremental effect of a new contract on expected default losses can be negative. This tends to happen when the value of the new contract is highly negatively correlated with the value of existing contracts. A firm may get different quotes in a well-functioning capital market. The company is likely to get the most favorable quote from a financial institution it has done business with in the past particularly if that business gives rise to exposures for the financial institution that are opposite to the exposure generated by the new transaction.
- Collateralization
- Downgrade triggers. This is a clause stating that if the credit rating of the counterparty falls below a certain level, say Baa, then the financial institution has the option to close out a derivatives contract at its market value.
- 4. Credit VaR. Whereas the time horizon for market risk is usually between one day and one month that for credit risk is usually much longer often one year.

5. Vasicek's modela:

- We are X% certain that the default rate will not exceed V(T,X).

V(T, X) =
$$N\left[\frac{N^{-1}[Q(T)] + \sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}}\right]$$
, where Q(T) is the cumulative

probability of each loan defaulting by time T.

- 6. **Credit risk plus**: the probability of n defaults follows the Poisson distribution and this is combined with a probability distribution for the losses experienced on a single counterparty default to obtain a probability distribution for the total default losses from the counterparties.
- 7. **CreditMetrics.** It is based on an analysis of credit migration. This is the probability of a firm moving from one rating category to another during a certain period of time. Calculating a one-year VaR for the portfolio using CreditMetrics involves carrying out Monte Carlo simulation of ratings transitions for bonds in the portfolio over a one-year period. On each simulation trial the final credit rating of all bonds is calculated and the bonds are revalued to determine total credit losses for the year. The 99% worst result is the one-year 99% VaR for the portfolio.
- The credit rating changes for different counterparties should not be assumed to be independent. \rightarrow use Gaussian copula model \rightarrow The copula correlation between the

rating transitions for two companies is typically set equal to the correlation between their equity returns using a factor model.

Chapter 13 Credit Derivatives

- 1. Credit derivatives are contracts where the payoff depends on the creditworthiness of one or more companies or countries. Banks have been the largest buyers of CDS credit protection and insurance companies have been the largest sellers.
- <u>The n-year CDS spread should be approximately equal to the excess of the par</u> <u>yield on an n-year corporate bond over the par yield on an n-year risk-free bond</u>. If it is markedly less than this, an investor can earn more than the risk-free rate by buying the corporate bond and buying protection. If it is markedly great than this, an investor can borrow at less than the risk-free rate by shorting the corporate bond and selling CDS protection.
- The payoff from a CDS in a credit event is notional principal*(1-recovery rate). Usually a CDS specifies that a number of different bonds can be delivered in the credit event. This gives the holder of CDS a <u>cheapest-to-deliver bond option</u>. Therefore, recovery rate should be the lowest recovery rate applicable to a deliverable bond.

2. Valuation of credit default swaps

- <u>CDS can be analyzed by calculating the present value of the expected payments</u> (including accrued interests) and the present value of the expected payoff.
- The default probabilities used to value a CDS should be risk-neutral **default probabilities**, which can be estimated from bond prices or asset swaps. An alternative is to imply them from CDS quotes.
- Binary CDS: it is similar to a regular CDS except that the payoff is a fixed dollar amount.
- Is the recovery rate important? whether we use CDS spreads or bond prices to estimate default probabilities, we need an estimate of the recovery rate. However, provided that we use the same recovery rate for (a) estimating risk-neutral default probabilities and (b) valuing a CDS, the value of the CDS is not very sensitive to the recovery rate. This is because the implied probabilities of default are approximately proportional to 1/(1-R) and the payoffs from a CDS are proportional to 1-R.
 - 3. <u>A total return swaps</u> is an agreement to exchange the total return on a bond (or any portfolio of assets) for LIBOR plus a spread. The total return includes <u>coupons</u>, interest, and the **gain or loss** on the asset over the life of the swap. The spread over LIBOR received by the payer is compensation for bearing the risk that the receiver will default. The payer will lose money if the receiver defaults at a time when the reference bond's price has decline. The spread therefore depends on the credit quality of the receiver and of the bond

issuer, and on the default correlation between the two.

- 4. Basket CDS
- <u>Add-up CDS</u> provides a payoff when any of the reference entities default.
- An $\underline{n}^{\underline{\text{th}}}$ -to-default CDS provides a payoff only when the nth default occurs.

5. CDOs

- Cash CDO (based on bonds)
- Synthetic CDO: the creator sells a portfolio of CDSs to third parties. It then passes the default risk on to the synthetic CDO's tranche holders. The first tranche may be responsible for the payoffs on the CDS until they have reached 5% of the total notional principal;...; The income from the CDS is distributed to the tranches in a way that reflects the risk they are bearing.

6. <u>Valuation of a basket CDS and CDO</u>

- The spread for an nth-to-default CDS and the tranche of a CDO is critically dependent on <u>default correlation</u>. As the default correlation increases the probability of one or more defaults declines and the probability of ten or more defaults increases.
- The one-factor Gaussian copula model of time to default has become the standard market model for valuing an nth-to-default CDS or a tranche of a CDO.
- Consider a portfolio of N firms, each having a probability Q(T) of defaulting by time T. \rightarrow the probability of default, conditional on the level of the factor F, is

$$Q(T \mid F) = N\left[\frac{N^{-1}[Q(T)] - \sqrt{\rho}F}{\sqrt{1 - \rho}}\right]$$

- The trick here is to calculate expected cash flows conditional on F and then integrate over F. The advantage of this is that, conditional on F, defaults are independent. The probability of exactly k defaults by time T, conditional on F, is

$$\frac{n!}{k!(n-k)!}Q(T \mid F)^{k}(1-Q(T \mid F))^{n-k}$$

- Derivates dealers calculate the <u>implied copula correlation rho from the spreads</u> <u>quoted in the market for tranches of CDOs and tend to quote these rather than</u> <u>the spreads themselves</u>. This is similar to the practice in options markets of quoting B-S implied volatilities rather than dollar prices.
- <u>Correlation smiles</u>: the compound correlation is the correlation that prices a particular tranche correctly. The base correlation is the correlation that prices all tranches up to a certain level of seniority correctly. In practice, we find that compound correlations exhibit a 'smile' with the correlations for the most junior (equity) and senior tranches higher than those for intermediate tranches. The base correlations exhibit a 'skew' where the correlation increases with the level of seniority considered.

Chapter 14 Operational Risk

- 1. **Operational risk**: the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events.
- It includes legal risk, but does not include reputation risk or the risk resulting from strategic decisions.

2. 7 Categories of operational risk

- <u>Internal fraud</u> (Barings),
- <u>External fraud (Republic NY corp. Lost \$611 million because of fraud committed</u> by a custodial client,
- <u>Employment practices and workplace safety</u> (Merrill Lynch lost \$250 million in a gender discrimination lawsuit),
- <u>Clients, products, & business practices</u> (Household International lost \$484 million from improper lending practices),
- Damage to physical assets (911 attacks),
- <u>Business disruption and system failures</u> (Solomon Brothers lost \$303 million from a change in computing technology).
- <u>Execution, delivery, and process management</u>: failed transaction processing or process management, and relations with trade counter-parties and vendors. E.g., Bank of America and Wells Fargo Bank lost \$225 and \$150 million, respectively, from systems integration failures and transactions processing failures.

3. Loss severity and loss frequency

- **Loss frequency distribution** the distribution of the number of losses observed during the time horizon (a Poisson distribution is usually used).
- **Loss severity distribution** the distribution of the size of a loss, given that a loss occurs. (usually assume the two are independent). (a lognormal probability distribution is often used)
- The two distributions must be used for each loss type and business line to determine a total loss distribution. Monte Carlo simulation can be used here:
 - a. Sample from the frequency distribution to determine the number of loss events (=n).
 - b. Sample n times from the loss severity distribution to determine the loss experienced for each loss event (L1, L2, ... Ln)
 - c. Determine the total loss experienced (=L1 + L2 + ... + Ln).
 - d. Repeat this many times.
- Data: the frequency distribution should be estimated from the bank's own data as far as possible. For the loss severity distribution, regulators encourage banks to use their own data in conjunction with external data (through sharing arrangements between banks or from data vendors. Both internal and external data must be adjusted for inflation. A scale adjustment should be made to external data).
 - 4. <u>Operational risk capital should be allocated to business units in a way that</u> <u>encourages them to improve their operational risk management</u>. The overall

result of operational risk assessment and operational risk capital allocation should be that business units become more sensitive to the need for managing operational risk.

- 5. The power law holds well for the large losses experienced by banks. This makes the calculation of VaR with high degree of confidence such as 99.9% possible. When loss distributions are aggregated, the distribution with the heaviest tails tends to dominate → it may only be necessary to consider one or two business-line/loss-type combinations.
- 6. Insurance two problems (moral hazard [deductibles, coinsurance provisions, and policy limits can be employed] and adverse selection). Some insurance companies do offer rogue trader insurance policies. These companies tend to specify carefully how trading limits are implemented. They may also require that the existence of the insurance policy not be revealed to anyone on the trading floor. They are likely to want to retain the right to investigate the circumstances underlying any loss.
- 7. <u>Sarbanes-Oxley</u> → It requires boards of directors to become much more involved with day-to-day operations. They must monitor internal controls to ensure risks are being assessed and handled well.
- 8. Two sorts of operational risk: high-frequency low-severity risks and low-frequency high-severity risks. The former are relatively easy to quantify, but operational risk VaR is largely driven by the latter.

Chapter 15 Model Risk and Liquidity Risk

- 1. <u>Model risk</u> is the risk related to the models a financial institution uses to value derivatives. <u>Liquidity risk</u> is the risk that there may not be enough buyers/sellers in the market for a financial institution to execute the trades it desires. <u>The two</u> risks are related. Sophisticated models are only necessary to price products that are relatively illiquid. When there is an active market for a product, prices can be observed in the market and models play a less important role.
- 2. Exploiting the weakness of a competitor's model: a LIBOR-in-arrears swap is an interest rate swap where the floating interest rate is paid on the day it is observed, not one accrual period later. Whereas a plain vanilla swap is correctly valued by assuming that future rates will be today's forward rates, a LIBOR-in-arrears swap should be valued on the assumption that the future rate is today's forward interest rate plus a 'convexity adjustment'.
- 3. Models for actively traded products.
- Traders frequently quote implied volatilities rather than the dollar prices of options, because implied volatility is more stable than the price.
- Volatility smiles: the volatility implied by B-S as a function of the strike price for a particular option maturity is known as a volatility smile. The relationship between strike price and implied volatility should be exactly the same for calls

and puts in the case of European options and approximately the same in the case of American options.

- <u>Volatility smile for foreign currency options</u>. The reason for the volatility smile is that B-S assumes: a. the volatility of the asset is constant; b. the price of the asset changes smoothly with no jumps. Neither of these conditions is satisfied for an exchange rate.
- <u>Volatility skew equity options</u>. The volatility decreases as the strike price increases. The volatility used to price a low-strike-price option (i.e., a deep-out-of-the-money put or a deep in the money call) is significantly higher than that used to price a high strike-price option (i.e., a deep in the money put or a deep out of the money call). **One explanation for the skew in equity options** concerns leverage. As a firm's equity declines in value, \rightarrow leverage increase \rightarrow equity becomes more risky and its volatility increases.

Crashophobia: the volatility smile for equities has existed only since the stock market crash of Oct 1987. Prior to Oct 1987 implied volatilities were much less dependent on strike price. \rightarrow Mark Rubinstein suggested that one reason for the equity volatility smile may be 'crashophobia'. Traders are concerned about the possibility of another crash similar to Oct 1987 and assign relatively high prices (and therefore relatively high implied volatilities) for deep out of the money puts.

- <u>Volatility surfaces</u> → implied volatility as a function of both strike price and time to maturity → volatility smile becomes less pronounced as the time to maturity increases. To value a new option, traders look up the appropriate volatility in the volatility surface table using interpolation.
- Models play a relatively minor role in the pricing of actively traded products. Models are used in a more significant way when it comes to hedging.
- a. <u>Within-model hedging</u> is designed to deal with the risk of changes in variables that are assumed to be uncertain by the model.
- b. <u>Outside-model hedging</u> deals with the risk of changes in variables that are assumed to be constant/deterministic by the model. When B-S is used, hedging against movements in the underlying stock price (delta and gamma hedging) is with-model hedging because the model assumes that stock price changes are uncertain. However, hedging against volatility (vega hedging) is outside-model hedging.
- 4. <u>Models for structured products</u>: exotic options and other nonstandard products that are tailored to the needs of clients are referred to as structured products. <u>Usually they do not traded actively and a financial institution must rely on a model to determine the price it charges the client</u>. A financial institution should, whenever possible, use several different models. → get a price range and a better understanding of the model risks being taken.
- 5. Detecting model problems:
- The risk management function should keep track of the following:
 - a. The type of trading the financial institution is doing with other financial institutions.
 - b. How competitive it is in bidding for different types of structured transactions.

- c. The profits being recorded from the trading of different products.
- Getting too much of a certain type of business or making huge profits from relatively simple trading strategies, or the financial institution is unable to unwind trades at close to the prices given by its models, can be a warning sign.
- 6. **Liquidity risk**: as the quantity increases, the price paid by the buyer increases and the price received by the sell decreases.
- The percentage bid-offer spread: s = (offer price bid price)/mid-price
- <u>Liquidity-adjusted VaR</u> = VaR + $\sum_{i=1}^{n} s_i \alpha_i / 2$, where α_i is the amount of money invested in the ith position. As the number of position, n, grows, VaR benefits from

diversification but the liquidity adjustment does not. Consequently, the percentage difference between VaR and liquidity-adjusted VaR grows as n grows.

- Liquidity black holes is created when a price decline causes more market participants to want to sell, driving prices well below where they will eventually settle. During the sell-off, liquidity dries up and the asset can be sold only at a fire-sale price. <u>Reasons for herd behavior and the creation of liquidity black holes are:</u>
- The computer models used by different traders are similar.
- All financial institutions are regulated in the same way and respond in the same way to changes in volatilities and correlations. E.g., when volatilities and correlations increase, market VaR and the capital required for market risks increase → banks tend to reduce their exposures. Since banks often have similar positions to each other, they try to do similar trades. → liquidity black hole. E.g., consider credit risk. During economic recessions, default probabilities are relatively high and capital requirements for loans under Basel II internal ratings based models tend to be high → banks may be less willing to make loans, creating a liquidity black hole for small and medium-sized businesses. → to solve this, Basel Committee requires that the probability of default should be <u>an average of the probability of default through the economic or credit cycle, rather than an estimate applicable to one particular point in time.</u>
- There is a natural tendency to feel that if other people are doing a certain type of trade then they must know something that you do not.

Chapter 17 Weather, Energy, and Insurance Derivatives

- 1. <u>Weather derivatives</u> (US department of Energy estimated that 1/7 of the US economy is subject to weather risk): we will see contracts on rainfall, snow and similar variables become more commonplace.
- <u>HDD: heating degree days</u> a measure of the volume of energy required for heating during the day.

HDD = max (0, 65 - A), where A is the average of the highest and lowest temperature during the day at a specified weather station, measured in degrees Fahrenheit.

- <u>CDD: cooling degree days</u> a measure of the volume of energy required for cooling during the day.
 - $CDD = \max(0, A 65)$
- A typical over-the-counter product is a forward or option contract providing a payoff dependent on the cumulative HDD or CDD during a month.
- 2. Energy Derivatives
- Oil: virtually any derivative that is available on common stocks is now available with oil as the underlying asset.
- Natural gas and electricity.
- An energy producer faces two risks: price risk and volume risk. Although prices do adjust to reflect volumes, there is a less-than-perfect relationship between the two. <u>The price risk can be hedged using the energy derivative contracts. The</u> volume risks can be hedged using the weather derivatives.
 - 3. <u>Insurance Derivatives</u> are beginning to become an alternative to traditional reinsurance as a way for insurance companies to manage the risks of catastrophic events: <u>CAT (catastrophic) bond</u>: a bond issued by a subsidiary of an insurance firm that pays a higher-than-normal interest rate. In exchange for the extra interest, the holder of the bond agrees to provide an excess-of-cost reinsurance contract. Depending on the terms of the CAT bond, the interest or principal (or both) can be used to meet claims. <u>There is no statistically significant correlation between CAT risks and market returns. CAT bonds are therefore an attractive addition to an investor's portfolio.</u>

Chapter 18 Big Losses and What We Can Learn From Them

- 1. <u>Risk Limits</u>: it is essential that all companies define in a clear and unambiguous way limits to the financial risks that can be taken. Ideally, overall risk limits should be set at board level. These should then be converted to limits applicable to the individuals responsible for managing particular risks.
- It is tempting to ignore violations of risk limits when profits result. <u>The penalties</u> for exceeding risk limits should be just **as great** when profits result as when losses result. Otherwise, traders that make losses are liable to keep increasing their bets in the hope that eventually a profit will result and all will be forgiven.
- Do not underestimate the benefits of diversification
- Carry out scenario analyses and stress tests: The calculation of risk measures such as VaR should always be accompanied by scenario analyses and stress testing to obtain an understanding of what can go wrong.
- 2. Managing the trading room
- There is a tendency to regard high-performing traders as 'untouchable' and to not subject their activities to the same scrutiny as other traders. It is important that all

traders – particularly those making high profits – be fully accountable.

- Separate the front (consists of the traders who are executing trades, taking positions, etc.), middle (consists of risk managers who are monitoring the risks being taken) and back office (record-keeping and accounting)
- Do not blindly trust models: if large profits are reported when relatively simple trading strategies are followed, there is a good chance that the models are wrong. Similarly, if a financial institution appears to be particularly competitive on its quotes for a particular type of deal, there is a good chance that it is using a different model from other market participants. Getting too much business of a certain type can be just as worrisome as getting too little business of that type.
- <u>Be conservative in recognizing inception profits</u>: it is much better to recognize inception profits slowly so that traders are motivated to investigate the impact of several different models and several different sets of assumptions before committing themselves to a deal.
- 3. <u>Liquidity risk</u>
- Financial engineers usually base the pricing of exotic instruments and other instruments that trade relatively infrequently on the prices of actively traded instruments.
 - a. Often calculate a zero curve from actively traded government bonds and uses it to price bonds that trade less frequently (off-the-run bonds).
 - b. Often implies the volatility of an asset from actively traded options and uses it to price less actively traded options.
 - c. Often implies information about the behavior of interest rates from actively traded interest rate caps and swap options and uses it to price products that are highly structured.

These practices are not unreasonable. However, it is dangerous to assume that less actively traded instruments can always be traded at close to their theoretical price.

- Beware when everyone is following the same trading strategy: In the months leading up to the crash of October 1987, many portfolio managers were attempting to insure their portfolios by creating synthetic put options. They bought stocks or stock index futures after a rise in the market and sold them after a fall. This created an unstable market.
- 4. lessons for nonfinancial corporation:
- Make sure you fully understand the trades you are doing. If a trade and the rationale for entering into it are so complicated that they cannot be understood by the manager, it is almost certainly inappropriate for the corporation.
- Make sure a hedger does not become a speculator.
- Be cautious about making the treasury department a profit center
- 5. INTERNAL CONTROLS!!!

T1 Bootstrap

- 1. The procedure:
- We start with a sample of n.
- Draw a new sample of the same size from the original sample.
- Obtain a new parameter estimate from the new sample.
- Repeat the process B times and obtain a bootstrapped sample of B parameter estimates;
- We use this sample to estimate a confidence interval.
- 2. Dealing with data dependency: the main limitation of the bootstrap is that standard boostrap procedures presuppose that observations are independent, and they can be unreliable if this assumption does not hold.
- If we are prepared to make parametric assumptions, we can model the dependence parametrically (e.g., using a GARCH procedure). We can then bootstrap from the residuals. However, the drawback of this solution is that is requires us to make parametric assumptions and of course presupposes that those assumptions are valid.
- An alternative is to use a block approach: we divide sample data into non-overlapping blocks of equal length, and select a block at random. However, this approach can be tricky to implement and can lead to problems because it tends to 'whiten' the data.
- A third solution is to modify the probabilities with which individual observations are chosen. Instead of assuming that each observation is chosen with the same probability, we can make the probabilities of selection dependent on the time indices of recently selected observations.

T2 Principal Component Analysis

- 1. The theory:
- Assume x is an mx1 random vector, with covariance matrix Σ , and let Λ be a diagonal matrix whose diagonal elements are eigenvalues of Σ , let A is the

matrix of eigenvectors of Σ . Then $\Sigma = A^T \Lambda A$

- The principal component of x are the linear combinations of the individual x-variables produced by pre-multiplying x by A: $p = Ax \rightarrow$ the variance-covariance matrix of p, VC(p), is then:

$$VC(p) = VC(Ax) = A\Sigma A^T = \Lambda$$

- since Λ is a diagonal matrix. \rightarrow the different principal components are uncorrelated with each other. And the variances of principal components are given

by the diagonal elements of Λ , the eigenvalues.

- In addition, we can choose the order of our principal components so that the eigenvalues are in declining order. The first principal component therefore 'explains' more of the variability of our original data than the second principal component, and so on.
- In short, the principal components of our m original variables are m artificial variables constructed so that the first principal component 'explains' as much as it can of the variance of these variables; the second principal component 'explains' as much as it can of the remaining variance, but it uncorrelated with the first component; and so forth.
- 2. Estimates of principal components based on historical data can be quite unstable. Some simple rules of thumb – such as taking moving averages of our principal components – can help to mitigate this instability. We also should be careful about using too many principal components. We only want to add principal components that represent stable relationships that are good for forecasting, and there will often come a point where additional principal components merely lead to the model tracking noise – and so undermine the forecasting ability of our model.

T3 Monte Carlo Simulation Methods

- 1. The steps
- Select a model for the stochastic variables of interest.
- Estimate its parameters –volatilities, correlations, and so on on the basis of whatever historical or market data are available.
- Construct simulated paths for the stochastic variables using 'random' numbers. Each set of 'random' numbers then produces a set of hypothetical terminal price(s) for the instrument(s) in our portfolio.
- Repeat these simulations enough times.
- 2. Monte Carlo Simulation with single risk factors
- if stick price S follows a geometric Brownian motion process then, lnS:

$$d \ln S = (\mu - \sigma^2 / 2)dt + \sigma dz \qquad \Rightarrow$$

$$S(\Delta t) = S(0) \exp\left[(\mu - \sigma^2 / 2)\Delta t + \sigma \phi(\Delta t)\sqrt{\Delta t}\right]$$

$$S(T) = S(0) \exp\left[(\mu - \sigma^2 / 2)T + \sigma \phi(T)\sqrt{T}\right] \Rightarrow \text{ provided we are only}$$

interested in the terminal stock value, this approach is both more accurate and less time-consuming than the Euler method (get stock price for each incremental dt).