Part I: Physics

Section – I (Single Correct Answer Type)

This section contains 10 **multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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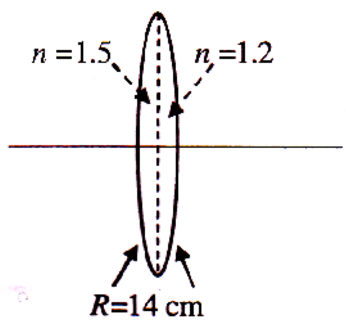
1. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) –280.0 cm **(B)** 40.0 cm **(C)** 21.5 cm **(D)** 13.3 cm

Sol: **[B]** The focal length of each part

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{14}-\frac{1}{\infty}\right) = \frac{1}{28}\text{cm}^{-1}$$

$$\frac{1}{f_2} = (1.2-1)\left(\frac{1}{\infty}-\frac{1}{14}\right) = \frac{1}{70}\text{cm}^{-1}$$

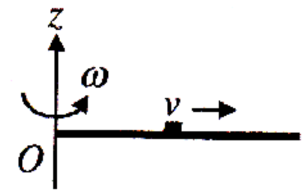
Hence equivalent focal length

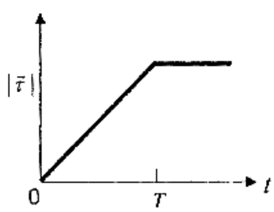
$$\frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2} = \frac{1}{28}+\frac{1}{70} = \frac{1}{20}\text{cm}^{-1}$$

$\Rightarrow$ $f_{eq} = 20$ cm

For $u = -40\ cm = -2f \Rightarrow v = 2f = 40$ cm

2. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot?



(A)


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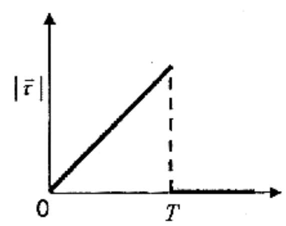
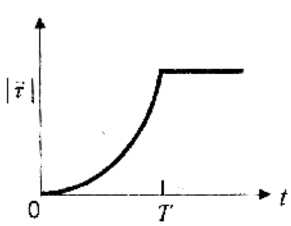
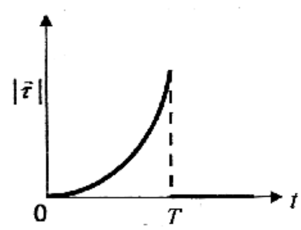


(C)


(D)


Sol: **[B]** The angular momentum of the system about the rotational axis is

$$L = L_{Rod} + \left(Mx^2\right)\omega = L_{Rod} + M(vt)^2\omega$$

or $$L = L_{Rod} + Mv^2\omega t^2$$

$$\tau = \frac{dL}{dt} = 2\ Mv^2\omega t$$

Hence $|\tau| \alpha$ t, till the particle reaches the end and stops, and then becomes zero

3. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ **(B)** $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$ **(C)** $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ **(D)** $(97)^{\frac{1}{4}} T$

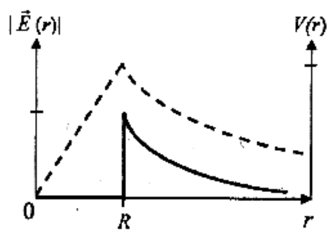
Sol: **[C]** The net heat received by the middle plate from the hotter plate is equal to the net heat given to the cooler plate hence

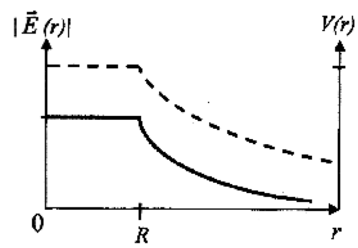
$$\sigma A\left((3T)^4 - T_0^4\right) = \sigma A\left(T_0^4 - (2T)^4\right)$$

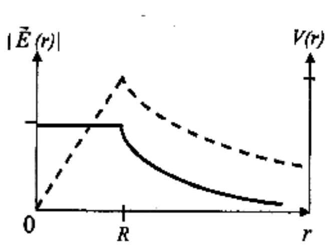
or $\quad 2T_0^4 = 81T^4 + 16T^4$

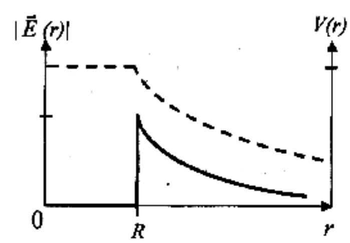
or $\quad T_0^4 = \dfrac{97\,T^4}{2} \qquad \Rightarrow \quad T_0 = \left(\dfrac{97}{2}\right)^{1/4} T$

4. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $\left|\vec{E}(r)\right|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph ?

(A)


(B)


(C)


(D)


Sol: **[D]** For a spherical shell

$E = 0 \qquad\qquad (r < R)$

$E = \dfrac{kQ}{r^2} \qquad\qquad (r > R)$

whereas the electric potential

$V = \dfrac{kQ}{R}$ (constant) for $r < R$

$V = \dfrac{kQ}{r}$ for $r > R$

The relevant graph is (D).

5. In the determination of Youngs modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length $L = 2$m and diameter $d = 0.5$ mm is used. For a load M = 2.5 kg, an extension $l = 0.25$ cm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. the number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and l are the same

(B) due to the error in the measurement of d is twice that due to the error in the measurement of l

(C) due to the error in the measurement of l is twice that due to the error in the measurement of d

(D) due to the error in the measurement of d is four times that due to the error in the measurement of l

Sol: [A] $\left.\frac{\Delta Y}{Y}\right|_{max} = \left.\frac{\Delta \ell}{\ell}\right|_{max} + 2\left.\frac{\Delta d}{d}\right|_{max}$

Here both screw gauge and micrometer used have same least count and hence maximum probable error in measurement of ℓ and d will be same i.e.,

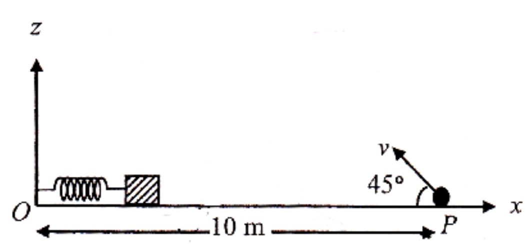
$$\Delta \ell_{max} = \Delta d_{max} = \Delta x$$

Thus, $\left.\frac{\Delta c}{\Delta \ell}\right|_{max} = \frac{\Delta x}{0.25 \text{ mm}}$

and $2\left.\frac{\Delta d}{d}\right|_{max} = \frac{2 \times \Delta x}{0.50 \text{ mm}} = \frac{\Delta x}{0.25}$

and hence contributions due to the errors in the measurement of d and ℓ are the same

6. A small block is connected to one end of a massless spring of un -stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad / s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45^0 as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$s, the value of v is (take g = 10 m / s^2)



(A) $\sqrt{50}$ m/s **(B)** $\sqrt{51}$ m/s **(C)** $\sqrt{52}$ m/s **(D)** $\sqrt{53}$ m/s

Sol: **[A]** The block starts its oscillation from its positive exrtreme and hence its position x_{block} from O as a function of time t can be written as

$x_{block} = 4.9 + 0.2\cos\omega t$

$\therefore$ $x_{block}(t = 1s) = 4.9 + 0.2\cos\left(\frac{\pi}{3}\times 1\right) = 5.0$ m

Hence range of the pebble R $= 10 - 5 = 5$ m

So $\frac{v^2 \sin 2\theta}{g} = 5 \Rightarrow v = \sqrt{50}$ m / s

7. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

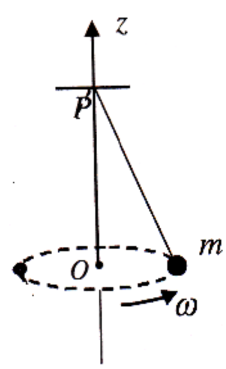
(A) $\beta_G > \beta_B > \beta_R$ **(B)** $\beta_B > \beta_G > \beta_R$ **(C)** $\beta_R > \beta_B > \beta_G$ **(D)** $\beta_R > \beta_G > \beta_B$

Sol: **[D]** Fringe width, $\beta = \frac{\lambda D}{d}$ and hence $\beta \propto \lambda$

Since $\lambda_{red} > \lambda_{green} > \lambda_{blue}$

So $\beta_{red} > \beta_{green} > \beta_{blue}$

8. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x - y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

Sol: **[C]** The net force acting on the mass m is $m\omega^2 r$ acting towards point O and hence net torque about point O is zero but about point P is non-zero. Hence $\vec{L}_0$ remains constant while $\vec{L}_P$ varies with time.

9. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the r m s speed $\left(\frac{v_{rms}(helium)}{v_{rms}(argon)}\right)$ is

(A) 0.32 **(B)** 0.45 **(C)** 2.24 **(D)** 3.16

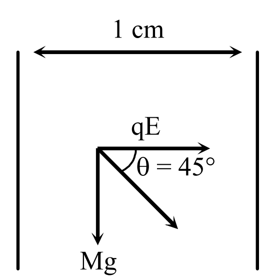
Sol: **[D]** Both gases are at same temperature and thus $v_{rms} \propto \frac{1}{\sqrt{M}}$ where M is molar mass of the gas

$$\therefore \quad \frac{v_{rms(Helium)}}{v_{rms(Argon)}} = \sqrt{\frac{M_{Argon}}{M_{Helium}}} = \sqrt{\frac{40}{4}} = 3.16$$

10. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A Proton is released at rest midway between the two plates. It is found to move at 45^0 to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V **(B)** 1×10^{-7} V **(C)** 1×10^{-9} V **(D)** 1×10^{-10} V

Sol: **[C]** As the net force is at 45^0 to the vertical so



$$qE = mg$$

or $$e\frac{X}{d} = mg$$

or $$X = \frac{mgd}{e} = \frac{1.6\times10^{-27}\times10\times10^{-2}}{1.6\times10^{-19}} = 1\times10^{-9}\text{ V}$$

Section–II (Multiple Correct Answer Type)

This section contains 5 **multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE or MORE THAN ONE** is / are correct.

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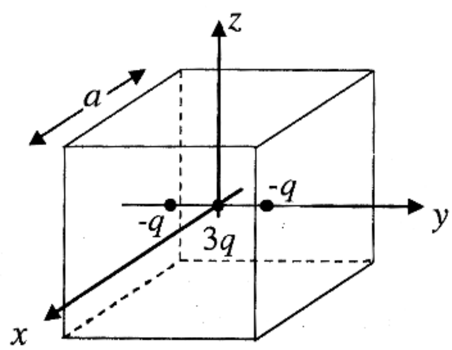
11. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at (0, 0, 0) and $-q$ at $(0, +a/4, O)$. Choose the correct option(s)



(A) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$.

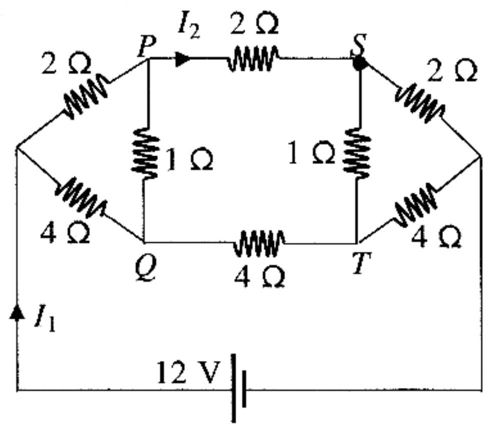
(B) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(D) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

Sol: **[A, B, C, D]** The given distribution is symmetric about planes $x = \pm\frac{a}{2}$ and $z = \pm\frac{a}{2}$. Further the distribution is separately symmetric about planes $y = \pm\frac{a}{2}$. The total charge enclosed is q so accordingly all options are correct.

12. For the resistance network shown in the figure, choose the correct option (s)



(A) The current through PQ is zero

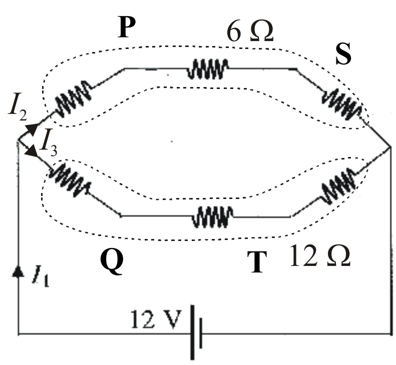
(B) $I_1 = 3A$

(C) The potential at S is less than that at Q

(D) $I_2 = 2A$

Sol: **[A, B, C, D]** Due to the symmtry potentials $V_P = V_Q$ and $V_S = V_T$

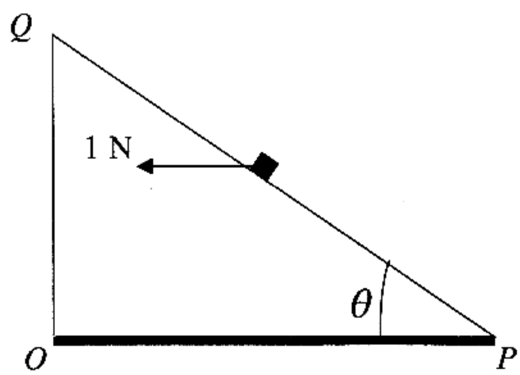
So current through PQ is zero.



Further $I_1 = \dfrac{12V}{(6 \| 12)\Omega} = 3\text{A}$ and $I_2 = \dfrac{12\ V}{6\Omega} = 2\text{A}$

Across each resistor potential decreases by 4V in the directin of current so $V_S < V_Q$

13. A small block of mass of 0.1 kg lies on a fixed inclined plane *PQ* which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take $g = 10$ m/s^2)



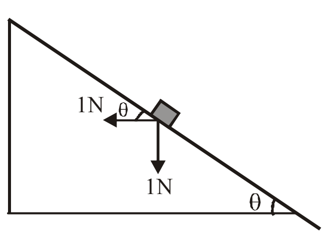
(A) $\theta = 45^0$

(B) $\theta > 45^0$ and a fricitional force acts on the block towards *P*.

(C) $\theta > 45^0$ and a frictional force acts on the block towards *Q*.

(D) $\theta < 45^0$ and a fricitional force acts on the block towards *Q*.

Sol: **[A, C]**



The resultant of weight (1 N dowanrd) and applied force (1N) is

$F_{net} = (1)(\sin\theta - \cos\theta)$

For $\theta > 45^0, F_{net}$ is down the incline so friction acts towards Q where as for $\theta < 45^0, F_{net}$ is up the incline so friction acts towards P.

For $\theta = 45^0$, the block remains at rest without friction

14. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x - y plane, making an angle θ with the x axis. Which of the following option(s) is(are) correct for time $t > 0$.

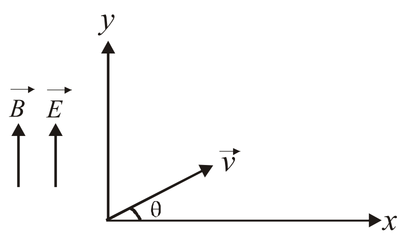
(A) If $\theta = 0^0$, the charge moves in a circular path in the x-z plane

(B) If $\theta = 0^0$, the charge undergoes helical motion with constant pitch along the y- axis.

(C) If $\theta = 10^0$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(D) If $\theta = 90^0$, the charge undergoes linear but accelerated motion along the y-axis.

Sol: [C, D]



For $0^0 \le \theta < 90^0$, charge particle, undergoes helical motion with increasing pitch. For $\theta = 90^0$, charge undergoes linear but accelerated motion along the y-axis.

15. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe

(A) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is open.

(B) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(C) a low-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

(D) a high-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed.

Sol: [B, D] Pressure wave undergoes a phase charge of π rad on refletion from open end and no phase change occurs on reflection from closed end. Accordingly B, D are correct.

Section – III (Integer Answer Type)

This section contains **5 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. (both enclusive)

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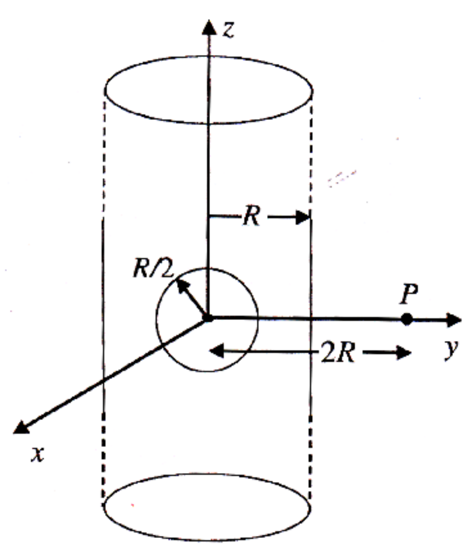
16. A ninfinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\dfrac{23\rho R}{16k\varepsilon_0}$. The value of k is



Sol. **[6]** Let us assume that the spherical cavity is filled with same charge density then

$$\vec{E} = \vec{E}_{complete\ cylinder} - \vec{E}_{spherical\ cavity}$$

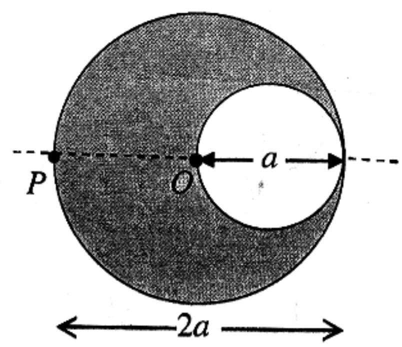
$$E = \frac{Q_1}{2\pi\varepsilon_0(2R)\ell} - \frac{Q_2}{4\pi\varepsilon_0(2R)^2}$$

$$= \frac{\rho\,\pi R^2\ell}{2\pi\varepsilon_0\ell \times 2R} - \frac{\frac{4}{3}\pi\frac{R^3}{8}\rho}{4\pi\varepsilon_0\,(4R)^2}$$

$$= \frac{\rho R}{\varepsilon_0}\left[\frac{1}{4} - \frac{1}{96}\right] = \frac{23\rho R}{96\,\varepsilon_0} = \frac{23\rho R}{16\,k\,\xi_0}$$

$\Rightarrow$ $k = 6$

17. A cylindrical cavity of diameter *a* exists inside a cylinder of diameter 2*a* as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



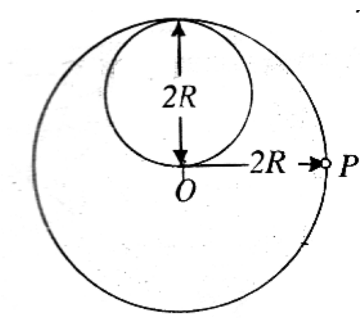
Sol: **[5]** Let us assume that cylindrical cavity is also carrying current with same current density

$$\vec{B}_{net} = \vec{B}_{complete\ cylinder} - \vec{B}_{cavity}$$

or $$B = \frac{\mu_0(\pi a^2 J)}{2\pi a} - \frac{\mu_0\left[\frac{\pi a^2}{4} J\right]}{2\pi\left(\frac{3a}{2}\right)}$$

$$= \mu_0 Ja\left[\frac{1}{2} - \frac{1}{12}\right] = \frac{5\mu_0 Ja}{12}$$

18. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_0 and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_0}$ to the nearest integer is



Sol: [2] Let us assume that the smaller disk is also filled with same surface mass density. Then as mass is propotional to area so if

$$m_{bO} = m \text{ then } m_{SD} = \frac{m}{4}$$

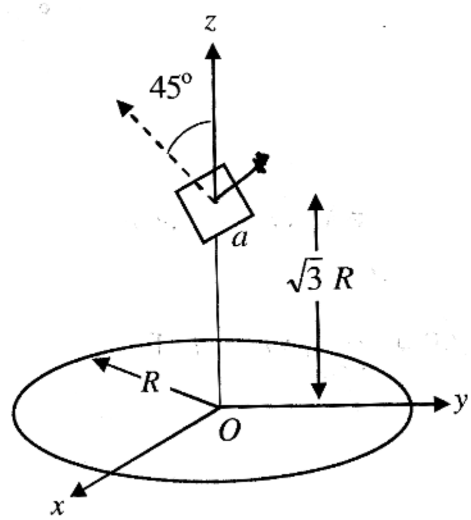
$$I_0 = \frac{m(2R)^2}{2} - \frac{m}{4}\left[\frac{R^2}{2} + R^2\right] = mR^2\left(2 - \frac{3}{8}\right) = \frac{13\, mR^2}{8}$$

$$I_{P'} = m\left[\frac{(2R)^2}{2} + (2R)^2\right] - \frac{m}{4}\left[\frac{R^2}{2} + 5R^2\right] = mR^2\left(6 - \frac{11}{8}\right)$$

$$= \frac{37}{8} mR^2$$

$$\frac{I_P}{I_O} = \frac{37/8}{13/8} = 2.8 \simeq 3$$

19. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side $a(a << R)$ having two turns is placed with its center at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of $45°$ with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is



Sol: [7] The mutual inductance

$$M = \frac{N_S\left(B_C A_S \cos 45°\right)}{i_e}$$

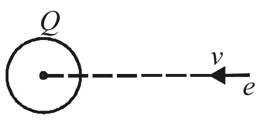
Here N_s = no of turns in square coil and B_C be the magnetic field of circular coil at the position of square coil.

$$m = (2)\frac{\frac{\mu_0 i_C R^2}{2\left(R^2 + 3a^2\right)^{3/2}} \times a^2 \times \frac{1}{\sqrt{2}}}{i_C} \simeq \frac{\mu_0 a^2}{8\sqrt{2}\,R} \simeq \frac{\mu_0 a^2}{(2)^{7/2} R}$$

$\therefore$ $p = 7$

20. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F ; 1 fm $= 10^{-15}$ m)

Sol: **[7]**



At closest approach

$$\Delta U_e = -\Delta K$$

$$\frac{kQe}{r_{min}} = \frac{p^2}{2m} = \frac{h}{2m\lambda^2}$$

$$\lambda = \sqrt{\frac{h^2 r_{min}}{2mkQe}} = \frac{h}{e}\sqrt{\frac{r_{min}}{2mk \times 120}}$$

$$= 4.2 \times 10^{-15}\sqrt{\frac{10 \times 10^{-15} \times 3}{2 \times 5 \times 10^{-27} \times 9 \times 10^9 \times 120}}$$

$$= 4.2 \times 10^{-15} \times 10 = 7 \text{ fm}$$

Part II

Section–I : Straight Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE is correct.**

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21. As per IUPAC nomenclaure, the name of the complex $\left[Co(H_2O)_4(NH_3)_2\right]Cl_3$ is

(A) Tetraaquadiaminecobalt (III) chloride
(B) Tetraaquadiamminecobalt (III) chloride
(C) Diaminetetraaquacoblat (III) chloride
(D) Diamminetetraaquacobalt (III) chloride

Ans. (D)

Sol. Diamminetetraaquacobalt(III) chloride

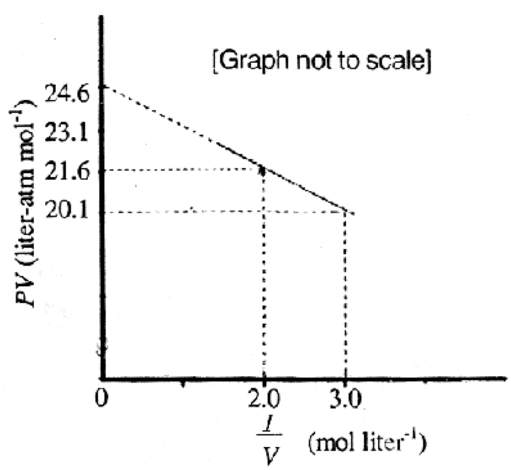
22. In allene (C_3H_4), the type(s) of hybridisationof the carbon atoms is(are)

(A) sp and sp^3 **(B)** sp and sp^2 **(C)** only sp^2 **(D)** sp^2 and sp^3

Ans. (B)

Sol. $\overset{sp^2}{H_2C} = \overset{sp}{C} = \overset{sp^2}{CH_2}$

23. For one mole of a vander Waals gas when $b = 0$ and $T = 300\,K$, the PV vs. 1 / V plot is shown below. The value of the van der Waals constant a (atm. liter^2 mol^{-2}) is



(A) 1.0 **(B)** 4.5 **(C)** 1.5 **(D)** 3.0

Ans. (C)

Sol. Vander Walls equation for 1 mol of gas

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT$$

given $b=0$

$$\left(p+\frac{a}{V^2}\right)V=RT$$

$$pV+\frac{a}{V}=RT$$

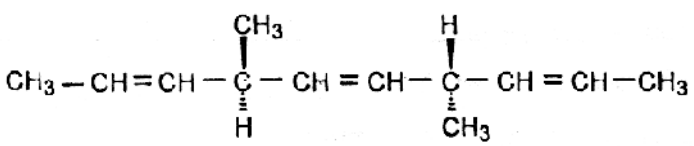
$$pV=RT-\frac{a}{V}$$

pV vs $\frac{1}{V}$ graph is a straight line have RT intercept and –a as slope

$$\text{Slope}=\frac{20.1-21.6}{3-2}=-1.5$$

so $a=1.5$

24. The number of optically active products obtained from the **complete** ozonolysis of the given compound is



(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

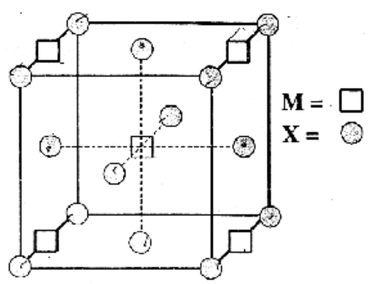
Ans. (A)

Sol. Ozonolysis products are

$$CH_3CHO \text{ and } OHC-CH(CH_3)CHO$$

none is optically active

25. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empircial formula fo the compound is



(A) MX **(B)** MX_2 **(C)** M_2X

Sol. 1 particle of M is located at bodycentre and 4 are on edge centre so total particle of M is

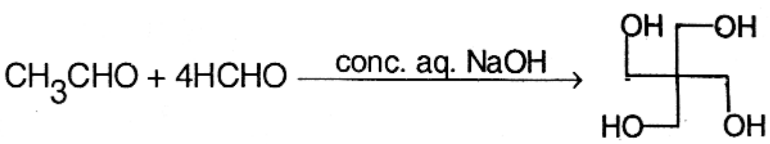
$$\frac{1}{4}\times 4 + 1 \times 1 = 2$$

X is located at all corner and face centre

$$\frac{1}{8}\times 8 + \frac{1}{2}\times 6 = 4$$

MX_2

26. The number of aldol reaction(s) that occurs in the given transformation is



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Ans. (C)

Sol. CH_3CHO has $3\alpha -$ hydrogenatoms. First three steps are Alodol reactions. Last step is cannizaro reaction.

27. The colour of light abosrbed by an aqeuous solution of $CuSO_4$ is

(A) oragne-red **(B)** blue-green **(C)** yellow **(D)** violet

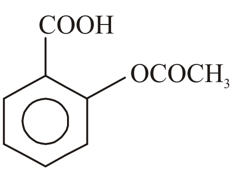
Ans. (A)

Sol. The complenentary colour of organe-red is blue

28. The carboxyl functional group (–COOH) is present in

(A) picric acid **(B)** barabituric acid **(C)** ascorbic acid **(D)** aspirin

Ans. (D)

Sol. Aspirin 

29. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]

(A) $\frac{h^2}{4\pi^2 ma_0^2}$ **(B)** $\frac{h^2}{16\pi^2 ma_0^2}$ **(C)** $\frac{h^2}{32\pi^2 ma_0^2}$ **(D)** $\frac{h^2}{64\pi^2 ma_0^2}$

Ans. (C)

Sol. according to Bohr's postulate

$$mvr = \frac{nh}{2\pi}$$

squaring both side

$$m^2v^2r^2 = \frac{n^2h^2}{(2\pi)^2}$$

$$\frac{1}{2}mv^2 = \frac{n^2h^2}{8\pi^2mr^2}$$

placing $r^2 = 16a_0^2$ for $n = 2$

$$K.E. = \frac{(2)^2h^2}{8\pi^2m16a_0^2} = \frac{h^2}{32\pi^2ma_0^2}$$

30. Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen?

(A) HNO_3, NO, NH_4Cl, N_2

(B) HNO_3, NO, N_2, NH_4Cl

(C) HNO_3, NH_4Cl, NO, N_2

(D) NO, HNO_3, NH_4Cl, N_2

Ans. (B)

Sol. In $H\underline{N}O_3$ oxidation state of $N = +5$

NO oxidation state of $N = +2$

NH_4Cl oxidation state of $N = -3$

N_2 oxidation state of $N = 0$

So order HNO_3, NO, N_2, NH_4Cl

Section–II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions.** Each question four 4 choices (A), (B), (C) and (D), out of which **ONE or MORE are correct**.

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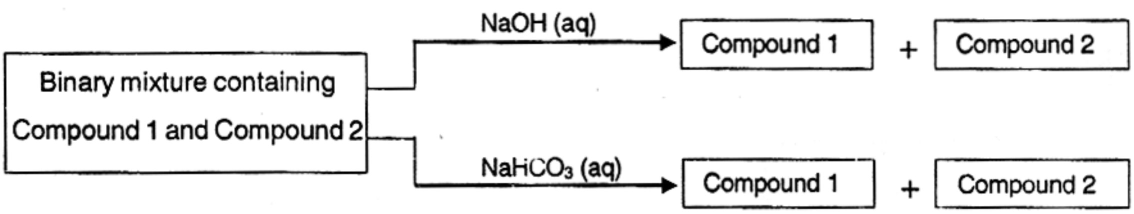
31. Identify the binary mixture(s) that can bse separated into individual compound, by differential extraction, as shown in the given scehem.



(A) C_6H_5OH and C_6H_5COOH **(B)** C_6H_5COOH and $C_6H_5CH_2OH$

(C) $C_6H_5CH_2OH$ and C_6H_5OH **(D)** $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

Ans. (B, D)

Sol. Both these mixtures can be separated by $NaOH(aq)$ as well as $NaHCO_3(aq)$

32. Choose the correct reason(s) for the stability of the **lyophobic** colloidal particles.

(A) Preferential adsorption of ions on their surface from the solution

(B) Preferential adsorptionof solvent on their surface from the solution

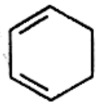
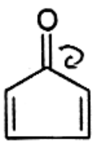
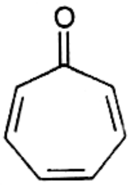
(C) Attraction between different particles having opposite charges on their surface

(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles

Ans. (A, D)

Sol. Preferential adsorption of ions and potential difference betwen fixed and diffused layer of oppsite charges.

33. Which of the following molecules, in pure form, is(are) **unstable** at room temperature?

(A)  **(B)**  **(C)**  **(D)** 

Ans. (B, C)

Sol: Both are antiaromatic.

34. Which of the following hydrogen halides react(s) with $AgNO_3(aq)$ to give a precipitate that dissolves in $Na_2S_2O_3(aq)$?

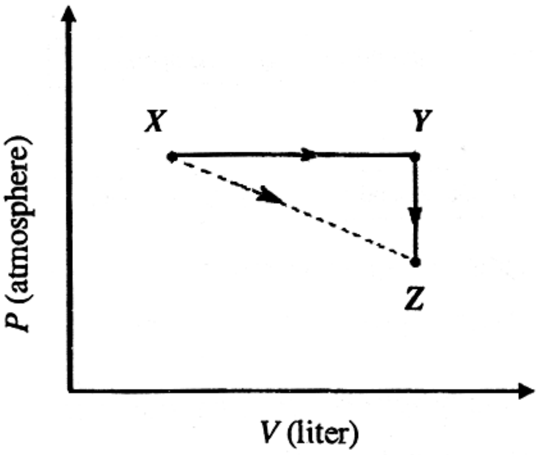
(A) HCl **(B)** HF **(C)** HBr **(D)** HI

Ans. (A, C)

Sol. Bothe AgCl and AgBr precipitates dissolve in sodium thisulphate solution.

35. For an ideal gas, consider only P-V work in going from an initial state **X** to the final state **Z**. The final state **Z** can be reached by either of the two paths shown int he figure. Which of the following choice(s) is(are) correct?

[taks ΔS as change in entropy and *w* as workd done]



(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$ **(B)** $w_{x\to z} = w_{x\to y} + w_{y\to z}$

(C) $w_{x\to y\to z} = w_{x\to y}$ **(D)** $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

Ans. (A, C)

Sol. Entropy is state function

so $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$

and $W_{x\to y\to z} = w_{x\to y}$

because process $y \to z$ is constant volume so work done is zero.

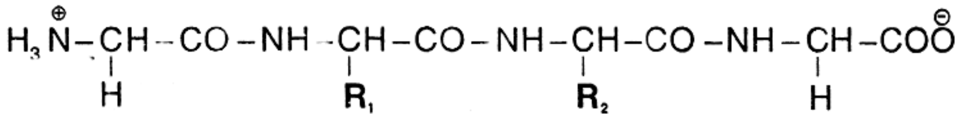
Section–III

Integer Answer Type

This section contains **5 questions**. The answer to each is a **single-digit integer**, ranging from 0 to 9 (both inclusive).

---

36. The substitutents $\mathbf{R_1}$ and $\mathbf{R_2}$ for nine peptides are listed int he table given below. How many of these peptides are positively charged at pH = 7.0?



Peptide	R_1	R_2
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$	CH_3

Ans. (4)

37. The periodic table consists of 18 groups.An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element **X** as shown below. To which group, element **X** belongs int he periodic table?

$${}^{63}_{29}Cu + {}^{1}_{1}H \rightarrow 6\,{}^{1}_{0}n + \alpha + 2\,{}^{1}_{1}H + X$$

Ans. (8)

Sol. ${}^{63}_{29}Cu + {}^{1}_{1}H \longrightarrow 6\,{}^{1}_{0}n + \alpha + 2\,{}^{1}_{1}H + X$

mass no of reactant side

$63 + 1 = 64$

mass no of product side

$6 \times 1 + 4 + 2 \times 1 + M_x$

So $M_x = 52$

atomic no $\rightarrow$

$$29 + 1 = 2 + 2 + Z_x$$

$$Z_x = 26$$

group no = 8

38. When the following aldohexose exits in its **D-**configuration, the total number of stereoisomers in its pyranose form is

CHO
|
CH_2
|
CHOH
|
CHOH
|
CHOH
|
CH_2OH

Ans. (8)

Sol. Three chiral centres $2^3 = 8$

39. 29.2% (w/w) HCl stock solution has a density of $1.25\ g\ mL^{-1}$. The molecular weight of HCl is $36.5\ g\ mol^{-1}$. The volume (mL) os stock solution required to prepare a 200 mL solution of 0.4 M HCl is

Ans. (8)

Sol. 29.2% (w/w) of HCl solution means
29.2 g of HCl = 100 g of solution

$$= \frac{100g}{1.25} mL \text{ of solution (given } d = 1.25 g / mL)$$

Now 200 mL of 0.4 M HCl is to be prepared
means = 0.08 mol
= 2.92 g of HCl
= 8 mL of given solution

40. An organic compound undergoes first-order decomposition. The time taken for its decomposition to 1 / 8 and 1 / 10 of its intial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\frac{[t_{1/8}]}{[t_{1/10}]} \times 10$?

(take $\log_{10} 2 = 0.3$)

Ans. (9)

Sol. $t_{1/8} \times k = -2.303 \log \frac{1/8a_0}{a_0}$

$$t_{1/10} \times k = -2.303 \log \frac{1/10a_0}{a_0}$$

$$\frac{t_{1/8}}{t_{1/10}} = 0.9$$

$$\frac{t_{1/8}}{t_{1/10}} \times 10 = 9$$

PART III

Section–I Single Correct Answer Type

This section contains **10 multiple choice questions** Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

---

41. The locus of the mid-point of the chord of contact of tangents drawn form points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2 + y^2) - 36x + 45y = 0$ **(B)** $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ **(D)** $36(x^2 + y^2) + 20x - 45y = 0$

Sol: [A] Consider a point on the line $(5t + 5, 4t)$ equation of the chord of contact of tangents drawn from the point $(5t + 5, 4t)$:

$$x(5t + 5) + y(4t) = 9 \quad \ldots(1)$$

Let mid -pt of the chord is (h, k). Its equation is

$$hx + ky = h^2 + k^2 \quad \ldots(2)$$

(1) and (2) represent the same line

$$\frac{5t+5}{h} = \frac{4t}{k} = \frac{9}{h^2 + k^2} = \frac{20t + 20}{4h} = \frac{20t}{5k}$$

$$= \frac{(20t + 20) - (20t)}{4h - 5k} = \frac{20}{4h - 5k}$$

$$\Rightarrow \frac{9}{h^2 + k^2} = \frac{20}{4h - 5k}$$

$$\Rightarrow 20(h^2 + k^2) - 9(4h - 5k) = 0$$

$$\Rightarrow 20h^2 + 20k^2 - 36h + 45k = 0$$

$\therefore$ Equation of locus is $20x^2 + 20y^2 - 36x + 45y = 0$

42. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 **(B)** 150 **(C)** 210 **(D)** 243

Sol: [B] Possible combination are 1, 1, 3 and 2, 2, 1

The number of required ways $= \frac{5!}{(1!)^2 \times 3!(2!)} \times 3! + \frac{5!}{(2!)^2 \times 1!(2!)} \times 3! = 60 + 90 = 150$

43. Let $f(x) = \begin{cases} x^2 \left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}, \; x \in IR$

then f is

(A) differentiable both at $x = 0$ and $x = 2$

(B) differentiable at $x = 0$ but not differentiable at $x = 2$

(C) not differentiable at $x = 0$ but differentiable at $x = 2$

(D) differentiable neither at $x = 0$ nor at $x = 2$

Sol: [B] $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$

$$= \lim_{x \to 0} \frac{x^2 \left|\cos\frac{\pi}{x}\right| - 0}{x} = \lim_{x \to 0} x\left|\cos\frac{\pi}{x}\right| = 0$$

$\therefore$ f is differentiable at $x = 0$

$$Rf'(2) = \lim_{x \to 2+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2+} \frac{x^2 \left|\cos\frac{\pi}{x}\right| - 0}{x - 2}$$

$$= \lim_{x \to 2+} \frac{x^2 \cos\frac{\pi}{x}}{x - 2} \qquad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to 2} \frac{2x\cos\frac{\pi}{x} + x^2\left(-\sin\frac{\pi}{x}\right)\left(-\frac{\pi}{x^2}\right)}{x} \text{ (Applying L' Hospital rule)}$$

$= 0 + \pi = \pi$

$$L\,f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{x^2 \left|\cos\frac{\pi}{x}\right| - 0}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{-x^2 \cos\frac{\pi}{x}}{x-2} = -\pi \qquad \left[\frac{0}{0} \text{ form}\right]$$

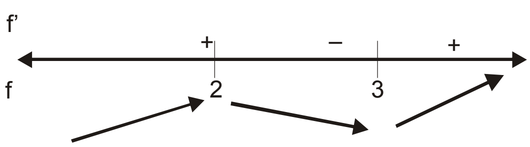
$\therefore$ f is not differentiable at $x = 2$

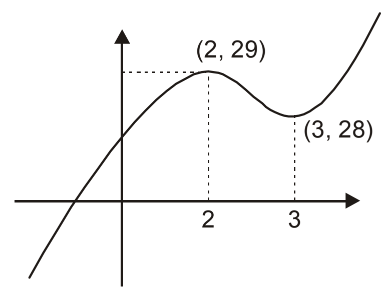
44. The function $f:[0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one-one and onto.
(B) onto but not one-one.
(C) one-one but not onto
(D) neither one-one nor onto.

Sol: [B] $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$





$f(0) = 1,\ f(2) = 29,\ f(3) = 28$

$\therefore$ f is not one-one but onto.

45. If $\lim_{x \to \infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$ then

(A) $a = 1, b = 4$ **(B)** $a = 1, b = -4$ **(C)** $a = 1, b = -3$ **(D)** $a = 2, b = 3$

Sol: [B] $\lim_{x \to \infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$

$$\Rightarrow \lim_{x \to \infty} \frac{x^2+x+1-(ax+3)(x+1)}{x+1} = 4$$

$$\Rightarrow \lim_{x \to \infty} \frac{(1-a)x^2+(1-a-b)x+(1-b)}{x+1} = 4$$

$$\Rightarrow \lim_{x \to \infty} \frac{(1-a)x+(1-a-b)+\frac{1-b}{x}}{1+\frac{1}{x}} = 4$$

$\Rightarrow 1-a = 0$ AND $1-a-b = 4$ $\qquad \Rightarrow a = 1$ AND $b = -4$

46. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a **cannot** take the value

(A) -1 **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{4}$

Sol: [D] Let $z = x + iy, y \neq 0$

$$a = z^2 + 2 + 1 = (x + iy)^2 + x + iy + 1$$

$$(x^2 - y^2 + x + 1) + i(2 \times y + y)$$

$\Rightarrow$ $2 + y + y = 0$ $\Rightarrow$ $y(2x + 1) = 0$

$\Rightarrow$ $2x + 1 = 0$ $\because$ $y \neq 0$

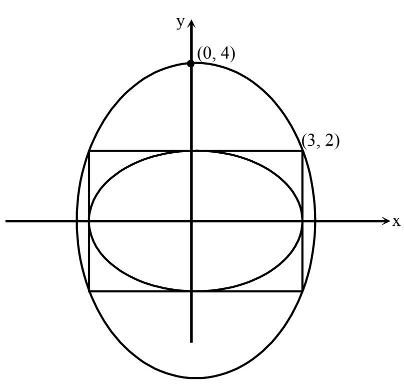
$\Rightarrow$ $x = -\frac{1}{2}$

Now $a = x^2 - y^2 + x + 1 = \frac{3}{2} - y^2 < \frac{3}{2}$ $\therefore$ $a \neq \frac{3}{4}$

47. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a reactangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{4}$

Sol: [C]



Let equation of ellipse E_2 is $\frac{x^2}{a^2} + \frac{y^2}{16} = 1$

This passes through (3, 2) $= \frac{9}{a^2} + \frac{4}{16} = 1$

This passes through (3, 2)

$\therefore \quad \frac{9}{a^2}+\frac{4}{16}=1 \quad \Rightarrow a^2=12 \quad \Rightarrow \quad e=\sqrt{1-\frac{12}{16}}=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

48. Let $P=\left[a_{ij}\right]$ be a 3×3 matrix and let $Q=\left[b_{ij}\right]$, where $b_{ij}=2^{i+j}a_{ij}$ for $1\le i, j\le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} **(B)** 2^{11} **(C)** 2^{12} **(D)** 2^{13}

Sol: [D] $|Q|=\begin{vmatrix} 2^3a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{vmatrix}$

$=(2^2)(2^3)(2^4)=\begin{vmatrix} a_{11} & 2a_{12} & 2^2a_{13} \\ a_{21} & 2a_{22} & 2^2a_{23} \\ a_{31} & 2a_{32} & 2^2a_{33} \end{vmatrix}=2^9\cdot(2)(2^2)=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$=2^{12}\cdot|P| \quad =2^{12}\cdot(2)$

$=2^{13}$

49. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(B) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

Sol: [C] $I=\int \frac{\sec^2 x(\sec x+\tan x)}{(\sec x+\tan x)^{11/2}}dx$

Let $\sec x + \tan x = t$ $\Rightarrow$ $\sec x(\sec x + \tan x)\, dx = dt$

Also $\sec x - \tan x = \frac{1}{t} \Rightarrow 2\sec x = t + \frac{1}{t}$

$$I = \int \frac{(2\sec x)\cdot \sec x(\sec x + \tan x)}{2(\sec x + \tan x)^{11/2}} dx = \int \frac{\left(t + \frac{1}{t}\right) dt}{2\, t^{11/2}}$$

$$= \frac{1}{2}\int (t^{-9/2} + t^{-13/2})\, dt$$

$$= \frac{1}{2}\left(\frac{t^{-7/2}}{-7/2} + \frac{t^{-11/2}}{-11/2}\right) + k \quad = -\frac{1}{t^{11/2}}\left(\frac{t^2}{7} + \frac{1}{11}\right) + k$$

$$= -\frac{1}{(\sec x + \tan x)}\left(\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right) + k$$

50. The point P is the intersection of the striaght line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ **(B)** $\sqrt{2}$ **(C)** 2 **(D)** $2\sqrt{2}$

Sol: [A] Equation of the line QR $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1} = \lambda(\text{say})$

$P \equiv (\lambda + 2, 4\pi + 3, \lambda + 5)$

P lies on the plane $5(\lambda + 2) - 4(4\lambda + 3) - (\lambda + 5) = 1$

$\Rightarrow$ $\lambda = -\frac{2}{3}$

$\therefore$ $P \equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Dr's of QR $= (2-1, 3+1, 5-4) = (1, 4, 1)$

Dr's of PT $= \left(\frac{4}{3} - 2, \frac{1}{3} - 1, \frac{13}{3} - 4\right) = \left(\frac{-2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

$\therefore$ Angle between TP and QR $= \theta \Rightarrow \cos\theta = \frac{(-2)\,(1) + (-2)\,(4) + (1)\,(1)}{\sqrt{1+16+1}\,\sqrt{4+4+1}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^{\circ}$

$\therefore \quad \text{Also TP} = \sqrt{\frac{4}{9}+\frac{4}{9}+\frac{1}{9}} = 1$

$\therefore \quad TS = TP\ \cos 45^{o} = 1 \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

SECTION II :

Multiple Correct Answer(s) Type

The section contains **5 multiple choice questions**. Each questions has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**

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51. Let $\theta, \varphi \in [0, 2\pi]$ be such that

$$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi - 1,$$

$\tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$

Then φ **cannot** satisfy

(A) $0 < \varphi < \frac{\pi}{2}$ **(B)** $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ **(C)** $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ **(D)** $\frac{3\pi}{2} < \varphi < 2\pi$

Sol: [A, B, C] $2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\frac{1+\tan^2\frac{\theta}{2}}{\tan\frac{\theta}{2}}\right)\cos\varphi - 1$

$= \sin^2\theta \cdot \frac{2}{\sin\theta}\cos\varphi - 1 \quad = 2\sin\theta\cos\varphi - 1$

$\Rightarrow \quad 2\cos\theta + 1 = 2\sin\theta\cos\varphi + 2\cos\theta\sin\varphi \quad = 2\sin(\theta+\varphi)$

$\sin(\theta+\varphi) = \cos\theta + \frac{1}{2}$

Now $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$ AND $\tan\theta < 0$

$\Rightarrow \quad 0 < \cos\theta < \frac{1}{2} \quad \left(\frac{3\pi}{2} < \theta < \frac{5\pi}{3}\right)$

$\therefore \quad \frac{1}{2} < \sec(\theta+\varphi) < 1$

For $0 < \varphi < \frac{\pi}{2} \Rightarrow \frac{3\pi}{2} < \theta + \varphi < \frac{13\pi}{6}$

$\Rightarrow \quad -1 < \sin(\theta+\varphi) < \frac{1}{2}$ (A) is an answer

For $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ $\Rightarrow 2\pi < \theta + \varphi < 3\pi$

$\Rightarrow$ (B) is Not an answer

$\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ $\Rightarrow \frac{17\pi}{6} < \theta + \varphi < \frac{19\pi}{6}$ $\Rightarrow$ (C) is an answer

$\frac{3\pi}{2} < \varphi < 2\pi$ $\Rightarrow 3\pi < \theta + \varphi < \frac{11\pi}{3}$ $\Rightarrow$ (D) is an answer

52. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then

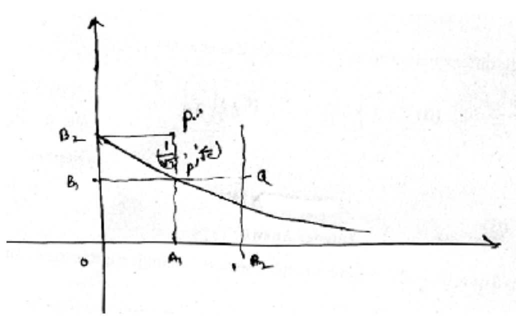
(A) $S \ge \frac{1}{e}$

(B) $S \ge 1 - \frac{1}{e}$

(C) $S \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

(D) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Sol: [A, B, D]



$S \ge (1)\left(\frac{1}{e}\right)$ $\Rightarrow S \ge \frac{1}{e}$

$e^{-x^2} \ge e^{-x}$ for $x \in [0,\ 1]$

$\Rightarrow \int e^{-x^2} dx \ge \int e^{-x}\, dx$ $\Rightarrow S \ge 1 - \frac{1}{e}$

$\Rightarrow \quad S \ge 1 - \frac{1}{e} > \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

$S <$ area of $OA_1\ PB_2$ + area of A_1A_2QP' $= \frac{1}{\sqrt{2}}(1) + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

53. A ship is fitted with three engiens E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

(A) $P\left[X_1^c \mid X\right] = \frac{3}{16}$ **(B)** P [Exactly two engines of the ship are functioning | X]= $\frac{7}{8}$

(C) $P\left[X \mid X_2\right] = \frac{5}{16}$ **(D)** $P\left[X \mid X_1\right] = \frac{7}{16}$

Sol: [B,D] $P(X_1) = \frac{1}{2}, P(X_2) = \frac{1}{4}, \ P(X_3) = \frac{1}{4}$

$$P(X) = P(X_1 \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap \overline{X}_3) + P(X_1 \cap \overline{X}_2 \cap X_3) + P(\overline{X}_1 \cap X_2 \cap X_3)$$

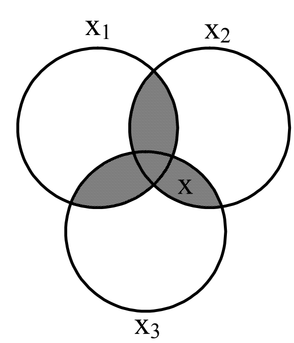
$$= \frac{1}{2} \times \frac{1}{4} \times \frac{1}{9} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{8}{32}$$

$$P(X_1^c / X) = 1 - P(X_1 / X)$$

$$= 1 - P\frac{(X_1 \cap X)}{P(X)}$$

$$= 1 - \frac{P(X_1 \cap X_2 \cap X_3') + P(X_1 \cap X_2' \cap X_3) + P(X_1 \cap X_2 \cap X_3)}{P(X)}$$

$$= 1 - \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{8}{32}} = 1 - \frac{7}{8} = \frac{1}{8}$$



P (Exactly two functioning / X) =

$$= \frac{P(X_1 \cap X_2 \cap X_3') + P(X_1 \cap X_2' \cap X_3) + P(X_1' \cap X_2 \cap X_3)}{P(X)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{8}{32}} = \frac{7}{8}$$

$$P(X/X_2) = \frac{P(X\cap X_2)}{P(X_2)} = \frac{P(X_1\cap X_2\cap X_3') + P(X_1'\cap X_2\cap X_3) + P(X_1\cap X_2\cap X_3)}{P(X_2)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}}{\frac{1}{4}} = \frac{5}{8}$$

$$P(X/X_1) = \frac{P(X_1\cap X_2'\cap X_3) + P(X_1\cap X_2\cap X_3') + P(X_1\cap X_2\cap X_3)}{P(X_1)}$$

$$= \frac{\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}}{\frac{1}{2}} = \frac{7}{16}$$

54. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ **(B)** $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ **(C)** $\left(3\sqrt{3},-2\sqrt{2}\right)$ **(D)** $\left(-3\sqrt{3},2\sqrt{2}\right)$

Sol: [A,B] Equation of Tangent $y = 2x \pm \sqrt{9(2)^2-4}$

$$y = 2x \pm 4\sqrt{2} \quad \ldots (1)$$

tangent at point (x_1, y_1) $\frac{xx_1}{9}-\frac{yy_1}{4}=1$

$$\pm\frac{x}{2\sqrt{2}} \mp \frac{y}{4\sqrt{2}} = 1 \qquad \Rightarrow \frac{x\left(\pm\frac{9}{2\sqrt{2}}\right)}{9} - \frac{y\left(\pm\frac{1}{\sqrt{2}}\right)}{4} = 1$$

55. If y(x) satisfies the differential equation $y' - y\tan x = 2x\sec x$ and $y(0)=0$, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ **(B)** $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$ **(C)** $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ **(D)** $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Sol: [A,D] $y' + y(-\tan x) = 2x\sec x$

Integration factor $I(x) = e^{\int -\tan x dx} = \cos x$

Solution is given by $y(\cos x) = \int (2x \sec x) \cos x dx = x^2 + K$

$y(0) = 0 \Rightarrow K = 0 \qquad \Rightarrow \quad y = x^2 \sec x$

SECTION III :

Integer Answer Type

This section contains **5 questions.** The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive)

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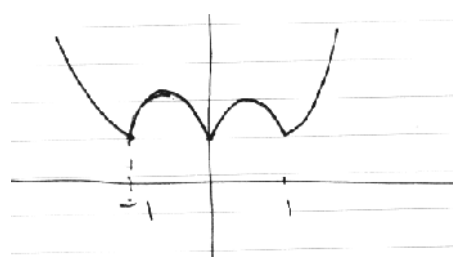
56. Let $f: IR \to IR$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Sol: [5] $f(x) = (x) + |x^2 - 1|$

$$= \begin{cases} -x + (x^2 - 1) & -\infty < x \le -1 \\ -x + (1 - x^2) & 1 < x \le -0 \\ x + (1 - x^2) & 0 < x \le 1 \\ x + x^2 - 1 & 1 < x \le \infty \end{cases}$$



$$= \begin{cases} x^2 - x - 1 & -\infty < x \le -1 \\ -x^2 + x + 1 & -1 < x \le 0 \\ -x^2 + x + 1 & 0 < x \le 1 \\ x^2 + x - 1 & 1 < x \le \infty \end{cases}$$

57. The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\ldots}}}\right)$ is

Sol: Let $x = \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{3}{3}\sqrt{4}\ldots}}$

$\Rightarrow \quad x = \frac{1}{3\sqrt{2}}\sqrt{4 - x} \qquad \Rightarrow 3\sqrt{2}\,x = \sqrt{4 - x}$

$\Rightarrow \quad 18x^2 = 4 - x \qquad \Rightarrow 18x^2 + x - 4 = 0$

$\Rightarrow \quad (9x-4)(2x+1)=0 \qquad \Rightarrow x=\frac{4}{9} \quad [\because\ x>0]$

$\therefore \quad 6+\log_{3/2}\left(\frac{4}{9}\right)=6+\log_{3/2}\left(\frac{3}{2}\right)^{-2}=6-2=4$

58. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x=1$ and a local minimum at $x=3$. If $p(1)=6$ and $p(3)=2$, then $p'(0)$ is

Sol: [9] Least degree the polynomial having a local maximum and a local minimum is 4.

$$P'(x)=A(x-1)(x-3)=A(x^2-4x+3)$$

$$P(x)=A\left(\frac{x^3}{3}-2x^2+3x\right)+B$$

$$P(1)=A\left(\frac{1}{3}-2+3\right)+B$$

$\frac{4}{3}A+B=6 \qquad \Rightarrow 4A+3B=18$

$P(3)=2 \Rightarrow A(9-18+9)+B=2 \qquad \Rightarrow B=2$

$P(B)=2 \Rightarrow A(9-18+9)+B=2 \qquad \Rightarrow B=2$

$\therefore \quad A=3 \qquad \Rightarrow P'(0)=3A=9$

59. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

Sol: [3] $2\left(|a|^2+|b|^2+|c|^2\right)-2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=9$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\frac{3}{2}$$

$$|2\vec{a}+5\vec{b}+5\vec{c}|^2=4|\vec{a}|^2+25|\vec{b}|^2+25|\vec{c}|^2$$

$$+20\,\vec{a}\cdot\vec{b}+50\,\vec{b}\cdot\vec{c}+50\,\vec{c}\cdot\vec{a}=54-30\,\vec{a}\cdot\vec{b}+50(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$=54-30\,\vec{a}\cdot\vec{b}+50\left(-\frac{3}{2}\right)=-21-30\,\vec{a}\cdot\vec{b}$$

$$|2\vec{a}+5\vec{b}+5\vec{c}|^2\le -21-30(-1)=9$$

$$\Rightarrow \quad |2\vec{a}+5\vec{b}+5\vec{c}|\le 3$$

60. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Sol: [4] $x^2 + y^2 - 2x - 4y = 0$

$\Rightarrow$ $x(x-2) + y(y-4) = 0$

$\Rightarrow$ $(0,\ 0)$ and $(2, 4)$ are diametric points

Also $(2, 4)$ lies on the parabola

$\therefore$ $P \equiv (0, 0)$, $Q \equiv (2, 4)$

$\Rightarrow$ $S \equiv (2, 0)$

$\therefore$ Area of Δ PQS $= \frac{1}{2} \times 2 \times 4 = 4$