UNIVERSITY OF MYSORE

ENTRANCE TEST FOR M.Sc. COURCE IN MATHEMATICS

Eligibility criteria for writing the Entrance Test : Those candidates who are appearing or have appeared for Final semester/Year of B.Sc./B.Sc. Ed. (RIE) course with Mathematics as Major/Optional subject are eligible to write the entrance test.

Eligibility criteria for Admission: The eligibility for admission is 45% of marks (40% for SC, ST and Cat. I candidates) after deducting 3% for each extra year over normal duration of the course, if any in Mathematics of B.Sc./B.Sc. Ed. (RIE) Examination.

ENTRANCE TEST SYLLABUS FOR M.Sc. COURSE IN MATHEMATICS

UNIT - I

Analytical Geometry:

Cartesian coordinates in three dimensional space – Relation between cartesian co-ordinates and position vector – Distance formula (cartesian and vector form) – Division formula (cartesian and vector form) – Direction cosines – Direction ratios – Projection on a straight line – Angle between two lines – Area of triangle – volume of a tetrahedron.

Straight line – Equations of straight lines (cartesian and vector form) - Planes – Equations of planes (cartesian and vector form) - Normal form – Angle between planes – Coaxial planes – Parallel and perpendicular planes – length of a perpendicular form a point to a plane – Bisectors of angles between two planes – Mutual position of a lines and planes – Shortest distances between two skew lines.

Quadric Curves:

Translation and rotation of cartesian axes in a plane – Curves of second degree – Discriminant and trace - theorem on discriminant and trace – removing the mixed term – removing linear terms – proof of the theorem. The set of points (x, y) satisfying equation $Ax^2 + 2Bxy + Cy^2 + Ey + F = 0$ is either empty or a point or consists of one or two lines or is a parabola, an ellipse or a hyperbola – problems there on – Polar equations of a conic – problems there on – Quadratic Surfaces – Sphere – Cylinder – Cone - Ellipsoid – Hyperboloids – Paraboloids - Ruled Surfaces.

UNIT - II

Differential Calculus:

Real Numbers – Inequalities – Absolute Value – Intervals – Functions – Graphs – Limit of a function – Left hand and right hand limits – $\varepsilon - \delta$ definition of continuity of a function - problems. Differentiation – Linear approximation theorem – derivatives of higher order – Leibnitz's theorem – Monotone functions - Maxima and Minima – Concavity, Convexity and points of inflection. Polar coordinates- angel between the radius vector and the tangent at a point on a curve – angle of intersection between two cnurves – Pedal equations – Derivative of arc length in cartesian, parametric and polar coordinates, curvature – radius of curvature – circle of curvature – evolutes.

Differentiability and its applications:

Differentiability- Theorems – Rolle's theorem – Lagranges's Mean valve theorem – Cauchy's mean value theorem – Taylor's theorem – Maclaurin's theorem – Generalized mean value theorem – Taylor's infinite series and power series expansion – Maclaurin's infinite series – Indeterminate forms.

Asymptotes – Envelopes – Singular points – Multiple points – cusp, nodes and conjugate points – Tracing of standard curves with Cartesian and polar equations.

Partial Derivatives:

Functions of two or more variables – Explicit and implicit functions – The neighborhood of a point – The limit of a function – Continuity – Partial derivatives – Differentiable functions – Linear approximation theorem – Homogeneous functions – Euler's theorem – Chain rule – Change of variables – Directional derivatives – Partial derivatives of higher order – Taylor's theorem – Derivatives of implicit functions – Jacobian – Some illustrative examples.

UNIT - III

Theory of Numbers:

Division Algorithm - Divisibility - Prime and composite numbers - Proving the existence and uniqueness of GCD and the Euclidean Algorithm - Fundamental theorem of Arithmetic - The least common multiple – congruences - linear congruences - Wilson's theorem - Simultaneous congruences - Theorem of Euler, Fermat and Lagrange.

Theory of Equations:

Theory of Equations – Euclid's algorithm - Polynomials with integral coefficients – Remainder theorem – Factor theorem – Fundamental theorem of algebra (statement only) – Irrational and complex roots occur in conjugate pairs – Relation between roots and coefficients of a polynomial equation – symmetric functions – Transformations – Reciprocal equations – Descartes rule of signs – Multiple roots - Solving cubic equations by Cardon's method – solving quartic equations by Descarte's and Ferrari's Method.

Group Theory:

Definition and examples of groups – Some general properties of Groups Permutations - group of permutations, cyclic permutations, Even and odd permutations. Powers of an element of a group – Subgroups – Cyclic groups, Z_n and Z. Cosets, Index of a group, Lagrange's theorem – consequences. Normal subgroups, Quotient groups – Homomorphism, Isomorphism, Automorphism. Fundamental theorem of homomorphism – Isomomorphism – Direct product of groups – Cayley's theorem.

UNIT - IV

Real Numbers:

Introduction – Field structure – Order structure - Bounded and unbounded sets – Supremum and infimum – Completeness - Some important subsets of R – Archimedean Property of real numbers – countable and uncountable sets.

Limits and continuity:

Limits - Continuous functions - discontinuous functions - theorems on continuity - Functions continuous on closed interval - Uniform continuity (explaining the idea).

Real sequences:

Sequences of real numbers – Bounded and unbounded sequences – Infimum and supremum of a sequence – Limit of a sequence – Sum, product and quotients of limits – Standard theorems on limits – Convergent, divergent and oscillatory sequences – Standard properties – Subsequences – monotonic sequences and their properties – Limit point of a sequences – Cauchy's general principle of convergence.

Infinite Series:

Infinite series of real numbers – Convergence – divergence and oscillation of series – properties of convergence – Positive term series – Geometric series – Comparison tests – Cauchy's root test – D'Alembert's ratio test, Raabe's test, Integral test – Absolute and conditional convergence - D'Alembert's test for absolute convergence – Leibnitz's test for alternating series. Summation of Binomial, Exponential and logarithmic series.

Fourier series:

Introduction – Periodic functions – Fourier series and Euler formulae – Even and odd functions – Half range series – Change of interval.

UNIT - V

Riemann Integration:

The Riemann integral – Upper and lower sums – Criterion for integrability – Integrability of continuous functions and monotonic functions – Fundamental theorem of Calculus – Change of variables – integration by parts –First and Second mean value theorems of integral calculus.

Integral Calculus:

Techniques of integrations – Integrals of Algebraic and transcendental functions – Reduction formulae - Definite integrals – properties.

Improper Integrals:

Improper integrals of the first and second kinds – Convergence – Gamma and Beta functions and results – Connection between Beta and gamma functions – Applications to evaluation of integrals – Duplication formula – Sterling formula.

Laplace Transforms:

Definition and basic properties – Laplace transforms of $\exp kt, \cos kt, \sin kt, t^n, \cosh kt$ and $\sinh kt$ - Laplace transform of $e^{at}F(t)t^{1/2}$ - problems - Theorems on the derivative of Laplace transform and the transform of derivatives - Inverse Laplace transforms – problems – alpha function – theorem on the Laplace transform of integrals – Laplace transform of $F(t)/t$.

Convolution theorem – Simple initial value problems – Special integral equations – Solution of first and second order differential equations with constant coefficients

by Laplace transform method – Systems of equations – Laplace transforms of Periodic functions.

UNIT - VI

Rings and Fields:

Rings – Examples – Integral domains – Division rings – Fields – Subrings – subfields - Characteristic of a ring – Ordered integral domain – Imbedding of a ring into another ring – The field of quotients – Ideals – Algebra of Ideals – Principle ideal ring – Divisibility in an integral domain – Units and Associates – Prime Elements – Polynomial rings – Divisibility - Irreducible polynomials – Division Algorithm – Greatest Common Divisors – Euclidean Algorithm – Unique factorization theorem – Prime fields – Quotient rings – Homomorphism of rings – Kernel of a ring homomorphism – Fundamental theorem of homomorphism – Maximal ideals – Prime Ideals – Properties - Unique Factorization domain – Eisenstein's Criterion of irreducibility.

UNIT - VII

Differential Equations:

Definition and examples of differential equations. The elimination of arbitrary constants - Families of curves - Differential equations of first order, separation of variables - equations with homogeneous coefficients – Exact equations - Linear equations of order one. The general solution of a linear equation – Integrating factors found by inspection. The determination of Integrating factors. Substitution suggested by the equation. Bernoulli's equation. Coefficients linear in two variables .

Equations of first order and higher degree Equations - solvable for x, solvable for y, solvable for P, Clairaut's equation – Singular solutions and geometrical meaning.

Ordinary Linear differential equations with constant coefficients – complementary function – particular integral – Inverse differential operators.

Linear Differential Equations:

Cauchy – Euler differential equations – Simultaneous differential equations (two variables with constant coefficients) - Solution of ordinary second order linear differential equations by the following methods

i. Reduction of order method and variation of parameters.
ii. Changing the independent variable.

iii. Changing the dependent variable.
iv. Exact equations.

Total differential equations – Necessary and sufficient condition for the equation $Pdx + Qdy + Rdz = 0$ to be exact (proof only for the necessary part) – Simultaneous equations of the form $dx/P = dy/Q = dz/R$.

Partial Differential Equations:

Basic concepts – Formation by elimination of arbitrary constants – Formation by eliminations of arbitrary functions – Solutions of partial differential equations – Solutions by direct integration – Lagranges's linear equations - $Pp + Qq = R$ - Standard types of first order non-linear partial differential equations – Charpit's method - Homogeneous linear equations with constant coefficients – Rules for finding the complimentary function – Rules for finding the particular integral method of separation of variables (product method).

UNIT - VIII

Line and Multiple Integrals:

Definition of a line integral and basic properties – Examples on evaluation of line integrals – Definitions of double integral – Conversion to iterated integrals - Evaluation of double integrals

i. Under given limits
ii. In regions bounded by given curves – change of variables – surface areas.

Definition of a triple integral – Evaluation – Change of variables - Volume as a triple integral.

Vector Calculus:

Vectors – Scalars – Vector field – Scalar field – Vector differentiation – The vector differential operator - del – Gradient – Divergence – Curl – standard derivations – Vector integrations – The divergence theorem of Gauss – Stoke's theorem, Green's theorem in the plane.

Numerical Analysis:

Numerical solutions of Algebraic and transcendental equation – Bisection method – The method of false position - Iteration method – Newton – Raphson method – Secant method.

Numerical solutions of a first order linear differential equations – Euler – Cauchy method – Euler's modified method – Runge –Kutta fourth order method – Picard's method.

Finite differences –Forward and backward differences – Shift operator – Derivatives operator - Weirstrass theorem (statement) – Interpolations – Newton – Gregory – forward and backward difference formulae – Lagrange's interpolations formula – Finding first and second derivatives using interpolation formulae – Difference equations.
Numerical integrations – General quadrature formula – Trapezoidal Rule – Simpson's $1/3$ rule – Simpson's $3/8$ th rule – Weddle's rule.

UNIT - IX

Matrices:

Matrices of order $m \times n$ - Algebra of Matrices – Symmetric and skew symmetric - Hermitian and skew Hermitian matrices, symmetric matrices and their standard properties – Determinants – Adjoint of a square matrix – Singular and non-singular matrices – Rank of a matrix – Elementary row/column operations – Invariance of rank under elementary operations – Inverse of a non-singular matrix by elementary operations.

System of m linear equations in n unknowns – matrices associated with linear equations – trivial and non-trivial solutions – Criterion for existence of non-trivial solution of homogeneous and non-homogeneous systems – Criterion for uniqueness of solutions – Problems.

Eigen values and Eigen vectors of a square matrix – Characteristic equation of a square matrix – Eigen values and eigen vectors of a real symmetric matrix - Properties – Diagonalization of a real symmetric matrix – Caley – Hamilton theorem – Applications to determine the power of square matrices and inverses of non-singular matrices.

Vector Spaces:

Vector spaces – Introduction – Examples – Vector subspaces – Criterion for a subset to be a subspace – Algebra of subspace – Linear combinations – Linear spans – Linear dependence and linear independence of vectors – Theorems on linear dependence and linear independence – Basis of a vector space – Dimension

of a vector space - Finite dimensional vector spaces – Some properties – Coordinates system – Quotient space – Homomorphism of vector spaces or linear transformations – Isomorphism of vector spaces – Direct sums – Inner product spaces – Euclidean vector spaces – Distance – length- Properties – Normal orthogonal vectors – Gram-Schmidt othogonalization process – Orthogonal complement.

Linear Transformations:

Linear transformations – Linear maps as matrices – Change of basis and effect of associated matrices – Kernel and image of a linear transformation – Rank and nullity theorem – Singular and non-singular linear transformations – Elementary matrices and transformations – Similarity – Eigen values and eigen vectors - Diagonalisation - Charateristic polynomial – Cayley –Hamilton theorems – Minimal polynomial. Automorphism.

UNIT - X

Complex Analysis:

The complex number system – Absolute value and conjugate of a complex number – Geometrical representation – Polar form of complex numbers – De Moiver's theorem – Euler's formula – Dot and cross product.

Neighbourhoods – Limit point – Interior, Exterior, Isolated and boundary points – Open sets – Closed sets - Bounded sets – Compact sets – Connected sets – Domain – Simply Connected regions.

Equation to a circle and a straight lines in complex form – Jordan arc – Closed Contour – The extended complex plane.

Functions of a Complex Variable:

Functions of a complex variable – Limit of a function – Continuity and differentiability – Analytic functions – Singular points – Cauchy-Riemann equations in cartesian and polar forms – Necessary and sufficient condition for f to be analytic – Harmonic functions – Real and Imaginary parts of an analytic functions are harmonic – Construction of analytic functions

i. Milne Thomson Method.
ii. Using the concept of Harmonic function.

Complex Integration:

The Complex Line integral – Examples and Properties – Proof of Cauchy's Integral theorem using Green's theorem – Direct consequences of Cauchy's theorem – The Cauchy's Integral formula for the function and the derivatives – Applications to the evaluations of simple line integrals – Cauchy's inequality – Liouville's theorem – Fundamental theorem of Algebra.

Transformations:

Definitions – Jacobian of a transformation - Identity transformation – Reflections – Translation – Rotation – stretching - Inversion - Linear Transformations – Definitions - The Bilinear transformation – Cross Ratio of four points – Cross Ratio Preserving property – Preservation of the family of straight lines and circles – Conformal mappings – Discussion of the transformations $w = z^2, w = \sin z, w = e^z, w = \frac{1}{2}\left(z + \frac{1}{z}\right)$.

Calculus of Residues:

Zeros and Singularities, Residues – The residue theorem – Evaluation of definite integrals.

Books for Reference

1. Natarajan - Manicavachogam Pillay and Ganapathy – Algebra
2. Lipman Bers – Calculus, Volumes 1 and 2
3. Courant and John – Introduction to Calculus and Analytical Geometry
4. Grosswald – Topics from the Theory of Numbers
5. N. Piskunov – Differential and integral Calculus
6. F. Ayers – Matrices, Schaum Series
7. Ranville and Bedient – A Short course in Differential equations
8. I. N. Herstein – Topics in Algebra
9. B. S. Grewal – Higher Engineering Mathematics
10. S. C. Mailk – Real Analysis
11. E. Kreyszig – Advanced Engineering Mathematics
12. Murray R Spiegel – Theory and Problems of Vector Analysis
13. S. S. Shastry – Introductory Methods of Numerical Analysis
14. Stewart – Introduction to Linear Algebra
15. Gopalakrishna – University Algebra
16. S. Ponnuswamy – Foundations of Complex Analysis