# Test Code: PQB (Short Answer Type) 2013

## M.Tech. in Quality, Reliability and Operations Research

The candidates applying for M. Tech. in Quality, Reliability and Operations Research will have to take two tests: **Test MMA** (objective type) in the forenoon session and **Test PQB** (short answer type) in the afternoon session.

For Test **MMA**, see a different Booklet. For Test **PQB**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**

If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups**: **S1**: **Statistics and S2: Probability – each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, there will be **SIX Groups: E1 to E6**. **E1** will contain **THREE [3]** questions from **Mathematics** and each other group will contain **TWO [2]** questions from **Engineering and Technology**. You will be required to answer a total of **SIX [6]** questions taking **AT LEAST TWO [2]** from group **E1**.

*Syllabus*

## PART I: STATISTICS / MATHEMATICS STREAM

### Statistics (S1)
- Descriptive statistics for univariate, bivariate and multivariate data.
- Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.
- Theory of estimation and tests of statistical hypotheses.
- Multiple linear regression and linear statistical models, ANOVA.
- Principles of experimental designs and basic designs [CRD, RBD & LSD].
- Elements of non-parametric inference.
- Elements of sequential tests.
- Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

### Probability (S2)
- Classical definition of probability and standard results on operations with events, conditional probability and independence.
- Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.
- Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.
- Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.
- Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].
- Distributions of functions of random variables.
- Multivariate normal distribution [density, marginal and conditional distributions, regression].
- Weak law of large numbers, central limit theorem.
- Basics of Markov chains and Poisson processes.

## PART II: ENGINEERING STREAM

### Mathematics (E1)
- Elementary theory of equations, inequalities.
- Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.
- Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].
- Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.
- Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (upto second order), complex numbers and De Moivre's theorem.

### Engineering Mechanics (E2)
- Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.

### Electrical and Electronics Engineering (E3)
- DC circuits, AC circuits (1-$\phi$), energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.
- Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

### Thermodynamics (E4)
- Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines. Principles of refrigeration.

**Engineering Properties of Metals (E5)**

- Structures of metals, tensile and torsional properties, hardness, impact properties, fatigue, creep, different mechanism of deformation.

**Engineering Drawing (E6)**

- Concept of projection, point projection, line projection, plan, elevation, sectional view ($1^{st}$ angle / $3^{rd}$ angle) of simple mechanical objects, isometric view, dimensioning, sketch of machine parts.
  (Use of set square, compass and diagonal scale should suffice).

## PART I: STATISTICS / MATHEMATICS STREAM

### GROUP S1: Statistics

1. Let $X_1$ and $X_2$ be independent $\chi^2$ variables, each with $n$ degrees of freedom. Show that $\dfrac{\sqrt{n}\left(X_1 - X_2\right)}{2\sqrt{X_1 X_2}}$ has the $t$ distribution with $n$ degrees of freedom and is independent of $X_1 + X_2$.

2. Let $[\{x_i \; ; \; i = 1, 2, ..., p\}; \{y_j \; ; \; j = 1,2,..., q\}; \{z_k \; ; \; k = 1, 2, ...,r\}]$ represent random samples from $N(\alpha + \beta, \sigma^2)$, $N(\beta + \gamma, \sigma^2)$ and $N(\gamma + \alpha, \sigma^2)$ populations respectively. The populations are to be treated as independent.

   (a) Display the set of complete sufficient statistics for the parameters ($\alpha$, $\beta$, $\gamma$, $\sigma^2$).
   (b) Find unbiased estimator for $\beta$ based on the sample means only. Is it unique?
   (c) Show that the estimator in (b) is uncorrelated with all error functions.
   (d) Suggest an unbiased estimator for $\sigma^2$ with maximum d.f.
   (e) Suggest a test for $H_0$: $\beta = \beta_0$.

3.a) A neighborhood expresses frequent concerns about the dangers of a traffic intersection. Over the last two years there have been 16 accidents at this uncontrolled intersection. The municipality has finally responded and has put up stop signs on each of the four roads that enter the problematic intersection. After one year it was noted that there has been three accidents. From this data would you conclude that the stop signs have really helped in reducing the rate of accidents? Explain stating your assumptions clearly.

   b) In a software development organization the project value ($X$) and productivity ($Y$) are known to be related. You have heard that the joint density of these two variables could probably be given by:

$$f(x, y) = \begin{cases} x \, exp(\, -x \, (1 + y)\,); & x, y > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

   Note that the value of the project can be well approximated before execution. The organization wants to develop a regression equation of $Y$ on $X$ to get an idea about the productivity on the basis of the value of a project. The organization wanted to develop the equation empirically and accordingly paired observations on ($x$, $y$) were collected for a number of completed projects. Do you think that

the values of $X$ and / or $Y$ needs to be transformed before fitting the equation? If yes, what is the transformation? Explain.

4. A town has $N$ taxis numbered 1 through $N$. A person standing on roadside notices the taxi numbers on $n$ taxis that pass by. Let $M_n$ be the largest number observed. Assuming independence of the taxi numbers and sampling with replacement, show that

$$\hat{N} = (n + 1) M_n / n$$

is an approximately unbiased estimator of $N$ for large $N$.

5.a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} \dfrac{1}{\theta}, & 0 < x < \theta \\ 0, & otherwise \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \ldots, x_n$. What is the associated probability of type I error and what is the power function?

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population having p.d.f.

$$f(x, \theta) = \begin{cases} \dfrac{\theta^3}{\Gamma(3)} e^{-\theta x} x^2, & 0 < x < \infty \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimate of $\theta$ and also obtain the Cramer Rao lower bound to the variance of an unbiased estimator of $\theta$.

6.(a) Give an example of a Latin Square Design of order 4 involving 4 rows, 4 columns and 4 treatments. Give the general definition of "treatment connectedness" in the context of a Latin Square Design and show that the Latin Square Design that you have given is indeed treatment connected.

(b) In a CRD set-up involving 5 treatments, the following computations were Made:
$n = 105$, Grand Mean $= 23.5$, SSB $= 280.00$, SSW $= 3055.00$

    (i) Compute the value of the F-ratio and examine the validity of the null hypothesis.

    (ii) It was subsequently pointed out that there was one additional treatment that was somehow missed out! For this treatment, we are given sample size $= 20$, Sum $= 500$ and Sum of Squares (corrected) $=$

560.00. Compute revised value of F-ratio and draw your conclusions.

7. If $X_1$, $X_2$, $X_3$ constitute a random sample from a Bernoulli population with mean $p$, show why $[X_1 + 2X2 + 3X_3] / 6$ is *not* a sufficient statistic for $p$.

8. If $X$ and $Y$ follow a trinomial distribution with parameters $n$, $\theta_1$ and $\theta_2$, show that

(a) $E(Y / X = x) = \dfrac{(n-x)\theta_2}{1-\theta_1}$,

(b) $V(Y / X = x) = \dfrac{(n-x)\theta_2(1-\theta_1-\theta_2)}{(1-\theta_1)^2}$

9. Life distributions of two independent components of a machine are known to be exponential with means $\mu$ and $\lambda$ respectively. The machine fails if at least one of the components fails. Compute the chance that the machine will fail due to the second component. Out of $n$ independent prototypes of the machine $m$ of them fail due to the second component. Show that $m / (n - m)$ approximately estimates the odds ratio $\theta = \lambda/\mu$.

## GROUP S2: Probability

1. A boy goes to his school either by bus or on foot. If one day he goes to the school by bus, then the probability that he goes by bus the next day is 7/10. If one day he walks to the school, then the probability that he goes by bus the next day is 2/5.

   (a) Given that he walks to the school on a particular Tuesday, find the probability that he will go to the school by bus on Thursday of that week.

   (b) Given that the boy walks to the school on both Tuesday and Thursday of that week, find the probability that he will also walk to the school on Wednesday.

   [You may assume that the boy will not be absent from the school on Wednesday or Thursday of that week.]

2. Suppose a young man is waiting for a young lady who is late. To amuse himself while waiting, he decides to take a random walk under the following set of rules:

He tosses an imperfect coin for which the probability of getting a head is 0.55. For every head turned up, he walks 10 meters to the north and for every tail turned up, he walks 10 meters to the south.

That way he has walked 100 meters.

(a) What is the probability that he will be back to his starting position?

(b) What is the probability that he will be 20 meters away from his starting position?

3. (a) A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

(b) For successful operation of a machine, we need at least three components (out of five) to be in working phase. Their respective chances of failure are 7%, 4%, 2%, 8% and 12%. To start with, all the components are in working phase and the operation is initiated. Later it is observed that the machine has stopped but the first component is found to be in working phase. What is the likelihood that the second component is also in working phase?

(c) A lot contains 20 items in which there are 2 or 3 defective items with probabilities 0.4 and 0.6 respectively. Items are tested one by one from the lot unless all the defective items are tested. What is the probability that the testing procedure will continue up to the twelfth attempt?

4. (a) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let $U = \min\{S, T\}$ and $V = \max\{S, T\}$. Find $E(U)$ and $E(U+V)$.

(b) Let $X$ be a random variable with $U(0,1)$ distribution. Find the p.d.f. of the random variable $Y = X / (1 + X)$.

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5. (a) Let $U$ and $V$ be independent and uniformly distributed random variables on [0, 1] and let $\theta_1$ and $\theta_2$ (both greater than 0) be constants.

Define $X = -\dfrac{1}{\theta_1}\ln U$ and $Y = -\dfrac{1}{\theta_2}\ln V$. Let $S = \min\{X,\ Y\}$, $T = max\{X,Y\}$ and $R = T - S$.

     (i)   Find $P[S=X]$.

     (ii)  Show that $S$ and $R$ are independent.

  (b) A sequence of random variables $\{X_n \mid n = 1,\ 2,\ ...\}$ is called a *martingale* if

     (i)     $E(|X_n|) < \infty$

     (ii)    $E(X_{n+1} \mid X_1, X_2,\ ...,\ X_n) = X_n$ for all $n = 1,\ 2,\ ...$

Let $\{Z_n \mid n = 1,\ 2,\ ...\}$ be a sequence of iid random variables with $P[Z_n = 1] = p$ and $P[Z_n = -1] = q = 1-p$, $0 < p < 1$. Let $X_n = Z_1 + Z_2 + ... + Z_n$ for $n = 1,\ 2,\ ....$ Show that $\{X_n \mid n = 1,\ 2,\ ...\}$, is a martingale if and only if $p = q = \frac{1}{2}$.

6. A manufacturer sells a bottle of mineral water at a fixed price of Rs.10. If the volume of water in the bottle is less than 800 ml then he is unable to sell it and it represents a total loss. The filled bottles have a normally distributed volume with mean $\mu$ ml and standard deviation 100 ml. The cost of filling per bottle is Rs. $c$, where $c = 0.002\ \mu + 1$. Determine the mean volume $\mu$ which will maximize the expected profit of the manufacturer. [Use $\sqrt{-\ln(0.0008\pi)} = 2.447.$]

7. (a) In a recent study, a set of $n$ randomly selected items is tested for presence of colour defect. Let $A$ denote the event "colour defect is present" and $B$ denote the event "test reveals the presence of colour defect". Suppose $P(A) = \alpha$, $P(B|A) = 1-\beta$ and $P(Not\ B | Not\ A) = 1-\delta$, where $0 < \alpha,\ \beta,\ \delta < 1$. Let $X$ be the number of items in the set with colour defects and $Y$ be the number of items in the set detected having colour defects.

     (i) Find $E(X \mid Y)$.

     (ii) If the colour defect is very rare and the test is a very sophisticated one such that $\alpha = \beta = \delta = 10^{-9}$, then find the probability that an item detected as having colour defect is actually free from it.

  (b) Consider the following bivariate density function

$$f(x,y) = \begin{cases} c.xy, & x > 0, y > 0, x+y < 1 \\ 0, & otherwise \end{cases}$$

     (i) Find $c$.

     (ii) Find the conditional expectation, $E(Y \mid X = x)$, for $0 < x < 1$.

8.    Suppose in a big hotel there are $N$ rooms with single occupancy and also suppose that there are $N$ boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \to \infty$?

9. (a) Consider a Markov Chain with state space $I = \{1, 2, 3, 4, 5, 6\}$ and transition probability matrix $P$ given by

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1/8 & 7/8 & 0 & 0 \\ 1/4 & 0 & 1/2 & 0 & 1/8 & 1/8 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \end{bmatrix}$$

Find the various classes of this chain and classify them as recurrent or transient.

(b) Pulses arrive at a Geiger counter according to a Poisson Process with parameter $\lambda > 0$. The counter is held open only a random length of time $T$ (independent of the arrival time of the pulses), where $T$ is exponentially distributed with parameter $\beta > 0$. Find the distribution of $N$ = Total number of pulses registered by the counter.

## PART II: ENGINEERING STREAM

### GROUP E1: Mathematics

1. (a) Let $f(x)$ be a polynomial in $x$ and let $a, b$ be two real numbers where $a \neq b$. Show that if $f(x)$ is divided by $(x - a)(x - b)$ then the remainder is

$$\frac{(x - a)f(b) - (x - b)f(a)}{b - a}.$$

(b) Find $\frac{dy}{dx}$ if $x^{\cos y} + y^{\sin x} = 1$.

2.(a) Let A be the fixed point (0,4) and B be a moving point (2*t*, 0). Let M be the mid-point of AB and let the perpendicular bisector of AB meets the y-axis at R. Find the equation of the locus of the mid-point P of MR.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm. The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively. Find the ratio PM : MQ.

3.(a) Evaluate the value of $3.9^{\frac{1}{2}}.27^{\frac{1}{4}}.81^{\frac{1}{8}}......\infty$.

(b) Let *f* be a twice differentiable function such that

$$f''(x) = -f(x); f'(x) = g(x) \text{ and } h(x) = f^2(x) + g^2(x).$$

Given that *h(5) = 11*, find *h(10)*.

4.(a) Show that

$$\underset{n\to\infty}{Lim}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots (upto\ [n/2]\ terms)\right]=\frac{1}{2}.$$

(b) Test the convergence of the series $x+\dfrac{2^2x^2}{2!}+\dfrac{3^3x^3}{3!}+\dfrac{4^4x^4}{4!}+\cdots\infty$.

Assume *x* > 0 and examine all possibilities.

5.(a) Find the limit of the following function as $x\to 0$.

$$\frac{|x|}{\sqrt{(x^4+4x^2+7)}}\sin\left(\frac{1}{3\sqrt{x}}\right).$$

(b) If the line *ax* + *by* + *c* =0 is a normal to the curve *xy* =1 then show that $a \cdot b < 0$.

6.(a) If ω is a complex cube root of unity then show that

$$a^3+b^3+c^3-3abc=\left(a+b+c\right)\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right).$$

(b) Show that $\left[\dfrac{\sum_{r>s}x^r}{r!}\right]\div\left[\dfrac{\sum_{r>s}y^r}{r!}\right]>\dfrac{x^s}{y^s}$, whenever *x* > *y* > 0.

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7.(a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them. Show that

(i) $\sin\left(\dfrac{C}{2}\right) \le \dfrac{1}{2}$

(ii) Hence, or otherwise, show that $\sin\left(\dfrac{A}{2}\right)\sin\left(\dfrac{B}{2}\right)\sin\left(\dfrac{C}{2}\right) < \dfrac{1}{4}$.

8.(a) Evaluate the following two integrals directly and compare them.

$$\iint\limits_{ax^2+by^2\le1} dxdy \qquad \text{and} \qquad \iint\limits_{\sqrt{a}|x|\le1,\sqrt{b}|y|\le1} dxdy.$$

(b) Determine $x$, $y$ and $z$ so that the $3 \times 3$ matrix with the following row vectors is orthogonal : $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $(1/\sqrt{2}, -1/\sqrt{2}, 0)$, $(x, y, z)$.

## GROUP E2: Engineering Mechanics

1.(a) A screw jack has a thread of 12 mm pitch. What effort needs to be applied the end of a handle of 450 mm to lift a load of 2.5 kN, if the corresponding efficiency is 50%?

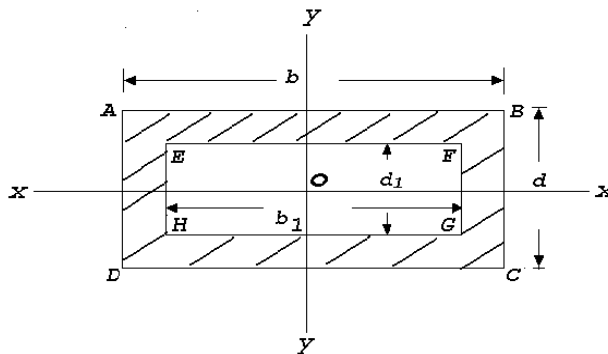(b) Derive the expression for moment of inertia $I_{YY}$ of the shaded hollow rectangular section (Fig. 1).



Fig. 1

2.(a) A turbine rotor weighs 20 tonnes and has a radius of gyration of 1.75 meter when running at 200 rpm. It is suddenly relieved of part of its load and its speed rises to 205 rpm in 1 sec. Find the unbalanced uniform turning moment.

(b) An Aluminium thin-walled tube (radius/thickness = 20) is closed at each end and pressurized by 6 MPa to cause plastic deformation. Neglect the elastic strain and find the plastic strain in the circumferential (hoop) direction of the tube. The plastic stress-strain curve is given by $\overline{\sigma} = 170 (strain\ rate)^{0.25}$.

3.(a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle $50^0$ to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm$^2$. The allowable angle of twist = $0.5^0$ per m, G = $8 \times 10^5$ kg/cm$^2$. What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

4.(a) For the beam and loading shown in Fig.2, determine the equation defining the shear and bending moment at any point and at point D.
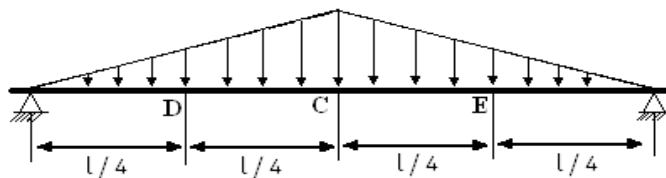


Fig. 2

(b) As illustrated in the given Fig.3 a metal punch (similar in principle to a paper punch) is used to punch holes in thin steel sheet that will be used to make a metal cabinet. To punch a 60 mm diameter disk or "slug" out of the sheet metal that is 10 mm thick requires a punch force of P=500 kg. Determine the average shear stress in the sheet metal resulting from the punching operation.
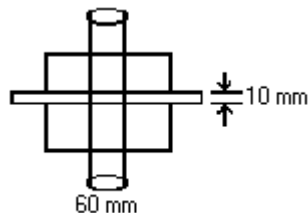


Fig. 3

5. (a) A tie rod in the suspension of a car is to be constructed from a grade of steel, which has 0.1% proof stress equal to 250 MN/m$^2$. The tie rod is to be constructed as a solid round bar of length 350 mm long. If the tie rod is subjected to a maximum axial force of 10 KN,

   (i) Determine the minimum diameter of the tie rod
   (ii) The extension of the tie rod under load (E= 2094 GN/m$^2$)
   (iii) The minimum diameter of the tie rod if a factor of safety of 2.5 is applied to the proof stress

   (b) Find the width of the belt necessary to transmit 11.25 KW power to a pulley of diameter 300mm when the pulley makes 1600 rpm. Assume the co-efficient of friction between the belt and the pulley is 0.22 and angle of contact is $210^0$. Maximum tension in the belt will not exceed 10N/mm width.


## GROUP E3: Electrical and Electronics Engineering

1.(a) A centrifugal pump, which is gear-driven by a DC motor, delivers 810 kg of water per minute to a tank of height 11 meter above the level of the pump. Draw the block diagram of the overall arrangement. Determine input power across the gearing and current taken by the motor operated at 220 volt provided the efficiency of the pump, gearing and motor respectively be 70%, 70% and 90% only. (Take g = 9.8 ms$^{-2}$).

  (b) The rms value of a sinusoidal alternating voltage at a frequency of 50 Hz is 155volt. If at t = 0 it crosses the zero axis in a positive direction, determine the time taken to attain the first instantaneous value of 155 volt. How much time it takes to fall from the maximum peak value to its half? Explain with suitable waveform.

2.(a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter have any inaccuracy? If so, find the percentage error.

  (b) Write down the transfer function of the given system (as shown in the following figure-4) and find the values of K for which the system will be stable but under damped.
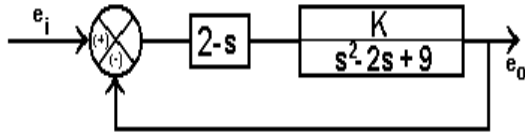
Fig - 4

3. (a) By intelligent selection of loop currents write down the mesh equations of the given circuit (as shown in Fig. 5) and determine the current flowing through that branch of the circuit containing capacitor. (All resistances/ reactance's are in ohms).
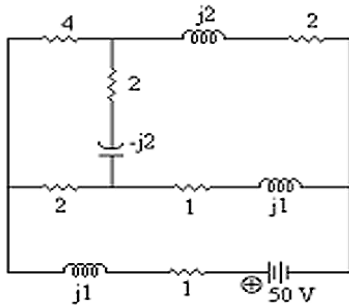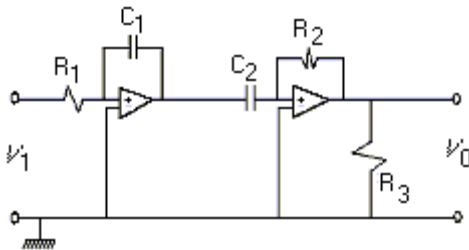


Fig. 5

(b)



Fig. 6

Refer Fig. 6. Find the expression for $V_0$. What would be the nature of $V_0$ when $R_1 = R_2$ and $C_1 = C_2$? (Consider the Op-amps to be identical).

4. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

   (b) Calculate the value of a capacitor which when connected across the circuit (as of Q. 4 (a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

5. (a) A 200/400 - V, 10KVA, 50Hz single phase transformer has, at full load, a Cu loss of 120W. If it has an efficiency of 98% at full load unity power factor,

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determine the iron losses. What would be the efficiency of the transformer at half load 0.8 power factor lagging?

(b) In the 2-port network given below, the parameters at two parts are related by the equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

   (i)  Find expressions for A, B, C and D
   (ii)  Show that AD - BC = 1
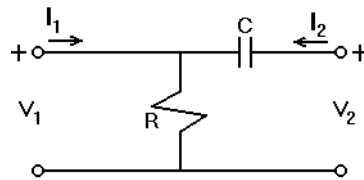   (iii) What are the physical interpretations of the above coefficients?



Fig 7

## GROUP E4: Thermodynamics

1.(a) In a thermodynamic system of a perfect gas, let $U = f(V,T)$ where $U$, $V$ and $T$ refer to internal energy, volume of a gram-molecule of the substance and temperature (in absolute scale) respectively. An amount of heat $\delta Q$ is added so that the volume expands by $\delta V$ against a pressure $P$. Prove that:

$$C_P - C_V = \left[ P + \left( \frac{\delta U}{\delta V} \right)_T \right] \left( \frac{\delta V}{\delta T} \right)_P$$

where $C_p$ and $C_V$ stand for specific heat at constant pressure and specific heat at constant volume respectively.

(b) 0.15 cu.m. of air at a pressure of 1.06 kg/cm$^2$ is compressed to a volume of 0.008 cu.m. at 361 kg/cm$^2$. Calculate (i) the quantity of heat rejected, (ii) change in internal energy if the process of compression is a) Adiabatic b) Polytropic with $n = 1.3$.

2.(a) A compression ignition engine has a stroke of 28 cm and a cylinder diameter of 18 cm. The clearance volume is 475 cm$^3$. The fuel injection takes place at constant pressure for 4.5% of the stroke. Find the air standard efficiency of

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the engine assuming that it works on diesel cycle. If the fuel injection takes place at 10% of the stroke, find the loss in air standard efficiency.

(b) A diesel engine has a compression ratio 14 to 1 and the fuel supply is cut off at 0.08 of the stroke. If the relative efficiency is 0.52, estimate the weight of fuel of a calorific value 10400 k.cal per kg that would be required per horsepower.

3.(a) Calculate the change in entropy of saturated steam at a given pressure such that the boiling point = 152.6 $^\circ$C and the latent heat at this temperature = 503.6 cal/gm. [Use $\log_e 1.56 = 0.445$.]

(b) Draw the pv and $T - \Phi$ diagrams for a diesel cycle in which 1 kg of air at 1 kg / cm$^2$ and 90 $^0$C is compressed through a ratio of 14 to 1. Heat is then added until the volume is 1.7 times the volume at the end of compression, after which the air expands adiabatically to its original volume. Take $C_v = 0.169$ and $\gamma = 1.41$.

4.(a) The approximated equation for adiabatic flow of super heated steam through a nozzle is given by $pv^n = \text{constant}.$ Show that

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/n+1}$$

where $p_1 = $ pressure of steam at entry ; $p_2 = $ pressure of steam at throat and $p_2 / p_1$ is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under cooling.

5. A mass of $m_1$ kg of a certain perfect gas at a temperature $T_1$ $^0$K is mixed at constant pressure with m$_2$ kg of mass of the same gas at a temperature $T_2$ $^0$K ($T_1 > T_2$). The system is thermally insulated. Find the change in entropy of the mixture and deduce the same for equal masses of the gas. Show that the change in entropy for equal masses of the gas is necessarily positive.

# GROUP E5: Engineering Properties of Metals

1. (a) Distinguish between modulus of rigidity and modulus of rupture. Give an expression for the modulus of rigidity in terms of the specimen geometry, torque, and angle of twist. Is the expression valid beyond the yield strength (torsion)?

   (b) A steel bar is subjected to a fluctuating axial load that varies from a maximum of 340 kN to a minimum of 120 kN compression. The mechanical properties of the steel are $\sigma_u$ = 1090 MPa, $\sigma_0$ = 1010 MPa and $\sigma_e$ = 510 MPa. Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5.

2 (a) A cylindrical bar is subjected to a torsional moment $M_T$ at one end. The twisting moment is resisted by shear stress μ set up in the cross section of the bar. The shear stress is zero at the centre of the bar and increases linearly with the radius. Find the maximum shear stress at the surface of the bar.

   Given $J = \dfrac{\pi D^4}{32}$ (assuming that the torsional deformation is restricted within the zone of elasticity)

   where, $J$ : Polar moment of inertia
   $D$ : Diameter of cylinder.

   (b) Consider a flat plane containing a crack of elliptical cross-section. The length of the crack is $2c$ and stress is perpendicular to the major axis of the ellipse. Show that

   $$\sigma = \sqrt{\frac{2\gamma E}{\pi c}}$$

   $\sigma$ : stress
   $\gamma$ : surface energy
   $E$ : Young's modulus of elasticity

3. (a) Consider a tension specimen, which is subjected to a total strain $\varepsilon$ at an elevated temperature where creep can occur. The total strain remains constant and the elastic strain decreases. Show that

$$\frac{1}{\sigma^{n-1}} = \frac{1}{\sigma_o^{n-1}} + BE(n-1)t$$

where,

$$\varepsilon = \varepsilon_e + \varepsilon_p \qquad\qquad \varepsilon_e : \text{elastic strain}$$
$$\varepsilon_e = \sigma/E \qquad\qquad\quad \varepsilon_p : \text{plastic strain}$$
$$\frac{d\varepsilon_p}{dt} = B\sigma^n \qquad\qquad\quad t : \text{time}$$
$$\sigma = \sigma_o \ \text{at} \ t = 0.$$

(b) Distinguish between slip and twinning with diagrams.

4. (a) Suppose a crystalline material has *fcc* structure with atomic radius of 1.278 Å Determine the density of the crystalline material. Assume number of atoms per unit cell and molecular weight are *n* and *M* gm respectively.

(b) Suppose there is an electron in an electric field of intensity 3200 volts/m. Estimate the force experienced by the electron. If it moves through a potential difference of 100 volts, find the kinetic energy acquired by the electron.

### GROUP E6: Engineering Drawing

1.(a) A hollow cube of 5cm side is lying on HP and one of its vertical face is touching VP. A slim rod, to be taken as its solid diagonal, is placed within it. Draw top and front / side views of solid diagonal and, from the drawn figure determine its true length.

(b) Two balls are vertically erected to 18 cm and 30 cm respectively above the flat ground. These balls are away from a 3 cm thick wall (on the ground) by 12 cm and 21 cm respectively but on either side of the wall. The distance between the balls, measured along the ground and parallel to the wall is 27 cm. Determine their approximate distance.

2. (a) Sketch the profile of a square thread, knuckle thread and a white-worth thread showing all relevant dimensions in terms of the pitch.

   (b) Sketch:

       (i) single riveted lap joint,
       (ii) double riveted lap joint chain-riveting,
       (iii) double riveted lap joint zigzag-riveting, and
       (iv) single cover single riveted butt joint.

3. (a) Draw the isometric view of an octahedron erected vertically up on one of its vertices. (Distinct free hand sketch only.)

   (b) You are given two square prisms of same height of 10 cm. Prism A has side 7 cm and prism B has side of 5 cm respectively. Longer face of B is lying on H.P. with its base perpendicular to V.P. Base of A is lying on H.P. but equally inclined to V.P. You are instructed to remove by cutting a portion of bottom base of A so that within the cavity maximum of B may be placed accordingly. Note that vertical face of B may be parallel to V.P. but just touch the central axis of A. Draw the sectional view of the combination and determine the volume of material to be removed from A.

4. A parallelepiped of dimension 100×60×80 is truncated by a plane which passes through 85, 45 and 65 unit distance on the associated edges from the nearest top point of the object. Draw the isometric view of the truncated solid object. In third angle projection method, draw its plan. (All dimensions are in mm).

**Note**: A copy of one of the previous year's TEST CODE: QR Question paper is appended in the following pages to give the candidate a rough idea.

BOOKLET No.            TEST CODE: PQB

*__Afternoon__*

**Time: 2 hours**

| Group | Questions | | Maximum marks |
|---|---|---|---|
| | Total | To be answered | |
| ***Part I (for Statistics/Mathematics Stream)*** | | | |
| S1 (Statistics) | 5 | A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP. | 120 |
| S2 (Probability) | 5 | | |
| ***Part II (for Engineering Stream)*** | | | |
| E1 (Mathematics) | 3 | A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1 | 120 |
| E2 (Engineering Mechanics) | 2 | | |
| E3 (Electrical and Electronics Engineering) | 2 | | |
| E4 (Thermodynamics) | 2 | | |
| E5 (Engineering Properties of Metals) | 2 | | |
| E6 (Engineering Drawing) | 2 | | |

*On the answer-booklet write your Name, Registration Number, Test Code, Number of this Booklet, etc. in the appropriate places.*

There are two parts in this booklet as detailed above. Candidates having **statistics background** are required to answer questions from **Part I** only as per instructions given. Those having **engineering background** are required to answer questions from **Part II** only as per instructions given.

**USE OF CALCULATORS IS NOT ALLOWED.**

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**STOP! WAIT FOR THE SIGNAL TO START**

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**PART I (FOR STATISTICS / MATHEMATICS STREAM)**

**ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP.** *(Note: Partial credit may be given for partially correct answer)*

## GROUP S1
### Statistics

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from a Poisson distribution with parameter $\lambda$. Define $T_1 = \dfrac{1}{n}\sum_{i=1}^{n} X_i$ and $T_2 = \dfrac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$.

   (a) Show that $E\left(T_1\right) = \lambda = E\left(T_2\right)$.

   (b) Explain giving reasons, which one of $T_1$ or $T_2$ will you use as an estimator for $\lambda$?

   $[8+12 = 20]$

2.(a) Examine whether a best critical region exists for testing the null hypothesis $H_0$: $\theta = \theta_0$ against the alternative hypothesis $H_1$: $\theta = \theta_1$ ($> \theta_0$) for the parameter $\theta$ of the distribution

   $$f(x, \theta) = \frac{1+\theta}{(x+\theta)^2}, \quad 1 \leq x < \infty.$$

   (b) Let $X_1, X_2, \cdots, X_n$ be a random sample of size $n$ from a distribution having the p.d.f.

   $$f(x; \alpha, \beta) = \frac{1}{2\beta}, \quad \alpha - \beta \leq x \leq \alpha + \beta$$

   Obtain the maximum likelihood estimator of $\alpha$.

   $[12+8 = 20]$

3.(a) An obstetrician claimed that more babies are born at night (6 PM to 6 AM) than during the day. In order to verify her claim, data on spontaneous births were maintained for a month and the following were observed.

| Time Period | Number of Births Observed |
|---|---|
| 12:00 noon to 3:00 P.M. | 9 |
| 3:00 P.M. to 6:00 P.M. | 10 |
| 6:00 P.M. to 9:00 P.M. | 12 |
| 9:00 P.M. to 12:00 midnight | 15 |
| 12:00 midnight to 3:00 A.M. | 16 |
| 3:00 A.M. to 6:00 A.M. | 17 |
| 6:00 A.M. to 9:00 A.M. | 12 |
| 9:00 A.M. to 12:00 noon | 9 |

Do you think that the obstetrician's claim is justified?

(b) A manufacturing organization believes that two different characteristics, say $X$ and $Y$, of one of its products are linearly related and the value of $X$ impacts the value of $Y$.

In order to verify the claim of linear relationship, data were collected on randomly selected products for two consecutive days. On each day, sample of size 11 were collected and the correlation coefficient as well as the line of best fit were computed. Surprisingly, in both the cases exactly same linear model described the data and the following results were obtained: $N = 11$, Mean of $X$'s $= 9.0$, Mean of $Y$'s $= 7.5$, equation of regression line $Y = 3.0 + 0.5X$, standard error of estimate of slope 0.118, correlation coefficient $r = 0.82$.

As the results tallied exactly across days, it was concluded that the production process is very repeatable and the hypothesis of linear relationship between $X$ and $Y$ is very much acceptable. However, one statistician wanted to see the data and the same is given subsequently.

| Day 1 | | Day 2 | |
|---|---|---|---|
| X | Y | X | Y |
| 10.0 | 8.04 | 7.0 | 7.26 |
| 8.0 | 6.95 | 11.0 | 9.26 |
| 13.0 | 7.58 | 6.0 | 6.13 |
| 9.0 | 8.81 | 12.0 | 9.13 |
| 11.0 | 8.33 | 5.0 | 4.74 |
| 14.0 | 9.96 | 8.0 | 8.14 |
| 6.0 | 7.24 | 14.0 | 8.10 |
| 4.0 | 4.26 | 4.0 | 3.10 |
| 12.0 | 10.84 | 13.0 | 8.74 |
| 7.0 | 4.82 | 10.0 | 9.14 |
| 5.0 | 5.68 | 9.0 | 8.77 |

Examine the data and give your comments.

[10+10 = 20]

4. (a) Explain with justification, the order in which *blocking* and *randomization* should be addressed during an experimental study.

(b) Describe how *randomization* is performed while using a Latin Square Design.

(c) An experimenter wanted to use a Latin Square Design (LSD) but instead, used the following design

$$\begin{bmatrix} A & B & C \\ B & A & C \\ C & A & B \end{bmatrix}$$

Are all treatment contrasts estimable? Give reasons for your answer.

[6+4+10 = 20]

5. (a) A finite lot of 100 resistors has $n_D$ defective (D) resistors and $n_N$ non-defective (N) ones. The overall number, i.e., the lot size, is known but the sizes of the two groups, i.e., $n_D$ and $n_N$ are unknown. If a random

sample of 5 resistors is taken without replacement from this lot, write down the likelihood for $n_D$ in the following three situations:

i) The observed sequence is *N, D, N, D, N*

ii) The observed sequence is *N, N, D, D, N*

iii) The precise sequence of the outcomes is not given, but it is known that three resistors are non-defective and two are defective.

Compare the likelihood functions and comment on your findings. Are the MLEs of $n_D$ same for the above three situations? Why?

(b) A number of spherical balls, all of the same material, give a symmetric distribution when classified according to their diameters. Do you think that their frequency distribution will exhibit symmetry when classified according to their weight? Justify.

[12+8 = 20]

## GROUP S2
## Probability

6. (a) Suppose that a product is produced in three factories X, Y and Z. It is known that factory X produces thrice as many items as factory Y, and factories Y and Z produce the same number of items. Assume that 3% of the items produced by each factory X and Z are defective, while 5% of those manufactured by factory Y is defective. All the items produced in the 3 factories are stacked and an item is selected at random.

   i) What is the probability that the selected item is defective?

   ii) If the item selected at random is found to be defective, what is the probability that it was produced by factory X, Y and Z respectively?

(b) Between 7:00 hours and 8:00 hours train leaves the Dadar station at 03, 05, 08, 10, 13, 15, 18, 20, ... ... minutes past 7:00 hours.

   What is the probability that a person arriving at the station will have to wait less than a minute for a train, assuming that the person's arrival at the station obeys a uniform distribution over the interval given below?

   i) 7:00 hours to 8:00 hours,

   ii) 7:15 hours to 7:45 hours.

[12+8 = 20]

7. (a) A pair of unbiased dice is thrown until the sum of two faces is either *four* or *six*. Find the probability that the sum *six* occurs first.

(b) Consider all the 8-letter words that can be formed by arranging the letters in BACHELOR in all possible ways. Any two such words are called equivalent if those two words maintain the same relative order of the letters A, E and O. For example, BACOHELR and CABLROEH are equivalent. Find the number of words that are equivalent to BACHELOR.

$$[12+8 = 20]$$

8. Let $X = (x_1, x_2)^T \sim N_2(\mu, \Sigma)$ with $\mu = (0, 0)^T$ and $\Sigma = I_2$. Find an appropriate transformation of $X$ to a random vector $Y = (y_1, y_2)^T$ such that $Y \sim N_2(\theta, \Sigma^*)$, with $\theta = (7, 5)^T$ and $\Sigma^* = \begin{pmatrix} 9 & -12 \\ -12 & 25 \end{pmatrix}$.

Justify whether the transformation is unique. Obtain the correlation coefficient between $x_2$ and $y_2$. Determine the regression line of $x_2$ on $y_2$.

$$[10+3+3+4 = 20]$$

9. (a) Let $X$ be a non-negative random variable with distribution function $F$ and density function $f$.

Show that $E(X^r) = \int_0^\infty rx^{r-1} \bar{F}(x) dx$ is finite, for any $r \geq 1$.

[Note: $\bar{F}(x) = 1 - F(x)$]

(b) Suppose that $X$ and $Y$ are i.i.d. $U(0,1)$. Define $P = Min(X,Y)$ and $Q = Max(X,Y)$. Calculate $Covariance(P,Q)$.

$$[8+12 = 20]$$

10. Given that the joint density function of $X$ and $Y$ is

$$f(x,y) = \begin{cases} \frac{1}{2} xe^{-y}, & 0 < x < 2, \ y > 0 \\ 0, & elsewhere, \end{cases}$$

find the distribution of $X + Y$.

$$[20]$$

**ATTENTION:** **ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM GROUP E1.**
*(Note: Partial credit may be given for partially correct answer)*

## GROUP E1
### Mathematics

1. (a) Show that $\dfrac{1}{1.2.3}+\dfrac{3}{2.3.4}+\dfrac{5}{3.4.5}+\dfrac{7}{4.5.6}+\cdots\cdots=\dfrac{3}{4}$.

   (b) Suppose $Z$ and $Z_1$ are two complex numbers, such that $(Z+Z_1)$ and $ZZ_1$ are both real. Show that either both $Z$ and $Z_1$ are real or $Z_1=\bar{Z}$.

   $$[14+6 = 20]$$

2. (a) Solve the differential equation $(x+y)^2\dfrac{dy}{dx}=a^2$.

   (b) Find the number of distinct roots of
   $$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}=0 \text{ in } \left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right].$$

   $$[12+8 = 20]$$

3. (a) Find the equation of the line perpendicular to the tangent of the graph $y=\dfrac{\tan x}{1+\tan x}$ at $x=\dfrac{\pi}{4}$.

   (b) Suppose that there are $n$ necklaces. The first necklace contains 5 beads, the second contains 7 beads and in general, the $i^{th}$ necklace contains $i$ beads more than that in the $(i-1)^{th}$ necklace. Find the total number of beads contained in all the $n$ necklaces.

   $$[12+8 = 20]$$

## GROUP E2
## Engineering Mechanics

4.(a) A simple pendulum OA when released from rest in the horizontal position falls under gravity and strikes a vertical wall at B as shown in the following diagram. The coefficient of restitution between the wall and the ball is 0.5. Find the angle $\theta$ defining the total rebound of the ball.
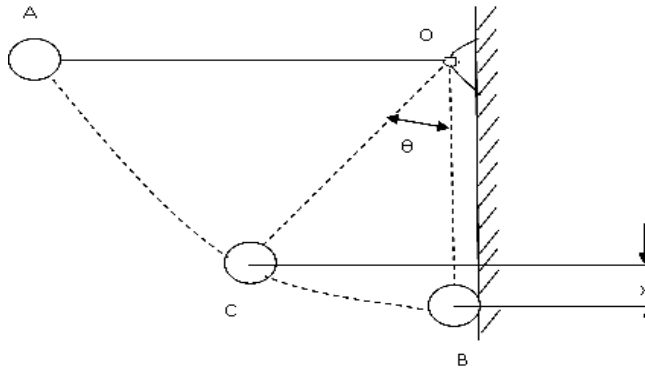


(b) An induction motor drives the main shaft of a workshop by means of a flat belt. The diameter of the pulleys on the motor shaft and the main shaft are 45 cm and 75 cm respectively. Another pulley of diameter 50 cm drives a countershaft having a pulley of diameter 70 cm keyed to it. If there is 3% slip on each drive, determine the speed of the countershaft if the motor runs at 1400 r.p.m. Neglect thickness of the belt.
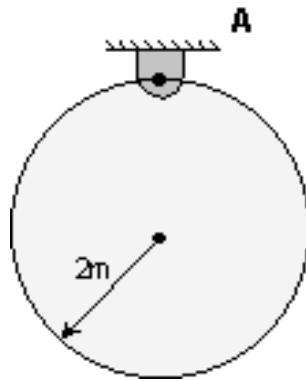
[10+10 = 20]

5. (a)
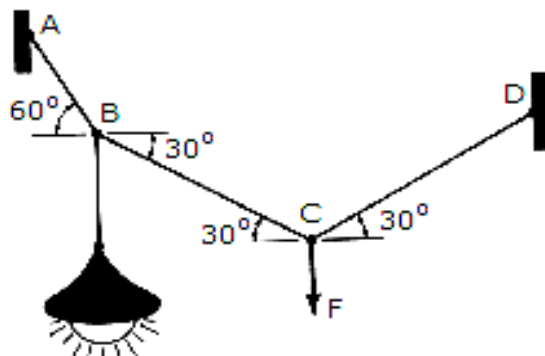
Determine the force F needed to hold the 4 kg lamp in the position shown.

(b) The moment of inertia of the 10kg disk about an axis passing through its mass centre is $I = mr^2/2$. Determine the moment of inertia of the disk about an axis passing through the pin at A.



[10+10 = 20]

## GROUP E3
## Electrical and Electronics Engineering

6. (a) In the circuit given below, $V_1 = 8$ V, $V_2 = 16$ V, $R_1 = 4$ Ohms, and $R_2 = 6$ Ohms. Find the Thevenin equivalent for the network to the left of terminals $a$, $b$. Assume that the internal resistances of the batteries are 0.
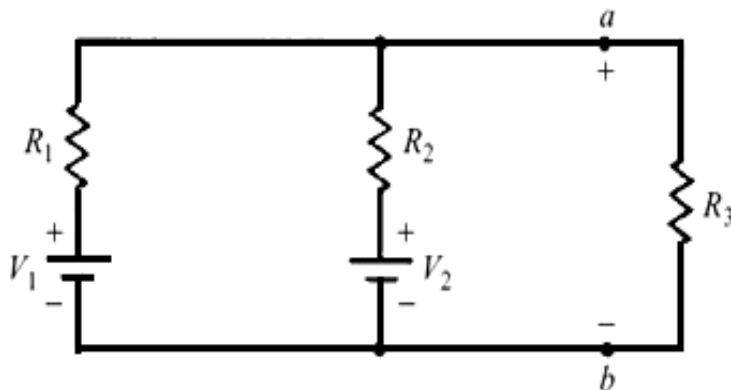
(b) For the circuit given in the next page, determine the output voltage $v_0$ for $v_S = 0.5$ V and $v_S = -0.5$ V. Assume that diode $D_2$ is ideal, $R_1 = 2$ KOhms, $R_2 = 8$ KOhms, $R = 2$ KOhms, $R_3 = 8$ KOhms and $v_{S2} = 0.5$ V.
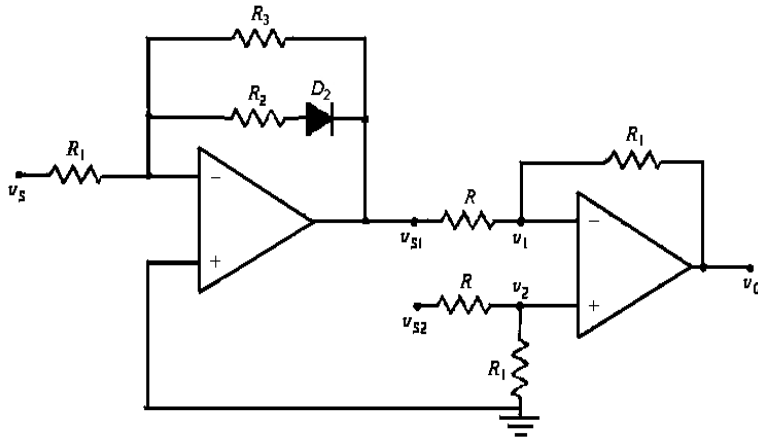


$$[8+12 = 20]$$

7. (a) A four-pole armature has 624 conductors and is lap-connected. The armature is driven at 1500 rpm. Calculate the useful flux per pole required to generate an emf of 220 V.

(b) Draw the complete phasor diagram of a transformer on-load. Explain briefly the parameters used in the diagram.

(c) A three-phase 50 Hz, 6-pole, 220 V wound rotor induction motor has its stator connected in delta and its rotor in star. There are 50 percent as many rotor conductors as stator conductors. For the rotor speed of 940 rpm, calculate i) the slip, ii) the blocked rotor voltage per phase, iii) the rotor generated voltage per phase, and iv) the rotor generated voltage between the terminals.

$$[4+8+8 = 20]$$

# GROUP E4
## Thermodynamics

8. A Carnot heat engine draws heat from a reservoir at temperature $T_A$ and rejects heat to another reservoir at temperature $T_B$. The Carnot forward cycle again drives a Carnot reversed cycle engine or Carnot refrigerator, which absorbs heat from the reservoir at temperature $T_C$ and rejects heat to the reservoir at temperature $T_B$ as depicted in the following diagram.
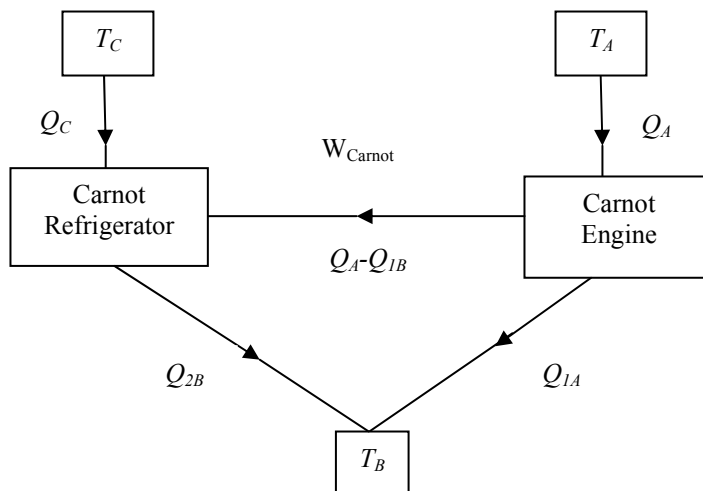


(a) Derive an expression for the ratio of heat absorbed from the reservoir at temperature $T_C$ to heat drawn from the reservoir at temperature $T_A$.

(b) If $T_A$ is 500 K and $T_C$ is 250 K, determine the temperature $T_B$, such that the heat supplied to the engine $Q_A$ is equal to the heat absorbed by the refrigerator $Q_C$.

(c) Also determine the efficiency of the Carnot engine and the COP of the Carnot refrigerator.

$$[10+4+6 = 20]$$

9. (a) Show that for $m$ kg of a certain gas, the change in entropy during the constant volume process is $\Delta S = mC_v \ln\left(\dfrac{p_2}{p_1}\right)$, where

$p_1$ = initial pressure of the gas

$v_1$ = initial volume of the gas

$T_1$ = initial temperature of the gas

$p_2, v_2, T_2$ = corresponding values for the final condition.

(b) A balloon filled with gas expands its volume by 2 L. If the pressure outside the balloon is 0.93 bar and the energy change of the gas is 450 J, how much heat did the surroundings give the balloon?

$$[10+10 = 20]$$

### GROUP E5
### Engineering Properties of Metals

10.  An aluminum thin walled tube (radius **:** thickness = 20 **:** 1) is closed at each end and pressurized to 7 MPa to cause plastic deformation.

(a) Derive the equations for the stresses acting on the tube.

(b) Based on the above equations, indicate the direction in which a gas cylinder is expected to fail.

(c) Indicate how the equations will get complicated in case of a thick walled tube.

(d) Find the plastic strain in the circumferential (hoop) direction. The plastic stress-strain curve is given by $\bar{\sigma} = 170(\bar{\varepsilon})^{0.25}$, where the stress is in MPa. Neglect elastic strain.

$$[4+2+4+10 = 20]$$

11.(a) Show schematically the basic surface features of a fatigue failure under plane bending. Describe in brief the significance of the surface features. Distinguish between progression and striation marks.

(b) How will the surface features look like in case of rotational bending? Assume that the stress concentration is low.

(c) By examining the fractured surface of a fatigue failure, how can you judge whether high load or high stress concentration is the cause of failure?

(d) Why does the fatigue limit occur? Give an example each of a metal that has/does not have fatigue limit. Draw the typical S-N curve of a polymer.
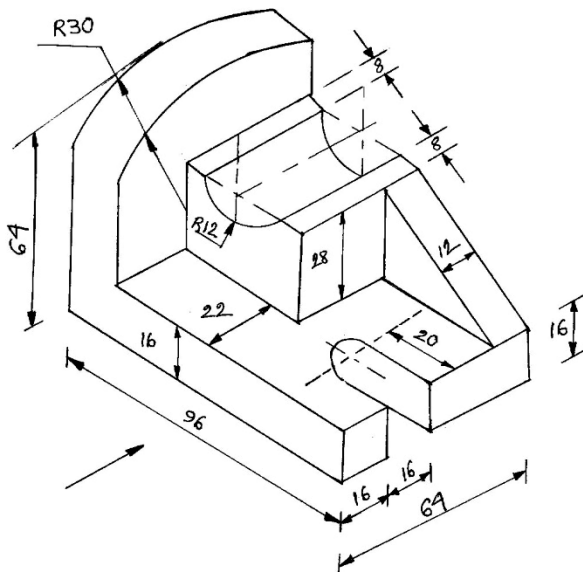
[6+3+4+7 = 20]


## GROUP E6
## Engineering Drawing

12. Draw freehand sketch for the following:

(a) An eye foundation bolt,

(b) A bent foundation bolt,

(c) A rag foundation bolt, and

(d) A single riveted single strap butt joint.

[20]


13. The following figure shows an isometric view of an object. Sketch the following views.

(a) Front view in the direction of the arrow,

(b) The top view, and

(c) The side view.



[20]