Stanford University Mathematics Camp (SUMaC) 2013

Application Packet

This packet contains the following:

- SUMaC 2013 Program Description
- SUMaC 2013 Personal Information Form
- SUMaC 2013 Teacher Recommendation Form
- SUMaC 2013 Financial Aid Form
- SUMaC 2013 Admission Exam

Important Information for SUMaC Applicants

- SUMaC 2013 will take place July 14, 2013 through August 10, 2013. July 14 is Arrival Day, and August 10 is Departure Day.
- Tuition for 2013 is **$6,150** per student. This covers room and board in University housing, course materials, expenses for organized activities, and transportation to and from the San Francisco Airport (if needed). Full and partial scholarships are available, both to U S and international students. To apply for financial aid, please submit the Financial Aid Form included in this packet.
- Applicants must be current high-school tenth or eleventh graders (going into eleventh or twelfth grade in Fall of 2013), and must be between 15 and 17 years old on the first day of the program (July 14, 2013).
- All applications received by **March 13, 2013** will be given equal priority in the first round of application review. Typically, the program is filled in this first round; however, additional applications may be considered if space remains. First round applicants will be notified of their acceptance by April 19.
- SUMaC offers two programs. Program I applicants must have taken a year of mathematics beyond a first course in algebra, by the start of the program. Program II applicants must have completed a one-year calculus course by the start of the program; however, Program I is also suitable for students who have taken calculus. Students are not required to have taken Program I in order to apply to Program II. Students can apply for both programs, but they can attend only one. More information on the two programs is provided later in this packet.
- Approximately 35 students will be admitted to SUMaC 2013, distributed between the two programs.
- A complete application consists of the following, sent to SUMaC by postal mail:
    i. Personal Information Form.
    ii. School transcript (report card). Unofficial transcripts are accepted.
    iii. Teacher Recommendation Form: Mailed by the applicant's teacher.
    iv. (Optional) Financial Aid Form.
    v. Solutions to the Admission Exam (*The admission exam consists of challenging problems to be completed by applicants on their own time. The only time restriction is the application deadline*.)
- Please mail all application materials to:

SUMaC
220 Panama St
Stanford, CA 94305-4101

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Program Description

Overview
SUMaC is designed for high-school tenth and eleventh graders who have exceptional interest and ability in mathematics. SUMaC is for those who seek to be challenged in mathematics and those who would enjoy four weeks of intensive, in-depth, mathematical pursuits. SUMaC provides an environment that fosters social and intellectual development centered on the study and enjoyment of mathematics.

Requirements
SUMaC participants must be current high-school tenth or eleventh graders (going into eleventh or twelfth grade in Fall of 2013), and must be between 15 and 17 years old on the first day of the program (July 14, 2013).

Admission
SUMaC students are selected based on grades in math courses, performance in mathematics contests, teacher recommendation, reasons for wanting to come to SUMaC as expressed on the personal information form of the SUMaC application, and performance on the SUMaC admission exam (a collection of challenging math problems for applicants to work on at home over an extended period). Special attention is paid to students who are in academic environments that do not offer the framework to excel in mathematics beyond the usual school curriculum, but who demonstrate exceptional mathematical talent. In recent years, about four times as many applications have been received as there are spaces in the program.

Content
The topics covered in SUMaC are central to mathematics through their historical significance and their relevance to current lines of mathematical research. The SUMaC courses focus on pure mathematics–mathematics that is motivated independently of ties to other sciences. Nonetheless, important applications are introduced and pursued throughout the program. SUMaC leaps ahead of the usual mathematics course sequence, giving a glimpse of advanced material not typically taught before college, yet it is presented here at a level that can be understood by mathematically motivated and talented high-school students. Topics covered at SUMaC are included in college math courses at about the sophomore and junior level; however, in order give a glimpse of such advanced topics in only four weeks, the material is streamlined, and not developed to the extent it would be in a university course. Assignments include problems in a range of difficulty so that every student is challenged, yet every student will have opportunities for accomplishment.

Programs
SUMaC includes two programs:

- Program I topics are introduced through five motivating problems: non-constructibility in geometry, classification of patterns in two dimensions, error-correcting codes, cryptography, and the analysis of the Rubik's cube. The mathematics that is central to solving these problems comes from the areas of abstract algebra and number theory. Abstract algebra originated in the early part of the 19th century through the study of polynomial equations. This branch of mathematics lies at the core of many areas of modern mathematical research. Number Theory is concerned with properties of the integers. Although number theory has its origins in ancient mathematics, it remains a very active field of study with interesting open problems and important applications in computer science.

- Program II contains an introduction to selected topics in combinatorial, differential and algebraic topology. The emphasis will be on developing ideas from, and problems in, geometric topology where methods from algebra and calculus have proved to be effective tools. These include the Euler characteristic as a way of distinguishing between various surfaces, the famous Koenigsberg bridge problem, and degree theory. Applicants to Program II need to be conversant with high school algebra, and must have taken a single-variable calculus course. Program II is designed for students who either have a special interest in the topics, or who have seen the Program I material before. Program II is thus especially suited for students who want to return to SUMaC having attended Program I in a prior year, or who have participated in a similar program somewhere else. Program II is more advanced, but advanced students will be challenged in Program I as well.

Coursework

SUMaC engages students by introducing the course material through problems that are of historical significance, that are important to the development of mathematics, that have applications to current lines of research in mathematics and the sciences, and that students who enjoy mathematics will find interesting. Additionally, the view is that one learns mathematics by doing mathematics–by working on, tackling, and solving interesting and challenging problems. The SUMaC program gives participants a variety of problems at different levels of abstraction, starting with problems that are relatively concrete and to some extent familiar. These initial problems are used to develop basic intuitions about a particular topic, as well as to strengthen general mathematics skills. Often these problems are followed by a series of more probing ones which dig successively deeper into the abstract, attempting to both fine-tune and generalize one's intuitions and knowledge about the underlying mathematical objects. The research project that students partake in as part of the program allows for the in-depth study of a focused portion of the course material. This is done with the assistance of a teaching assistant, and students use articles and books as sources of additional material. Students share what they have learned in the research project by giving a short presentation at the end of the program. Constant emphasis is placed on developing the ability to reason both formally and informally, and to express one's ideas in mathematically coherent and precise language.

Student Life

Many participants find the math-friendly social environment the best part of the program; students comment that it is great to be at a place where it is considered fun to discuss a math problem during dinner or during a game of basketball. Every student leaves with a deeper understanding of mathematics and an appreciation for parts of mathematics that they never knew existed. Some students leave with the plan to pursue mathematics further. Everyone makes new friends and contacts based in their common interests and abilities– friendships that will remain strong for many years to follow. Students return home with good memories and much to share with family, friends, and teachers. Although SUMaC does not give course credit or grades, a record of each student's performance is kept, and this information is readily provided for use in college applications.

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Daily Schedule

The daily routine at SUMaC is as follows. Monday through Thursday students spend the morning in class learning the central program topics and working on problems as a class, in small groups, and individually. After lunch students begin work on the daily problem sets. During that time, teaching assistants meet individually with students in 20-60 minute sessions to go over the problem set from the previous day (turned in that morning). Also, throughout the afternoon and evening, members of the teaching staff are available to help on that day's problem sets. On Fridays, students meet in groups of four or five to work on research topics. This activity allows students to pursue a more in-depth study of a focused aspect of the course material; in doing so they research books and articles that go beyond what is presented in class. This culminates in an in-class presentation in the final week of the program. Throughout the four weeks, there will be frequent guest lectures given by internationally renowned mathematicians. These talks will provide an introduction to some of areas of current mathematical research. And, of course, there will be a variety of non-math activities, including organized sports activities during the week and on the weekends, and field trips every Saturday. Although the SUMaC staff will be available for tutoring and other needs, there will be no mandatory organized activities on Sundays.

Residential Life

SUMaC students are housed in a residence on campus which is occupied by Stanford students during the academic year. SUMaC is the only group occupying the residence during the four weeks of the camp. Each student will have a roommate. Rooms for boys and girls are on separate floors. Dining takes place at a separate dining hall shared by other summer youth programs. Special dietary needs are accommodated.

Staff

SUMaC was founded in 1995 by Professors Rafe Mazzeo and Ralph Cohen of the Stanford Mathematics Department, and is currently directed by Prof. Mazzeo and Dr. Rick Sommer. Rick was an Assistant Professor in the Stanford Mathematics Department and is currently Managing Director of the Education Program for Gifted Youth (EPGY), at Stanford. Dr. Sommer designed the Program I course and has been teaching versions of it since the first SUMaC in 1995. The Program II course is designed by Prof. Rafe Mazzeo. The Program II instructor will be announced at a later date. Most of the SUMaC residential counselors and teaching assistants are Stanford mathematics graduate students and undergraduate math majors. SUMaC provides at least a 1-to-4 ratio of staff to students, with most of the teaching assistants serving in the role of live-in counselors. Some of the SUMaC teaching assistants and counselors will be returning from previous years, and some attended SUMaC in high school. The counselors and teaching assistants play a key role in creating the social and academic atmosphere enjoyed by the students.

More information is available at http://math.stanford.edu/sumac.

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Personal Information Form

Last Name:	First Name:
Mailing address:	Email address (Print Clearly!):
	Phone number:
Name, phone number, and email address (print clearly) of parent or legal guardian	
For which program(s) are you applying? ☐ Program I ☐ Program II *If you are applying for both Program I and Program II, please circle your first choice.*	
Date of birth:	Check one: ☐ Male ☐ Female
Current year in high school: ☐ Tenth grade ☐ Eleventh grade	
What math course(s) are you currently taking?	
(Optional) Are you a member of an under-represented group? If so, which one?	
Name and address of high school:	
Are you applying for financial aid? ☐ Yes ☐ No *(If yes, then you need to submit the financial aid form with your application.)*	
How did you find out about SUMaC?	
Answer the following on a separate sheet of paper: 1. List any extra-curricular mathematics activities in which you have participated (e.g., math clubs and camps). 2. List all mathematics courses taken in the last two academic years, including the current term, and the grades you received in the courses completed. 3. List all math contests and competitions in which you have participated, giving your score and/or ranking if possible. Be sure to include your scores on the AMC10, AMC12, PSAT, or SAT, if you have taken any of these exams. 4. What aspects of mathematics do you like the most? 5. What aspects of mathematics do you like the least? 6. Describe a positive experience that you have had with mathematics. 7. What do you hope to gain by participating in SUMaC? 8. What aspect of SUMaC appeals to you the most? 9. Which of the problems on the application exam did you find the most interesting? Explain. 10. Which of the problems on the application exam did you find the most difficult? Explain. 11. Which of the problems on the application exam did you find the easiest? Explain. 12. On which of the problems on the application exam did you spend the most time? 13. Tell us anything else about yourself that you feel is relevant, or describe some of your interests outside of mathematics.	
I have answered all questions on this form honestly, and the work on the admission exam is my own, except where indicated otherwise. Applicant's signature: _____ Date: _____	

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Teacher Recommendation Form

This form is to be completed by the applicant's mathematics teacher and then mailed to:

SUMaC
220 Panama St.
Stanford, CA 94305-4101

Or send via fax to (866) 835-3312, or for international 1-650-721-9383. Alternatively, email this information to sumac@epgy.stanford.edu. The applicant should provide a copy of the SUMaC Program Description, from the application packet, to the teacher filling out this form.

IMPORTANT PRIVACY NOTICE: Under the terms of the Family Educational Rights and Privacy Act (FERPA), if you are accepted you *will* have access to this form and all other supporting documents submitted by you and on your behalf, unless you waive this right.

- ☐ Yes, I do waive my right to access, and I understand I will never see this form or any other recommendations submitted by me or on my behalf.
- ☐ No, I do *not waive* my right to access, and I may someday choose to see this form or any other recommendations or supporting documents submitted by me or on my behalf.

1. Applicant's name: _____

2. Teacher's name, phone number, and email address:

_____

3. Name and address of school:

_____

4. How long have you known the applicant?

_____

5. For each of the following indicate how the applicant compares to other students in similar classes by giving the applicant's approximate rank in terms of "top 1%", "top 5%", etc.

a. Mathematical ability.
b. Mathematical interest.
c. Ability to work with others.
d. Ability to work independently.
e. Creativity.
f. Analytical ability.
g. Personal initiative.
h. Perseverance.
i. Emotional maturity.

6. Please indicate the level to which you would recommend the applicant for the SUMaC program. Circle one:

Very highly recommended Highly recommended Recommended Not recommended

7. On the back of this form, or on a separate sheet of paper, please comment further on the degree to which you have recommended the student as indicated in Question 6, and on the various qualities listed in Question 5. We appreciate any information that you feel is relevant to determining whether the applicant is suited for SUMaC. Note that this is an intensive program where the students will be living as a close group for four weeks, so your impressions of the student's emotional maturity and ability to live and work with other students are especially relevant. Note also that we seek students whose mathematical talent is exceptionally high. If you have written letters for former SUMaC participants, please mention them by name. It will be useful if you compare this applicant to any former SUMaC participants that you have known.

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Financial Aid Application Form

To be considered for financial aid this form must be completed by the parent or guardian who claims the applicant as a dependent, and mailed along with a copy of the main pages of the parent's or guardian's 2012 (if possible, 2011 otherwise) federal income tax forms together with the student's application form. For any tax documentation not in English, please provide an English language version and covert all amounts to U.S. Dollars.

Admitted applicants will be notified of financial aid decisions along with their admission decision.

Applicant's name: _____

Parent's or guardian's name: _____

Parent's or guardian's address: _____

_____

Parent's or guardian's telephone number: _____

What portion of the $6,150 SUMaC fee can you afford to pay? _____

_____

Describe any special circumstances you would like us to consider when reviewing the application for financial aid. Attach a separate sheet if needed.

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Admission Exam

- Solve as many of the following problems as you can.
- You do not need to provide correct answers to every problem in order to be admitted to SUMaC. Your work on these problems together with your grades, teacher recommendations, and answers to questions on the Personal Information Form, are all used to evaluate your application and compare it to other applicants to the program.
- There is no time limit for this exam other than the application deadline of March 13.
- Feel free to report partial progress toward a solution, in the event you are unable to solve a problem completely.
- We expect detailed explanations to be included with ***all of your answers***; numerical answers or formulas with no explanation will receive little or no credit.
- You are expected to do your own work. If you receive help from someone, or if you use any outside source (books, articles, the web, etc.), you should indicate to what extent you received help. For example, if true, you should write, "After several hours of work on this problem, I received the following hint from my math teacher. . . ."

1. Find all positive integers *x* and *y* such that $x^2 - y^2 = 2013$.

2. Prove that every set of 100 integers includes a subset of 10 integers such that each pair of integers in that subset differ by a multiple of 11.

3. Pennies are placed on an 8 × 8 checkerboard in an alternating pattern of heads and tails.

   i. You are allowed to make moves where in each move you turn over exactly two pennies that lie next to each other in the same row or column. Can you take a sequence of moves that leaves just one penny face up? Explain your answer.

   ii. You are allowed to make moves where in each move you turn over exactly three neighboring pennies that lie in the same row or column of the checkerboard. Can you take a sequence of moves that leaves just one penny face up? Explain your answer.

4. Three lines can be drawn in the plane to form one enclosed region (a triangle).

   i. Show that four lines can be drawn to form three enclosed regions.

   ii. How many enclosed regions can be formed by five lines?

   iii. Find a formula that gives the maximum number of enclosed regions formed by *n* lines. Explain your answer.

   iv. Show that for any three points in the plane, four lines can be drawn that separate the three points into distinct enclosed regions.

   v. Can any six points be enclosed in distinct regions formed by five lines? Explain.

5. Show that if *n* is a positive power of 2 (that is, for some positive integer k, $n = 2^k$), and *m* is obtained from *n* by rearranging the digits of *n*, then *m* is not a power of 2 (that is, for all positive integers l, $m \neq 2^l$).

6. Let *S* be a set of real numbers satisfying the following conditions:

   i. 0 is in *S*.

   ii. Whenever *x* is in *S* then $2^x + 3^x$ is in *S*.

   iii. Whenever $x^2 + x^3$ is in *S* then *x* is in *S*.

   Prove:

   (a) *S* is unbounded, and

   (b) *S* contains at least two distinct real numbers between 0 and 1.

7. Suppose $p(x)$ is a polynomial with integer coefficients. Show that if $p(a) = 1$ for some integer *a* then $p(x)$ has at most two integer roots (that is, there are at most two integers *b* and *c* such that $p(b) = 0$ and $p(c) = 0$).

8. Let a, b, and c denote 3 different colors, and if x and y are among a, b, and c, let xy denote the result of "mixing" the colors x and y; in each case xy is among the colors a, b, and c. This is a hypothetical situation that is not meant to pertain to actually mixing light, or paint, or any other physical entity. Suppose that mixing these colors satisfies the following rules.

   i. $xx = x$, for each color x. (Mixing a color with itself results in that color.)

   ii. $xy = yx$, for all colors x and y. (Mixing x with y is the same as mixing y with x.)

   iii. $x(yz) = (xy)z$, for all colors x, y, and z. (The order in which you mix the colors doesn't matter.)

   iv. $ax = x$, for each color x. (Color a is "clear"; that is, a mixed with another color is that color.)

   Show that there is a color x among a, b, and c that absorbs all colors in the sense that x mixed with anything is x ($ax = x$, $bx = x$, and $cx = x$.) Note that you are not being asked to show which of the colors absorbs all colors, but only that one of the colors has this property.

9. Prove that if $p(x)$ is polynomial with integer coefficients and $p(\sqrt{2}) = 0$ then $p(-\sqrt{2}) = 0$.

10. Suppose for a positive integer *n* both 5*n* + 1 and 7*n* + 1 are perfect squares. Show that *n* is divisible by 24.