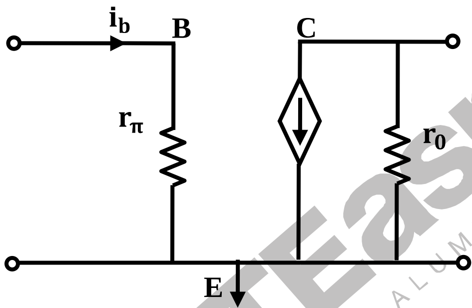
GATE 2012

Electronics & Communication Engineering

Set - A

Q. 1 – Q. 25 carry one mark each.

1. The current i_b is through the base of a silicon *npn* transistor is $1 + 0.1 \cos(10000 \pi t)$ mA. At 300 K, the r_π in the small signal model of the transistor is



(A) 250 Ω
(B) 27.5 Ω
(C) 25 Ω
(D) 22.5 Ω

[Ans. C]

Given base current is a sum of DC and AC (small signal currents),

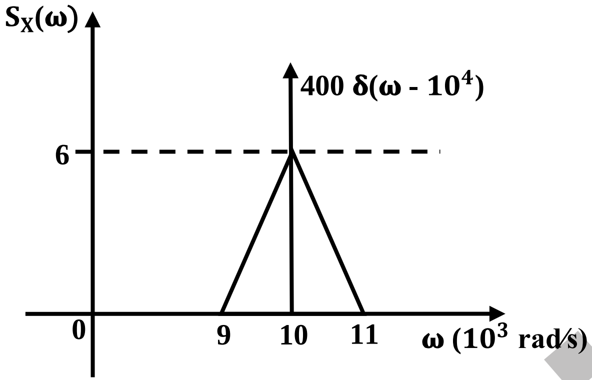
So $I_B(dc) = 1mA$ and we know $V_T = \frac{KT}{q}$ at 300 K $V_T \simeq 25$ mV

So $I_B(dc) = 1$ mA

$r_\pi = \frac{V_T}{I_B(dc)} = \frac{25m}{1m}\ \Omega$

$\Rightarrow r_\pi = 25\,\Omega$

2. The power spectral density of a real process X(t) for positive frequencies is shown below. The values of $E[X^2(t)]$ and $|E[X(t)]|$, respectively, are



(A) $6000/\pi, 0$

(B) $6400/\pi, 0$

(C) $6400/\pi, 20/(\pi\sqrt{2})$

(D) $6000/\pi, 20/(\pi\sqrt{2})$

[Ans. B]

$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f)\, df$

$= 2\int_0^{\infty} S_X(f)\, df \quad (\because \text{PSD is even})$

$= 2 \times \frac{1}{2\pi}\int_0^{\infty} S_X(\omega) d\omega$

$= \frac{1}{\pi}$ [Area under the triangle + Integration of delta function]

$= \frac{1}{\pi}\left[\frac{1}{2} \times 2 \times 10^3 \times 6 + 400\right] = \frac{6400}{\pi}$

$|E[X(t)]|$ = Absolute value of mean of signal X(t)

= Average value of X(t)

= D.C. Component present in X(ω)

= Value of X(ω) at ω = 0

From given PSD $S_X(\omega)$: $S_X\Big|_{\omega = 0} = 0$

We know that $S_X(\omega) = |X(\omega)|^2$

$\therefore |X(\omega)|^2\Big|_{\omega = 0} = 0 \Rightarrow |X(\omega)| = 0$

From inequality: $|E[X(\omega)]| \le E[|X(\omega)|]$

As $|X(\omega)| = 0 \Rightarrow E[|X(\omega)|] = 0$

$\therefore |E[X(\omega)]| = 0$

Logically, If PSD doesn't contain value at ω = 0 then the dc component in the signal is zero.

3. In a baseband communications link, frequencies upto 3500 Hz are used for signaling. Using a raised cosine pulse with 75% excess bandwidth and for no inter-symbol interference, the maximum possible signaling rate in symbols per second is

(A) 1750
(B) 2625
(C) 4000
(D) 5250

[Ans. C]

In the problem, it is given that, frequencies upto 3500 Hz $\Rightarrow$ W = 3500 Hz
Raised Cosine Pulse with 75% excess BW $\Rightarrow$ $\alpha = 0.75$
We know that,

$$(1+\alpha)R_b = 2W$$

$\alpha \Rightarrow$ Roll off factor (excess BW)
$R_b \Rightarrow$ Signalling rate (Symbols per second)
W $\Rightarrow$ Highest signalling frequency or signal BW
Using above formula,

$$R_b = \frac{2W}{1+\alpha} = \frac{2\times3500}{1+0.75} = \frac{7000}{1.75} = 4000$$

4. A plane wave propagation in air with $\vec{E} = (8\,\hat{a}_x + 6\,\hat{a}_y + 5\,\hat{a}_z)\,e^{j(\omega t+3x+4y)}$ V/m is incident on a perfectly conducting slab positioned at $x \le 0$. The $\vec{E}$ field of the reflected wave is

(A) $(-8\,\hat{a}_x - 6\,\hat{a}_y - 5\,\hat{a}_z)\,e^{j(\omega t+3x+4y)}$ V/m
(B) $(-8\,\hat{a}_x + 6\,\hat{a}_y - 5\,\hat{a}_z)\,e^{j(\omega t+3x+4y)}$ V/m
(C) $(-8\,\hat{a}_x - 6\,\hat{a}_y - 5\,\hat{a}_z)\,e^{j(\omega t+3x+4y)}$ V/m
(D) $(-8\,\hat{a}_x - 6\,\hat{a}_y - 5\,\hat{a}_z)\,e^{j(\omega t+3x+4y)}$ V/m

[Ans. Marks to All*] (*Ambiguous options)

5. The electric field of a uniform plane electromagnetic wave in free space, along the positive x direction, is given by $\vec{E} = 10\,(\hat{a}_y + j\hat{a}_z)\,e^{-j25x}$. The frequency and polarization of the wave, respectively, are

(A) 1.2 GHz and left circular
(B) 4 Hz and left circular
(C) 1.2 GHz and right circular
(D) 4 Hz and right circular

[Ans. A]

<u>Approach – 1</u>

$\frac{2\pi}{\lambda} = 25$

$\Rightarrow \lambda = \left(\frac{2\pi}{25}\right)$

$f = \frac{3\times10^8}{\frac{2\pi}{25}} = 1.2$ GHz

Let $E_y = \cos \omega t$

then $E_z = \cos\left(\omega t + \frac{\pi}{2}\right)$

Now if we increase ‘t’, we will see that it in left circular polarization.

<u>Approach – 2</u>

$\vec{E} = 10\,(\hat{a}_y + j\,\hat{a}_z)\,e^{-j25x}$

$\beta = \frac{2\pi}{\lambda}$

$\vec{E}(x,t) = \text{Re}\{\vec{E}(x)\cdot e^{j\omega t}\}$

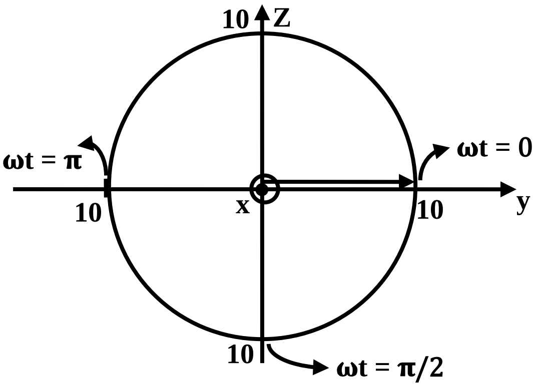
$\beta = \frac{2\pi}{\lambda}$

$\Rightarrow 25 = \frac{2\pi f}{C}$

$\Rightarrow f = \frac{25C}{2\pi} = \left(\frac{25\times3\times10^8}{2\pi\times10^9}\right)$ GHz

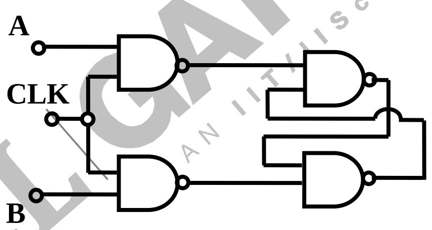
$\cong 1.2$ GHz

To know the polarization we convert $\vec{E}$ into time-domain from phasor form



$\vec{E}(x,t) = \text{Re}\{\vec{E}(x)\cdot e^{j\omega t}\}$
$= \text{Re}\{10(\hat{a}_y + j\,\hat{a}_z)e^{j(\omega t - 25x)}\}$
$= 10\cos(\omega t - 25x)\ \hat{a}_y - 10\sin(\omega t - 25x)\ \hat{a}_z$
$\vec{E}(0,t) = (10\cos\omega t)\ \hat{a}_y - (10\sin\omega t).\hat{a}_z$
∴ It is left – circularly polarized.

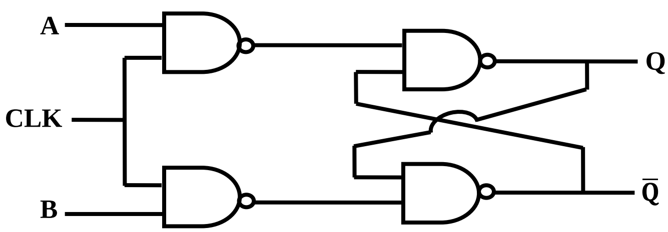
6. Consider the given circuit.



In this circuit, race around

(A) does not occur
(B) occurs when CLK = 0
(C) occurs when CLK = 1 and A = B = 1
(D) occurs when CLK = 1 and A = B = 0

[Ans. A]



$Q_{next} = \overline{\overline{A.CLK.\bar{Q}}}$

= A.CLK + Q

$\overline{Q}_{next} = A.CLK + \overline{Q}$

If CLK = 1 and A and B = 1

then $\left.\begin{matrix} Q_{next} = 1 \\ \overline{Q}_{next} = 1 \end{matrix}\right\}$ No race around

If CLK = 1 and A = B = 0

$\left.\begin{matrix} Q_{next} = Q \\ \overline{Q}_{next} = \overline{Q} \end{matrix}\right\}$ No race around

Thus race around does not occur in the circuit.

7. The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is

(A) 4 (C) 8

(B) 6 (D) 10

[Ans. B]

Approach – 1

A > B   A & B are 2 bit

01 00 –1

10 00 01 –2

11 00 01 10 $\frac{-3}{6}$

Approach – 2

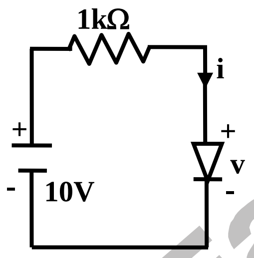
Input A		Input B		Y
A_2	A_1	B_2	B_1	
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0

1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Thus for 6 combinations output in logic 1.

8. The i-v characteristics of the diode in the circuit given below are

$$i = \begin{cases} \frac{v-0.7}{500} A, & v \geq 0.7V \\ 0\ A, & v < 0.7\ V \end{cases}$$



The current in the circuit is

(A) 10 mA
(B) 9.3 mA
(C) 6.67 mA
(D) 6.2 mA

[Ans. D]

Approach – 1

I_D(diode current) = 500 i + 0.7

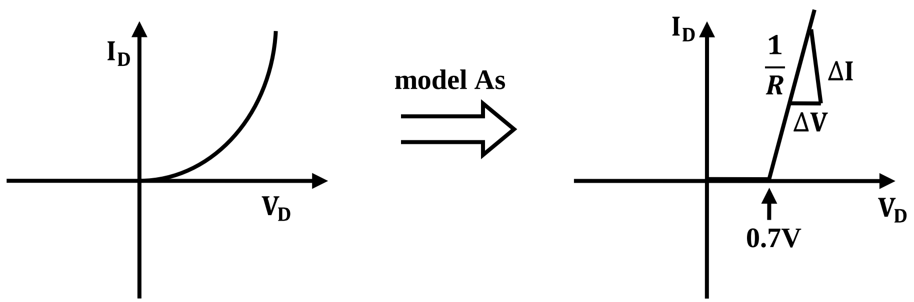
Applying KCL,

10 = 1000 i + 500 i + 0.7

$\Rightarrow i = \frac{9.3}{1.5} = 6.2mA$

Approach – 2

Here diode equivalent circuit model is given which is graphically presented as →



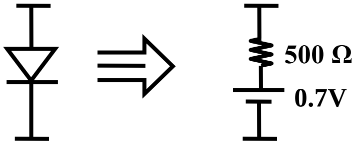
$V \geq 0.7V$

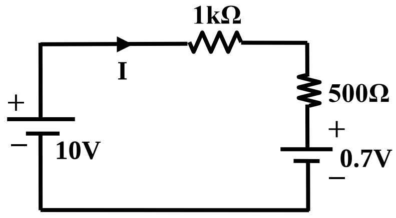
$$I = \frac{V - 0.7}{500}\ A$$

Slope $= \frac{1}{R} = \frac{1}{500}$

R = 500 Ω

Equivalent circuit Model:





Now applying KVL for Linear circuit

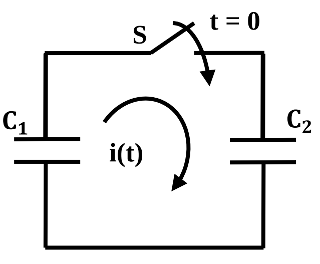
$$-10V + 1K\,\Omega \times I + 500\,\Omega \times I + 0.7V = 0$$

$$\Rightarrow -9.3V + 1500\,\Omega\ \mathrm{I} = 0$$

$$\Rightarrow I = \frac{9.3}{1500}\frac{V}{\Omega} = 6.2 \times 10^{-3}\ A$$

$I = 6.2$ mA

9. In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at t = 0. The current i(t) for all t is



(A) zero

(B) a step function

(C) an exponentially decaying function

(D) an impulse function

[Ans. D]

Initially when switch is closed, impulse current flows from C1 to C2 till voltages of C1 and C2 becomes equal. This happens due to the fact that there is a potential difference initially between C_1 and C_2, but resistance in the circuit is zero leading to an infinite current. Once charge is equal in C1 and C2, current i(t) will be zero.

10. The average power delivered to an impedance (4 – j3) Ω by a current 5 cos(100 π t + 100) A is

(A) 44.2 W
(B) 50 W
(C) 62.5 W
(D) 125 W

[Ans. B]

$$P_{avg} = \frac{1}{2}\text{Re}\{VI^*\}$$
$$= \frac{1}{2}\text{Re}\{IZI^*\}$$
$$= \frac{1}{2}\text{Re}\{|I|^2 Z\}$$
$$= \frac{1}{2}|I|^2\text{Re}\{Z\}$$
$$= \frac{1}{2}\times 5^2 \times 4 \qquad (\because \text{Re}\{Z\} = 4)$$
$$= 50\text{ W}$$

11. The unilateral Laplace transform of f(t) is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of tf(t) is

(A) $-\frac{s}{(s^2+s+1)^2}$
(B) $-\frac{2s+1}{(s^2+s+1)^2}$
(C) $\frac{s}{(s^2+s+1)^2}$
(D) $\frac{2s+1}{(s^2+s+1)^2}$

[Ans. D]

$$L\{t.f(t)\} = (-1)^1.\frac{d}{ds}F(s)$$
$$= -\frac{d}{ds}\left(\frac{1}{s^2+s+1}\right)$$
$$= 1$$

12. With initial condition x(1) = 0.5, the solution of the differential equation.

$t\frac{dx}{dt} + x = t$ is

(A) $x = t - \frac{1}{2}$
(B) $x = t^2 - \frac{1}{2}$
(C) $x = \frac{t^2}{2}$
(D) $x = \frac{t}{2}$

[Ans. D]

Approach – 1

Just substitute, $x = \frac{t}{2}$, or divide by t, and take integrating fact.

Approach – 2

Given DE is $t\frac{dx}{dt} + x = t \Rightarrow \frac{dx}{dt} + \frac{x}{t} = 1$

$IF = e^{\int \frac{1}{t}dt} = e^{\log t} = t$; solution is x (IF) = $\int (IF)$ tdt

$xt = \int t.tdt \Rightarrow xt = \frac{t^2}{2} + c$; Given that $x(1) = 0.5 \Rightarrow 0.5 = \frac{1}{2} + c \Rightarrow c = 0$

$\therefore$ the required solution is $xt = \frac{t^2}{2} \Rightarrow x = \frac{t}{2}$

Approach – 3

Given: $t\frac{dx}{dt} + x = t, x(1) = 0.5$

$t\frac{dx}{dt} + x = t$

$t\,dx + xdt = tdt$

$d(xt) = t\,dt$

$xt = \frac{t^2}{2} + C$

Using initial condition, at t = 1, x = 0.5

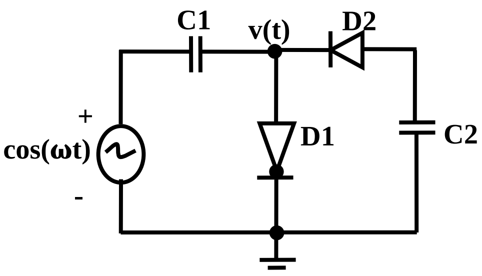
$0.5 \times 1 = \frac{1}{2} + C$

$C = 0$

$\therefore xt = \frac{t^2}{2}$

$x = \frac{t}{2}$

13. The diodes and capacitors in the circuit shown are ideal. The voltage v(t) across the diode D1 is



(A) cos (ωt) – 1
(B) sin (ωt)
(C) 1 − cos (ωt)
(D) 1 – sin (ωt)

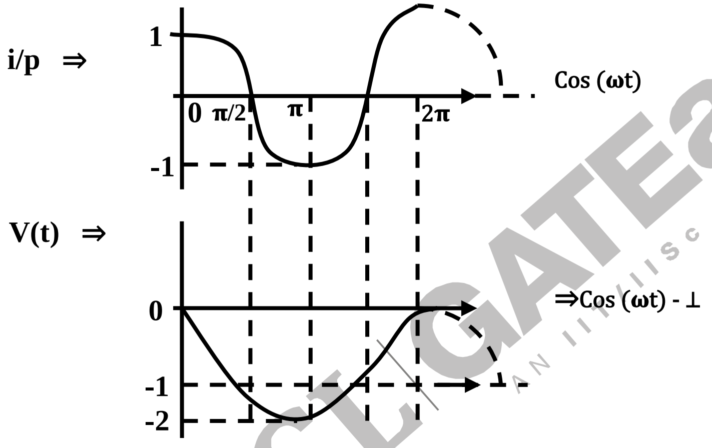
[Ans. A]

Approach – 1

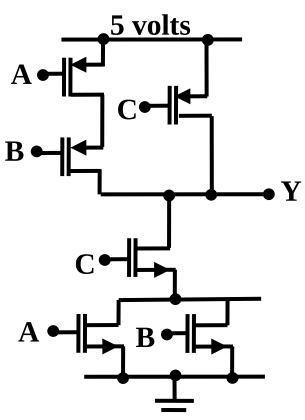
C_1 will trap the peak voltage that will be – 1V, so the voltage across D_1 is $-1 + \cos\omega t = (\cos\omega t - 1)$

Approach – 2

Circuit composed of two sections in cascade: A clamper form by C_1 and D_1 and peak rectifier formed by D_2 and C_2. When excited by sinusoidal of [cos (ω t)], the clamping section clam the positive peak to 0 volt and height peak reach to -2 volt. So the whole i/p signal lower down by -1 volt level.



14. In the circuit shown



(A) $Y = \bar{A}\,\bar{B} + \bar{C}$
(B) $Y = (A + B)\,C$
(C) $Y = (\bar{A} + \bar{B})\,\bar{C}$
(D) $Y = AB + C$

[Ans. A]

Since A & B are in parallel so those represent (A + B) & C is in sense, so it represents 'dot' operation and the whole function should be inverted or it is complementary logic.

So

$\gamma = \overline{(A+B)C} = \overline{A+B} + \bar{C} = \bar{A}\,\bar{B} + \bar{C}$

15. A source alphabet consists of N symbols with the probability of the first two symbols being the same. A source encoder increase the probability of the first symbol by a small amount ε and decreases that of the second by ε. After encoding, the entropy of the source

(A) increases
(B) remains the same
(C) increases only if N = 2
(D) decreases

[Ans. D]

Approach – 1

Entropy is maximum when all symbols are equiprobable.

If the probability of symbols at different, then entropy in going to decrease.

Approach – 2

We know that Entropy is maximum when symbols are equally likely.

Before encoding $p_1 = p_2 = p$

$$H = -\left(p_1 \log p_1 + p_2 \log p_2 + \sum_{k=3}^{N} p_k \log p_k\right)$$

$$= -\left(2p \log p + \sum_{k=3}^{N} p_k \log p_k\right)$$

After encoding $p_1 = p + \varepsilon \quad p_2 = p - \varepsilon$

$$H' = -\left((p+\varepsilon)\log(p+\varepsilon) + (p-\varepsilon)\log(p-\varepsilon) + \sum_{k=3}^{N} p_k \log p_k\right)$$

Comparing H and H′ and using the concept given above $2p\log p > (p+\varepsilon)\log(p+\varepsilon) + (p-\varepsilon)\log(p-\varepsilon)$

Thus, $H' < H$

Hence, entropy of the source will decrease.

16. A coaxial cable with an inner diameter of 1 mm and outer diameter of 2.4 mm is filled with a dielectric of relative permittivity 10.89. Given $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m, the characteristic impedance of the cable is

(A) 330 Ω
(B) 100 Ω
(C) 143.3 Ω
(D) 43.4 Ω

[Ans. Marks to All*] (*Ambiguous options)

17. The radiation pattern of an antenna in spherical co-ordinates is given by

$$F(\theta) = \cos^4 \theta;\ 0 \le \theta \le \pi / 2$$

The directivity of the antenna is

(A) 10 dB
(B) 12.6 dB
(C) 11.5 dB
(D) 18 dB

[Ans. B]

18. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its Z-transform in the Z-plane will be

(A) $\frac{1}{3} < |z| < 3$
(B) $\frac{1}{3} < |z| < \frac{1}{2}$
(C) $\frac{1}{2} < |z| < 3$
(D) $\frac{1}{3} < |z|$

[Ans. C]

Given: $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$

$x[n] \rightleftharpoons x(z)$

$x(z) = \sum_{n=-\infty}^{\infty} x[n]\ z^{-n}$

First consider $\left(\frac{1}{3}\right)^{|n|}$

$$\sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|n|} z^{-n}$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{n} z^{-n}$$

$$= \sum_{n=-\infty}^{-1} \left(3/z\right)^{n} + \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^{n}$$

$$= \left(\frac{3}{z}\right)^{-1} + \left(3/z\right)^{-2} + ------ + 1 + \frac{1}{3z} + \left(\frac{1}{3z}\right)^{2} + -------$$

$\left|\frac{3}{z}\right| > 1$ $\qquad$ $\left|\frac{1}{3z}\right| < 1$

$\left|\frac{z}{3}\right| < 1$ $\qquad\qquad$ $|z| > {}^1/_3$

$|z| < 3$

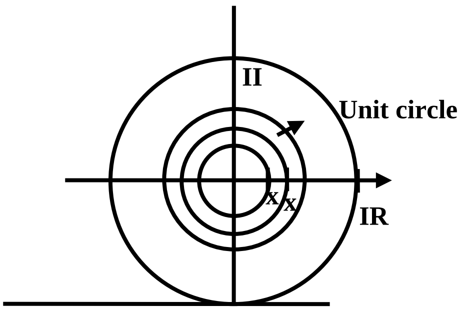
Consider $\left(\frac{1}{2}\right)^n u(n)$

$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$

$= 1 + \frac{1}{2z} + \left(\frac{1}{2z}\right)^2 +$ -------------

$\left|\frac{1}{2z}\right| < 1 \quad |z| > {}^1/_2$

ROC is $\frac{1}{2} < |z| < 3$



19. In the sum of products function f(X, Y, Z) = $\sum(2, 3, 4, 5)$, the prime implicants are

(A) $\overline{X}$ Y, X $\overline{Y}$

(B) $\overline{X}$ Y, X $\overline{Y}$ $\overline{Z}$, X $\overline{Y}$ Z

(C) $\overline{X}$ Y $\overline{Z}$, $\overline{X}$Y Z, X $\overline{Y}$

(D) $\overline{X}$ Y $\overline{Z}$, $\overline{X}$ Y Z, X $\overline{Y}$ $\overline{Z}$, X $\overline{Y}$ Z

[Ans. A]

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

f(x, y, z) = $\bar{x}y + x\bar{y}$

20. A system with transfer function

$$G(s) = \frac{(s^2 + 9)(s + 2)}{(s + 1)(s + 3)(s + 4)}$$

is excited by sin (ωt). The steady-state output of the system is zero at

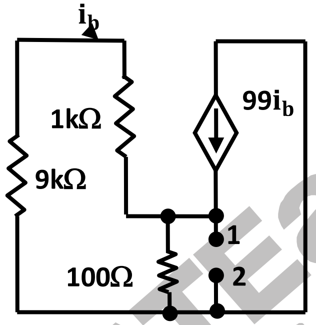
(A) ω = 1 rad/s
(C) ω = 3 rad/s
(B) ω = 2 rad/s
(D) ω = 4 rad/s

[Ans. C]

$G(s) = \frac{(S^2+9)\,(S+2)}{(S+1)(S+3)(S+4)}$

By substituting s = jw and equating |G(s)|=0 we get w=3

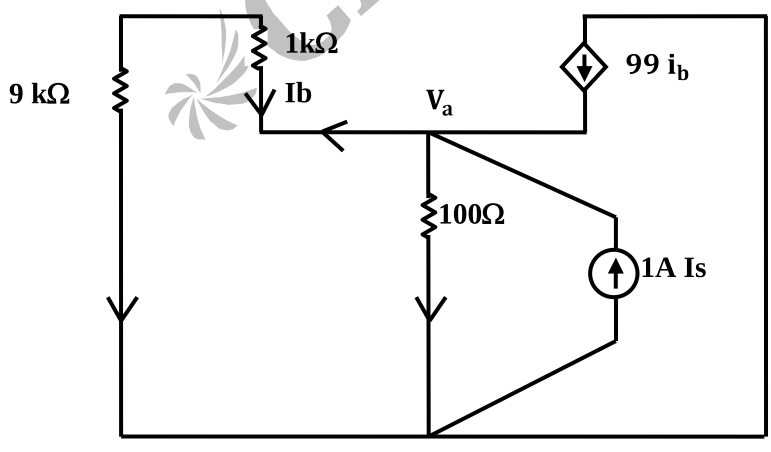
21. The impedance looking into nodes 1 and 2 in the given circuit is



(A) 50 Ω
(C) 5 kΩ
(B) 100 Ω
(D) 10.1kΩ

[Ans. A]

Approach – 1



By nodal analysis at node a,

$\frac{V_a - 0}{10k} + \frac{V_a}{100} - 99\, i_b = 1$

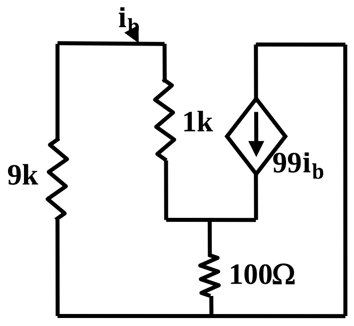
$\frac{V_a - 0}{10k} + \frac{V_a}{100} - 1 + \frac{99\, V_a}{10k} = 0$

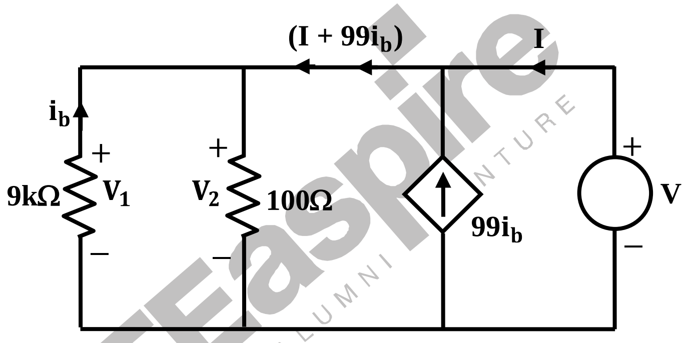
$\Rightarrow V_a \left[\frac{100}{10k} + \frac{100}{10k}\right] = 1.$

$\Rightarrow V_a = 50V$

$\therefore$ R (thevenin) $= \frac{V_a}{I_s} = 50\Omega$

Approach – 2





After connecting a voltage source of V

$V_1 = V_2 \Rightarrow (10k)(-i_b) = 100(I + 99i_b + i_b);$

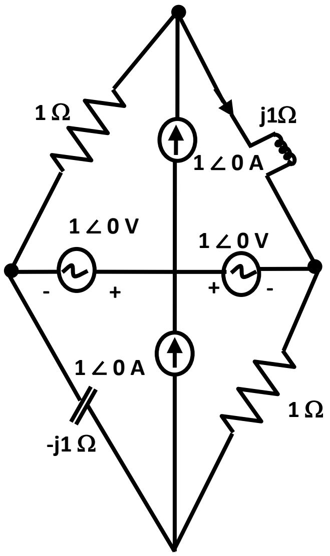
$-10000i_b = 100I + 100 \times 100i_b = 100I + 10000i_b$

$$-20000i_b = 100I \Rightarrow i_b = -\left(\frac{100}{20000}\right)I = \left[-\frac{I}{200}\right]$$

$$V = 100[I + 99i_b + i_b] = 100\left[I + 100\left(\frac{-I}{200}\right)\right] = 50I$$

$$R_{th} = \frac{V}{I} = \frac{50I}{I} = 50\Omega$$

22. In the circuit shown below, the current through the inductor is



(A) $\frac{2}{1+j}$ A

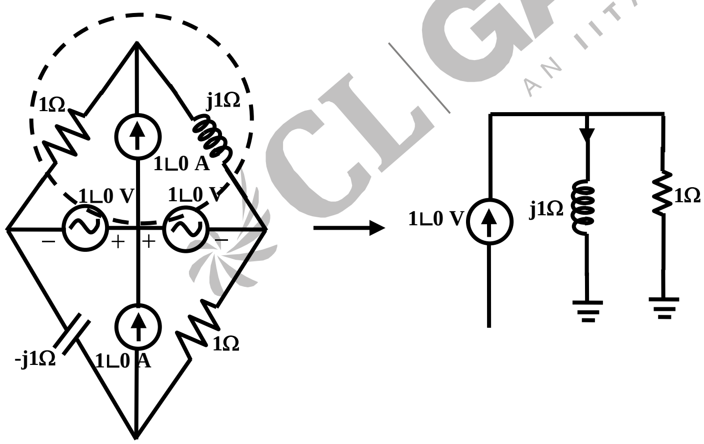
(B) $\frac{-1}{1+j}$ A

(C) $\frac{1}{1+j}$ A

(D) 0 A

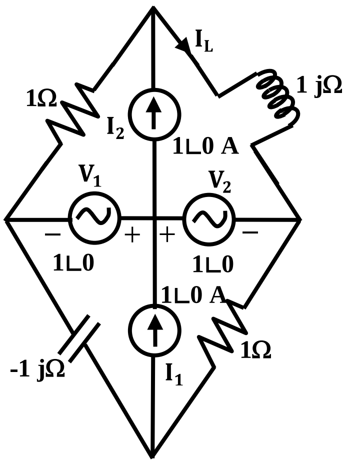
[Ans. C]

<u>Approach – 1</u>



$$I_L = 1\lfloor\underline{0} \times \frac{1}{1+j1} = \frac{1}{1+j1}\ A$$

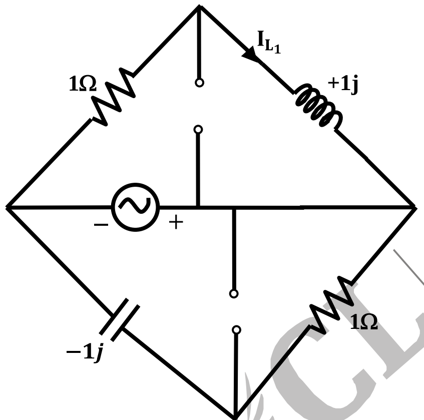
<u>Approach – 2</u>



Apply Superposition theorem,

V_1 only: Short circuit

V_2 open circuit I_1 and I_2



$I_{L_1} = \frac{-1}{1+1j}$ --------------(1)

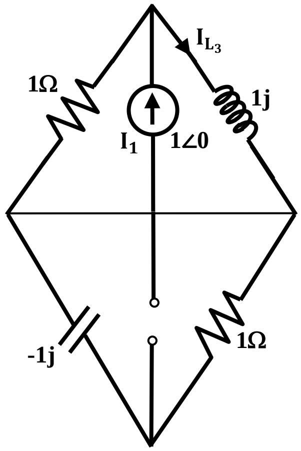
$\left[I_{L_1} = \frac{-V_1}{Total\ Impedance}\right]$

Similarly for V_2 only

$I_{L_2} = \frac{1}{1+1j}$ --------------(2)

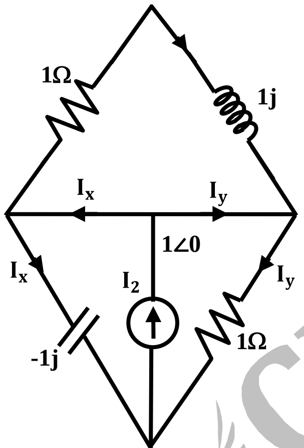
For I_1 only

Using current divider rule



$I_{L_3} = \frac{1}{1+1j} \times 1 < 0 = \frac{1}{1+1j}$ -------------- (3)

For I_2 only



$I_2 = I_x + I_y$

Current in Inductor = 0

So $I_{L_4} = 0$ ------------ (4)

From (1), (2), (3), (4), Total current,

$$I_L = \frac{-1}{1+1j} + \frac{1}{1+1j} + \frac{1}{1+1j} + 0$$

$\Rightarrow I_L = \frac{1}{1+1j}$

23. Given

$f(z) = \frac{1}{z+1} - \frac{2}{z+3}$. If C is a counterclockwise path in the z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi j}\oint_C f(z)dz$ is

(A) – 2
(B) – 1
(C) 1
(D) 2

[Ans. C]

Z = –3 is outside circle

Z = 1 is inside circle

$\lim_{z\to -1} (z+1).\frac{1}{(z+1)} = 1$

24. Two independent random variables X and Y are uniformly distributed in the interval $[-1, 1]$. The probability that max[X, Y] is less than 1/2 is

(A) 3/4
(B) 9/16
(C) 1/4
(D) 2/3

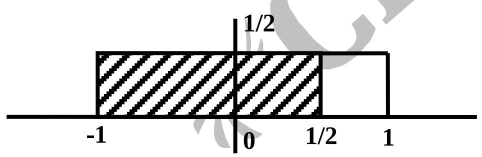
[Ans. B]

$P\left[\max[X,Y] < \frac{1}{2}\right] = P\left[X < \frac{1}{2}, Y < \frac{1}{2}\right]$ (If maximum is $< \frac{1}{2}$ then both are less than $\frac{1}{2}$)

$P\left[x < \frac{1}{2}, y < \frac{1}{2}\right]$

$P\left[x < \frac{1}{2}\right].P\left[y < \frac{1}{2}\right]$ (since independent events)

Probability density function of X and Y



$= \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

25. If $x = \sqrt{-1}$, then the value of x^x is

(A) $e^{-\pi/2}$
(B) $e^{\pi/2}$
(C) x
(D) 1

[Ans. A]

Approach – 1

Given, $x = \sqrt{-1}$; $x^x = \left(\sqrt{-1}\right)^{\sqrt{-1}} = i^i$

We know that $e^{i\theta} = \cos\theta + i\sin\theta \Rightarrow e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$\therefore\ (i)^i = \left(e^{i\pi/2}\right)^i = e^{-\pi/2}$

Approach – 2

$x = \sqrt{-1} = i = e^{i\pi/2}$

Let $y = x^x$

Taking Natural logarithm on both sides

$\log y = \log x^x$

$\log y = x \log x$

$= e^{i\pi/2} \log e^{i\pi/2}$

$= i(i\pi/2)$

$\log y = -\pi/2$

$y = e^{-\pi/2}$

$\therefore\ x^x = e^{-\pi/2}$

Q. 26 To Q.55 carry two marks each.

26. The source of a silicon ($n_i = 10^{10}$ per cm^3) n-channel MOS transistor has an area of 1 sq μm and a depth of 1μm. If the dopant density in the source is $10^{19}/cm^3$, the number of holes in the source region with the above volume is approximately

(A) 10^7

(B) 100

(C) 10

(D) 0

[Ans. D]

Given ⟹ Si $\left[n_i^2\ 10^{10}/cm^3\right]$

n – MOS – Source – Area = 1μm × 1 μm

Area = 10^{-4}cm × 10^{-4}cm

Depth = 14m = 10^{-4}cm

Do pant density (n) = $10^{19}/cm^3$

To find ⟹ no. of holes (p) = p in volume

Formula ⟹ np = n_i^2 [Law of Mass Act]

$p = \frac{n_i^2}{n} \Rightarrow \frac{10^{10} \times 10^{10}}{10^{19}} \frac{cm^3}{cm^3 \times cm^3}$

$p \Rightarrow 10/cm^3$

Volume of source = $10^{-4} \times 10^{-4} \times 10^{-4} \times cm \times cm$

$\Rightarrow 10^{-12} cm^3$

no. of holes in source volume $\Rightarrow$ (p) = $10 \times 10^{-12} \frac{cm^3}{cm^3}$

(p) = 10^{-11}

thus no. of holes in source approx ~ zero [0]

27. A BPSK scheme operating over and AWGN channel with noise power spectral density of $N_0/2$, uses equiprobable signals $s_1(t) = \sqrt{\frac{2E}{T}} \sin(\omega_c t)$ and $s_2(t) = -\sqrt{\frac{2E}{T}} \sin(\omega_c t)$ over the symbol interval (0, T). If the local oscillator in a coherent is ahead in phase by 45^0 with respect to the received signal, the probability of error in the resulting system is

(A) $Q\left(\sqrt{\frac{2E}{N_0}}\right)$

(B) $Q\left(\sqrt{\frac{E}{N_0}}\right)$

(C) $Q\left(\sqrt{\frac{E}{2N_0}}\right)$

(D) $Q\left(\sqrt{\frac{E}{4N_0}}\right)$

[Ans. B]

Equiprobable signals

$s_1(t) = \sqrt{\frac{2E}{T}} \sin(\omega_c t)$; and $s_2(t) = -\sqrt{\frac{2E}{T}} \sin(\omega_c t)$

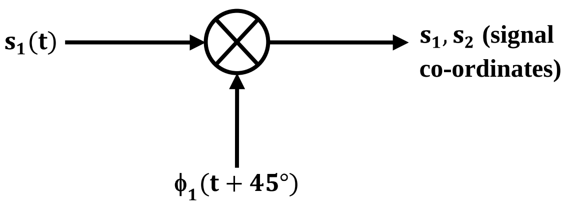
Local oscillator in-coherent receiver is ahead in phase by 45° with respect to the received signal.

Generally, we consider the local oscillator function of unit energy

$\phi_1(t) = \sqrt{\frac{2}{T}} \sin \omega_c t \; 0 \le t \le T$ but the local oscillator is ahead with 45°

$$\phi_1(t + 45^\circ) = \sqrt{\frac{2}{T}} \sin(\omega_c t + 45^\circ) \;\; 0 \le t \le T$$

Receiver structure:



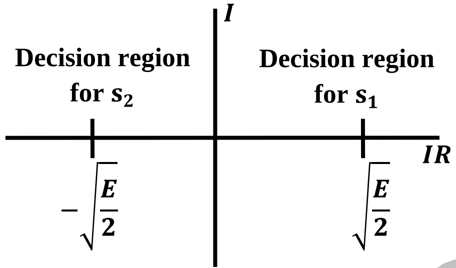
$$s_1 = \int_0^T s_1(t)\phi_1(t+45^\circ)dt = \int_0^T \sqrt{\frac{2E}{T}}\sin\omega_c t.\sqrt{\frac{2}{T}}\sin(\omega_c t+45^\circ)\ dt$$

$$s_1 = \sqrt{\frac{E}{2}};$$

Similarly $s_2 = -\sqrt{\frac{E}{2}}$

Probability of making an error when we transmit $\sqrt{\frac{E}{2}}$ i.e. s_1

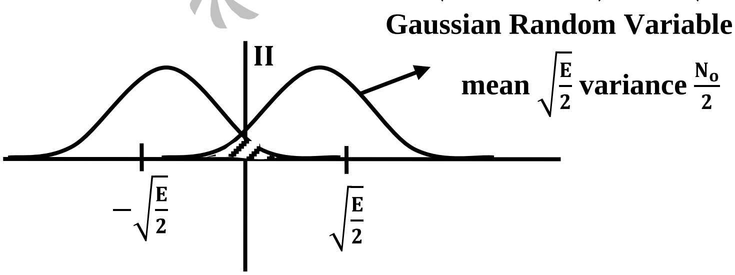
$$y = \sqrt{\frac{E}{2}} + N; P\left(\frac{s_{2received}}{s_{1transmitted}}\right) = P(\gamma < 0) = P\left(\sqrt{\frac{E}{2}} + N < 0\right) = P\left(N < -\sqrt{\frac{E}{2}}\right)$$



Probability of making an error when we transmit $\sqrt{\frac{E}{2}}$ $i.e.$ s_1

$$y = \sqrt{\frac{E}{2}} + N;$$

$$P\left(\frac{s_{2received}}{s_{1transmitted}}\right) = P(\gamma < 0) = P\left(\sqrt{\frac{E}{2}} + N < 0\right) = P\left(N < -\sqrt{\frac{E}{2}}\right)$$



$$P\left(N<\sqrt{\frac{E}{2}}\right)=\int_{-\infty}^{0}\frac{1}{\sqrt{2\pi\frac{N_0}{2}}}e^{-\frac{\left(x+\sqrt{\frac{E}{2}}\right)^2}{2\frac{N_0}{2}}}dx=\int_{-\infty}^{0}\frac{1}{\sqrt{\pi N_0}}e^{-\frac{\left(x+\sqrt{\frac{E}{2}}\right)^2}{N_0}}dx;$$

$$Let\ \frac{\left(x+\sqrt{\frac{E}{2}}\right)}{\sqrt{\frac{N_0}{2}}}=t;dx=\sqrt{\frac{N_0}{t}}dt\Rightarrow P\left(N<\sqrt{\frac{E}{2}}\right)=\int_{\sqrt{\frac{E}{N_0}}}^{\infty}\frac{1}{2\pi}e^{-\frac{t^2}{2}}dt=Q\left(\sqrt{\frac{E}{N_0}}\right)$$

(∵ sign in the limit is removed since Area of Gaussian Pulse is same)

Symbols are equiprobable $P(e)=\frac{1}{2}\left(P\left(\frac{s_{1received}}{s_{2transmitted}}\right)+P\left(\frac{s_{2received}}{s_{1transmitted}}\right)\right)$

$$\therefore P(e)=\frac{1}{2}Q\left(\sqrt{\frac{E}{N_0}}\right)+\frac{1}{2}Q\left(\sqrt{\frac{E}{N_0}}\right)=Q\left(\sqrt{\frac{E}{N_0}}\right)$$

28. A transmission line with a characteristic impedance of 100 Ω is used to match a 50 Ω section to a 200 Ω section. If the matching is to be done both at 429 MHz and 1 GHz, the length of the transmission line can be approximately

(A) 82.5 cm
(B) 1.05 m
(C) 1.58 m
(D) 1.75 m

[Ans. C]

29. The input x(t) and output y(t) of a system are related as $y(t)=\int_{-\infty}^{t}x(\tau)\cos(3\tau)\ d\tau$. The system is

(A) time-invariant and stable
(B) stable and not time-invariant
(C) time-invariant and not stable
(D) not time-invariant and not stable

[Ans. D]

- Assume a bounded input x(t) = cos (3t)

$$y(t)=\int_{-\infty}^{t}cos^2\ (3\tau)\,d\tau$$

Thus, y(t) is unbounded, hence, system is not stable.

- Assume $x(t)=\delta(t)$

$$y(t) = \int_{-\infty}^{t} \delta(\tau)\cos(3\tau)\ d\tau$$

$= u(t)\cos(0) = u(t)$

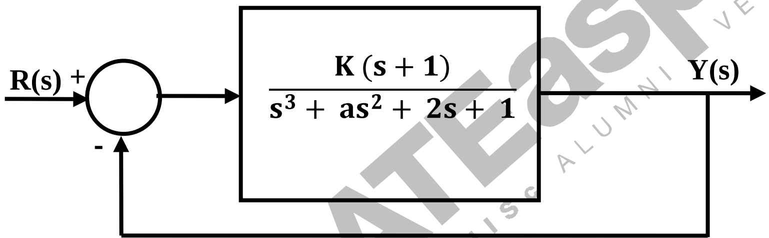
Time shifted input $x\left(t - \frac{\pi}{6}\right) = \delta\left(t - \frac{\pi}{6}\right)$

$$y'(t) = \int_{-\infty}^{t} \delta\left(\tau - \frac{\pi}{6}\right)\cos(3\tau)\ d\tau$$

$= u(t)\cos\left(3 \times \frac{\pi}{6}\right) = 0$

$y'(t) \neq y\left(t - \frac{\pi}{6}\right) \Longrightarrow$ System is not time – invariant

30. The feedback system shown below oscillates at 2 rad/s when



(A) K = 2 and a = 0.75
(B) K = 3 and a = 0.75
(C) K = 4 and a = 0.5
(D) K = 2 and a = 0.5

[Ans. A]

$1 + G(S)H(S) = S^3 + as^2 + (2+k)s + 1 + k$

s^3	$1\ (2+k)$
s^2	$a\ (2+k)$
s	$a(2+k) - (2+k)0$
s^0	$(1+k)^a$

For system to oscillate,

$a(2+k) - (1+k) = 0$

$a = \left(\frac{1+k}{2+k}\right)$

$$A.E \Rightarrow as^2 + (1+k) = 0 \Rightarrow s = \sqrt{\frac{1+k}{a}} = 2 \Rightarrow \left(\frac{1+k}{a}\right) = a \Rightarrow 2+k = 4 \Rightarrow k = 2$$

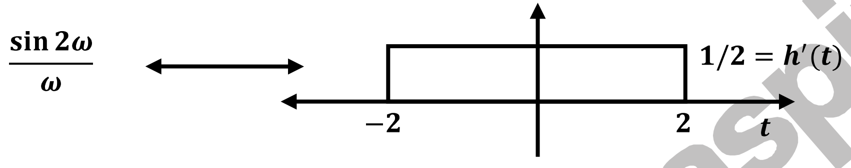
Thus a = 0.75

31. The Fourier transform of a signal h(t) is H(jω) = (2 cos ω) (sin 2ω)/ ω. The value of h(0) is

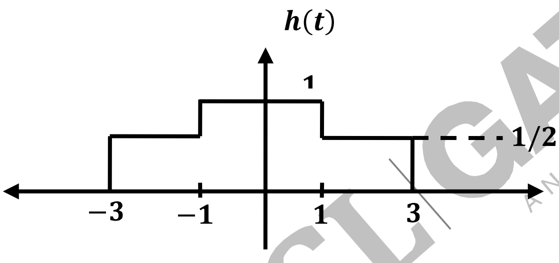
(A) 1/4
(B) 1/2
(C) 1
(D) 2

[Ans. C]

Approach – 1



$$2\cos\omega\left(\frac{\sin 2\omega}{\omega}\right) = \left[e^{j\omega} + e^{-j\omega}\right]\left(\frac{\sin 2\omega}{\omega}\right) \leftrightarrow h(t) = h'(t-1) + h'(t+1)$$

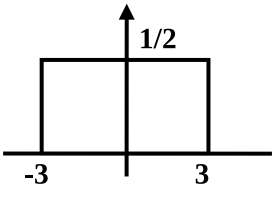


Approach – 2

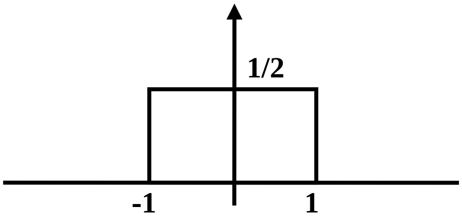
Given: H(j ω) = (2 cos ω) (sin 2 ω)/ ω

Solution: $H(j\omega) = \frac{2\cos\omega \sin 2\omega}{\omega}$

$$H(j\omega) = \frac{\sin 3\omega}{\omega} + \frac{\sin \omega}{\omega}$$

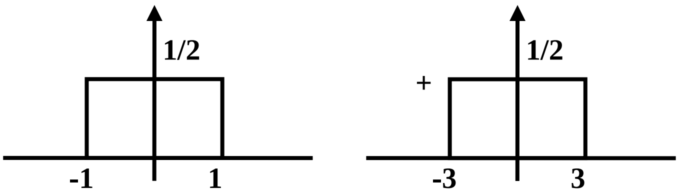


$\rightleftharpoons \quad \frac{\sin 3\omega}{\omega}$



$\rightleftharpoons \quad \dfrac{\sin\omega}{\omega}$

$h(t) =$



$h(0) = \dfrac{1}{2} + \dfrac{1}{2}$

$h(0) = 1$

32. The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = (1 \quad 0 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

(A) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

(B) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(C) $a_1 = 0, a_2 \neq 0, a_3 = 0$

(D) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

[Ans. D]

The controllability matrix

$= [B \quad AB \quad A^2B]$

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow$ Controllability matrix $= \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

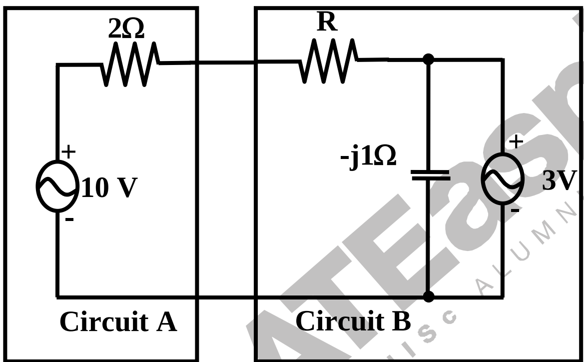
$\Rightarrow a_1 \neq 0$

$a_2 \neq 0$

a_3 can be zero

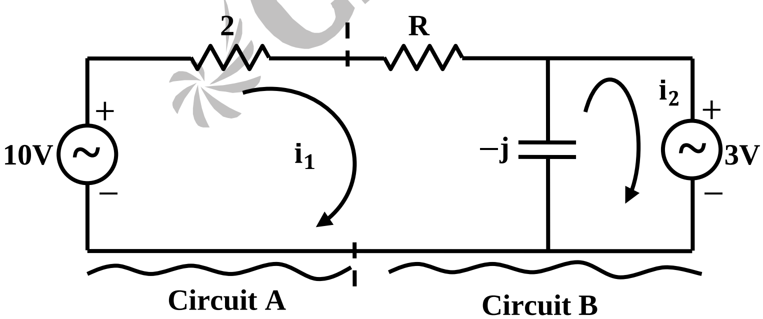
For system to be controllable, determinant of control ability matrix should not be zero.

33. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(A) 0.8 Ω
(B) 1.4 Ω
(C) 2 Ω
(D) 2.8 Ω

[Ans. A]



From KVL: $10 = (2 + R)i_1 + (i_1 - i_2)(-j)$

$10 = (2 + R - j)i_1 + ji_2$ -------------- (1)

$3 = -j(i_1 - i_2) \Rightarrow 3 = -ji_1 + ji_2$ ------------- (2)

From (1) & (2): $i_1 = \frac{7}{2+R}$; $i_2 = \frac{7}{2+R} - 3j$

From (2) $i_1 - i_2 = 3j$

Power transfer from circuit A to circuit B

$P = i_1^2 R + (i_1 - i_2)^2(-j) + 3i_2$

$= \frac{49R}{(2+R)^2} + 9j + 3\left(\frac{7}{2+R} - 3j\right) = \frac{7(10R+6)}{(2+R)^2}$

$\frac{dP}{dR} = 0$ for max power $\Longrightarrow R = 0.8\ \Omega$

34. Consider the differential equation

$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$ with $y(t)|_{t=0^-} = -2$ and $\frac{dy}{dt}\Big|_{t=0^-} = 0$

The numerical value of $\frac{dy}{dt}\Big|_{t=0^+}$ is

(A) −2
(B) −1
(C) 0
(D) 1

[Ans. D]

<u>Approach – 1</u>

$$\frac{d^2y\,(t)}{dt^2} + \frac{2\,dy\,(t)}{dt} + y\,(t) = \delta(t)$$

Converting to s – domain,

$s^2 y(s) - sy(0) - y'(0) + 2[sy(s) - y(0)] + y(s) = 1$

$[s^2 + 2s + 1]y(s) + 2s + 4 = 1$

$$y(s) = \frac{-3 - 2s}{(s^2 + 2s + 1)}$$

Find inverse Laplace transform

$y(t) = [-2e^{-t} - te^{-t}]\,u(t)$

$$\frac{dy(t)}{dt} = 2e^{-t} + te^{-t} - e^{-t}$$

$$\frac{dy(t)}{dt}\Big|_{t=0^+} = 2 - 1 = 1$$

<u>Approach – 2</u>

$$\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = \delta(t)$$

Applying Laplace Transform on both sides

$$s^2y(s) - sy(0^-) - \frac{dy}{dt}\Big|_{t=0^-} + 2\big(sy(s) - y(0^-)\big) + y(s) = 1$$

$s^2y(s) + 2s + 2sy(s) + y(s) = 1 - 4$

$y(s) = \frac{-3-2s}{s^2+2s+1} = \frac{-3}{(s+1)^2} - \frac{2s}{(s+1)^2}$

$y(t) = -3te^{-t} - 2\frac{d}{dt}(te^{t})$

$= -3te^{-t} - 2(-te^{-t} + e^{-t})$

$y(t) = -te^{-t} - 2e^{-t}$

$\frac{dy(t)}{dt} = +te^{-t} + (e^{-t}) + 2e^{-t}$

$= te^{-t} + e^{-t}$

$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 1$

35. The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla \cdot A = 0$ is

(A) −2
(B) 2
(C) 1
(D) 0

[Ans. A]

We know that, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r)$

Now, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r)$

$= \frac{1}{r^2}\frac{\partial}{\partial r}(kr^{rH2}) = \frac{k}{r^2}(n+2)\, r^{n+1}$

$= k(n+2)\, r^{n+1}$

$\therefore$ For, $\nabla.\vec{A} = 0, \Rightarrow (n+2) = 0 \Rightarrow n = -2$

36. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

(A) 1/3
(B) 1/2
(C) 2/3
(D) 3/4

[Ans. C]

If required tosses is odd the possible sequence of heads and tails will be:

H, TTH, TTTTH, TTTTTT H, ……………

Since, these events are mutually exclusive, we can add the prob. of each event.

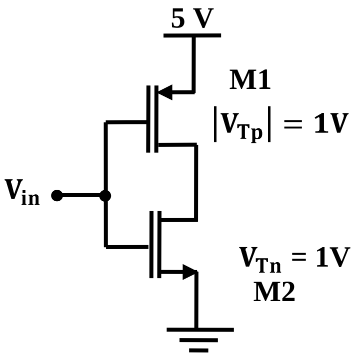
Thus, the required prob. is given by

$P = \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \ldots\ldots\ldots\ldots\ldots$

This is a geometric series with $a = \frac{1}{2}, r = \frac{1}{4}$

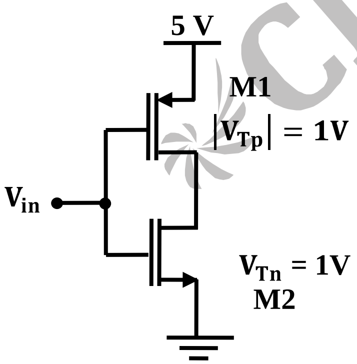
$$P = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

37. In the CMOS circuit shown, electron and hole mobilities are equal, and M1 and M2 are equally sized. The device M1 is in the linear region if



(A) $V_{in} < 1.875\,V$
(B) $1.875\,V < V_{in} < 3.125\,V$
(C) $V_{in} > 3.125\,V$
(D) $0 < V_{in} < 5\,V$

[Ans. A]



$$\left(\frac{W}{L}\right)_{M_1} = \left(\frac{W}{L}\right)_{M_2}$$

$$(\mu_n)\,M_2 = (\mu_p)M_1$$

$$M_1 \xrightarrow{\text{is in}} \text{Linear Region}$$

Concept: When M_1 is in Linear Region, M_2 will be with cutoff or saturation.

If M_2 is in cutoff $I_2 = 0$ hence $I_1 = 0$

$$I_1 = \mu_p C_0 \left(\frac{W}{L}\right)_{M_1} \left[(V_{SG} - V_{TP})V_{SD} - \frac{V_{SD}^2}{2}\right] = 0$$

$$\Rightarrow V_{SD} = 2\,(V_{SG} - V_{TP})$$

$$\Rightarrow 5 - V_D = 2(5 - V_{in} - 1) \Rightarrow V_{in} = \frac{3 + V_D}{2}$$

Now $I_2 = 0 \Rightarrow V_D = 5V$ hence $V_{in} = 4V$

If $V_{in} = 4V \Rightarrow V_{GS} < V_{T_n}$ for cutoff $4 \nless IV$ (false so M_2 must be in saturation then $I_1 = I_2$

$$\mu_p c_0 \left(\frac{W}{L}\right)_{M_1} \left[(V_{SG} + V_{TP})V_{SD} - \frac{V_{SD}^2}{2}\right] = \frac{\left(V_{GS} - V_{T_n}\right)^2}{2} \mu_n c_0 \left(\frac{W}{L}\right)_{M_2}$$

$$\therefore (5 - V_{in} - 1)\,(5 - V_D) - \frac{(5 - V_D)^2}{2} = \frac{(V_{in} - 1)^2}{2}$$

$$\therefore V_{in} = 1.833\,V$$

So the device M_1 is in the linear region if

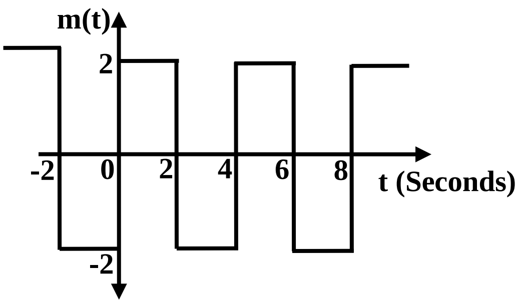
$V_{in} < 1.833\,V$

38. A binary symmetric channel (BSC) has a transition probability of 1/8. If the binary transmit symbol X is such that P(X=0) ≈ 9/10, then the probability of error for an optimum receiver will be

(A) 7/80
(B) 63/80
(C) 9/10
(D) 1/10

[Ans. D]

39. The signal *m(t)* as shown is applied both to a phase modulator (with k_p as the phase constant) and a frequency modulator (with k_f as the frequency constant) having the same carrier frequency.



The ratio k_p/k_f (in rad/Hz) for the same maximum phase deviation is

(A) 8π (C) 2π
(B) 4π (D) π

[Ans. B]

Approach – 1

In phase modulation,

Maximum Phase deviation = $K_p\ |m(t)|_{max} = K_p.2$

In Frequency modulation,

Maximum Phase deviation $= 2\pi\ k_f\ \int_a^2 2\ dt = 2\pi\ k_f \times 4$

Now $K_p.2 = 2\ \pi\ K_f \times 4 \Rightarrow \frac{k_p}{k_f} = 4\pi$

Approach – 2

$$S_{FM}(t) = A_c \cos\left(2\ \pi\ f_c t + 2\ \pi\ k_f \int_0^t m(\tau)\ d\tau\right)$$

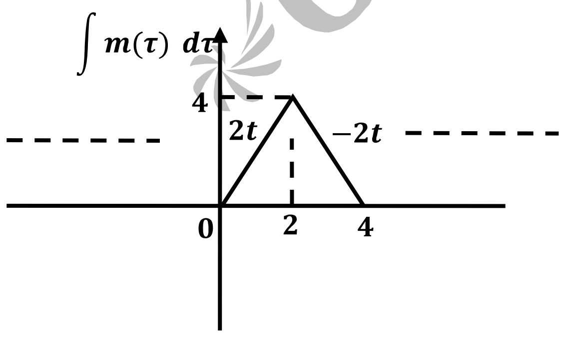
$$S_{PM}(t) = A_c \cos\left(2\ \pi\ f_c t + k_p m(t)\right)$$

Given same maximum phase deviation

$$k_p(m(t))_{Max} = 2\pi k_f \left(\int_0^t m(\tau)\ d\tau\right)_{Max}$$

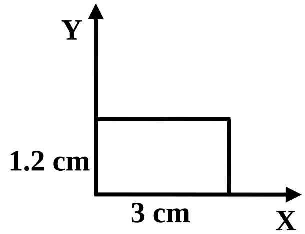
$k_p(2) = 2\ \pi\ \ k_f\ (4)$ (Explanation given belo)

$$\frac{k_p}{k_f} = 4\pi$$



40. The magnetic field along the propagation direction inside a rectangular waveguide with the cross-section shown in the figure is

$H_z = 3\cos(2.094 \times 10^2 x)\cos(2.618 \times 10^2 y)\cos(6.283 \times 10^{10} t - \beta z)$

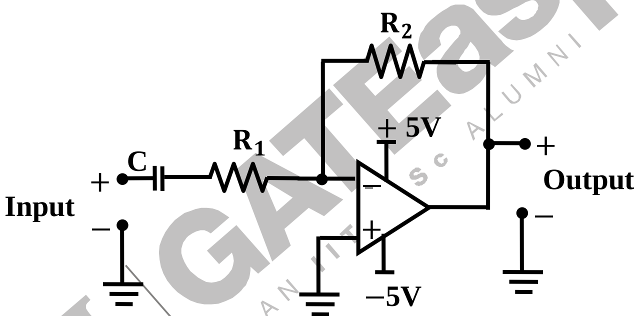


The phase velocity v_p of the wave inside the waveguide satisfies

(A) $v_p > c$
(C) $0 < v_p < c$
(B) $v_p = c$
(D) $v_p = 0$

[Ans. Marks to All*] (*Ambiguous options)

41. The circuit shown is a



(A) low pass filter with $f_{3dB} = \frac{1}{(R_1+R_2)C}$ rad/s

(B) high pass filter with $f_{3dB} = \frac{1}{R_1C}$ rad/s

(C) low pass filter with $f_{3dB} = \frac{1}{R_1C}$ rad/s

(D) high pass filter with $f_{3dB} = \frac{1}{(R_1+R_2)C}$ rad/s

[Ans. B]

The transfer function of the N/W is $\frac{V_0(s)}{V_i(s)} = \frac{-R_2}{R_1+\frac{1}{CS}} = -\frac{R_2CS}{R_1CS+1}$

This represents H.P filter with cutoff frequency at $\frac{1}{R_1C}$

42. Let y[n] denote the convolution of h[n] and g[n], where h[n] – $(1/2)^n$ u[n] and g[n] is a causal sequence. If y[0] = 1 and y[1] = 1/2, then g[1] equals

(A) 0
(B) 1/2
(C) 1
(D) 3/2

[Ans. A]

$$y[n]=\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{k}\ g(n-k)$$

$$y[0]=\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{k}\ g(-k)=1$$

$$\Rightarrow\left(\frac{1}{2}\right)^{O}\ g(0)=1$$

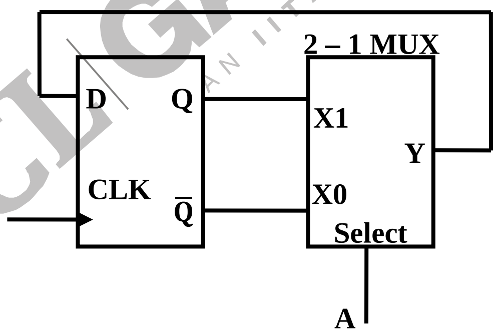
$\Rightarrow g(0)=1$ since g(n) is Causal sequence $g(-1), g-2),$………….. = 0

$$y[1]=\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{k}\ g[1-k]$$

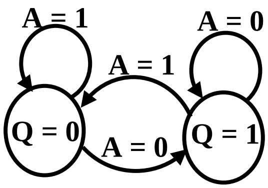
$$\Rightarrow\left(\frac{1}{2}\right)^{0}\ g[1]+\left(\frac{1}{2}\right)^{1}\ g(0)={}^{1}/_{2}$$

$$g[1]=0$$

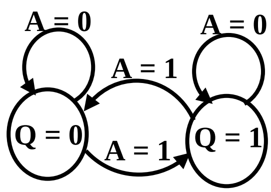
43. The state transition diagram for the logic circuit shown is

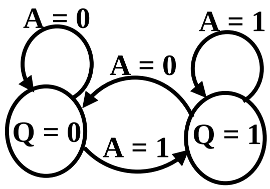


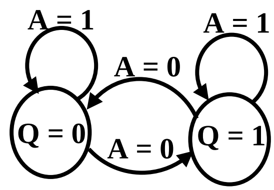
(A)



(B)

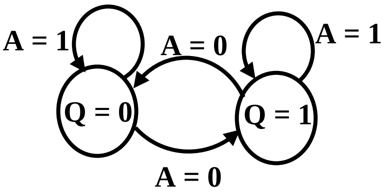


(C) 

(D) 

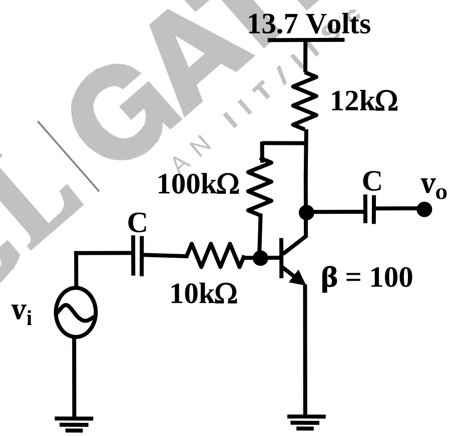
[Ans. D]

$\left.\begin{array}{l} A = 0, \quad Y = Q \\ A = 1, \quad Y = \overline{Q} \end{array}\right\}$ whenever A = 1, output gets into same state



Whenever A = 0, output gets toggled

44. The voltage gain A_v of the circuit shown below is



(A) $|A_v| \approx 200$
(B) $|A_v| \approx 100$
(C) $|A_v| \approx 20$
(D) $|A_v| \approx 10$

[Ans. D]

Approach – 1

This is voltage shunt feedback if we neglect base to emitter voltage. Feedback factor $P_f = \frac{1}{100 \times 103}$

$= \frac{1}{105}\Omega - 1$

Now with F/B,

$A_2 = \frac{V_0}{I_i} = 12 \times 10^5 \Omega$

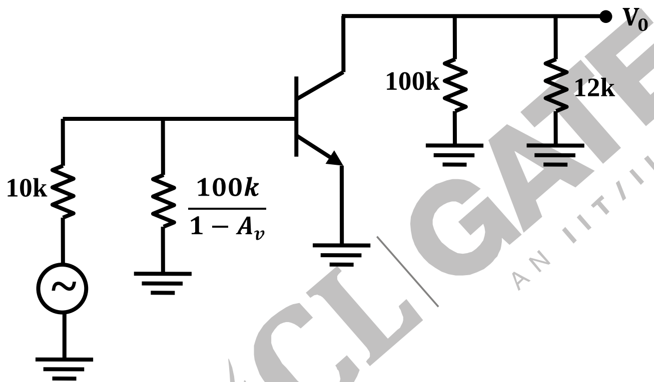
So with feedback

$\frac{V_0}{I_f} = A_{2_f} = \frac{A_2}{1+\beta\ A_2} = \frac{12 \times 105}{\beta} \simeq 10^5$

But $I_i = \frac{V_L}{10 \times 10^3} = \frac{V_L}{10^4}$

$\frac{V_0}{V_L} \simeq \frac{10^5}{10^4} \simeq 10$

Approach – 2



KVL in input loop, $13.7 - (I_C + I_B)12k - 100k\ (I_B) - 0.7 = 0$

$\Rightarrow\ I_B - 9.9\mu A;\ I_C = \beta I_B = 0.99\text{mA};\ I_E = 1\text{mA}$

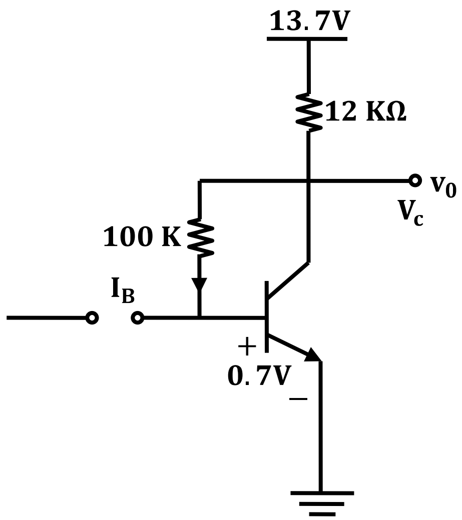
$\therefore\ r_e = \frac{26mA}{I_E} = 26\,\Omega;\ z_i = \beta\, r_e = 2.6k\,\Omega;\ \therefore A_v = \frac{(100k||12k)}{26} = 412$

$z_i' = z_i || \left(\frac{100k}{1+412}\right) = 221\,\Omega;\ A_{vs} = A_v \frac{z_i'}{z_i' + R_s} = (412)\left(\frac{221}{221+10k}\right)$

$|A_{vs}| \approx 10$

Approach – 3

This is a shunt – shunt feedback amplifier output voltage is sampled and current is feedback , , , DC circuit reduces to



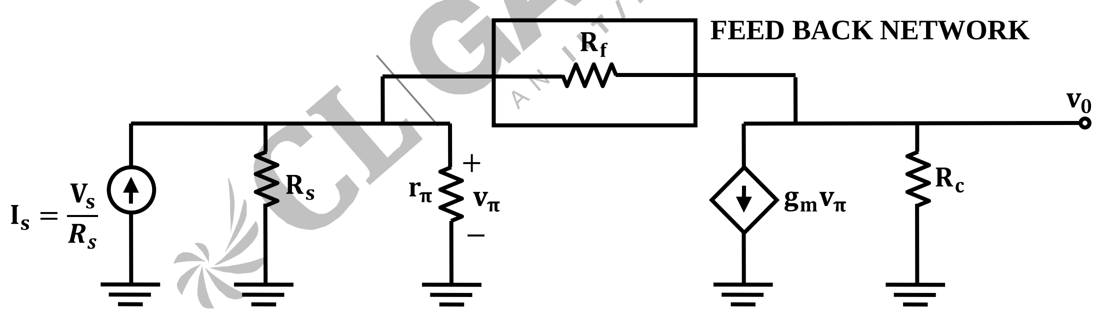
$V_C = 100\, I_B + 0.7$ ---------------- (1)

$\frac{13.7 - V_C}{12} = (\beta + 1)\, I_B$ -------------- (2)

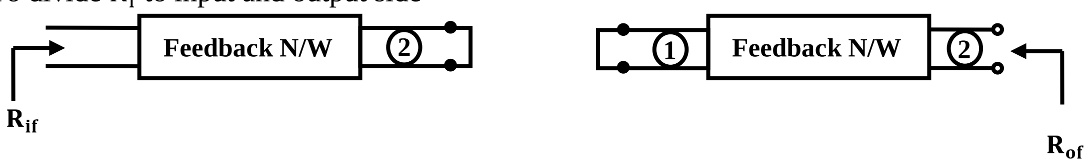
From (1) and (2)

$$\frac{13.7 - 100\, I_B - 0.7}{12} \simeq \beta I_B = 100\, I_B$$

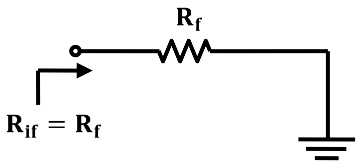
$\Rightarrow I_B = 0.01\, mA$ and $I_C = \beta\, I_B = 1$ mA

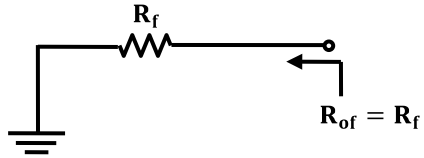


To divide R_f to input and output side

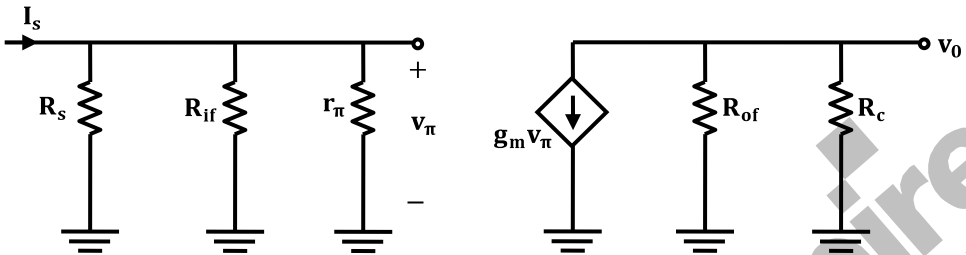


So





So, our circuit redeces to,



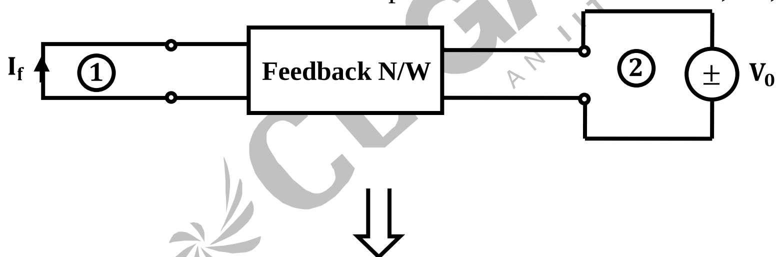
$$A = \frac{V_0}{I_S} = -\frac{g_m\left(R_f||R_c\right)v_\pi}{\dfrac{v_\pi}{\left(R_s||R_{if}||r_\pi\right)}} = -g_m\left(R_{of}||R_c\right)\left(R_s||R_{if}||r_\pi\right)$$

Now $R_{of} = R_{if} = 100k\Omega$, $R_c = 12k\Omega$, $r_\pi = \frac{V_T}{I_B} = 2.5k\Omega$, and $R_s = 10$ k

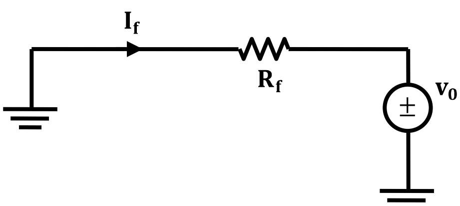
Also, $g_m = \frac{I_C}{V_T} = \frac{40\ mA}{V}$

$A = -840.336\ k\Omega$ ---------------- (1)

Note: This is a trans-resistance amplifier. Now feedback factor, for, shunt – shunt configuration is



$$\beta \equiv \left.\frac{I_f}{V_0}\right|_{V_I=0}$$



$$\frac{v_0}{I_f} = \frac{1}{\beta} \Rightarrow \beta = \frac{I_f}{v_0} = -\frac{1}{R_f}$$

So here $\beta = \frac{-1}{100k\Omega}$ ---------- (2)

So gain after feedback

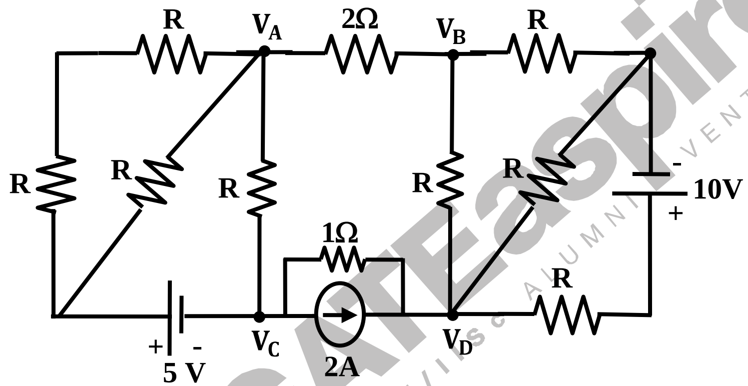
$\frac{V_0}{I_s} = \frac{A}{1+A\beta} \Rightarrow \frac{v_0}{I_s} = \frac{-840.336k}{1+8.40336} \approx 10^5$ (Approx.)

So,

$$\frac{V_0}{V_I} = \frac{V_0}{I_s\, R_s} = \frac{1}{R_s} \times \frac{v_0}{I_s} = \frac{1}{10k} \times 10^5 = 10$$

So $|A_v| \approx 10$

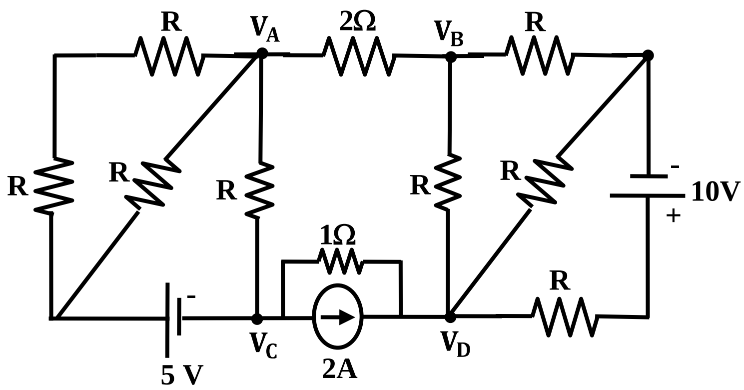
45. If $V_A - V_B = 6$ V, then $V_C - V_D$ is

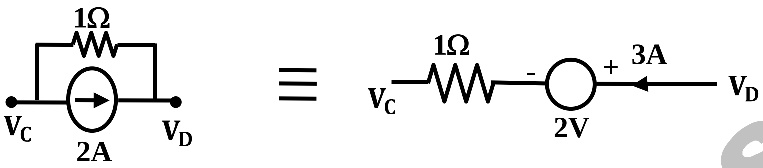


(A) −5V
(B) 2V
(C) 3V
(D) 6V

[Ans. A]

$I = \frac{V_A - V_B}{2} = \frac{6}{2} = 3A$; Since current entering any network is same as leaving in $V_C - V_D$ branch also it is I = 3A





$V_D = 2 + 3 + V_C = 5 + V_C; V_C - V_D = -5V$

46. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

(A) 21
(B) 25
(C) 41
(D) 46

[Ans. C]

$f(x) = x^3 - 9x^2 + 24x + 5$

$f(x) = x^3 - 9x^2 + 24x + 5$

$\frac{df(x)}{dx} = 3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

x = 2, x = 4 (critical points)

$\frac{d^2f(x)}{dx^2} = 6x - 18$

$= 6(2) - 18 < 0$ (for x=2)

$\frac{d^2f(x)}{dx^2} = 6(4) - 18 > 0$ (for x=4)

∴ Maximum at x = 2

$f(2) = 2^3 - 9(2)^2 + 24(2) + 5$

$= 8 - 36 + 48 + 5$

$= 25$

We have to find the maximum in the close interval [1, 6]

Hence, we have to check at end points also (as extremum exists at the critical points or end points)

f(6) = $(6)^3 - 9(6)^2 + 24(6) + 5$

= 41

∴ Maximum value = 41

47. Given that

$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is

(A) 15 A + 12 I

(B) 19 A + 30 I

(C) 17 A + 15 I

(D) 17 A + 21 I

[Ans. B]

Given: $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

We know that Every characteristic equation satisfies its own matrix

$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$

$+5\lambda + \lambda^2 + 6 = 0$

$\Rightarrow \lambda^2 + 5\lambda + 6 = 0$

We know by Cayley Hamilton Theorem that every characteristic equation satisfies its own matrix.

$\therefore A^2 + 5A + 6I = 0$

$\Rightarrow A^3 + 5A^2 + 6A = 0$

$A^3 + 5(-5A - 6I) + 6A = 0$

$\therefore A^3 = 19A + 30I$

Common Data Questions

Common Data for Question 48 and 49:

With 10 V dc connected at port A in the linear nonreciprocal two-port network shown below, the following were observed:

(i) 1 Ω connected at port B draws a current of 3 A

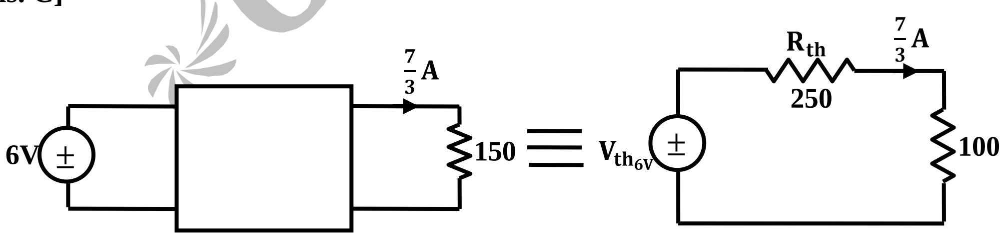
(ii) 2.5 Ω connected at port B draws a current of 2 A

48. With 10 V dc connected at port A, the current drawn by 7 Ω connected at port B is
(A) 3/7 A
(B) 5/7 A
(C) 1 A
(D) 9/7 A

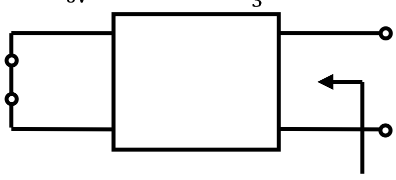
[Ans. C]
The given network can be replaced by a Thevenin equivalent with V_{th} and R_{th} as Thevenin voltage and Thevenin Resistance.
Now we can write two equations for this
$V_{th} = 3(R_{th} + 1)$
$V_{th} = 2(R_{th} + 2.5)$
Solving these two equations we get $R_{th} = 2$ and $V_{th} = 9$.
Now using the same equation with current unknown,
$9 = I \times 2 + 7 \times I \Rightarrow I = 1A$

49. For the same network, with 6 V dc connected at port A, 1 Ω connected at port B draws 7/3 A. If 8 V dc is connected to port A, the open circuit voltage at port B is
(A) 6 V
(B) 7 V
(C) 8 V
(D) 9 V

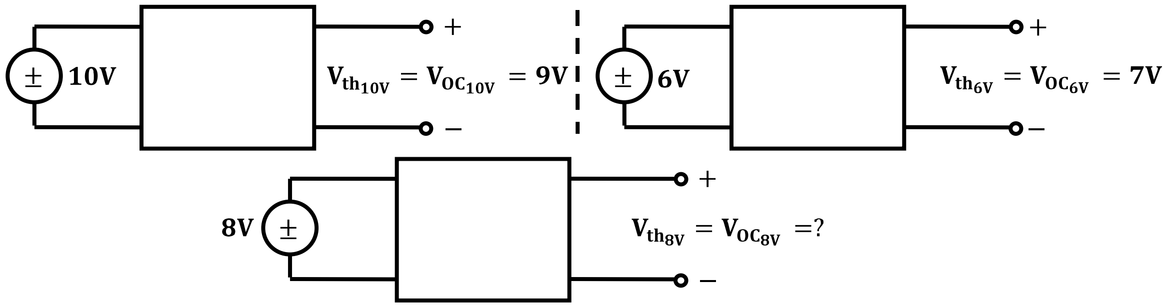
[Ans. C]



$\therefore V_{th_{6V}} = (R_{th} + 1)\frac{7}{3} = 7V$



R_{th} is same whatever the input voltage.



Since, two port network is linear non reciprocal, output (at port 2) can be expressed as a linear function of input.

i.e. $V_{OC} = aV_{in} + b$

9 = 10a + b -------------------(I)

7 = 6a + b -------------------(II)

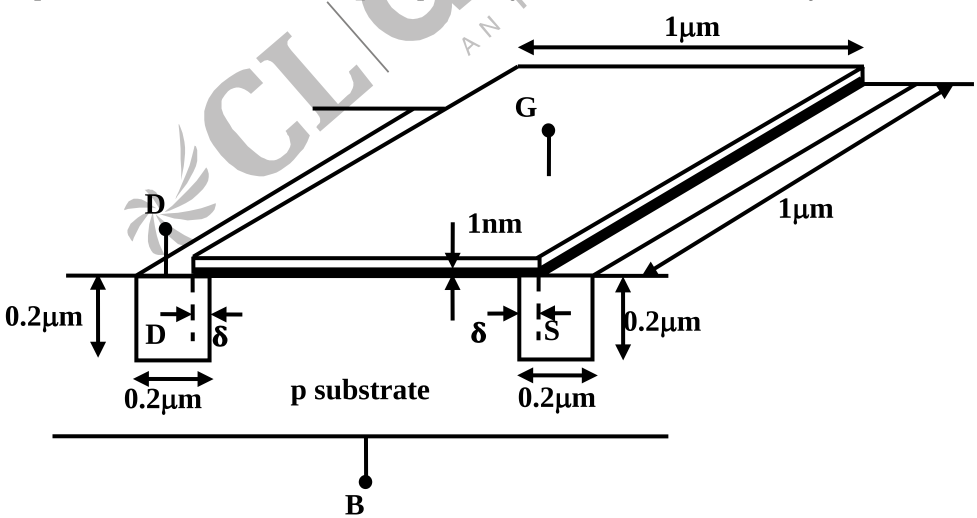
From (I) & (II),

a = 0.5, b = 4

$\therefore V_{OC_{8V}} = 0.5(8) + 4 = 8V$

Common Data for Questions 50 and 51:

In the three dimensional view of a silicon n-channel MOS transistor shown below, δ = 20 nm. The transistor is of width 1 μm. The depletion width formed at every p-n junction is 10 nm. The relative permittivities of Si and SiO_2, respectively, are 11.7 and 3.9, and $\varepsilon_0 = 8.9 \times 10^{-12}$ F/m.



50. The gate-source overlap capacitance is approximately

(A) 0.7 fF

(B) 0.7 pF

(C) 0.35 fF

(D) 0.24 pF

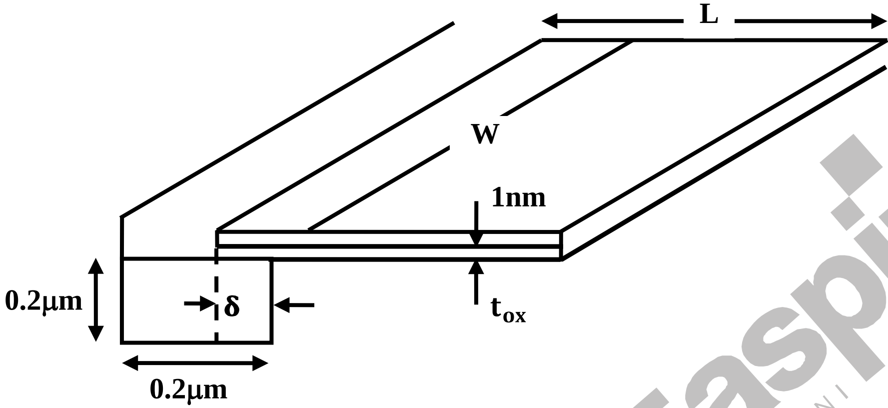
[Ans. A]

δ = 20 nm

Depletion width at each junction = 10 nm

t_{ox} = 1nm

$\in_{rsi}$ = 11.7 $\in_{r,Sio_2}$ = 3.9



Gate source overlap capacitance,

Formulae: $C = \frac{A\in_r\in_0}{d}$

Overlap area will be,

A = $\delta \times$ w and medium Si O_2

So

$$C = \frac{A \in_{ox} \in_0}{d} = \frac{\delta w \times \in_{ox} \times \in_0}{d}$$

Here d = t_{ox} i.e. oxide thickness

So

$$C_{ov} = \frac{\delta \text{ w } \in_{ox} \in_0}{t_{ox}} = \frac{20 \times 10^{-9} \times 1 \times 10^{-6} 3.9 \times 8.9 \times 10^{-12}}{1 \times 10^{-9}} F$$

$\Rightarrow C_{ov} = 0.69$ fF

Or $C_{ov} \simeq 0.7$fF

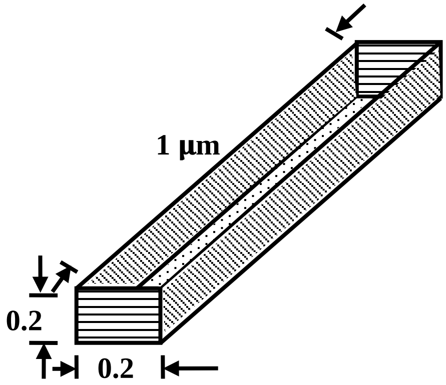
51. The source-body junction capacitance is approximately

(A) 2 fF
(B) 7 fF
(C) 2 pF
(D) 7 pF

[Ans. B]

Formulae: $C = \frac{A\in_r\in_0}{d}$

Find source body junction area



Area will be, four wall shown in red + area of base in blue.

So A = (0.2 μ m + 0.2 μ m + 0.2 μ m) $\times$1 μ m + (0.2 μ m * 0.2 μ m)

Side wall along length + base Two ends of parallel piped

So, A = 0.68×10^{-12} m^2

Now, Given, depletion width at all junctions is 10nm, and here material is silicon, so $\epsilon_r = 11.7$

So,

$$C_{SB} = \frac{0.68 \times 10^{-12} \times 11.7 \times 8.9 \times 10^{-12}}{10 \times 10^{-9}} F$$

$\Rightarrow C_{SB} = 7 \times 10^{-15}$ F or $C_{SB} = 7$fF

Linked Answer Questions

Statement for Linked Answer Question 52 and 53:

An infinitely long uniform solid wire of radius *a* carries a uniform dc current of density $\vec{J}$.

52. The magnetic field at a distance r from the center of the wire is proportional to

(A) r for r < a and l/r^2 for r > a

(B) 0 for r < a and l/r for r > a

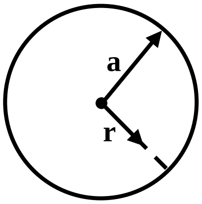
(C) r for r < a and l/r for r > a

(D) 0 for r < a and l/r^2 for r > a

[Ans. C]

$$\oint \vec{H}.d\vec{l} = I_{enclosed}$$

When we take contour for r > a,



$I_{enclosed} = J.\pi a^2 = I_o(\text{say})$

$\oint \vec{H}.d\vec{l} = H.2\pi r$

$\Rightarrow H.2\pi r = I_o$ for $r > a$

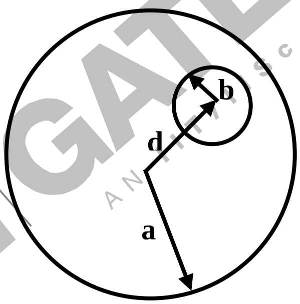
$\therefore \boxed{H = \frac{I_o}{2\pi r}}$; $r > a$

For Contour $r < a$, $I_{enclosed} = \frac{J\pi r^2}{\pi a^2} = J\left(\frac{r}{a}\right)$

Hence, $H.2\pi r = J.\frac{r^2}{a^2}$

$\Rightarrow \boxed{H \propto r}$; $r < a$

53. A hole of radius b (b < a) is now drilled along the length of the wire at a distance *d* from the center of the wire as shown below.



The magnetic field inside the hold is
(A) Uniform and depends only on *d*
(B) Uniform and depends only on *b*
(C) Uniform and depends on both *b* and *d*
(D) Non uniform

[Ans. A]

Statement for Linked Answer Questions 54 and 55:

Transfer function of a compensator is given as

$$G_c(s) = \frac{s + a}{s + b}$$

54. $G_c(s)$ is a lead compensator if

(A) $a = 1, b = 2$
(B) $a = 3, b = 2$
(C) $a = -3, b = -1$
(D) $a = 3, b = 1$

[Ans. A]

$\phi = tan^{-1}\frac{\omega}{a} - tan^{-1}\frac{\omega}{\beta}$

For phase lead ϕ should be positive

$\Rightarrow tan^{-1}\frac{\omega}{a} > tan^{-1}\frac{\omega}{\beta}$

$\Rightarrow a < b$

Both option (A) and (C) satisfies,

It may be observed that option (C) will have poles and zero in RHS of s – plane, thus not possible (not a practical system).

Therefore, it can be concluded that Option (A) is right.

55. The phase of the above lead compensator is maximum at

(A) $\sqrt{2}$ rad/s
(B) $\sqrt{3}$ rad/s
(C) $\sqrt{6}$ rad/s
(D) $1/\sqrt{3}$ rad/s

[Ans. A]

ω = geometric mass of two carrier frequencies

$= \sqrt{2 \times 1} = \sqrt{2}\, rad/sec$

General Aptitude (GA) Questions (Compulsory)

Q. 56 – Q. 60 carry one mark each.

56. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(A) 2.23
(B) 4.33
(C) 11.37
(D) 27.64

[Ans. D]

$(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{1259} \times (1.001)^{2062} = 3.52 \times 7.85$ or $(1.001)^{(1259 + 2062)} = 3.52 \times 7.85$ or $(1.001)^{3321} = 27.64$

57. Choose the most appropriate alternative from the options given below to complete the following sentence:

If the tired soldier wanted to lie down, he _______ the mattress out on the balcony.

(A) should take
(B) shall take
(C) should have taken
(D) will have taken

[Ans. A]

58. Choose the most appropriate word from the options given below to complete the following sentence:

Given the seriousness of the situation that he had to face, his _____ was impressive

(A) beggary
(B) nomenclature
(C) jealousy
(D) nonchalance

[Ans. D]

Nonchalance means behaving in a calm and relaxed way; giving the impression that you are not feeling any anxiety.

59. Which one of the following options is the closest in meaning to the word given below?

Latitude

(A) Eligibility
(B) Freedom
(C) Coercion
(D) Meticulousness

[Ans. B]

Latitude means freedom to choose what you do or the way that you do it.

60. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?

I requested that he should be given the driving test today instead of tomorrow.

(A) requested that
(B) should be given
(C) the driving test
(D) instead of tomorrow

[Ans. B]

The correct statement should be -" i requested that he be given the driving test today instead of tomorrow."

Q. 61 – Q. 65 carry two marks each.

61. **One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**

Which one of the following statements best sums up the meaning of the above passage?

(A) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.

(B) The legions were treated inhumanly as if the men were animals.

(C) Discipline was the armies' inheritance from their seniors.

(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

[Ans. A]

The passage states that the strict discipline kept the armies intact even when condition were against them.

62. Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money value of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

(A) 5
(B) 6
(C) 9
(D) 10

[Ans. A]

Let the number of Rs. 10 notes = x

And the number of Rs. 20 notes = y

Now,

$x + y = 14$ and $10x + 20y = 230$.

Solving them, we get $x = 5$ and $y = 9$.

63. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using the balance, the minimum number of weighing required to identify the heavier bag is

(A) 2
(B) 3
(C) 4
(D) 8

[Ans. A]

64. The data given in the following table summarizes the monthly budget of an average household.

Category	Amount (Rs.)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget NOT spent on savings is

(A) 10%
(B) 14%
(C) 81%
(D) 86%

[Ans. D]

Total Income = 10500

Savings =1500

Percentage of budget spent on savings = $\frac{1500}{10500} \times 100 = 14.28\%$

Percentage of budget not spent on savings = 86%

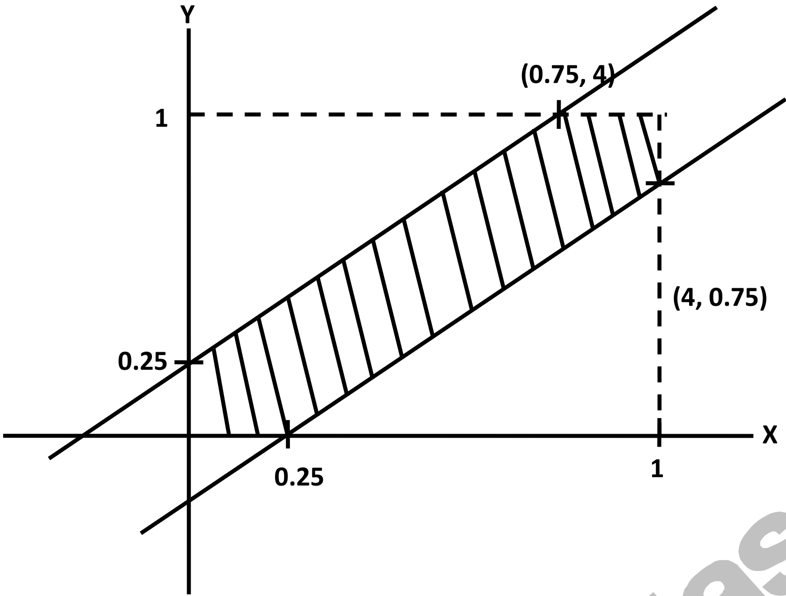
65. A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(A) 1/4
(B) 1/16
(C) 7/16
(D) 9/16

[Ans. C]

We can solve this question graphically.

Let x axis represents the time when A reaches the meeting place and y axis represents the time when B reaches the same place.



Given conditions are

Total area is represented by

$0 \le x \le 1$ and $0 \le y \le 1$.

Total area = 1 × 1 = 1 unit

Desired area is represented by

$0 \le x \le 1,\ 0 \le y \le 1,\ x - y \le 1/4$ and $y - x \le 1/4$

$$\text{Desired area} = 1 - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right) - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right) = \frac{7}{16}$$

$$\text{Required probability} = \frac{desired\ area}{total\ area} = \frac{7}{16\times 1} = \frac{7}{16}$$