Name: ________________________________

Registration No.(CME/RME):_______________

Indian Institute of Technology, Bombay
Department of Mechanical Engineering

Admission Test for Ph.D. Candidates

Dec, 2012 **Time Limit: 90 minutes**

Note:

1. This test has several applied mathematics problems and they do not require any specialized field knowledge. It is designed only to test elementary comprehension and simple math skills. There are no complicated formulae to be used from memory as all the required information is given in the problem statements. So, read your problems carefully and you should be able to solve all of them.
2. All questions carry equal marks.
3. No calculators are allowed
4. Please write in the space provided after each problem and use the back side of the page if necessary.
5. An additional sheet has been provided in the end for rough work. THIS SHEET WILL NOT BE GRADED

DO NOT WRITE IN THE SPACE BELOW

Problem	Marks
1	
2	
3	
4	
5	
6	
TOTAL	

1. A stepped circular rod of equal step size l and cross-section A and $2A$ connects two rigid and fixed plates, which form the walls of a channel carrying fluid in a process plant. The material modulus of elasticity of the rod is E and coefficient of thermal expansion is α. The fluid flowing raises the temperature of the rod uniformly to level T above the room temperature. At room temperature, the rod is free of any load.Using the equilibrium and compatibility conditions, it is possible to calculate the stresses in each segment of the rod. The equilibrium condition means that sum of all forces acting on the component is zero. The compatibility refers to the constraint on the deformation.

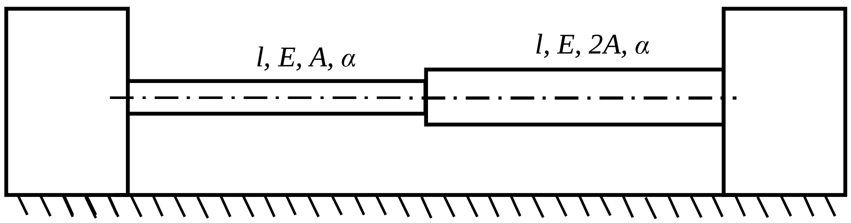

Fig. Q.1

a) Draw the free body diagram of the rod when its temperature is T.
b) Write the compatibility condition.
c) Determine nature and magnitude of stresses in each segment of the rod.
d) If the segment lengths are changed to $1.5l$ (of area A) and $0.5l$ (of area $2A$) respectively, what will happen to the stresses?

2. A circular bar with fixed ends carries a wheel of mass *m* (kg) with polar mass momentof inertia *J* (kgm^2). The polar area moment of inertia of the bar cross-section is *I* (m^4). The wheel islocated at the centre of the bar of length *l* (m); the modulus of rigidity of the bar material is *G* (N/m^2). The thickness of the wheel *b* (m) is very small compared to 0.5*l*. The wheel is given a small rotation $\theta=\theta_o$ from its rest position ($\theta=0^o$) and let go.Applying D'Alembert's principle and the condition of equilibrium, it is possible to obtain theequation of motion of the wheel.

According to the D'Alembert's principle, the inertia force acts in the direction opposite to the direction of acceleration. In the above case that inertia torque is equal to $J\ddot{\theta}$.

As per the theory of torsion of circular shaft, the angle θ of twist for a given torque *T* is given by$\theta = \frac{T\,L}{G\,I}$, where *L* is the length of the shaft, and *G* and *I* are already defined earlier.

a) Draw the free body diagram of the wheel and write the equation of motion in terms of instantaneous rotation θ from the position ($\theta=0^o$).
b) Obtain the solution of this equation.
c) What is the nature of this motion? Has the motion any frequency? If so, what's the value?
d) What will happen if segment lengths are changed to 0.75*l* and 0.25*l*.

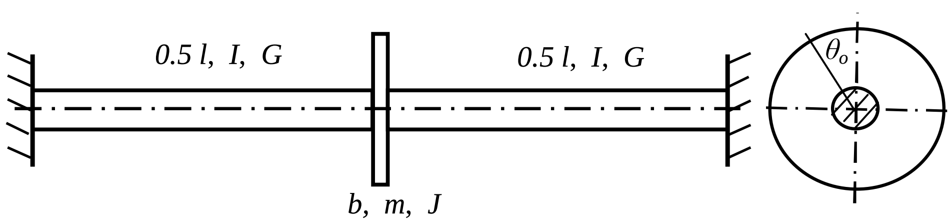

Fig. Q.2

3. The temperature of a body that is heated by a heat source can be written as

$$mC\frac{dT}{dt} - Q = 0$$

where **m** is the mass of the body, **C** is the specific heat and **T** is the temperature of the body and **Q** is the total heat supplied to the body by the external source and **t** is the time. The properties of the body are such that, there are no gradients of temperature within the body.

An available heat source is characterized by $\mathbf{Q=kT^2}$ where **k** is a constant with appropriate units and the temperature **T** is in deg. **K**. In a certain application, $\mathbf{k=5X10^{-2}W\ /K^2}$, **m= 1kg** and **C=0.125 J/gK**. The heating is started at time **t=0secs** and it was observed that at time **t=5sec**, the temperature of the body is 500K. Determine the temperature of the body when the heating was started, i.e. at **t=0**.

4. An incompressible fluid(ρ=constant) flows through a conical pipe, with inlet radius r_i and exit radius r_e as shown in the figure 3 given below. The inlet and exit velocity profiles $\mathbf{u_i}$ and $\mathbf{u_e}$ respectively are $u_i = 5\left[1-\left(\frac{r}{r_i}\right)^2\right]; u_e = k\left[1-\left(\frac{r}{r_e}\right)^2\right]$ where r is the distance from the pipe centerline as shown in Fig 4 and k is a constant. The mass flow rate, at any section $r_s(r_i<r_s<r_e)$ can be evaluated as $\dot{m} = \iint \rho u r d\theta dr$ where ρ is the density, u is the local velocity over an infinitesimally small area **rdθdr** at the section as shown in Figure 4. Determine **k** in terms of $\mathbf{r_i}$ and $\mathbf{r_e}$using the fact that at any section of the pipe the mass flow rate is constant

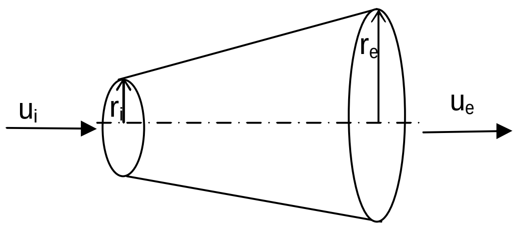

Fig. 3

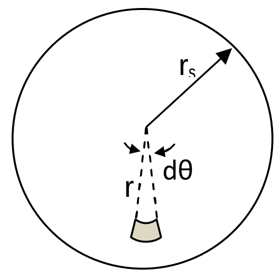

Fig. 4

5. The surface roughness can be modeled as a sinusoidal or a square wave. The average surface roughness is given by, $R_a = \frac{1}{\theta}\int_0^{\theta} |y| d\theta$ and root mean square surface roughness is given by, R_q = $\left(\frac{1}{\theta}\int_0^{\theta} y^2 d\theta\right)^{0.5}$. Plot the surface profiles and find the ratio of root mean square surface roughness, R_q and average surface roughness, R_a for both cases.

6. The solution of temperature rise at any position (x, y, z) and time t, $T-T_0$, during laser surface hardening with a Gaussian beam having a scan velocity, v, on a semi-infinite body is given by:

$$T-T_0=\frac{4P}{\rho C\pi\sqrt{4a\pi}}\int_{t'=0}^{t'=t} e^{-\left[\frac{2\left((x-vt')^2+y^2\right)}{\sigma^2+8a(t-t')}+\frac{z^2}{4a(t-t')}\right]}\times\frac{dt'(t-t')^{-0.5}}{\sigma^2+8a(t-t')}$$

where P= power of the heat source, C = sp. heat capacity, α = diffusivity, ρ = density, t = time, K = thermal conductivity. t' is a variable that varies from 0 to t and v.t' represents the instantaneous location of moving heat center. The velocity v is imparted along x-axis. The semi-infinite body extends from z=0 to z=-∞.

Prove that the maximum temperature rise possible is:

$$T-T_0=\frac{4P}{\rho C\pi\sqrt{4a\pi}}\int_{t'=0}^{t'=t}\frac{dt'(t-t')^{-0.5}}{\sigma^2+8a(t-t')}$$

Rough Work