Application Form Number:	
Name:	
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M.A./M.Sc ADMISSION ENTRANCE TEST — 2014

INSTRUCTIONS TO THE CANDIDATES

(1) Duration: 3 Hours
(2) Enter your name , application number and also sign in the space provided above. Also write your application form number on the coding sheet.
(3) Do not open the question paper booklet until the Invigilator gives the signal for the commencement of the examination.
(4) The question paper shall be of 250 marks.
(5) The paper will consist of two parts: Part I (150 marks / 50 questions of 3 marks each) and Part II (100 marks / 20 questions of 5 marks each).
(6) **Part-I contains 50 Multiple Choice Questions having exactly one correct answer. Shade exactly one option. For each correct answer, three marks will be given and for an incorrect answer one mark will be deducted.**
(7) **Part-II consists of 20 Multiple Choice Questions. Questions may have multiple correct answers and carry five marks. Shade all correct option(s). Five marks will be given only if all correct choice(s) are shaded. There will be no negative marking in this part.**
(8) The answers should be given only in the coding sheet. Do not write or mark anything in the question paper booklet.
(9) Four rough work sheets are provided.
(10) You are advised to complete answering 10 minutes before the end of the examination and verify all your entries.
(11) **Every question paper and OMR sheet has one of the four codes: A, B, C, D. Check whether the answer booklet code matches with question paper code.**
(12) At the end of the examination when the Invigilator announces **Stop writing**, you must stop immediately and place the coding sheet, question paper booklet, rough work sheets, and acknowledgement letter on the table. You should not leave the hall until all the above sheets are collected.

Part I: For each correct answer 3 marks will be given and for an incorrect answer one mark will be deducted. The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ respectively denote the sets of integers, rational numbers, real numbers and complex numbers.

(1) How many elements are there in $\mathbb{Z}[i]/\langle 3+i\rangle$?

A) infinite
B) 3
C) 10
D) finite but not 3 or 10.

(2) Let P be the set of all $n \times n$ complex Hermitian matrices. Then P is a vector space over the filed of

A) $\mathbb{C}$
B) $\mathbb{R}$ but not $\mathbb{C}$
C) both $\mathbb{R}$ and $\mathbb{C}$
D) $\mathbb{C}$ but not $\mathbb{R}$.

(3) Which one of the following is true?

A) There are infinitely many one-one linear transformations from $\mathbb{R}^4$ to $\mathbb{R}^3$
B) The dimension of the vector space of all 3×3 skew-symmetric matrices over the field of real numbers is 6
C) Let F be a field and A a fixed $n \times n$ matrix over F. If $T : M_n(F) \to M_n(F)$ is a linear transformation such that $T(B) = AB$ for every $B \in M_n(F)$, then the characteristic polynomial for A is the same as the characteristic polynomial for T
D) A two-dimensional vector space over a field with 2 elements has exactly 3 different basis.

(4) Let V and W be vector spaces over a filed F. Let $S : V \to W$ and $T : W \to V$ be linear transformations. Then which one of the following is true?

A) If ST is one-to-one, then S is one-to-one
B) If $V = W$ and V is finite-dimensional such that $TS = I$, then T is invertible
C) If $\dim V = 2$ and $\dim W = 3$, then ST is invertible
D) If TS is onto, then S is onto.

(5) The order of the automorphism group of Klein's group is

A) 3
B) 4
C) 6
D) 24.

(6) Which one of the following group is cyclic?

A) The group of positive rational numbers under multiplication

B) The dihedral group of order 30
C) $\mathbb{Z}_3 \oplus \mathbb{Z}_{15}$
D) Automorphism group of $\mathbb{Z}_{10}$.

(7) Which one of the following is a field?

A) An infinite integral domain
B) $\mathbb{R}[x]/\langle x^2-2\rangle$
C) $\mathbb{Z}_3 \oplus \mathbb{Z}_{15}$
D) $\mathbb{Q}[x]/\langle x^2-2\rangle$.

(8) Which one of the following is true for the transformation $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ defined by $T(f) = f + f' + f''$?

A) T is one-to-one but not onto
B) T is onto but not one-to-one
C) T is invertible
D) the matrix of T with respect to the basis $\{1, x, x^2\}$ is upper triangular.

(9) In $\mathbb{Z}[x]$, the ideal of $\langle x\rangle$ is

A) maximal but not prime
B) prime but not maximal
C) both prime and maximal
D) neither prime not maximal.

(10) Which one of the following is true for the transformation $T : M_n \to \mathbb{C}$ defined by $T(A) = \operatorname{tr} A = \sum_{i=1}^{n} A_{ii}$?

A) Nullity of T is n^2-1
B) Rank of T is n
C) T is one-to-one
D) $T(AB) = T(A)T(B)$ for all $A, B \in M_{n\times n}$.

(11) Let $W_1 = \{A \in M_n(\mathbb{C}) : A_{ij} = 0 \ \forall\ i \le j\}$ and W_2 is the set of symmetric matrices of order n. Then the dimension of $W_1 + W_2$ is

A) n
B) $2n$
C) n^2
D) $n^2 - n$.

(12) The logarithmic map from the multiplicative group of positive real numbers to the additive group of real number is

A) a one-to-one but not an onto homomorphism
B) an onto but not a one-to-one homomorphism
C) not a homomorphism
D) an isomorphism.

(13) If f is a group homomorphism from $(\mathbb{Z}, +)$ to $(\mathbb{Q} - \{0\}, \cdot)$ such that $f(2) = 1/3$, then the value $f(-8)$ is

A) 81
B) 1/81
C) 1/27
D) 27.

(14) The quotient group $\mathbb{Q}_8/\{1,-1\}$ is isomorphic to

A) $(\mathbb{Q}_8,\cdot)$
B) $(\{1,-1\},\cdot)$
C) $(V_4,+)$
D) $(\mathbb{Z}_4,+)$.

(15) The converse of Lagrange's theorem does not hold in

A) A_4, the alternating group of degree 4
B) $A_4\times\mathbb{Z}_2$
C) the additive group of integers modulo 4
D) Klein's four group.

(16) The ring $(R,+,\cdot)$ is an integral domain when R is

A) $M_2(\mathbb{Z})$
B) $\mathbb{Z}_7$
C) $\mathbb{Z}_6$
D) $C[0,1]$ of all continuous functions from $[0,1]$ to $\mathbb{R}$.

(17) The polynomial ring $\mathbb{Z}[x]$ is

A) a field
B) a principal ideal domain
C) unique factorization domain
D) Euclidian domain.

(18) An algebraic number is a root of a polynomial whose coefficients are rational. The set of algebraic numbers is

A) finite
B) countably infinite
C) uncountable
D) none of these.

(19) Let $f:A\to A$ and $B\subset A$. Then which one of the following is always true?

A) $B\subset f^{-1}(f(B))$
B) $B=f^{-1}(f(B))$
C) $B\subset f(f^{-1}(B))$
D) $B=f(f^{-1}(B))$.

(20) Which one of the following does not imply $a=0$?

A) For all $\epsilon>0$, $0\le a<\epsilon$
B) For all $\epsilon>0$, $-\epsilon<a<\epsilon$
C) For all $\epsilon>0$, $a<\epsilon$
D) For all $\epsilon>0$, $0\le a\le\epsilon$.

(21) Let X and Y be metric spaces and $f:X\to Y$ be continuous. Then f maps

A) open sets to open sets and closed sets to closed sets

B) compact sets to bounded sets
C) connected sets to compact sets
D) bounded sets to compact sets.

(22) Suppose $f : [0, 1] \to \mathbb{R}$ is bounded. Then

A) f is Riemann integrable on $[0, 1]$
B) f is continuous on $[0, 1]$ except for finitely many points implies f is Riemann integrable on $[0, 1]$
C) f is Riemann integrable on $[0, 1]$ implies f is continuous on $[0, 1]$
D) f is Riemann integrable on $[0, 1]$ implies f is monotone function.

(23) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sin x^3$, then f is

A) uniformly continuous
B) not differentiable
C) continuous but not uniformly continuous
D) not continuous.

(24) Consider the sequence $\langle f_n \rangle$ defined by $f_n(x) = 1/(1 + x^n)$ for $x \in [0, 1]$. Let $f(x) = \lim_{n\to\infty} f_n(x)$. Then

A) For $0 < a < 1$, $\langle f_n \rangle$ converges uniformly to f on $[0, a]$
B) the sequence $\langle f_n \rangle$ converges uniformly to f on $[0, 1]$
C) the sequence $\langle f_n \rangle$ converges uniformly to f on $[1/2, 1]$
D) the sequence $\langle f_n \rangle$ converges uniformly to f on $[0, 1]$.

(25) The open unit ball $B((0, 0), 1)$ in the metric space $(\mathbb{R}^2, d)$ where the metric d is defined by $d((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$ is the inside portion of

A) the circle centered at the origin and radius 1
B) the rectangle with vertices at $(0, 1)$, $(1, 0)$, $(-1, 0)$, $(0, -1)$
C) the rectangle with vertices at $(1, 1)$, $(1, -1)$, $(-1, 1)$, $(-1, -1)$
D) the triangle with vertices $(0, 1)$, $(-1, -1)$, $(1, -1)$.

(26) Let $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences of real numbers such that $a_n = b_n - b_{n+1}$ for $n \in \mathbb{N}$. If $\sum b_n$ is convergent, then which of the following is true?

A) $\sum a_n$ may not converge
B) $\sum a_n$ is convergent and $\sum a_n = b_1$
C) $\sum a_n$ is convergent and $\sum a_n = 0$
D) $\sum a_n$ is convergent and $\sum a_n = a_1 - b_1$.

(27) Let f be a real-valued function on $[0, 1]$ such that $f(0) = -1$ and $f(1) = 1/2$, then there always exists a $t \in (0, 1)$ such that

A) $f'(t) = -2$ B) $f'(t) = 1$
C) $f'(t) = 3/2$ D) $f'(t) = -1/2$.

(28) Let S and T be subsets of $\mathbb{R}$. Select the incorrect statement:

A) $(\operatorname{int} S) \cap (\operatorname{int} T) = \operatorname{int}(S \cap T)$
B) $(\operatorname{int} S) \cup (\operatorname{int} T) \subset \operatorname{int}(S \cup T)$
C) $\overline{S}$ is closed in $\mathbb{R}$
D) $\overline{T}$ is the largest closed set containing T.

(29) The number of solutions of the equation $3^x + 4^x = 5^x$ in the set of positive real numbers is exactly

A) 1 B) 2 C) 3 D) 5.

(30) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable and f' be bounded. Then

A) f has a local maximum at exactly one point of $\mathbb{R}$
B) f has a local maximum at exactly two point of $\mathbb{R}$
C) f is uniformly continuous on $\mathbb{R}$
D) $f + f'$ is uniformly continuous on $\mathbb{R}$.

(31) Let $a_n = 2^n + n^2$ for $n \leq 100$ and $a_n = 3 + (-1)^n \frac{n^2}{2^n+1}$ for $n > 100$. Then

A) $\langle a_n \rangle$ is a Cauchy sequence
B) $\langle a_n \rangle$ is an unbounded sequence
C) $\langle a_n \rangle$ has exactly three limit points
D) $\langle a_n \rangle$ has two convergent subsequences converging to two different points.

(32) Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(0,0) = 0$ and $f(x,y) = \frac{x^3-y^3}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Then

A) f is continuous on $\mathbb{R}^2$
B) f is continuous at all points of $\mathbb{R}^2$ except at $(0,0)$
C) $f_x(0,0) = f_y(0,0)$
D) f is bounded.

(33) Let $f : [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1/2 & \text{if } x = 1/4 \\ 1/4 & \text{if } x = 1/2 \\ 0 & \text{if } x \in [0,1] \setminus \{1/4, 1/2\}. \end{cases}$$

Then

A) f is Riemann integrable and $\int_0^1 f(x)dx = 3/4$

B) f is Riemann integrable and $\int_0^1 f(x)dx = 1/4$
C) f is Riemann integrable and $\int_0^1 f(x)dx = 0$
D) f is not Riemann integrable.

(34) Let $f : [0, \pi/2] \to \mathbb{R}$ be continuous and satisfy $\int_0^{\sin x} f(t)dt = \sqrt{3}x/2$ for $0 \leq x \leq \pi/2$. Then $f(1/2)$ equals

A) $1/2$ B) $1/\sqrt{2}$ C) $1/\sqrt{3}$ D) 1.

(35) For $n \in \mathbb{N}$, let $f_n(x) = \frac{\sin x}{x} + \frac{\cos x}{\sqrt{n}}$ for $x \in (0, \pi/2]$. Then

A) $\langle f_n \rangle$ converges uniformly on $(0, \pi/2)$ but not on $(0, \pi/2]$
B) $\langle f_n \rangle$ converges uniformly on $(0, \pi/2]$
C) $\langle f_n \rangle$ converges uniformly on $(0, \pi/4)$ but not on $(0, \pi/4]$
D) none of these.

(36) Define a metric d on $\mathbb{R}$ by $d(x, x) = 0$ for any x and $d(x, y) = 1$ for any x, y with $x \neq y$. Let $\langle a_n \rangle$ be a Cauchy sequence in $\langle \mathbb{R}, d \rangle$. Then

A) $\langle a_n \rangle$ is a constant sequence
B) $\langle a_n \rangle$ contains infinitely many points
C) $\langle a_n \rangle$ contains at most finite number of distinct points
D) none of these.

(37) The singular solution of $y = px + p^3$, $p = dy/dx$ is

A) $4y^3 + 27x^2 = 0$ B) $4x^2 + 27y^3 = 0$
C) $4y^2 - 27x^3 = 0$ D) $4x^3 + 27y^2 = 0$.

(38) Consider the following initial value problem: $(x + 1)^2 y'' - 2(x + 1)y' + 2y = 1$ subject to the condition $y(0) = 0$ and $y'(0) = 1$. Given that $x + 1$ and $(x + 1)^2$ are linearly independent solutions of the corresponding homogeneous equation, the value of $y(1/2)$ is equal to

A) 5/16 B) 7/8 C) 0 D) 1/24.

(39) Assume that all the roots of the polynomial equation $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$ have negative real parts. If $u(t)$ is any solution to the differential equation: $a_n u^{(n)} + a_{n-1}u^{(n-1)} + \cdots + a_1 u' + a_1 u = 0$, the value of the limit $\lim_{t\to\infty} u(t)$ is

A) 0 B) n C) ∞ D) 1.

(40) The initial value problem $y' = y^{2/3}$ with $0 \leq x \leq a$ for any positive real number a and $y(0) = 0$ has

A) infinitely many solutions
B) more than one but finitely many solutions
C) unique solution
D) no solution.

(41) One of the particular integrals of the partial differential equation $r - 2s + t = \cos(2x + 3y)$ is

A) $-\cos(2x + 3y)$ B) $\cos(2x + 3y)$
C) $\sin(2x + 3y)$ D) none of these.

(42) The region in which the equation $xu_{xx} + u_{yy} = x^2$ is hyperbolic is

A) the whole plane $\mathbb{R}^2$ B) the half plane $x > 0$
C) the half plane $y > 0$ D) the half plane $x < 0$.

(43) The solution of Cauchy problem $u_t + uu_s = x$, $u(x, 0) = 1$ is $u(x, t) =$

A) $x \tanh t + \operatorname{sech} t$ B) $\tanh t + \operatorname{sech} t$
C) $(x^2 + t^2) \sin t$ D) none of these.

(44) The integral surface that satisfies the first order partial differential equation:

$$(x^2 - y^2 - z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz$$

is given by

A) $\phi(xy/z, y^2/(x^2 + z^2)) = 0$
B) $\phi(y/z, (x^2 + y^2 + z^2)/x) = 0$
C) $\phi(y/z, (x^2 + y^2 + z^2)/z) = 0$
D) $\phi(y/(zx), x^2/(y^2 + z^2)) = 0$.

(45) Consider the diffusion equation $u_{xx} = u_t$ with $0 < x < \pi$ and $t > 0$, subject to the initial and boundary conditions: $u(x, 0) = 4 \sin 2x$ for $0 < x < \pi$ and $u(0, t) = 0 = u(\pi, t)$ for $t > 0$. Then, $u(\pi/8, 1)$ is equal to:

A) $4e^{-4}/\sqrt{2}$ B) $4e^{-9}/\sqrt{2}$ C) $4/e^2$ D) $4/\sqrt{e}$.

(46) The general solution to the second order partial differential equation $u_{xx} + u_{xy} - 2u_{yy} = (y + 1)e^x$ is given by

A) $\phi_1(y - x) + \phi_2(y + 2x) + xe^y$
B) $\phi_1(y + x) + \phi_2(y - 2x) + ye^x$
C) $\phi_1(y + x) + \phi_2(y - 2x) + xe^{-y}$
D) $\phi_1(y - x) + \phi_2(y + 2x) + ye^{-x}$.

(47) The trajectories of the system of differential equations $dx/dt = y$ and $dy/dt = -x$ are

A) ellipses B) hyperbolas C) circles D) spirals.

(48) The backward Euler method for solving the differential equation $y' = f(x, y)$ is

A) $y_{n+1} = y_n + hf(x_n, y_n)$
B) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$
C) $y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$
D) $y_{n+1} = (1 + h)f(x_{n+1}, y_{n+1})$.

(49) The Newton-Raphson formula for finding approximate root of the equation $f(x) = 0$ is

A) $x_{n+1} = f(x_n)/f'(x_n),\ f'(x_n) \neq 0$
B) $x_{n+1} = x_n + f(x_n)/f'(x_n),\ f'(x_n) \neq 0$
C) $x_{n+1} = x_{n-1} - f(x_n)/f'(x_n),\ f'(x_n) \neq 0$
D) $x_{n+1} = x_n - f(x_n)/f'(x_n),\ f'(x_n) \neq 0$.

(50) If Euler's method is used to solve the initial value problem $y' = -2ty^2$, $y(0) = 1$ numerically with step size $h = -0.2$, the approximate value of $y(0.6)$ is

A) 0.7845 B) 0.8745 C) 0.8754 D) 0.7875.

Part II: Questions may have multiple correct answers and carry five marks. Five marks will be given only if all correct choices are marked. There will be no negative marks.

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(51) There exists a finite field of order

A) 6 B) 12 C) 81 D) 121.

(52) If S_3 and A_3 respectively denote the permutation group and alternating group, then

A) A_3 is the Sylow 3-subgroup of S_3
B) Sylow 2-subgroup of S_3 is unique
C) $\{I, (12)\}$, $\{I, (12)\}$, $\{I, (23)\}$ are Sylow 2-subgroup of S_3
D) A_3 is not a normal subgroup of S_3.

(53) Let G be a group of order 105 and H be its subgroup of order 35. Then

A) H is a normal subgroup of G

B) H is cyclic
C) G is simple
D) H has a normal subgroup K of order 5 and K is normal in G.

(54) The quotient group $\mathbb{R}/\mathbb{Z}$ is

A) an infinite Abelian group
B) cyclic
C) the same as $\{r + \mathbb{Z} : 0 \leq r < 1\}$
D) isomorphic to the multiplicative group of all complex numbers of unit modulus.

(55) Which of the following pairs of groups are isomorphic to each other?

A) $\langle \mathbb{Z}, + \rangle$, $\langle \mathbb{Q}, + \rangle$
B) $\langle \mathbb{Q}, + \rangle$, $\langle \mathbb{R}^+, \cdot \rangle$
C) $\langle \mathbb{R}, + \rangle$, $\langle \mathbb{R}^+, \cdot \rangle$
D) $\mathrm{Aut}(\mathbb{Z}_3)$, $\mathrm{Aut}(\mathbb{Z}_4)$.

(56) Let V and W be finite-dimensional vector spaces and $T : V \to W$ be a linear transformation. Then

A) $\dim V < \dim W \Rightarrow T$ cannot be onto
B) $\dim V > \dim W \Rightarrow T$ cannot be one-to-one
C) $\dim V + \operatorname{null} T = \operatorname{rank} T$
D) $\dim V = \dim W \Rightarrow T$ is invertible .

(57) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by the formula $T(x, y, z) = A(x\ y\ z)^t$ where A is a 3×3 real orthogonal matrix of determinant 1. Then

A) T is an isometry of $\mathbb{R}^3$
B) the matrix of T with respect to the usual basis of $\mathbb{R}^3$ is A^t
C) the eigenvalues of T are either 1 or -1
D) T is surjective.

(58) Choose the correct statements

A) $\cup_{n=1}^{\infty}[1/n, 2] = [0, 2]$
B) $\cup_{n=1}^{\infty}(1/n, 2] = (0, 2]$
C) $\cap_{n=1}^{\infty}(1 - 1/n, 2] = (1, 2]$
D) $\cap_{n=1}^{\infty}[1 - 1/n, 2] = [1, 2]$.

(59) If $\mathbb{Q} \subset A \subset \mathbb{R}$, which of the following must be true?

A) If A is open, then $A = \mathbb{R}$
B) If A is closed, then $A = \mathbb{R}$
C) If A is uncountable, then A is closed
D) If A is countable, then A is closed.

(60) The function $f : [0, 1] \to [0, 1]$ defined by $f(0) = 0$ and $f(x) = x^2 \sin(1/x)$ for $x \neq 0$, is

A) differentiable on $(0, 1)$
B) is continuous on $[0, 1]$
C) is continuous on $[0, 1]$ but not differentiable at 0
D) is uniformly continuous.

(61) Let $f : [a, b] \to [a, b]$ be a continuous function. Then

A) $\lim_{n\to\infty} \int_a^b f(x) \sin nx dx = \pi$
B) $\lim_{n\to\infty} \int_a^b f(x) \cos nx dx = \pi$
C) $\lim_{n\to\infty} \int_a^b f(x) \sin nx dx = 0$
D) $\lim_{n\to\infty} \int_a^b f(x) \cos nx dx = 0$.

(62) Which of the following statements about a sequence of real numbers are true?

A) Every bounded sequence has a convergent subsequence
B) Every sequence has a monotonic subsequence
C) Every sequence has a limit point
D) Every sequence has a countable number of terms.

(63) Let $\langle a_n \rangle = \langle 1, 1, 1/2, 1, 1/2, 1/3, 1, 1/2, 1/3, 1/4, \ldots \rangle$ be a sequence of real numbers. Then

A) $\langle a_n \rangle$ has infinite number of limit points
B) $\limsup_{n\to\infty} a_n = 1$
C) $\liminf_{n\to\infty} a_n = 0$
D) $\langle a_n \rangle$ has infinite number of convergent subsequences.

(64) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \min\{x, x + 1, |x - 2|\}$. Then

A) f is continuous on $\mathbb{R}$
B) f is not differentiable at exactly two points
C) f increases on the interval $(-\infty, 1]$
D) f decreases on the interval $[1, 2]$.

(65) Let $F_n = [-1/n, 1/n]$ for each $n \in \mathbb{N}$ and let $F = \cap_{n=1}^{\infty} F_n$. Then

A) F contains finite number of points
B) $\sup\{|x - y| : x, y \in F\} = 0$
C) $\inf\{|x - y| : x, y \in F\} = 0$
D) F is a closed set.

(66) Let $d_1, d_2, d_3, d_4 : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$d_1(x,y) = \sqrt{|x-y|},$$
$$d_2(x,y) = |x^2 - y^2|,$$
$$d_3(x,y) = |\sin x - \sin y|,$$
$$d_4(x,y) = |\tan^{-1} x - \tan^{-1} y|.$$

Then which of the following is/are metric on $\mathbb{R}$?

A) d_1 B) d_2 C) d_3 D) d_4.

(67) Which of the following is/are true for the initial value problem: $xy' = 2y$, $y(a) = b$:

A) there is unique solution near (a, b) if $b \neq 0$
B) there is no solution if $a = 0$ but $b \neq 0$
C) there are infinitely many solutions if $a = b = 0$
D) the function $y = x^2$ if $y \leq 0$ and $y = cx^2$ if $x \geq 0$ is one of the solutions.

(68) The solution of the partial differential equation $z = pq$ where $p = \partial z/\partial x$ and $q = \partial z/\partial y$ is

A) $z = (x+a)(x+b)$ B) $4z = (ax + y/a + b)^2$
C) $z = ax + a^2 + by$ D) none of these.

(69) Consider the second order Sturm - Liouville problem: $x^2y'' + xy' + \lambda y = 0$ where $\lambda \geq 0$, subject to the conditions: $y'(1) = y'(e^{2\pi}) = 0$. Pick out the true statements

A) For $\lambda = 1$, the given problem has infinitely many solutions
B) For $\lambda = 0$, only solution to the given problem is the trivial solution
C) The characteristic values λ_n of the given problem can be arranged in a monotonically increasing sequence
D) For $\lambda = 1/16$, a non-trivial solution exists.

(70) For any integer $n \geq 2$, let $S_n = \{(x,y) \in \mathbb{R}^2 : (x - \frac{1}{2})^2 + y^2 = \frac{1}{n^2}\}$ and $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. The second order partial differential equation: $(x^2 - 1)u_{xx} + 2yu_{xy} - u_{yy} = 0$ is

A) elliptic on $\cup_{n=2}^{\infty} S_n$
B) elliptic on $\cup_{n=3}^{\infty} S_n$ and parabolic on S_2
C) hyperbolic in $\mathbb{R}^2 - S$
D) parabolic on $S \cap (\cup_{n=2}^{\infty} S_n)$.

Rough Work

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