M 2012 — 68010

MULTIPLE CHOICE QUESTIONS

SUBJECT : MATHEMATICS

Duration : Two Hours — **Maximum Marks : 100**

[Q. 1 to 60 carry one mark each]

1. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^9 + y^9 + z^9 - \frac{1}{x^9 y^9 z^9}$ is equal to

   A. 0 B. 1 C. 2 D. 3

2. Let p, q, r be the sides opposite to the angles P, Q, R respectively in a triangle PQR. If $r^2 \sin P \sin Q = pq$, then the triangle is

   A. equilateral
   B. acute angled but not equilateral
   C. obtuse angled
   D. right angled

3. Let p, q, r be the sides opposite to the angles P, Q, R respectively in a triangle PQR. Then $2pr\sin\left(\frac{P-Q+R}{2}\right)$ equals

   A. $p^2 + q^2 + r^2$ B. $p^2 + r^2 - q^2$ C. $q^2 + r^2 - p^2$ D. $p^2 + q^2 - r^2$

4. Let P (2, –3), Q (–2, 1) be the vertices of the triangle PQR. If the centroid of ΔPQR lies on the line 2x + 3y = 1, then the locus of R is

   A. 2x + 3y = 9 B. 2x – 3y = 9 C. 3x + 2y = 5 D. 3x – 2y = 5

5. $\lim\limits_{x \to 0} \frac{\pi^x - 1}{\sqrt{1+x} - 1}$

   A. does not exist B. equals $\log_e(\pi^2)$ C. equals 1 D. lies between 10 and 11

6. If f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, then

   A. f (x) must be an increasing function
   B. f (x) must be a decreasing function
   C. |f (x)| must be an increasing function
   D. |f (x)| must be a decreasing function

7. Rolle's theorem is applicable in the interval [–2, 2] for the function

   A. $f(x) = x^3$ B. $f(x) = 4x^4$ C. $f(x) = 2x^3 + 3$ D. $f(x) = \pi|x|$

8. The solution of $25\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + y = 0, \quad y(0) = 1, \quad y(1) = 2e^{\frac{1}{5}}$ is

   A. $y = e^{5x} + e^{-5x}$ B. $y = (1 + x)e^{5x}$ C. $y = (1+x)e^{\frac{x}{5}}$ D. $y = (1+x)e^{-\frac{x}{5}}$

9. Let P be the midpoint of a chord joining the vertex of the parabola $y^2 = 8x$ to another point on it. Then the locus of P is

A. $y^2 = 2x$ B. $y^2 = 4x$ C. $\frac{x^2}{4} + y^2 = 1$ D. $x^2 + \frac{y^2}{4} = 1$

10 .The line $x = 2y$ intersects the ellipse $\frac{x^2}{4} + y^2 = 1$ at the points P and Q. The equation of the circle with PQ as diameter is

A. $x^2 + y^2 = \frac{1}{2}$ B. $x^2 + y^2 = 1$ C. $x^2 + y^2 = 2$ D. $x^2 + y^2 = \frac{5}{2}$

11. The eccentric angle in the first quadrant of a point on the ellipse $\frac{x^2}{10} + \frac{y^2}{8} = 1$ at a distance 3 units from the centre of the ellipse is

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$

12. The transverse axis of a hyperbola is along the x-axis and its length is 2a. The vertex of the hyperbola bisects the line segment joining the centre and the focus. The equation of the hyperbola is

A. $6x^2 - y^2 = 3a^2$ B. $x^2 - 3y^2 = 3a^2$ C. $x^2 - 6y^2 = 3a^2$ D. $3x^2 - y^2 = 3a^2$

13. A point moves in such a way that the difference of its distance from two points (8,0) and (–8,0) always remains 4. Then the locus of the point is

A. a circle B. a parabola C. an ellipse D. a hyperbola

14. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

A. 0 B. 2 C. 4 D. 1

15. If a straight line passes through the point (α,β) and the portion of the line intercepted between the axes is divided equally at that point, then $\frac{x}{\alpha} + \frac{y}{\beta}$ is

A. 0 B. 1 C. 2 D. 4

16. The maximum value of $|z|$ when the complex number z satisfies the condition $\left|z + \frac{2}{z}\right|$ is

A. $\sqrt{3}$ B. $\sqrt{3} + \sqrt{2}$ C. $\sqrt{3} + 1$ D. $\sqrt{3} - 1$

17. If $\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$, where x and y are real, then the ordered pair (x,y) is

A. (–3, 0) B. (0, 3) C. (0, –3) D. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

18. If $\frac{z-1}{z+1}$ is purely imaginary, then

A. $|z|=\frac{1}{2}$ B. $|z|=1$ C. $|z|=2$ D. $|z|=3$

19. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of the three subjects. Only one student passed in all the subjects. Then the number of students failing in all the three subjects.

A. is 12 B. is 4

C. is 2 D. cannot be determined from the given information

20. A vehicle registration number consists of 2 letters of English alphabet followed by 4 digits, where the first digit is not zero. Then the total number of vehicles with distinct registration numbers is

A. $26^2 \times 10^4$ B. ${}^{26}P_2 \times {}^{10}P_4$ C. ${}^{26}P_2 \times 9 \times {}^{10}P_3$ D. $26^2 \times 9 \times 10^3$

21. The number of words that can be written using all the letters of the word 'IRRATIONAL' is

A. $\frac{10!}{(2!)^3}$ B. $\frac{10!}{(2!)^2}$ C. $\frac{10!}{2!}$ D. 10!

22. Four speakers will address a meeting where speaker Q will always speak after speaker P. Then the number of ways in which the order of speakers can be prepared is

A. 256 B. 128 C. 24 D. 12

23. The number of diagonals in a regular polygon of 100 sides is

A. 4950 B. 4850 C. 4750 D. 4650

24. Let the coefficients of powers of x in the 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^n$, where n is a positive integer, be in arithmetic progression. Then the sum of the coefficients of odd powers of x in the expansion is

A. 32 B. 64 C. 128 D. 256

25. Let $f(x) = ax^2 + bx + c, g(x) = px^2 + qx + r$ such that $f(1) = g(1)$, $f(2) = g(2)$ and $f(3) - g(3) = 2$. Then $f(4) - g(4)$ is

A. 4 B. 5 C. 6 D. 7

26. The sum $1 \times 1! + 2 \times 2! + ... + 50 \times 50!$ equals

A. 51! B. 51! – 1 C. 51! + 1 D. 2 × 51!

27. Six numbers are in A.P. such that their sum is 3. The first term is 4 times the third term. Then the fifth term is

A. –15 B. –3 C. 9 D. –4

28. The sum of the infinite series $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+\cdots$ is equal to

A. $\sqrt{2}$ B. $\sqrt{3}$ C. $\sqrt{\frac{3}{2}}$ D. $\sqrt{\frac{1}{3}}$

29. The equations $x^2 + x + a = 0$ and $x^2 + ax + 1 = 0$ have a common real root

A. for no value of a
B. for exactly one value of a
C. for exactly two values of a
D. for exactly three values of a

30. If 64, 27, 36 are the P^{th}, Q^{th} and R^{th} terms of a G.P., then $P + 2Q$ is equal to

A. R
B. 2R
C. 3R
D. 4R

31. The equation $y^2 + 4x + 4y + k = 0$ represents a parabola whose latus rectum is

A. 1
B. 2
C. 3
D. 4

32. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is equal to

A. 2 or $-\frac{3}{2}$
B. -2 or $-\frac{3}{2}$
C. 2 or $\frac{3}{2}$
D. -2 or $\frac{3}{2}$

33. If four distinct points (2k, 3k), (2, 0), (0, 3), (0, 0) lie on a circle, then

A. $k < 0$
B. $0 < k < 1$
C. $k = 1$
D. $k > 1$

34. The line joining $A(b \cos \alpha, b \sin \alpha)$ and $B(a \cos \beta, a \sin \beta)$, where $a \neq b$, is produced to the point $M(x, y)$ so that $AM : MB = b : a$. Then $x \cos\frac{\alpha+\beta}{2} + y \sin\frac{\alpha+\beta}{2}$

A. 0
B. 1
C. -1
D. $a^2 + b^2$

35. Let the foci of the ellipse $\frac{x^2}{9} + y^2 = 1$ subtend a right angle at a point P. Then the locus of P is

A. $x^2 + y^2 = 1$
B. $x^2 + y^2 = 2$
C. $x^2 + y^2 = 4$
D. $x^2 + y^2 = 8$

36. The general solution of the differential equation $\frac{dy}{dx} = \frac{x + y + 1}{2x + 2y + 1}$ is

A. $\log_e|3x + 3y + 2| + 3x + 6y = c$
B. $\log_e|3x + 3y + 2| - 3x + 6y = c$
C. $\log_e|3x + 3y + 2| - 3x - 6y = c$
D. $\log_e|3x + 3y + 2| + 3x - 6y = c$

37. The value of the integral $\int_{\pi/6}^{\pi/2} \left(\frac{1 + \sin 2x + \cos 2x}{\sin x + \cos x}\right) dx$ is equal to

A. 16
B. 8
C. 4
D. 1

38. The value of the integral $\int_0^{\frac{\pi}{2}} \frac{1}{1 + (\tan x)^{101}} dx$ is equal to

A. 1
B. $\frac{\pi}{6}$
C. $\frac{\pi}{8}$
D. $\frac{\pi}{4}$

39. The integrating factor of the differential equation $3x\log_e x\frac{dy}{dx}+y=2\log_e x$ is given by

A. $(\log_e x)^3$ B. $\log_e(\log_e x)$ C. $\log_e x$ D. $(\log_e x)^{\frac{1}{3}}$

40. Number of solutions of the equation $\tan x+\sec x=2\cos x,\ x\in[0,\pi]$ is

A. 0 B. 1 C. 2 `D. 3

41. The value of the integral

$$\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{3+\sin 2x}\,dx$$ is equal to

A. $\log_e 2$ B. $\log_e 3$ C. $\frac{1}{4}\log_e 2$ D. $\frac{1}{4}\log_e 3$

42. Let $y=\left(\frac{3^x-1}{3^x+1}\right)\sin x+\log_e(2+x),\ x>-1$. Then at $x=0,\ \frac{dy}{dx}$ equals

A. 1 B. 0 C. –1 D. –2

43. Maximum value of the function $f(x)=\frac{x}{8}+\frac{2}{x}$ on the interval $[1,\ 6]$ is

A. 1 B. $\frac{9}{8}$ C. $\frac{13}{12}$ D. $\frac{17}{8}$

44. For $-\frac{\pi}{2}<x<\frac{3\pi}{2}$, the value of $\frac{d}{dx}\left\{\tan^{-1}\frac{\cos x}{1+\sin x}\right\}$ is equal to

A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. 1 D. $\frac{\sin x}{(1+\sin x)^2}$

45. The value of the integral $\int_{-2}^{2}(1+2\sin x)e^{|x|}\,dx$ is equal to

A. 0 B. e^2-1 C. $2(e^2-1)$ D. 1

46. If $(\alpha+\sqrt{\beta})$ and $(\alpha-\sqrt{\beta})$ are the roots of the equation $x^2+px+q=0$ where α, β, p and q are real, then the roots of the equation $(p^2-4q)(p^2x^2+4px)-16q=0$ are

A. $\left(\frac{1}{\alpha}+\frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\alpha}-\frac{1}{\sqrt{\beta}}\right)$ B. $\left(\frac{1}{\sqrt{\alpha}}+\frac{1}{\beta}\right)$ and $\left(\frac{1}{\sqrt{\alpha}}-\frac{1}{\beta}\right)$

C. $\left(\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\sqrt{\alpha}}-\frac{1}{\sqrt{\beta}}\right)$ D. $(\sqrt{\alpha}+\sqrt{\beta})$and $(\sqrt{\alpha}-\sqrt{\beta})$

47. The number of solutions of the equation $\log_2\left(x^2+2x-1\right)=1$ is

A. 0 B. 1 C. 2 D. 3

48. The sum of the series $1+\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2+....+\frac{1}{n+1}{}^nC_n$ is equal to

A. $\frac{2^{n+1}-1}{n+1}$ B. $\frac{3\left(2^n-1\right)}{2n}$ C. $\frac{2^n+1}{n+1}$ D. $\frac{2^n+1}{2n}$

49. The value of $\sum_{r=2}^{\infty}\frac{1+2+.....+(r-1)}{r!}$ is equal to

A. e B. 2e C. $\frac{e}{2}$ D. $\frac{3e}{2}$

50. If $P=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}$, $Q=PP^T$, then the value of the determinant of Q is equal to

A. 2 B. –2 C. 1 D. 0

51. The remainder obtained when $1!+2!+\ldots+95!$ is divided by 15 is

A. 14 B. 3 C. 1 D. 0

52. If P, Q, R are angles of triangle PQR, then the value of $\begin{vmatrix}-1 & \cos R & \cos Q\\ \cos R & -1 & \cos P\\ \cos Q & \cos P & -1\end{vmatrix}$ is equal to

A. –1 B. 0 C. $\frac{1}{2}$ D. 1

53. The number of real values of α for which the system of equations

$$x+3y+5z=\alpha x$$
$$5x+y+3z=\alpha y$$
$$3x+5y+z=\alpha z$$

has infinite number of solutions is

A. 1 B. 2 C. 4 D. 6

54. The total number of injections (one-one into mappings) from $\{a_1, a_2, a_3, a_4\}$ to $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ is

A. 400 B. 420 C. 800 D. 840

55. Let $(1+x)^{10} = \sum_{r=0}^{10} c_r x^r$ and $(1+x)^7 = \sum_{r=0}^{7} d_r x^r$. If $P = \sum_{r=0}^{5} c_{2r}$ and $Q = \sum_{r=0}^{3} d_{2r+1}$, then $\frac{P}{Q}$ is equal to

A. 4 B. 8 C. 16 D. 32

56. Two decks of playing cards are well shuffled and 26 cards are randomly distributed to a player. Then the probability that the player gets all distinct cards is

A. ${}^{52}C_{26} / {}^{104}C_{26}$ B. $2 \times {}^{52}C_{26} / {}^{104}C_{26}$

C. $2^{13} \times {}^{52}C_{26} / {}^{104}C_{26}$ D. $2^{26} \times {}^{52}C_{26} / {}^{104}C_{26}$

57. An urn contains 8 red and 5 white balls. Three balls are drawn at random. Then the probability that balls of both colours are drawn is

A. $\frac{40}{143}$ B. $\frac{70}{143}$ C. $\frac{3}{13}$ D. $\frac{10}{13}$

58. Two coins are available, one fair and the other two-headed. Choose a coin and toss it once ; assume that the unbiased coin is chosen with probability $\frac{3}{4}$. Given that the outcome is head, the probability that the two-headed coin was chosen is

A. $\frac{3}{5}$ B. $\frac{2}{5}$ C. $\frac{1}{5}$ D. $\frac{2}{7}$

59. Let R be the set of real numbers and the functions $f : R \to R$ and $g : R \to R$ be defined $f(x) = x^2 + 2x - 3$ and $g(x) = x + 1$. Then the value of x for which $f(g(x)) = g(f(x))$ is

A. –1 B. 0 C. 1 D. 2

60. If a, b, c are in arithmetic progression, then the roots of the equation $ax^2 - 2bx + c = 0$ are

A. 1 and $\frac{c}{a}$ B. $-\frac{1}{a}$ and $-c$ C. -1 and $-\frac{c}{a}$ D. -2 and $-\frac{c}{2a}$

[Q. 61 to 80 carry two marks each]

61. Let y be the solution of the differential equation $x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$ satisfying y(1) = 1. Then y satisfies

A. $y = x^{y-1}$ B. $y = x^y$ C. $y = x^{y+1}$ D. $y = x^{y+2}$

62. The area of the region, bounded by the curves $y = \sin^{-1} x + x(1-x)$ and $y = \sin^{-1} x - x(1-x)$ in the first quadrant, is

A. 1 B. $\frac{1}{2}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$

63. The value of the integral $\int_1^5 [|x-3|+|1-x|]dx$ is equal to

A. 4 B. 8 C. 12 D. 16

64. If f(x) and g(x) are twice differentiable functions on (0, 3) satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$, $g(2) = 9$, then $f(1) - g(1)$ is

A. 4 B. –4 C. 0 D. –2

65. Let [x] denote the greatest integer less than or equal to x, then the value of the integral $\int_{-1}^{1} (|x| - 2[x])dx$ is equal to

A. 3 B. 2 C. –2 D. –3

66. The points representing the complex number z for which $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$ lie on

A. a circle B. a straight line C. an ellipse D. a parabola

67. Let a, b, c, p, q, r be positive real numbers such that a, b, c are in G.P. and $a^p = b^q = c^r$. Then

A. p, q, r are in G.P. B. p, q, r are in A.P. C. p, q, r are in H.P. D. p^2, q^2, r^2 are in A.P.

68. Let S_k be the sum of an infinite G.P. series whose first term is k and common ratio is $\frac{k}{k+1}$ (k > 0). Then the value of $\sum_{k=1}^{\infty} \frac{(-1)^k}{S_k}$ is equal to

A. $\log_e 4$ B. $\log_e 2 - 1$ C. $1 - \log_e 2$ D. $1 - \log_e 4$

69. The quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite sign. Then

A. $a \le 0$ B. $0 < a < 4$ C. $4 \le a < 8$ D. $a \ge 8$

70. If $\log_e(x^2 - 16) \le \log_e(4x - 11)$, then

A. $4 < x \le 5$ B. $x < -4$ or $x > 4$ C. $-1 \le x \le 5$ D. $x < -1$ or $x > 5$

71. The coefficient of x^{10} in the expansion of $1 + (1+x) + \ldots + (1+x)^{20}$ is

A. ${}^{19}C_9$ B. ${}^{20}C_{10}$ C. ${}^{21}C_{11}$ D. ${}^{22}C_{12}$

72. The system of linear equations

$\lambda x + y + z = 3$

$x - y - 2z = 6$

$-x + y + z = \mu$

has

A. Infinite number of solutions for $\lambda \ne -1$ and all μ

B. Infinite number of solutions for $\lambda = -1$ and $\mu = 3$

C. No solution for $\lambda \ne -1$

D. Unique solution for $\lambda = -1$ and $\mu = 3$

73. Let A and B be two events with $P(A^C) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^C) = 0.5$. Then $P(B \backslash A \cup B^C)$ is equal to

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

74. Let p, q, r be the altitudes of a triangle with area S and perimeter 2t. Then the value of $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$ is

A. $\frac{s}{t}$ B. $\frac{t}{s}$ C. $\frac{s}{2t}$ D. $\frac{2s}{t}$

75. Let C_1 and C_2 denote the centres of the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 1$ respectively and let P and Q be their points of intersection. Then the areas of triangles C_1PQ and C_2PQ are in the ratio

A. 3 : 1 B. 5 : 1 C. 7 : 1 D. 9 : 1

76. A straight line through the point of intersection of the lines $x + 2y = 4$ and $2x + y = 4$ meets the coordinates axes at A and B. The locus of the midpoint of AB is

A. $3(x + y) = 2xy$ B. $2(x + y) = 3xy$ C. $2(x + y) = xy$ D. $x + y = 3xy$

77. Let P and Q be the points on the parabola $y^2 = 4x$ so that the line segment PQ subtends right angle at the vertex. If PQ intersects the axis of the parabola at R, then the distance of the vertex from R is

A. 1 B. 2 C. 4 D. 6

78. The incentre of an equilateral triangle is (1, 1) and the equation of the one side is $3x + 4y + 3 = 0$. Then the equation of the circumcircle of the triangle is

A. $x^2+y^2-2x-2y-2=0$ B. $x^2+y^2-2x-2y-14=0$

C. $x^2+y^2-2x-2y+2=0$ D. $x^2+y^2-2x-2y+14=0$

79. The value of $\lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n}$ is

A. 1 B. $\frac{1}{e^2}$ C. $\frac{1}{2e}$ D. $\frac{1}{e}$

80. The area of the region bounded by the curves $y = x^3$, $y = \frac{1}{x}$, $x = 2$ is

A. $4 - \log_e 2$ B. $\frac{1}{4} + \log_e 2$ C. $3 - \log_e 2$ D. $\frac{15}{4} - \log_e 2$

M 2012 — 68010

SUBJECT : MATHEMATICS — **KEY ANSWERS WITH EXPLANATIONS**

1. Given $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3.\frac{\pi}{2}$

   $\therefore \quad \sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

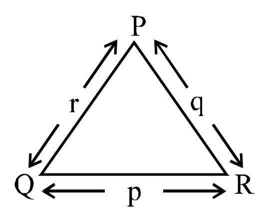
   $\therefore \quad x = y = z = 1$

   $x^9 + y^9 + z^9 - \frac{1}{x^9.y^9.z^9}$

   $= 1+1+1-\frac{1}{1} = 3-1 = 2$

   Ans. C.

2. $r^2.\sin P.\sin Q = pq$



   $r^2 \times \frac{p}{2R}.\frac{q}{2R} = pq$

   $r^2 = 4R^2$

   $r = 2R$

   side = 2 × circumradius = diameter

   So, triangle is right angled triangle.

   Ans. D.

3. Given P + Q + R = 180°

   $\therefore \quad P + R = 180° - Q$

   $\therefore \quad \sin\left(\frac{180° - 2Q}{2}\right) = \sin(90° - Q) = \cos Q$

   Now $2pr\cos Q$

   $= 2pr\,\frac{p^2 + r^2 - q^2}{2pr} = p^2 + r^2 - q^2$

   Ans. B.

4. Let R = (h, k)

   Centroid $= \left(\frac{2-2+h}{3}, \frac{1-3+k}{3}\right) = \left(\frac{h}{3}, \frac{k-2}{3}\right)$

   $\therefore \quad 2\frac{h}{3} + 3.\frac{k-2}{3} = 1$

   $\frac{2h}{3} + k - 2 = 1$

   $2h + 3k = 9$

   Required locus $2x + 3y = 9$

   Ans. A.

5. $\lim_{x\to 0}\frac{\pi^x-1}{\sqrt{1+x}-1}$

$= \lim_{x\to 0}\frac{(\pi^x-1)(\sqrt{1+x}+1)}{1+x-1}$

$= \lim_{x\to 0}\frac{\pi^x-1}{x}.(\sqrt{1+x}+1)$

$= (\log_2 \pi)\times 2 = \log_2 \pi^2$

Ans. B.

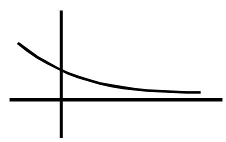
6. Given $f(x).f'(x) < 0 \quad \forall x \in R$

and f(x) is differentiable

$\therefore$ f(x) is continuous function and f(x) and $f'(x)$ are opposite of sign

$\therefore$ either f(x) > 0 or $f(x) < 0$ but

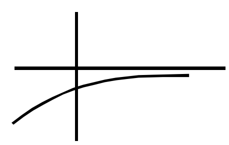
It can not cut the x-axis



$\therefore$ When f(x) > 0 then $f'(x) < 0$

$\therefore$ f(x) is decresing function

When $f(x) < 0$ then $f'(x) > 0$

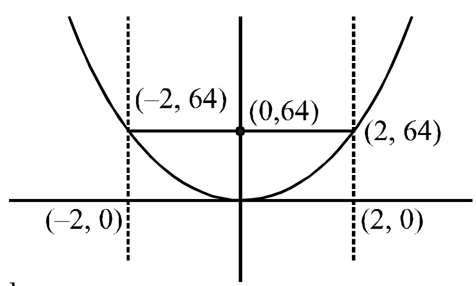


$\therefore$ f(x) is increasing function

$\therefore$ We can say that $|f(x)|$ is decreasing function.

Ans. D.

7. Check the options individually. Take option (b) $f(x) = 4x^4$



Now, Rolle's theorem is applicable.

Ans. B.

8. Let $y = e^{mx}$

$\frac{dy}{dx} = me^{mx}$

$\frac{d^2y}{dx^2} = m^2e^{mx}$

$25m^2e^{mx} - 10me^{mx} + e^{mx} = 0$

$25m^2 - 10m + 1 = 0$

$(5m-1)^2 = 0$

$m = \frac{1}{5} \cdot \frac{1}{5}$

$\therefore$ General solution is $y = (A + Bx)e^{\frac{1}{5}x}$

y (0) = 1

1 = A .1 $\Rightarrow$ A = 1

$y(1) = 2e^{\frac{1}{5}}$

$2e^{\frac{1}{5}} = (A + B)e^{\frac{1}{5}}$

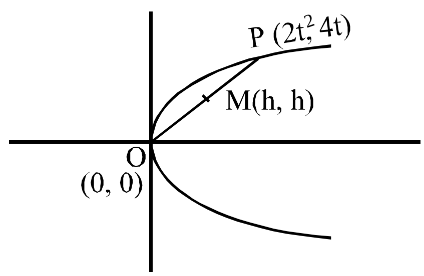
2 = (1 + B)

B = 1

$\therefore \quad y = (1 + x)e^{\frac{x}{5}}$

Ans. C.

9. $y^2 = 8x$



$y^2 = 4ax$

4a = 8

a = 2

$h = \frac{2t^2 + 0}{2} = t^2$ or $t^2 = h$

$k = \frac{4t + 0}{2} = 2t$ or $t = \frac{k}{2}$

$\therefore \quad \frac{k^2}{4} = h$

$\therefore\ k^2 = 4h \Rightarrow y^2 = 4x$

Ans. B.

10. $\frac{x^2}{4} + y^2 = 1$

$\frac{4y^2}{4} + y^2 = 1$

$2y^2 = 1$

$y^2 = \frac{1}{2}$

$y = \pm \frac{1}{\sqrt{2}}$

$x = \pm \sqrt{2}$

$P\left(\sqrt{2},\ \frac{1}{\sqrt{2}}\right)$

$Q\left(-\sqrt{2},\ -\frac{1}{\sqrt{2}}\right)$

equation of circle PQ as diameter is $\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)+\left(y-\frac{1}{\sqrt{2}}\right)\left(y+\frac{1}{\sqrt{2}}\right)=0$

$x^2-2+y^2-\frac{1}{2}=0$

$x^2+y^2=\frac{5}{2}$

Ans. D.

11. Let the pt on the first quadrant $\left(\sqrt{10}\cos\theta,\ \sqrt{8}\sin\theta\right)$

distance from the centre = 3

$\therefore\ 10\cos^2\theta+8\sin^2\theta=9$

$\Rightarrow\ 8+2\cos^2\theta=9$

$\Rightarrow\ \cos^2\theta=\frac{1}{2}\Rightarrow\cos\theta=+\frac{1}{\sqrt{2}}$ [$\because$ pt lie on lst quadrant]

$\therefore\ \theta=\frac{\pi}{4}$

Ans. B.

12. Let the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore$ Length of the transverse axis = 2a

vertex = (a, 0)

focus (ae, 0) $\therefore\ \left(\frac{ae}{2},\ 0\right)=(a,\ 0)$

centre = (0, 0) $\therefore\ \frac{ae}{2}=a$

$\Rightarrow\ e=2$

$\therefore\ e^2=4$

$\Rightarrow\ 1+\frac{b^2}{a^2}=4\ \Rightarrow\ b^2=3a^2$

$\therefore$ Equation : $\frac{x^2}{a^2}-\frac{y^2}{3a^2}=1$

Ans. D.

13. The distance between (8, 0) and (–8, 0) = 16 > 4.

$\therefore$ According to the defination of hyperbola the locus is a hyperbola.

Ans. D.

14. $m \in I$

$$\left.\begin{matrix} 3x + 4y = 9 \\ y = mx + 1 \end{matrix}\right\} \Rightarrow 3x + 4(mx + 1) = 9$$

$$\Rightarrow (3 + 4m)x = 5$$

$$\Rightarrow x = \frac{5}{3 + 4m} \in I$$

$\therefore$ $3 + 4m = -5, \ -1, \ 1, \ 5$ when $m \in I$

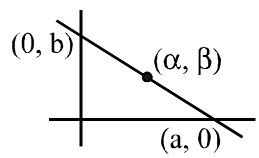
$\therefore$ m can take only two values

$m = -1, -2.$

Ans. B.

15. Let the equation of the line $\frac{x}{a} + \frac{y}{b} = 1$

Its passes through (α, β)



$\therefore \quad \frac{\alpha}{a} + \frac{\beta}{b} = 1$

(α, β) is the mid points of (a, 0) and (0, b)

$\therefore \quad \alpha = \frac{a}{2}$ and $\beta = \frac{b}{2}$

$\Rightarrow \quad a = 2\alpha, \ b = 2\beta$

$\therefore$ Equation of the line $\frac{x}{2\alpha} + \frac{y}{2\beta} = 1 \Rightarrow \frac{x}{\alpha} + \frac{y}{\beta} = 2$

Ans. C.

16. $\left|z + \frac{2}{z}\right| = 2$

Now, $\left|z + \frac{2}{z}\right| \le |z| + \frac{2}{|z|}$

$\Rightarrow |z| + \frac{2}{|z|} \ge 2$

$\Rightarrow |z|^2 - 2|z| + 2 \ge 0$

$\Rightarrow |z|^2 - 2|z| + 1 \le 3$

$\Rightarrow (|z| - 1)^2 \le 3$

$\Rightarrow -\sqrt{3} \le (|z| - 1) \le \sqrt{3}$

$\Rightarrow 1 - \sqrt{3} \le |z| \le 1 + \sqrt{3} \Rightarrow 1 - \sqrt{3}$

$\because \ |z| \ge 0$

Ans. C.

17. $\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}\ (x+iy)$

$\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)^{50} = x+iy$

$i^{50}\left(\frac{-1+i\sqrt{3}}{2}\right)^{50} = x+iy$

$\Rightarrow\ i^{50}.w^{50} = x+iy \Rightarrow\ x+iy\ = i^2\times w^2 \Rightarrow\ x+iy = -w^2\ \Rightarrow\ x+iy = \frac{1}{2}+\frac{i\sqrt{3}}{2}$

Ans. D.

18. $\frac{z-1}{z+1} = ik, \quad k\in R, \quad k\neq 0$

$\frac{2z}{-2} = \frac{ik+1}{ik-1}$; by comp.-div.

$z = \frac{1+ik}{1-ik}$

$|z| = \frac{|1+ik|}{|1-ik|} = \frac{\sqrt{1+k^2}}{\sqrt{1+k^2}} = 1$

Ans. B.

19. n(M) = 50 = No. of failed in maths.

n(P) = 45

n (B) = 40

$n(M\cap P)+n(M\cap B)+n(P\cap B)-3\,n(M\cap P\cap B) = 32$

We have to find $n(M\cap P\cap B)$

Total no of student = 100

$n(M\cup P\cup B) = 99$

$\Rightarrow\ n(M)+n(P)+n(B)-\{n(M\cap P)+n(M\cap B)+n(P\cap B)\}+n(M\cap P\cap B) = 99$

$\Rightarrow\ 50+45+40-\{32+3n(M\cap P\cap B)\}+n(M\cap P\cap B) = 99$

$\Rightarrow\ 135-32-2n(M\cap P\cap B) = 99$

$\Rightarrow\ 2n(M\cap P\cap B) = 4$

$\Rightarrow\ n(M\cap P\cap B) = 2$

Ans. C.

20. $\overline{\ 1\ }\ \overline{\ 2\ }\quad \overline{\ 3\ }\ \overline{\ 4\ }\quad \overline{\ 5\ }\ \overline{\ 6\ }$

two alphabet we can choose 26^2 ways.

and 1st number we can choose 9 ways.

next 3 numbers we can choose 10^3 ways.

Ans. D.

21. I - 2

R - 2

A - 2

T - 1

N - 1

O - 1

L - 1

Number of words $\frac{10!}{(2!)^3}$

Ans. A.

22. Required number of ways is which the order of speakers can be prepared

$= \frac{4!}{2!}$

$= \frac{24}{2}$

= 12 [Taking speakers P & Q as identical]

Ans. D.

23. No. of diagonals in a regular polygon

$= {}^{100}C_2 - 100$

$= \frac{100 \times 99}{2} - 100$

$= 50 \times 99 - 100$

$= 4950 - 100$

$= 4850$

Ans. B.

24. ${}^nC_1$, ${}^nC_2$, ${}^nC_3$ are in A.P.

$\therefore\ \cancel{2} \cdot \frac{\cancel{n(n-1)}}{\cancel{2}} = \frac{\cancel{n(n-1)}(n-2)}{6} + n$

or, $n - 1 = \frac{n^2 - 3n + 2}{6} + 1$

or, $6n - 6 = n^2 - 3n + 2 + 6$

or, $n^2 - 9n + 14 = 0$

$n = 7$, $n = 2$ not acceptable.

$$\text{sum} = \frac{2^n}{2} = \frac{2^7}{2} = 2^6 = 64$$

Ans. B.

25. $f(x) = ax^2 + bx + c$, $g(x) = px^2 + qx + r$

Now $f(1) = g(1) \Rightarrow a + b + c = p + q + r \Rightarrow \boxed{(a-b)+(b-q)+(c-r)=0}$ ...... (1)

$f(2) = g(2) \Rightarrow 4a + 2b + c = 4p + 2q + r \Rightarrow \boxed{4(a-b)+2(b-q)+(c-r)=0}$

$\Rightarrow \boxed{3(a-b)+(b-q)=0}$ .... (2)

[using (1)]

$f(3) - g(3) = 2$

$\Rightarrow 9(a - p) + 3(b - q) + (c - r) = 2$

$\Rightarrow 8(a - p) + 2(b - q) = 2$ [using (1)]

$\Rightarrow 4(a - p) + (b - q) = 1$

$\Rightarrow (a - p) = 1$ (using (2))

Now, $f(4) - g(4) = 16(a - p) + 4(b - q) + (c - r)$

$= 15\ (a - p) + 3(b - q)$ (using (1))

$= 15.(1) + 3(-3)$ $\left\{\begin{matrix}\because a - p = 1 \\ b - q = -3\end{matrix}\right\}$

$= 15 - 9$

$= 6$

Ans. C.

26. $1 \times 1! + 2 \times 2! + 3 \times 3! + ... + n \times n!$

$= (n + 1)! - 1$

$1 \times 1! + 2 \times 2! + ... + 50 \times 50!$

$= 51! - 1$

Ans. B.

27. Six numbers are in A.P.

$a - 5d$, $a - 3d$, $a - d$ $a + d$, $a + 3d$, $a + 5d$

$6a = 3$

$$\therefore \quad a = \frac{1}{2}$$

$$\frac{1}{2} - 5d = 4\left(\frac{1}{2} - d\right)$$

$\frac{1}{2} - 5d = 2 - 4d$

$d = -\frac{3}{2}$

Fifth term = a + 3d

$= \frac{1}{2} + 3\left(-\frac{3}{2}\right)$

$= \frac{1}{2} - \frac{9}{2}$

$= \frac{-8}{2}$

$= -4$

Ans. D.

28. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots.$

comparing, $nx = \frac{1}{3}$

$\frac{nx(nx - x)}{2} = \frac{1}{3} \cdot \frac{3}{6}$

$\frac{\frac{1}{3}\left(\frac{1}{3} - x\right)}{2} = \frac{1}{3} \cdot \frac{1}{2}$

$\frac{1}{3} - x = 1$

$x = -\frac{2}{3}$

$n\left(-\frac{2}{3}\right) = \frac{1}{3}$

$n = -\frac{1}{2}$

$\therefore \left(1 - \frac{2}{3}\right)^{-\frac{1}{2}}$

$= \left(\frac{1}{3}\right)^{-\frac{1}{2}} = \sqrt{3}$

Ans. B.

29. Let α be the common root

$$\alpha^2 + \alpha + a = 0$$
$$\alpha^2 + a\alpha + 1 = 0$$
$$(-)$$
$$\therefore \ \alpha(1-a) + a - 1 = 0$$

$(1-a)(\alpha - 1) = 0$

Either $a = 1$

or, $\alpha = 1$

Put $\alpha = 1$ in the 1st equation

$1 + 1 + a = 0$

$a = -2$

Put $a = 1$

$.^2 + x + 1 = 0$

$x^2 + x + 1 = 0$

They have no real common root.

Put $a = -2$

$x^2 + x - 2 = 0$ & $x^2 - 2x + 1 = 0$ or, $x = 1$

they have a one real common root.

Ans. B.

30. Let A be the 1st term & r be the c.r.

$A \,.\, r^{p-1} = 64 = 2^6$

$A \,.\, r^{q-1} = 27 = 3^3$

$A \,.\, r^{R-1} = 36 = 2^2 \cdot 3^2$

Now, $2 = A^{\frac{1}{6}} \cdot r^{\frac{p-1}{6}}$

$$3 = A^{\frac{1}{3}} \cdot r^{\frac{q-1}{3}}$$

$$2 \cdot 3 = A^{\frac{1}{2}} \cdot r^{\frac{R-1}{2}}$$

$$A^{\frac{1}{6}} \cdot r^{\frac{p-1}{6}} \cdot A^{\frac{1}{3}} \cdot r^{\frac{q-1}{3}} = A^{\frac{1}{2}} \cdot r^{\frac{R-1}{2}}$$

$$r^{\frac{p-1}{6}+\frac{2q-2}{6}} = r^{\frac{3R-3}{6}}$$

$$\frac{p-1+2q-2}{6} = \frac{3R-3}{6}$$

$\therefore \quad p + 2q = 3R$

Ans. C.

31. $y^2 + 4y = -4x - k$

$y^2 + 4y + 4 = -4x + 4 - k$

$(y+2)^2 = -4x - (k-4) = -4\left[x + \frac{k-4}{4}\right]$

$Y^2 = -4AX$

L.R. = 4A = 4 unit.

Ans. D.

32. Apply, $2(g_1g_2 + f_1f_2) = C_1 + C_2$

$2(1 \times 0 + k \cdot k) = 6 + k$

$\therefore \quad 2k^2 = 6 + k$

or, $2k^2 - k - 6 = 0$

or, $2k^2 - 4k + 3k - 6 = 0$

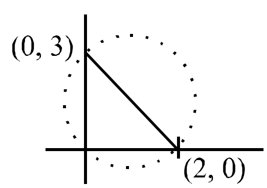
or, $2k(k-2) + 3(k-2) = 0$

or, $(k-2)(2k+3) = 0$

$k = 2, \ -\frac{3}{2}$

Ans. A.

33. Equation of circle is $(x-2)(x-0) + (y-0)(y-3) = 0$



$x^2 - 2x + y^2 - 3y = 0$

$x^2 + y^2 - 2x - 3y = 0$

$4k^2 + 9k^2 - 4k - 9k = 0$

$13k^2 = 13k$

$\therefore \quad k = 0, \ 1$

$k = 1$

Ans. C.

34. $x = \dfrac{ab\cos\beta - ab\cos\alpha}{b-a}$

$$= \frac{ab}{b-a} = (\cos\beta - \cos\alpha)$$

$$= \frac{ab}{b-a}\left[2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}\right]$$

$$y = \frac{ab\sin\beta - ab\sin\alpha}{b-a}$$

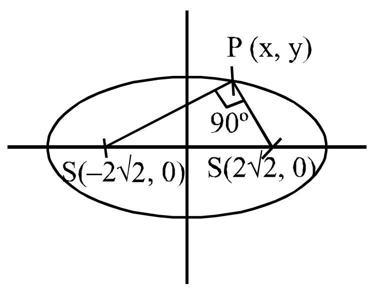
$$= \frac{ab}{b-a}\left[2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \sin\frac{\beta-\alpha}{2}\right]$$

$$\frac{x}{y} = -\frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$$

$$x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0$$

Ans. A.

35. $\dfrac{x^2}{9} + \dfrac{y^2}{1} = 1$



a = 3, b = 1

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$e = \frac{2\sqrt{2}}{3}$$

$$ae = 2\sqrt{2}$$

$$m_{ps} \times m_{ps'} = -1$$

$$\frac{y-0}{x+2\sqrt{2}} \times \frac{y-0}{x-2\sqrt{2}} = -1$$

$$\frac{y^2}{x^2-8} = -1$$

$$y^2 = -x^2 + 8$$

$$x^2 + y^2 = 8$$

Ans. D.

36. $\frac{dy}{dx} = \frac{x+y+1}{2(x+y)+1}$

Let $x + y = z$

$1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dy}{dx} = \frac{dz}{dx} - 1$

$\frac{dz}{dx} - 1 = \frac{z+1}{2z+1}$

$\frac{dz}{dx} = \frac{z+1}{2z+1} + 1$

$= \frac{z+1+2z+1}{2z+1}$

$= \frac{3z+2}{2z+1}$

or, $\frac{2z+1}{3z+1} dz = dx$

or, $\frac{2}{3}\int \frac{3z+\frac{3}{2}}{3z+1} dz = \int dx + c$

or, $\frac{2}{3}\int \frac{3z+2-\frac{1}{2}}{3z+2} dz = x + c$

or, $\frac{2}{3}z - \frac{1}{3}\cdot\frac{1}{3}\int \frac{d(3z+2)}{3z+2} = x + c$

or, $\frac{2}{3}z - \frac{1}{9}\log|3z+2| = x + c$

or, $\frac{2}{3}(x+y) - \frac{1}{9}\log|3x+3y+2| = x + c$

or, $6x + 6y - \log|3x+3y+2| = 9x + c'$

or, $3x - 6y + \log|3x+3y+2| = c$

Ans. D.

37. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2 + (\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)}$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[(\cos x + \sin x) + (\cos x - \sin x)\right] dx$$

$$= 2\sin x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 2\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right]$$

$$= 2\left[1 - \frac{1}{2}\right]$$

$$= 2\cdot\frac{1}{2} = 1$$

Ans. D.

38. $\displaystyle I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \left(\frac{\sin x}{\cos x}\right)^{101}} dx$

$$= \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{101}}{(\cos x)^{101} + (\sin x)^{101}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{101}}{(\sin x)^{101} + (\cos x)^{101}} dx \qquad \left[\text{Apply } \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$\therefore \quad 2I = \int_0^{\frac{\pi}{2}} dx = x\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

Ans. D.

39. $\frac{dy}{dx} + \frac{y}{3x\log_e x} = \frac{2}{3x}$

$I = e^{\int \frac{dx}{3x\log_e x}}$

$= e^{\frac{1}{3}\int \frac{d(\log_e x)}{\log_e x}}$

$= e^{\frac{1}{3}\log(\log_e x)}$

$= e^{\log_e (\log_e x)^{\frac{1}{3}}}$

$= (\log_e x)^{\frac{1}{3}}$

Ans. D.

40. $\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\sin x + 1 = 2\cos^2 x = 2(1 - \sin^2 x)$

$\sin x + 1 = 2(1 + \sin x)(1 - \sin x)$

$(1 + \sin x)[1 - 2(1 - \sin x)] = 0$

$(1 + \sin x)(1 - 2 + 2\sin x) = 0$

$(1 + \sin x)(2\sin x - 1) = 0$

$\sin x = -1$

$\sin x = \frac{1}{2}$

$\therefore \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

Ans. C.

41. $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

$= \int_0^{\pi/4} \frac{(\sin x + \cos x)dx}{4 - (\sin x - \cos x)^2}$

$= \int_0^1 \frac{dz}{2^2 - z^2}$ Putting $\sin x - \cos x = z \Rightarrow (\cos x + \sin x)dx = dz$, $\begin{array}{c|c|c} x & 0 & \frac{\pi}{4} \\ \hline z & 1 & 0 \end{array}$

$= \frac{1}{2 \times 2}\left[\log\left(\frac{2+z}{2-z}\right)\right]_0^1$

$= \frac{1}{4}[\log(3) - \log(1)]$

$= \frac{1}{4}\log(3)$

Ans. D.

42. $y = \left(\frac{3^x - 1}{3^x + 1}\right)\sin x + \log_e(1 + x)$

$= \frac{3^x + 1 - 2}{3^x + 1}(\sin x) + \log_e(1 + x)$

$= \left(1 - \frac{2}{3^x + 1}\right)\sin x + \log_e(1 + x)$

$= \sin x - \frac{2\sin x}{3^x + 1} + \log(1 + x)$

$\frac{dy}{dx} = \cos x - 2\frac{(3^x + 1)\cdot\cos x - (\sin x)3^x \cdot \log_e 3}{(3^x + 1)^2} + \frac{1}{1 + x}$

$\therefore \left.\frac{dy}{dx}\right|_{x=0} = 1 - 2\frac{2 - 0}{2^2} + \frac{1}{1 + 0}$

$= 1 - 1 + 1 = 1$

Ans. A.

43. $f(x) = \frac{x}{8} + \frac{2}{x}$

$f'(x) = \frac{1}{8} - \frac{2}{x^2}$

For max & min.

$f'(x) = 0$

$\frac{1}{8} - \frac{2}{x^2} = 0$

$\frac{1}{8} = \frac{2}{x^2}$

$x^2 = 16$

$x = +4 \quad [x \in [1, 6]]$

$f'(4^-) > 0$

$f'(4^+) < 0$

at $x = 4$ $f(x)$ is max.

$f(4) = \frac{4}{8} + \frac{2}{4} = \frac{1}{2} + \frac{1}{2} = 1$

Ans. A.

44. Exp. $= \frac{d}{dx}\left\{\tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right)\right\}$

$= \frac{d}{dx}\left\{\tan^{-1}\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right\}$

$= \frac{d}{dx}\left(\tan^{-1}\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right)$

$= \frac{d}{dx}\left\{\tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)\right\}$

$\Rightarrow -\frac{\pi}{2} < x < \frac{3\pi}{2}$

$\Rightarrow \ -\frac{\pi}{4} < \frac{x}{2} < \frac{3\pi}{4}$

$\Rightarrow \ \frac{\pi}{4} > -\frac{x}{2} > -\frac{3\pi}{4}$

$\Rightarrow \ \frac{\pi}{2} > \frac{\pi}{4} - \frac{\pi}{2} > -\frac{\pi}{2}$

$\therefore \ \text{Exp.} = \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = 0 - \frac{1}{2}$

Ans. B.

45. $\int_{-2}^{2}(1+2\sin x)e^{|x|}dx$

$= \int_{-2}^{2} e^{|x|}dx + 2\int_{-2}^{2}\sin x e^{|x|}dx$

$= 2 \times \int_{0}^{2} e^{|x|}dx + 0$

$= 2e^{x}\Big|_{0}^{2}$

$= 2(e^2 - 1)$

Ans. C.

46. $x^2 + px + q = 0 \ \rightarrow$ roots are $\alpha + \sqrt{\beta}$ and $\alpha - \sqrt{\beta}$

$\therefore 2\alpha = -p \ \Rightarrow \boxed{p = -2\alpha} \Rightarrow \alpha = -\frac{p}{2}$

$\alpha^2 - \beta = q \Rightarrow \beta = \alpha^2 - q = \frac{p^2}{4} - q = \left(\frac{p^2 - 4q}{4}\right) \ \Rightarrow \boxed{(p^2 - 4q) = 4\beta}$

Now, $4\beta(4\alpha^2x^2 - 8\alpha x) - 16(\alpha^2 - \beta) = 0$

$\Rightarrow 16\alpha^2\beta x^2 - 32\alpha\beta x - 16\alpha^2 + 16\beta = 0$

$\Rightarrow x^2 - \frac{2}{\alpha}x - \frac{1}{\beta} + \frac{1}{\alpha^2} = 0$

$\therefore$ Sum of the roots $= \frac{2}{\alpha} = \left(\frac{1}{\alpha} + \frac{1}{\sqrt{\beta}}\right) + \left(\frac{1}{\alpha} - \frac{1}{\sqrt{\beta}}\right)$

Product of the roots $= \frac{1}{\alpha^2} - \frac{1}{\beta} = \left(\frac{1}{\alpha} + \frac{1}{\sqrt{\beta}}\right) \cdot \left(\frac{1}{\alpha} - \frac{1}{\sqrt{\beta}}\right)$

Ans. A.

47. $x^2+2x-1=2$

$x^2+2x=3$

$x^2+2x+1=4$

$(x+1)^2=2^2$

$x+1=\pm 2$

$x = 1, -3$

Ans. C.

48. $C_0+\frac{C_1}{2}+\frac{C_2}{3}+\cdots+\frac{C_n}{n+1}$

$$\int_0^1 (1+x)^n dx = \int_0^1 C_0 dx + \int_0^1 C_1 x dx + \int_0^1 C_2 x^2 dx + \cdots + \int_0^1 C_n x^n dx$$

$$\left.\frac{(1+x)^{n+1}}{n+1}\right|_0^1 = C_0 \cdot x\Big|_0^1 + C_1 \left.\frac{x^2}{2}\right|_0^1 + C_2 \left.\frac{x^3}{3}\right|_0^1 + \cdots + C_n \left.\frac{x^{n+1}}{n+1}\right|_0^1$$

$$\frac{2^{n+1}-1}{n+1} = \frac{C_0}{1}+\frac{C_1}{2}+\cdots+\frac{C_n}{n+1}$$

Ans. A.

49. $\sum_{r=2}^{\infty} \frac{(r-1)r}{2\cdot r!}$

$$=\sum_{r=2}^{\infty} \frac{1}{2}\cdot\frac{1}{(r-2)!}$$

$$=\frac{1}{2}\cdot\sum_{r=2}^{\infty}\frac{1}{(r-2)!}$$

$$=\frac{1}{2}\cdot e$$

Ans. C.

50. $P=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}$

$$P^T=\begin{bmatrix}1 & 1\\ 2 & 3\\ 1 & 1\end{bmatrix}$$

$Q = PP^T$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+1 & 1+6+1 \\ 1+6+1 & 1+9+1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 \\ 8 & 11 \end{bmatrix}$$

$|Q| = 66 - 64 = 2$

Ans. A.

51. $1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33$

$$15\,\overline{)\;33\;(}\,2$$
$$\underline{30}$$
$$3$$

Required remainder = 3

Ans. B.

52. Putting $P = Q = R = \dfrac{\pi}{3}$

$$\begin{vmatrix} -1 & \cos R & \cos Q \\ \cos R & -1 & \cos P \\ \cos Q & \cos P & -1 \end{vmatrix} = \begin{vmatrix} -1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 \end{vmatrix}$$

$$= -\left(1 - \frac{1}{4}\right) - \frac{1}{2}\left(-\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{4} + \frac{1}{2}\right)$$

$$= -\frac{3}{4} + \frac{3}{4} = 0$$

Ans. B.

53. $(1-\alpha)x + 3y + 5z = 0$

$5x + (1-\alpha)y + 3z = 0$

$3x + 5y + (1-\alpha)z = 0$

$$\therefore \begin{vmatrix} 1-\alpha & 3 & 5 \\ 5 & 1-\alpha & 3 \\ 3 & 5 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow (1-\alpha)^3 + 27 + 125 - 45(1-\alpha) \qquad \left[\because \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = \left(a^3+b^3+c^3-3abc\right)\right]$$

$$\Rightarrow 1-\alpha^3 - 3\alpha(1-\alpha) + 152 - 45 + 45\alpha = 0$$

$$\Rightarrow 3\alpha^2 - 3\alpha - \alpha^3 + 108 + 45\alpha = 0$$

$$\Rightarrow -\alpha^3 + 3\alpha^2 + 42\alpha + 108 = 0$$

$$\Rightarrow \alpha^3 - 3\alpha^2 - 42\alpha - 108 = 0$$

It has one real solution and two imaginary solution.

Ans. A.

54. n(A) = 4

n(B) = 7

no. of mappings = $^7p_4 = \frac{7!}{3!}$

$= \frac{3! \times 4 \times 5 \times 6 \times 7}{3!} = 20 \times 6 \times 7 = 840$

Ans. D.

55. $2^{10} = C_0 + C_1 + \ldots + C_{10}$

$= C_0 + C_2 + C_4 + C_6 + C_8 + C_{10} = 2^9$

$2^7 = d_0 + d_1 + \ldots + d_7$

$= d_1 + d_3 + d_5 + d_7 = 2^6$

$\therefore \frac{P}{Q} = \frac{2^9}{2^6} = 2^3 = 8$

Ans. B.

56. Total no. of possible outcomes = $^{104}C_{26}$, which are equally likely.

Number of casses that the player gets all distinct cards = $\left(^2C_1\right)^{26} \times {}^{52}C_{26}$

$= 2^{26} \times {}^{52}C_{26}$

$\therefore$ Required probability = $\frac{2^{26} \times {}^{52}C_{26}}{^{104}C_{26}}$

Ans. D.

57. case I $\rightarrow$ 2R, 1W

8R
5W

case II $\rightarrow$ 1R, 2W

$$\frac{{}^{8}C_2 \times {}^{5}C_1 + {}^{5}C_2 \times {}^{8}C_1}{{}^{13}C_3}$$

$$= \frac{\frac{8.7}{2} \times 5 + \frac{5.4}{2} \times 8}{\frac{13 \times 12 \times 11}{6}}$$

$$= \frac{(140 + 80) \times 6}{13 \times 12 \times 11} = \frac{220}{13 \times 22} = \frac{10}{13}$$

Ans. D.

58. Let X = the event that outcame is head

Given that

A	B
H/T	H/H

$P(A) = \frac{3}{4}$ $\quad P = (B) = \frac{1}{4}$

$$\therefore \quad P\left(\frac{B}{X}\right) = \frac{1/4 \times 1}{\frac{3}{4} \times \frac{1}{2} + \frac{1}{4} \times 1} = \frac{1/4}{\frac{3}{8} + \frac{1}{4}}$$

$$= \frac{1/4}{\frac{3+2}{8}} = \frac{\frac{1}{4}}{\frac{5}{8}} = \frac{2}{5}$$

Ans. B.

59. $f : R \rightarrow R$

$g : R \rightarrow R$

$f(x) = x^2 + 2x - 3 = x^2 + 2x + 1 - 4 = (x+1)^2 - 4$

$g(x) = x + 1$

$f[g(x)] = f(x+1)$

$= (x+2)^2 - 4$

$= x^2 + 4x + 4 - 4 = x^2 + 4x$

$g[f(x)] = g\left[(x+1)^2 - 4\right]$

$= (x+1)^2 - 4 + 1 = (x+1)^2 - 3$

$= x^2 + 2x + 1 - 3 = x^2 + 2x - 2$

$\therefore \quad x^2 + 4x = x^2 + 2x - 2$

$\Rightarrow 2x = -2 \Rightarrow x = -1$

Ans. A.

60. $2b = a + c$

$ax^2 - 2bx + c = 0$

$ax^2 - (a + c)x + c = 0$

$ax^2 - ax - cx + c = 0$

$ax(x - 1) - c(x - 1) = 0$

$(ax - c)\,(x - 1) = 0$

$x = \frac{c}{a}, \quad x = 1$

Ans. A.

61. $x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$

$\frac{1}{x} \cdot \frac{dx}{dy} = \frac{1}{y^2} - \frac{1}{y}\log x \qquad \log x = t$

$\frac{1}{x} \cdot \frac{dx}{dy} = \frac{dt}{dy}$

or, $\frac{dt}{dy} = \frac{1}{y^2} - \frac{t}{y}$

or, $\frac{dt}{dy} + \frac{t}{y} = \frac{1}{y^2}$

$I.F. = e^{\int \frac{1}{y}dy} = e^{\log y} = y$

$\therefore \quad \int d(t.y) = \int \frac{1}{y}dy + k$

$t.y = \log y + k$

$y.\log x = \log y + k$

$1 \times 0 = \log 1 + k$

$\therefore \quad k = 0$

$\therefore \quad y.\log_e x = \log_e y \quad \therefore \; y = x^y$

Ans. B.

62. Solving, $\sin^{-1} x + x(1 - x) = \sin^{-1} x - x(1 - x)$

or, $2x(1 - x) = 0$

$\therefore \; x = 0, \; 1$

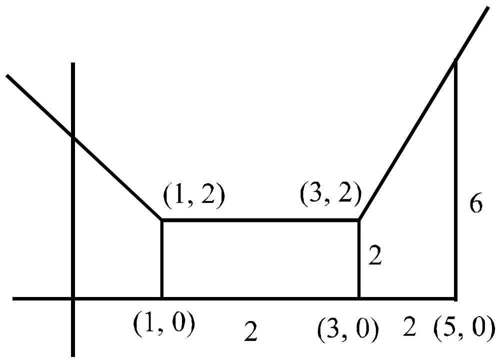
$\therefore \quad$ Required area $= \int_0^1 2x(1 - x)dx$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = 2 \times \frac{1}{6} = \frac{1}{3} \text{ sq. unit.}$$

Ans. C.

63.



Area $= 2 \times 2 + \frac{1}{2}(2+6).2$

$= 4 + \frac{1}{2} \times 8 \times 2$

$= 4 + 8 = 12$ sq. unit

Ans. C.

64. $h(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x)$

$h''(x) = f''(x) - g''(x)$

$h''(x) = 0$

$\therefore h'(x) = c$

$f'(x) - g'(x) = c$

$f'(1) - g'(1) = c$

$\therefore 4 - 6 = c$

$\therefore c = -2$

$f'(x) - g'(x) = -2$

$f(x) - g(x) = -2x + c'$

$f(2) - g(2) = -2.2 + c'$

$3 - 9 = -4 + c'$

$-6 = -4 + c \Rightarrow c' = -2$

$f(x) - g(x) = -2x - 2$

$f(1) - g(1) = -2.1 - 2$

$= -4$

Ans. B.

65. $\int_{-1}^{1} (|x| - 2[x])dx$

$= \int_{-1}^{1} |x|dx - 2\int_{-1}^{1} [x]dx$

$= \int_{-1}^{0} (-x)dx + \int_{0}^{1} xdx - 2\left[\int_{-1}^{0} (-1)dx + 0\right]$

$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} + 2[x]_{-1}^{0}$

$= -\left(-\frac{1}{2}\right) + \frac{1}{2} + 2$

$= 3$

Ans. A.

66. $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$

$\therefore$ z lies on a circle

Ans. A.

67. a, b, c are in G.P.

$\therefore\ b^2 = ac\ \ \therefore 2\log b = \log a + \log c$ .... (1)

Now, $a^p = b^q = c^r$

$\therefore \quad p \log a = q \log b = r \log c$

$\therefore \dfrac{\log a}{\log b} = \dfrac{q}{p}, \quad \dfrac{\log c}{\log b} = \dfrac{q}{r}$

From (1)

$\therefore \dfrac{q}{p} + \dfrac{q}{r} = 2$

$\Rightarrow \dfrac{1}{p} + \dfrac{1}{r} = \dfrac{2}{q}$

$\therefore$ p, q, r in H.P.

Ans. C.

68. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{s_k}$

$$= \sum_{k=1}^{\infty} \frac{\frac{(-1)^k}{k}}{1 - \frac{k}{k+1}}$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)}$$

$$= \sum_{k=1}^{\infty} (-1)^k \left[\frac{1}{k} - \frac{1}{k+1}\right]$$

$$= -1 + \frac{1}{2} + \frac{1}{2} - \frac{1}{3} - \frac{1}{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{5} - \frac{1}{5} + \frac{1}{6} + \frac{1}{6} \ldots\ldots.$$

$$= -\frac{2}{3} + \frac{2}{4} - \frac{2}{5} + \frac{2}{6} \ldots\ldots\ldots$$

$$= \ 2\left[-\frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} \ldots\ldots.\right]$$

$$= 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} \ldots\ldots\right] - 1$$

$= -2\left[-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}.....\right]-1$

$= -2\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}.....\right]+1$

$= -2\ \ln 2 + 1$

$= 1 - \ln 4$

Ans. D.

69. $\therefore$ O lies between the roots

$\therefore\ f(0) < 0$

$\Rightarrow a^2 - 4a < 0$

$\Rightarrow a(a-4) < 0$

$\Rightarrow 0 < a < 4$

Ans. B.

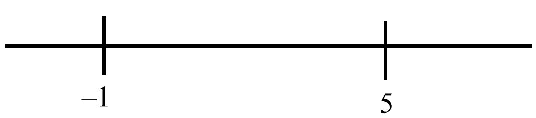
70. $\log_e\left(x^2-16\right) \le \log_e(4x-11)$

$x^2 - 16 \le 4x - 11$

$x^2 - 4x - 5 \le 0$

$x^2 - 5x + x - 5 \le 0$

$(x-5)(x+1) \le 0$



$-1 \le x \le 5$

$4x - 11 > 0 \ \Rightarrow x > \frac{11}{4}$

$x^2 - 16 > 0 \ \Rightarrow x > 16$

$\Rightarrow x > 4,\ x < -4$

Req. Answer $4 < x \le 5$

Ans. A.

71. $1+(1+x)+(1+x)^2+....+(1+x)^{20}$

$= 1.\frac{(1+x)^{21}-1}{(1+x)-1}$

$= \frac{(1+x)^{21}-1}{x}$

$\therefore$ Required coefficient = $^{21}C_{11}$

[From N^r found coefficien of x^{11}]

Ans. C.

72. To get infinite no of solution.

u = 3

Now $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix} = 0$

$\lambda\begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0$

or, $\lambda(-1+2)+1(+2-1)+1(1-1)=0$

or, $\lambda+1=0$

$\therefore$ $\lambda = -1$

Ans. B.

73. $P(A^C) = 0.3$ | $P(B) = 0.4$

$P(A) = 0.7$ | $P(B^C) = 0.6$

$P(A \cap B^C) = 0.5$

$P(B / A \cup B^C)$

$= \frac{P(B \cap (A \cup B^C))}{P(A \cup B^C)}$

$= \frac{P((B \cap A) \cup \phi)}{P(A \cup B^C)}$

$$= \frac{P(B \cap A)}{P(A \cup B^C)}$$

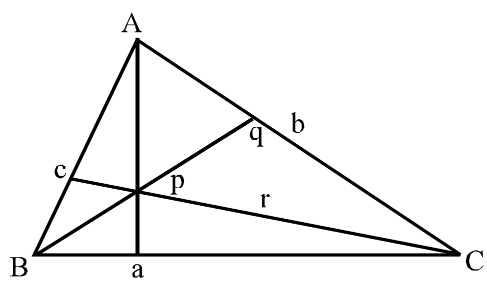
$$= \frac{P(A) - P(A \cap B^C)}{P(A \cup B^C)}$$

$$= \frac{0.7 - 0.5}{P(A) + P(B^C) - P(A \cap B^C)}$$

$$= \frac{0.2}{0.7 + 0.6 - 0.5} = \frac{1}{4}$$

Ans. A.

74.



$$S = \frac{1}{2}ap = \frac{1}{2}bq = \frac{1}{2}cr,$$

where a, b, c are the sides of the triangle

$\therefore$ a + b + c = 2t

Now $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{a}{2s} + \frac{b}{2s} + \frac{c}{2s} = \frac{2t}{2s} = \frac{t}{s}$

Ans. B.

75. $x^2 + y^2 = 4$

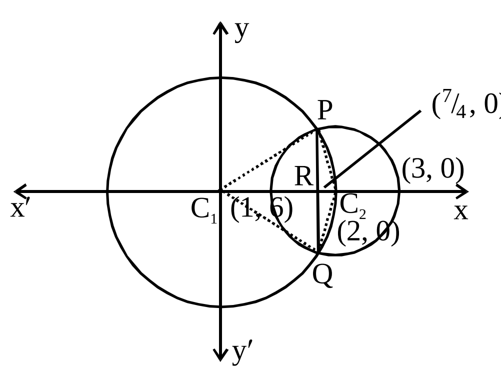
$(x-2)^2 + y^2 = 1$

equation of common chord.

$x^2 + y^2 - 4 - \left[(x-2)^2 + y^2 - 1\right] = 0$

$x^2 + y^2 - 4 - \left[x^2 - 4x + 4 + y^2 - 1\right] = 0$

$x^2 + y^2 - 4 - x^2 - y^2 + 4x - 3 = 0$



$4x = 7$

$x = \frac{7}{4}$

Here $R = \left(\frac{7}{4},\ 0\right)$

$C_1Q = 2$

$RQ = \sqrt{4 - \frac{49}{16}} = \frac{\sqrt{15}}{4}$

$\therefore\ PQ = \frac{\sqrt{15}}{2}$

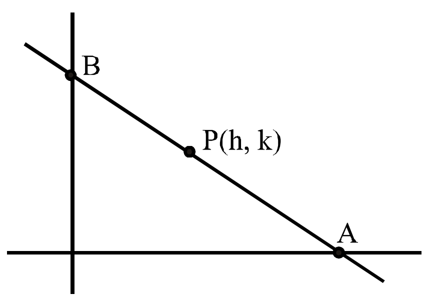
$\Delta C_1PQ = \frac{1}{2}.\frac{7}{4}.\frac{\sqrt{15}}{2} = \frac{7.\sqrt{15}}{16}$

$\Delta C_2PQ = \frac{1}{2}.\frac{1}{4}.\frac{\sqrt{15}}{2} = \frac{\sqrt{15}}{16}$

$\therefore\ \frac{\Delta C_1PQ}{\Delta C_2PQ} = \frac{7}{1}$

Ans. C.

76.



The point of intersectgion of the given lines is $\left(\frac{4}{3}, \frac{4}{3}\right)$.

Any line through this point is $y - \frac{4}{3} = m\left(x - \frac{4}{3}\right)$

Then coordinates of A & B are $A\left(\frac{4(m-1)}{3m}, 0\right)$ and $B\left(0, \frac{4(1-m)}{3}\right)$

Let P (h, k) be the mid-point of AB. Then $h = \frac{2(m-1)}{3m}$ and $k = \frac{2(1-m)}{3}$.

Eliminating 'm' from the above two repations, we get $2(h + k) = 3hk$.

$\therefore$ The required locus is $2(x+y) = 3xy$.

Ans. B.

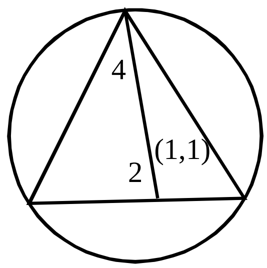
77. Any chord of a parabola $y^2 = 4ax$, which subtends right angle at the vertex, always pass through a fixed point (4a, 0) on the axis of the parabola.

$\therefore$ In this case R is (4, 0)

$\therefore$ The distance of R from the vertex is 4.

Ans. C.

78.



The incentre, circumcentre, centroid of an equlateral triangle are same.

$\therefore$ Inradius $= \dfrac{|3(1)+4(1)+3|}{\sqrt{3^2+4^2}} = 2$

$\therefore$ Circumradius = 4

$\therefore$ Circumcircle is $(x-1)^2 + (y-1)^2 = 4^2$

i.e., $x^2 + y^2 - 2x - 2y - 14 = 0$

Ans. B.

79. Let $L = \lim\limits_{n\to\infty}\left(\dfrac{n!}{n^n}\right)^{\frac{1}{n}}$

$\therefore \log L = \lim\limits_{n\to\infty}\dfrac{1}{n}\log\left(\dfrac{1}{n}\cdot\dfrac{2}{n}\ldots.\dfrac{n}{n}\right)$

$= \lim\limits_{n\to\infty}\dfrac{1}{n}\left[\log\dfrac{1}{n} + \log\dfrac{2}{n} + \ldots. + \log\dfrac{n}{a}\right]$

$= \lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{r=1}^{n}\log\left(\dfrac{r}{n}\right)$

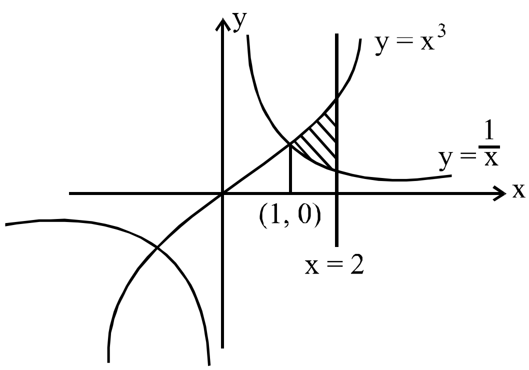
$= \int_0^1 \log x \, dx$

$= \left[(x\log x - x)\right]_0^1$

$= -1$

$\therefore L = e^{-1} = \dfrac{1}{e}$

Ans. D.

80. 

$\therefore$ Area $= \int_1^2 \left(x^3 - \frac{1}{x}\right)dx$

$= \left[\frac{x^4}{4} - \log x\right]_1^2$

$= \left(\frac{16}{4} - \frac{1}{4}\right) - (\log 2 - \log 1)$

$= \left(\frac{15}{4} - \log 2\right)$ sq. unit.

Ans. D.