

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

22 April 2014 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 You are given the following information in respect of a pension fund:

<i>Calendar Year</i>	<i>Value of fund at 1 January</i>	<i>Value of fund at 30 June</i>	<i>Net cash flow received on 1 July</i>
2011	£870,000	£872,000	£26,000
2012	£914,000	£902,000	£27,000
2013	£953,000	£962,000	£33,000
2014	£990,000		

Calculate, to the nearest 0.1%, the annual effective money-weighted rate of return earned by the fund during the period from 1 January 2011 to 1 January 2014. [4]

2 Describe the main features of:

- (a) debenture stocks.
- (b) unsecured loan stocks. [5]

3 £900 accumulates to £925 in four months.

Calculate the following:

- (i) the nominal rate of interest per annum convertible half-yearly [2]
 - (ii) the nominal rate of discount per annum convertible quarterly [2]
 - (iii) the simple rate of interest per annum [2]
- [Total 6]

4 A company issues a loan stock bearing interest at a rate of 8% per annum payable half-yearly in arrear. The stock is to be redeemed at 103% on any coupon payment date in the range from 20 years after issue to 25 years after issue inclusive, to be chosen by the company.

An investor, who is liable to income tax at 30% and tax on capital gains at 40%, bought the stock at issue at a price which gave her a minimum net yield to redemption of 6% per annum effective.

Calculate the price that the investor paid. [7]

- 5** On 25 October 2008 a certain government issued a 5-year index-linked stock. The stock had a nominal coupon rate of 3% per annum payable half-yearly in arrear and a nominal redemption price of 100%. The actual coupon and redemption payments were index-linked by reference to a retail price index as at the month of payment.

An investor, who was not subject to tax, bought £10,000 nominal of the stock on 26 October 2012. The investor held the stock until redemption.

You are given the following values of the retail price index:

	2008	-----	2012	2013
April	-----	-----	-----	171.4
October	149.2	-----	169.4	173.8

- (i) Calculate the coupon payment that the investor received on 25 April 2013 and the coupon and redemption payments that the investor received on 25 October 2013. [3]
- (ii) Calculate the purchase price that the investor paid on 25 October 2012 if the investor achieved an effective real yield of 3.5% per annum effective on the investment. [4]
- [Total 7]

- 6** An insurance company has liabilities of £10 million due in 10 years' time and £20 million due in 15 years' time. The company's assets consist of two zero-coupon bonds. One pays £7.404 million in 2 years' time and the other pays £31.834 million in 25 years' time. The current interest rate is 7% per annum effective.

- (i) Show that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this insurance company. [6]
- (ii) Calculate the present value of profit that the insurance company will make if the interest rate increases immediately to 7.5% per annum effective. [2]
- (iii) Explain, without any further calculation, why the insurance company made a profit as a result of the change in the interest rate. [2]
- [Total 10]

- 7** Six months ago, an investor entered into a one-year forward contract to purchase a non-dividend paying stock. The risk-free force of interest was 4% per annum. The value of the stock is now 98% of its original value.

Calculate the minimum value for the risk-free force of interest at which the original forward contract still has a positive value to the investor. [6]

8 An insurance company borrows £50 million at an effective interest rate of 9% per annum. The insurance company uses the money to invest in a capital project that pays £6 million per annum payable half-yearly in arrear for 20 years. The income from the project is used to repay the loan. Once the loan has been repaid, the insurance company can earn interest at an effective interest rate of 7% per annum.

- (i) Calculate the discounted payback period for this investment. [4]
- (ii) Calculate the accumulated profit the insurance company will have made at the end of the term of the capital project. [5]
- [Total 9]

9 The effective n -year spot rate of interest y_n , is given by:

$$y_n = 0.035 + \frac{n}{1000} \quad \text{for } n = 1, 2 \text{ and } 3$$

- (i) Determine the implied one-year forward rates applicable at times $t = 1$ and $t = 2$ to four significant figures. [4]
- (ii) Calculate, assuming no arbitrage:
- (a) The price at time $t = 0$ per £100 nominal of a bond which pays annual coupons of 4% in arrear and is redeemed at 105% per £100 nominal after three years.
- (b) The two-year par yield.

[6]
[Total 10]

10 A loan of £20,000 is repayable by an annuity payable annually in arrear for 25 years. The annual repayment is calculated at an effective interest rate of 8% per annum and increases by £50 each year.

- (i) Calculate the amount of the first payment. [3]
- (ii) Calculate the capital outstanding after the first three payments have been made. [2]
- (iii) Explain your answer to part (ii). [2]
- (iv) Calculate the total amount of interest paid over the term of the loan. [3]
- [Total 10]

- 11** An individual can obtain a force of interest per annum at time t , measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \leq t < 4 \\ 0.07 & 4 \leq t < 6 \\ 0.09 & 6 \leq t \end{cases}$$

- (i) Calculate the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of \$3,000 per annum from time $t = 4$ to $t = 10$. [6]
- (ii) Calculate the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]
- [Total 9]

- 12** An investor is considering investing £18,000 for a period of 12 years. Let i_t be the effective rate of interest in the t^{th} year, $t \leq 12$. Assume, for $t \leq 12$, that i_t has mean value of 0.08 and standard deviation 0.05 and that $1 + i_t$ is independently and lognormally distributed.

- (i) Determine the distribution of S_{12} where S_t is the accumulation of £1 over t years. [5]

At the end of the 12 years the investor intends to use the accumulated amount of the investment to purchase a 12-year annuity certain paying:

£4,000 per annum monthly in advance during the first four years;
£5,000 per annum quarterly in advance during the second four years;
£6,000 per annum continuously during the final four years.

The effective rate of interest will be 7% per annum in years 13 to 18 and 9% per annum in years 19 to 24 where the years are counted from the start of the initial investment

- (ii) Calculate the probability that the investor will meet the objective. [12]
- [Total 17]

END OF PAPER

