

---

<b>Course Code</b>	<b>: MCS-053</b>
<b>Course Title</b>	<b>: Computer Graphics and Multimedia</b>
<b>Assignment Number</b>	<b>: MCA (5)/053/Assign /2013</b>
<b>Maximum Marks</b>	<b>: 100</b>
<b>Weightage</b>	<b>: 25%</b>
<b>Last Dates for Submission</b>	<b>: 15th October, 2013 (For July 2013 Session) 15th April, 2014 (For January 2014 Session)</b>

**There are fifteen questions in this assignment. Answer all the questions. 20 Marks are for viva-voce. You may use illustrations and diagrams to enhance explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation.**

**Question 1:** **(6 Marks)**

Differentiate between following:

(i) Painting and Drawing

**Answer:**

**The differences between Painting and Drawing**

<b>Painting</b>	<b>Drawing</b>
<b>Painting</b> functions, on the other hand, don't create objects. If you look at a computer screen, you'll see that it's made up of millions of tiny dots called pixels. You'll see the same thing in a simpler form if you look at the colour comics in the Sunday newspaper—lots of dots of different colour ink that form a picture. Unlike a drawing function, a paint function changes the colour of individual pixels based on the tools you choose. In a photograph of a person's face, for example, the colours change gradually because of light, shadow and complexion. You need a paint function to create this kind of effect; there's no object that you can select or move the way you can with the drawn square, i.e., a painting program allows the user to paint arbitrary swaths using a brush various size, shape, colour and pattern. More painting programs allows placement of such predefined shapes as rectangles, polygon and canvas. Any part of the canvas can be edited at the pixel level.	<b>Drawing</b> is a software application means using tools that create "objects," such as squares, circles, lines or text, which the program treats as discrete units. If you draw a square in PowerPoint, for example, you can click anywhere on the square and move it around or resize it. It's an object, just like typing the letter "e" in a word processor, i.e., a drawing program allows a user to position standard shape (also called symbols, templates, or objects) which can be edited by translation, rotations and scaling operations on these shapes.

The reason why the differences are important is that, as noted earlier, many different kinds of programs offer different kinds of graphics features at different levels of sophistication, but they tend to specialise in one or the other.

(ii) Computer Graphics and Animation

**Answer:**

**The differences between Computer Graphics and Animation:**

Animation is a time based phenomenon for imparting visual changes in any scene according to any time sequence, the visual changes could be incorporated through translation of object, scaling of object, or change in colour, transparency, surface texture etc., whereas Graphics does not contain the dimension of time.

**Graphics + Dimension of Time = Animation**

(iii) Printer and Plotter

**Answer:**

**The differences between printers and plotters:**

(a) Plotters print their output by moving a pen across the surface of piece of a paper. This means that plotters are restricted to line art, rather than raster graphics as with other printers. They can draw complex line art, including text, but do so very slowly because of mechanical movement of pen.

(b) Another difference between plotter and printer is that the printer is aimed primarily at printing text. Thus, the printer is enough to generate a page of output, but this is not the case with the line art on a plotter.

(iv) Random Scan Display Devices and Raster Scan Display Devices

**Answer:**

**The differences between Random Scan Display Devices and Raster Scan Display Devices:**

(a) In Random Scan system the Display buffer stores the picture information, further the device is capable of producing pictures made up of lines but not of curves. Thus, it is also known as “Vector display device or Line display device or Calligraphic display device.

(b) In Raster Scan system the Frame buffer stores the picture information which is the bit plane (with m rows and n columns) because of this type of storage the system is capable of producing realistic images, but the limitation is that the line segments may not appear to be smooth.

**Question 2:**

**(5 Marks)**

Write a program in C/C++ to generate line segment between two points, by using DDA line generation Algorithm. Your program should map each and every step of pseudo algorithm, in the form of comments.

**Answer:**

```
#include <windows.h> // Header File For The Windows Library
#include <gl/glut.h> // Header File For The OpenGL32 Library
#include <math.h> // Header File For The Math Library
#include <stdio.h> // Header File For Standard Input/Output
const float PI=3.14;

void drawLine(int x0,int y0,int x1,int y1){ // input the line endpoint and
    glBegin(GL_POINTS); // store the left endpoint in (x0, y0)
    glColor3f(1.0,1.0,1.0); // and right endpoint (x1,y1)
    double m=(double)(y1-y0)/(x1-x0); // calculate the values of Δx and Δy
    double y=(double)y0; // using Δx = x1 - x0, Δy = y1 - y0
    double x=(double)x0;
    if(m<1) { // if the value of Δx <= Δx1 assign values of
        while(x<=x1) { // steps as Δx otherwise the values of steps as Δy

            glVertex2d(x,floor(y));
            printf("%f %f\n",floor(y),x);
            y=y+m; // calculate the values of x and y increment
            x++; // and assign the value x++ and y=y+m
        }
    }
}
```

```

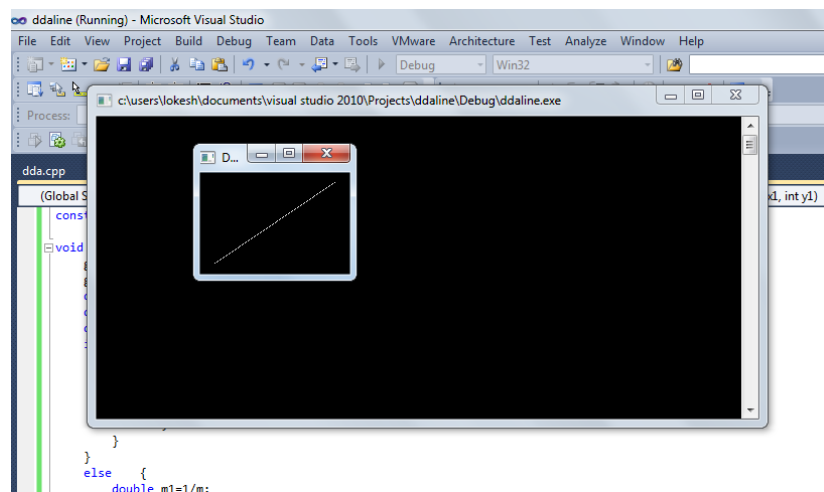
else {
    double m1=1/m;
    while(y<=y1) { // while (y <= y1) compute the pixel and
        glVertex2d(floor(x),y); // plot the pixel with y increment and x=x+m1
        y++;
        x=x+m1;
    }
}
glEnd();
}

void init(void){ // init() method to set the color and projection
    glClearColor(0.0,0.0,0.0,0.0);
    glMatrixMode(GL_PROJECTION);
    glLoadIdentity();
    glOrtho(0.0,100.0,0.0,100.0,0.0,100.0);
}
void display(void) // method to draw the line
{
    glClear(GL_COLOR_BUFFER_BIT);
    drawLine(10,10,90,90); // value of the line according to coordinates
    glutSwapBuffers();
}

int main(int argc, char** argv){ // main function
    glutInit(&argc,argv); // calling glutInit() method
    glutInitDisplayMode(GLUT_DOUBLE|GLUT_RGB); // calling glutInitDisplayMode() method
    glutInitWindowSize(100,100); // Setting of the windows size
    glutInitWindowPosition(200,100); // setting windows position
    glutCreateWindow("DDA Line Drawing!"); // creating windows header name
    init(); // calling init() method
    glutDisplayFunc(display); // calling display method to draw the line
    glutMainLoop();
    return 0;
}

```

**Output:**



**Question 3:****(5 Marks)**

Draw line segment joining (20, 10) and (25, 14) by using Bresenham Line Generation algorithm.

**Answer:**

$$(x_0, y_0) \rightarrow (20, 10) \quad ; \quad (x_1, y_1) \rightarrow (25, 14)$$

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{14 - 10}{25 - 20} = \frac{4}{5} < 1$$

$$\text{As, } m = \frac{\Delta y}{\Delta x} = \frac{4}{5} \Rightarrow \Delta y = 4$$

$$\Delta x = 5$$

→ plot point (20, 10)

$$p_i = 2\Delta y - \Delta x$$

$$i = 1: \quad p_1 = 2 * 4 - 5 = 3$$

as  $p_1 > 0$  so  $x_0 \leftarrow 21; y_0 \leftarrow 11$ 

now plot (21, 11)

$$i = 2 \text{ as } p_1 > 0$$

$$\therefore p_2 = p_1 + 2(\Delta y - \Delta x)$$

$$= 3 + 2(4 - 5) = 3 - 2 = 1$$

 $p_2 > 0$  so  $x_0 \leftarrow 22; y_0 \leftarrow 12$ 

plot (22, 12)

$$i = 3 \text{ as } p_2 > 0$$

$$\therefore p_3 = p_2 + 2(\Delta y - \Delta x) = 1 + 2(4 - 5) = -1$$

$$p_3 < 0 \quad \therefore x_0 \leftarrow 23$$

$$y_0 \leftarrow 12$$

plot (23, 12)

$$i = 4 \text{ as } p_3 < 0$$

$$\therefore p_4 = p_3 + 2\Delta y$$

$$= -1 + 2 * 4 = 7$$

$$\therefore x_0 \leftarrow 24; y_0 \leftarrow 13$$

plot (24, 13)

$i = 5$  as  $p_4 > 0$

$$\begin{aligned} \therefore p_5 &= p_4 + 2(\Delta y - \Delta x) \\ &= 7 + 2(4 - 5) = 5 \end{aligned}$$

$$x_0 \leftarrow 25; y_0 \leftarrow 14$$

plot (25, 14)

{for  $i = 6$ ,  $x_0$  will be  $> x_i$  so algorithm terminates

#### Question 4:

(5 marks)

Given a circle radius  $r = 5$ , determine positions along the circle octants in 1<sup>st</sup> Quadrant from  $x = 0$  to  $x = y$ .

**Answer:**

An initial decision parameter  $p_0 = 1 - r = 1 - 5 = -4$

For circle centred on coordinate origin the initial point  $(x_0, y_0) = (0, 5)$  and initial increments for calculating decision parameter are:

$$2x_0 = 0, 2y_0 = 10$$

Using mid point algorithm point are:

$k$	$p_k$	$(x_{k+1}, y_{k-1})$	$2x_{k+1}$	$2y_{k-1}$
0	-4	(1, 5)	2	10
1	-1	(2, 5)	4	10
2	4	(3, 5)	6	10
3	-3	(4, 4)	8	8
4	6	(5, 4)	10	8
5	-5	(6, 3)	12	6
6	8	(7, 2)	14	4

#### Question 5:

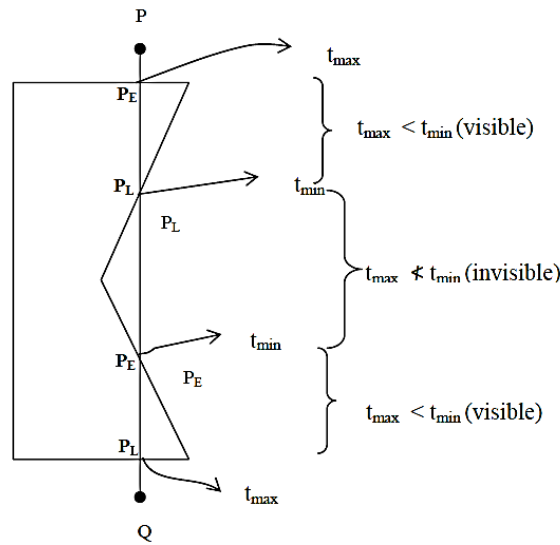
(5 Marks)

How Cyrus Back line clipping algorithm, clips a line segment, if the window is non-convex?

**Answer:**

Consider the *Figure below*, here, the window is non-convex in shape and PQ is a line segment passing through this window. Here too the condition of visibility of the line is  $t_{max} < t_{min}$  and the line is visible from  $P + t_{max}(Q - P)$  to  $P + t_{min}(Q - P)$ , if  $t_{max} \not< t_{min}$  then reject the line segment. Now, applying this rule to the *Figure 13*, we find that when PQ line segment passes through the non-convex window, it cuts the edges of the window at 4 points.  $1 \rightarrow PE$ ;  $2 \rightarrow PL$ ;  $3 \rightarrow PE$ ;  $4 \rightarrow PL$ . In this example, using the algorithm we reject the line segment PQ but it is not the correct result.

Condition of visibility is satisfied in region 1-2 and 3-4 only so the line exists there but in region 2-3 the condition is violated so the line does not exist.

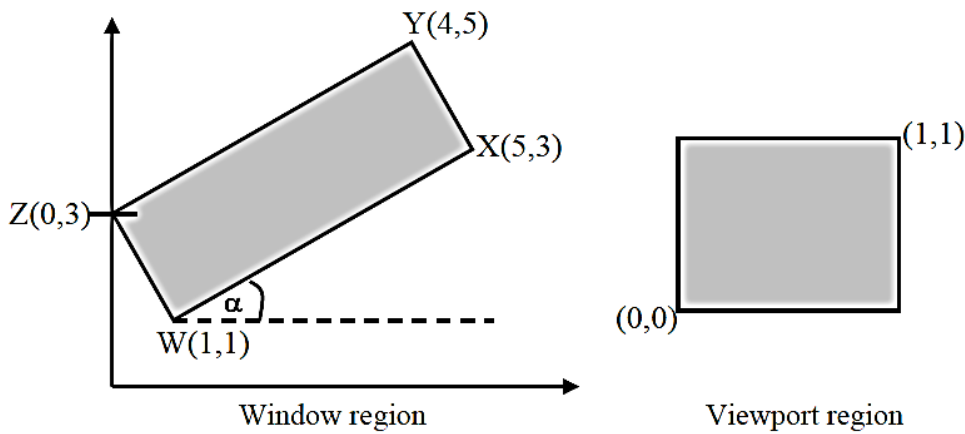


**Question 6:**

(5 Marks)

Find the normalization transformation  $N$ , which uses the rectangle  $W(1, 1)$ ;  $X(5, 3)$ ;  $Y(4, 5)$  and  $Z(0, 3)$  as a window and the normalized device screen as viewpoint.

**Answer:**



**Figure: Example Transformations**

Here, we see that the window edges are not parallel to the coordinate axes. So we will first rotate the window about W so that it is aligned with the axes.

$$\text{Now, } \tan \alpha = \frac{3-1}{5-1} = \frac{1}{2}$$

$$\Rightarrow \sin \alpha = \frac{1}{\sqrt{5}} \quad \cos \alpha = \frac{2}{\sqrt{5}}$$

Here, we are rotating the rectangle in clockwise direction. So  $\alpha$  is (-)ve i.e.,  $-\alpha$

The rotation matrix about W (1, 1) is,

$$[T_{R,\theta}]_W = \begin{bmatrix} \cos \alpha & -\sin \alpha & (1-\cos \alpha) x_p + \sin \alpha y_p \\ \sin \alpha & \cos \alpha & (1-\cos \alpha) y_p - \sin \alpha x_p \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T_{R,\theta}]_W = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \left(\frac{1-3}{\sqrt{5}}\right) \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \left(\frac{1-1}{\sqrt{5}}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

The x extent of the rotated window is the length of WX which is  $\sqrt{(4^2 + 2^2)} = 2\sqrt{5}$

Similarly, the y extent is length of WZ which is  $\sqrt{(1^2 + 2^2)} = \sqrt{5}$

For scaling the rotated window to the normalised viewport we calculate  $s_x$  and  $s_y$  as,

$$s_x = \frac{\text{viewport x extent}}{\text{window x extent}} = \frac{1}{2\sqrt{5}}$$

$$s_y = \frac{\text{viewport y extent}}{\text{window y extent}} = \frac{1}{\sqrt{5}}$$

As in expression (1), the general form of transformation matrix representing mapping of a window to a viewport is,

$$[T] = \begin{bmatrix} s_x & 0 & -s_x xw_{\min} + xv_{\min} \\ 0 & s_y & -s_y yw_{\min} + yv_{\min} \\ 0 & 0 & 1 \end{bmatrix}$$

In this problem  $[T]$  may be termed as  $N$  as this is a case of normalisation transformation with,

$$\begin{aligned} xw_{\min} &= 1 & xv_{\min} &= 0 \\ yw_{\min} &= 1 & yv_{\min} &= 0 \\ s_x &= \frac{1}{2\sqrt{5}} & s_y &= \frac{1}{\sqrt{5}} \end{aligned}$$

By substituting the above values in  $[T]$  i.e.  $N$ ,

$$N = \begin{pmatrix} \frac{1}{2\sqrt{5}} & 0 & \left(\frac{-1}{2}\right)\frac{1}{\sqrt{5}} + 0 \\ 0 & \frac{1}{\sqrt{5}} & \left(\frac{-1}{\sqrt{5}}\right)1 + 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Now, we compose the rotation and transformation  $N$  to find the required viewing transformation  $N_R$

$$N_R = N [T_{R,\theta}]_w = \begin{pmatrix} \frac{1}{2\sqrt{5}} & 0 & \frac{-1}{2\sqrt{5}} \\ 0 & \frac{1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{5} & 1 - \frac{3}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 1 - \frac{1}{\sqrt{5}} \\ 0 & 0 & 1 \end{pmatrix}$$

### Question 7:

(5 Marks)

Show that two successive reflections about either of the coordinate axes is equivalent to a single rotation about the coordinate origin.

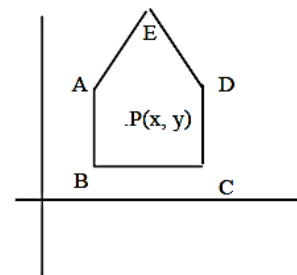
**Answer:**

Let  $(x, y)$  be any object point, as shown in *Figure (a)*. Two successive reflection of  $P$ , either of the coordinate axes, i.e., Reflection about  $x$ -axis followed by reflection about  $y$ -axis or *vice-versa* can be reprocuted as:

$$(x, y) \xrightarrow{M_x} (x, -y) \xrightarrow{M_y} (-x, -y) \quad \text{----(i)}$$

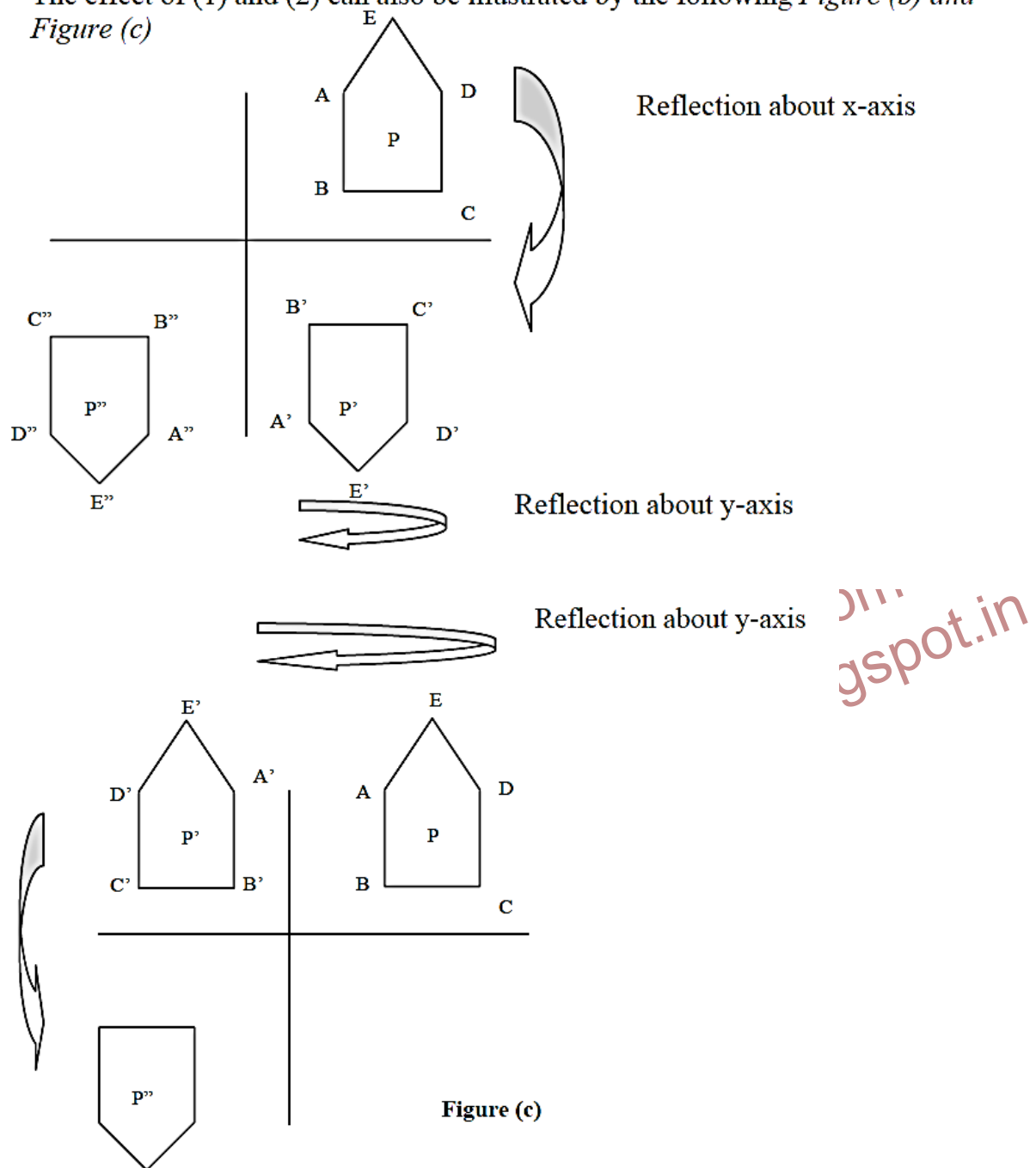
$$(x, y) \xrightarrow{M_y} (x, -y) \xrightarrow{M_x} (-x, -y) \quad \text{----(ii)}$$

**Figure (a)**





The effect of (1) and (2) can also be illustrated by the following *Figure (b)* and *Figure (c)*



**Figure (c)**

Reflection about x-axis

From equation (i) and (ii), we can write:

$$(x, y) \longrightarrow (-x, -y) = (x, y) \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{(iii)}$$

Equation (3) is the required reflection about the origin. Hence, two successive reflections about either of the coordinate axes is just equivalent to a single rotation about the coordinate origin.

**Question 8:****(6 Marks)**

Reject the Diamond-shaped polygon whose vertices are A(-1, 0), B(0, -2); C(1, 0) and D(0, 2) about (a) Horizontal line Y = 2; (b) the vertical line X = 2; (c) the line Y = X + 2.

**Answer:**

We can represent the given polygon by the homogeneous coordinate matrix as

$$V=[ABCD] = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

- a) The horizontal line  $y=2$  has an intercept (0,2) on y axis and makes an angle of 0 degree with the x axis. So  $m=0$  and  $c=2$ . Thus, the reflection matrix

$$M_L = T_{-v} \cdot R_{-\theta} \cdot M_x \cdot R_{\theta} \cdot T_v, \quad \text{where } v=0\mathbf{I}+2\mathbf{J}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

So the new coordinates A'B'C'D' of the reflected polygon ABCD can be found as:

$$[A'B'C'D'] = [ABCD] \cdot M_L$$

$$= \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 1 \\ 0 & 6 & 1 \\ 1 & 4 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Thus, A'=(-1,4), B'=(0,6), C'=(1,4) and D'=(0,2).

- b) The vertical line  $x=2$  has no intercept on y-axis and makes an angle of 90 degree with the x-axis. So  $m=\tan 90^\circ = \infty$  and  $c=0$ . Thus, the reflection matrix

$$M_L = T_{-v} \cdot R_{-\theta} \cdot M_y \cdot R_{\theta} \cdot T_v, \quad \text{where } v=2\mathbf{I}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

So the new coordinates A'B'C'D' of the reflected polygon ABCD can be found as:

$$[A'B'C'D'] = [ABCD] \cdot M_L$$

$$= \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 1 \\ 4 & -2 & 1 \\ 3 & 0 & 1 \\ 4 & 2 & 1 \end{pmatrix}$$

Thus, A'=(5,0), B'=(4,-2), C'=(3,0) and D'=(4,2)

- c) The line  $y=x+2$  has an intercept  $(0,2)$  on y-axis and makes an angle of  $45^\circ$  with the x-axis. So  $m=\tan 45^\circ=1$  and  $c=2$ . Thus, the reflection matrix

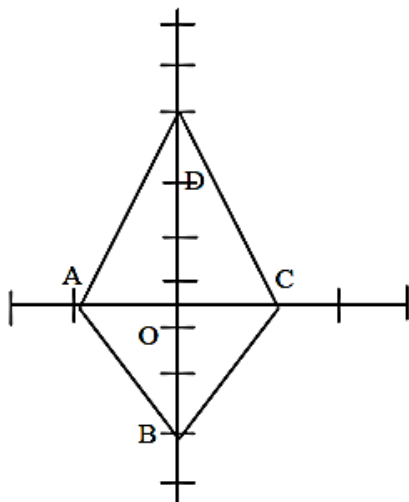
$$M_L = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{pmatrix}$$

The required coordinates  $A', B', C'$ , and  $D'$  can be found as:  
 $[A'B'C'D'] = [ABCD] \cdot M_L$

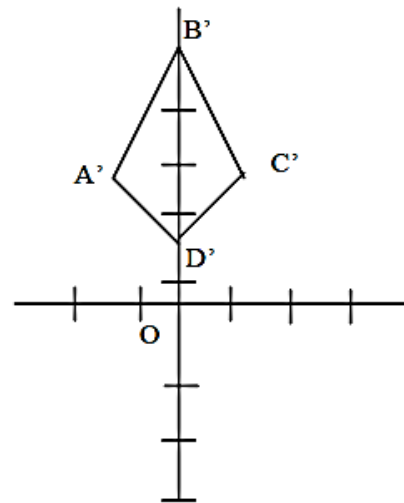
$$\begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 1 \\ -4 & 2 & 1 \\ -2 & 3 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Thus,  $A'=(-2,1)$ ,  $B'=(-4,2)$ ,  $C'=(-2,3)$  and  $D'=(0,2)$

The effect of the reflected polygon, which is shown in *Figure (a)*, about the line  $y=2$ ,  $x=2$ , and  $y=x+2$  is shown in *Figure (b) - (d)*, respectively.



**Figure (a)**



**Figure (b)**

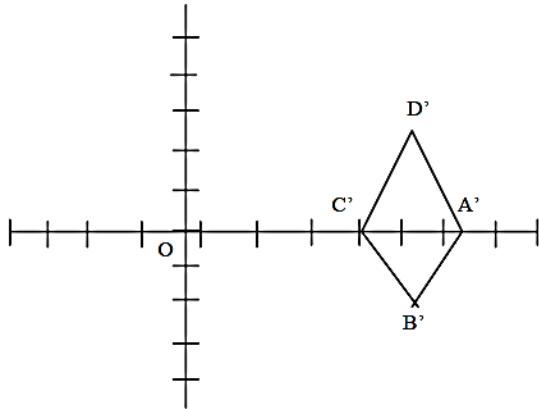


Figure (c)

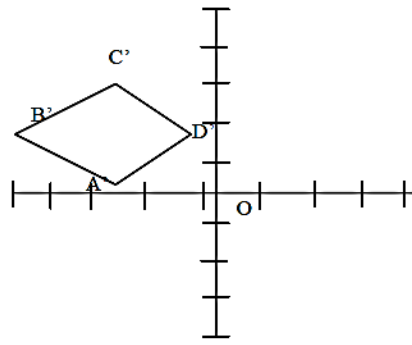


Figure (d)

**Question 9:****(5 Marks)**

Find the principal vanishing point, when the object is first rotated with respect to y-axis by  $-30^\circ$  and x-axis by  $45^\circ$ , and projected onto  $z = 0$  plane, with the centre of projection being  $(0, 0, -5)$ .

**Answer:**

Rotation about the y-axis with angle of rotation  $\theta = (-30^\circ)$  is

$$\begin{aligned}
 [R_y] &= [R_y]_{\theta=-30} = \begin{bmatrix} \cos(30^\circ) & 0 & -\sin(-30^\circ) \\ 0 & 1 & 0 \\ \sin(-30^\circ) & 0 & \cos(-30^\circ) \end{bmatrix} \\
 &= \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix}
 \end{aligned}$$

Similarly Rotation about the x-axis with angle of Rotation  $\phi 45^\circ$  is:

$$[R_x] = [R_x]_{45^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{aligned} \therefore [R_y] \cdot [R_x] &= \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{3}/2 & -1/2\sqrt{2} & 1/2\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1/2 & -3/2\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix} \text{-----(1)} \end{aligned}$$

**Projection:** Center of projection is E (0, 0, -5) and plane of projection is z = 0 plane.

For any point p (x, y, z) from the object, the Equation of the ray starting from E and passing through the point P is:

$$\begin{aligned} &E + t(P - E), t > 0 \\ \text{i.e.} \quad &(0, 0, -5) + t[(x, y, z) - (0, 0, -5)] \\ &= (0, 0, -5) + t(x, y, z + 5) \\ &= (t.x, t.y, -5 + t(z + 5)) \end{aligned}$$

for this point to be lie on z = 0 plane, we have:

$$-5 + t(z + 5) = 0$$

By: Mr. Lokesh Chandrasingh  
lokeshchandrasingh.com  
ignoumcaassignmentsolution.blogspot.in

$$\therefore t = \frac{5}{z+5}$$

$\therefore$  the projection point of  $p(x, y, z)$  will be:

$$P' = (x', y', z') = \left( \frac{5x}{z+5}, \frac{5y}{z+5}, 0 \right)$$

In terms of homogeneous coordinates, the projection matrix will become:

$$[P] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix} \text{-----(2)}$$

$$\therefore [R_y] \cdot [R_x] \cdot [P] = \begin{bmatrix} \sqrt{3}/2 & -1/2\sqrt{2} & 1/2\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1\sqrt{2} & -\sqrt{3}/2\sqrt{2} & \sqrt{3}/2\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5\sqrt{3}}{2} & \frac{-5}{2\sqrt{2}} & 0 & \frac{1}{2\sqrt{2}} \\ 0 & \frac{5}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-5}{2} & \frac{-5\sqrt{3}}{2\sqrt{2}} & 0 & \frac{\sqrt{3}}{2\sqrt{2}} \\ 0 & 0 & 0 & 5 \end{bmatrix} \text{-----(3)}$$

Let  $(x, y, z)$  be projected, under the combined transformation (3) to  $(x', y', z')$ , then

$$(x', y', z', 1) = (x, y, z, 1) \begin{bmatrix} \frac{5\sqrt{3}}{2} & \frac{-5}{2\sqrt{2}} & 0 & \frac{1}{2\sqrt{2}} \\ 0 & \frac{5}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-5}{2} & \frac{-5\sqrt{3}}{2\sqrt{2}} & 0 & \frac{\sqrt{3}}{2\sqrt{2}} \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$= x' = \frac{\left( \frac{5\sqrt{3}}{2}x - \frac{5}{2}z \right)}{\left( \frac{x}{2\sqrt{2}} + \frac{y}{\sqrt{2}} + \frac{\sqrt{3}z}{2\sqrt{2}} + 5 \right)}$$

and

$$y' = \frac{\left( \frac{-5}{2\sqrt{2}} \cdot x + \frac{5}{\sqrt{2}} \cdot y - \frac{5\sqrt{3}}{2\sqrt{2}} \cdot z \right)}{\left( \frac{x}{2\sqrt{2}} + \frac{y}{\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} \cdot z + 5 \right)} \quad \text{-----(4)}$$

**Case 1:** Principal vanishing point w.r.t the  $x$ -axis.

By considering first row of the matrix (Equation – (3)), we can claim that the principal vanishing point (w.r.t) the  $x$ -axis) will be:

$$\left( \frac{5\sqrt{3}}{2}, \frac{-5}{2\sqrt{2}}, 0 \right)$$

i.e.,  $(5\sqrt{6}, -5, 0)$  -----(I)

In order to verify our claim, consider the line segments AB, CD, which are parallel to the  $x$ -axis, where  $A = (0, 0, 0)$ ,  $B = (1, 0, 0)$ ,  $C = (1, 1, 0)$ ,  $D = (0, 1, 0)$

If  $A'$ ,  $B'$ ,  $C'$ ,  $D'$  are the projections of  $A$ ,  $B$ ,  $C$ ,  $D$ , respectively, under the projection matrix (3), then

$$A' = (0, 0, 0), B' = \left( \frac{5\sqrt{3}}{\frac{1}{2\sqrt{2}} + 5}, \frac{\frac{-5}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}} + 5}, 0 \right)$$

$$C' = \left( \frac{\frac{5\sqrt{3}}{2}}{\left( \frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}} + 5 \right)}, \frac{\left( \frac{-5}{2\sqrt{2}} + \frac{5}{\sqrt{2}} \right)}{\left( \frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}} + 5 \right)}, 0 \right)$$

$$D' = \left( 0, \frac{5/\sqrt{2}}{\left( \frac{1}{\sqrt{2}} + 5 \right)}, 0 \right) \quad \{\text{Using Equation (4)}\}$$

$$A' = (0,0,0), B' = \left( \frac{5\sqrt{6}}{1+10\sqrt{2}}, \frac{-5}{1+10\sqrt{2}}, 0 \right).$$

$$C' = \left( \frac{5\sqrt{6}}{3+10\sqrt{2}}, \frac{5}{3+10\sqrt{2}}, 0 \right) \text{ and}$$

$$D' = \left( 0, \frac{5}{1+5\sqrt{2}}, 0 \right)$$

Consider the line equation of A'B': The parametric Equation is:

$$A' + t(B' - A')$$

$$\begin{aligned} \text{i.e. } (0, 0, 0) + t \left( \frac{5\sqrt{6}}{1+10\sqrt{2}}, \frac{-5}{1+10\sqrt{2}}, 0 \right) \\ = \left( \frac{5t\sqrt{6}}{1+10\sqrt{2}}, \frac{-5t}{1+10\sqrt{2}}, 0 \right) \end{aligned}$$

we will verify that the vanishing point (I) lies on this line:

$$\text{i.e. } \left( \frac{5t\sqrt{6}}{1+10\sqrt{2}}, \frac{-5t}{1+10\sqrt{2}}, 0 \right) = (5\sqrt{6}, -5, 0)$$

$$= \frac{5t\sqrt{6}}{1+10\sqrt{2}} = 5\sqrt{6}$$

$$\text{and } \frac{-5t}{1+10\sqrt{2}} = -5 \quad \text{-----(5)}$$

must be true for some 't' value.

$$t = (1 + 10\sqrt{2})$$

then the equation (5) is true and hence (I) lies on the line A'B'.

Similarly consider the line equation C'D': The parametric Equation is:

$$C' + s(D' - C') \quad \text{i.e.}$$



$$\begin{aligned}
&= \left( \frac{5\sqrt{6}}{3+10\sqrt{2}}, \frac{5}{3+10\sqrt{2}}, 0 \right) + s \left[ \left( 0, \frac{5}{1+5\sqrt{2}}, 0 \right) - \left( \frac{5\sqrt{6}}{3+10\sqrt{2}}, \frac{5}{3+10\sqrt{2}}, 0 \right) \right] \\
&= \left( \frac{5\sqrt{6}}{3+10\sqrt{2}}, \frac{5}{3+10\sqrt{2}}, 0 \right) + s \left( \frac{-5\sqrt{6}}{3+10\sqrt{2}}, \frac{5}{1+5\sqrt{2}} - \frac{5}{3+10\sqrt{2}}, 0 \right) \text{ and} \\
&= \left( \frac{5\sqrt{6}}{3+10\sqrt{2}} - \frac{5s\sqrt{6}}{3+10\sqrt{2}}, \frac{5}{3+10\sqrt{2}} + \frac{5s(2+5\sqrt{2})}{(1+5\sqrt{2})(3+10\sqrt{2})}, 0 \right), \text{ but}
\end{aligned}$$

we have to verify that the vanishing point (I) lies on C'D'.

i.e. we have to show

$$\left( \frac{5\sqrt{6}}{3+10\sqrt{2}}(1-s) - \frac{5s\sqrt{6}}{3+10\sqrt{2}}, \frac{5}{3+10\sqrt{2}} \left( 1 + \frac{s(2+5\sqrt{2})}{(1+5\sqrt{2})} \right), 0 \right) = (5\sqrt{6}, -5, 0)$$

for some 's' value This holds true if

$$\left. \begin{aligned}
&\frac{5\sqrt{6}}{3+10\sqrt{2}}(1-s) = 5\sqrt{6} \\
\text{and } \frac{5}{3+10\sqrt{2}} \left( 1 + \frac{s(2+5\sqrt{2})}{(1+5\sqrt{2})} \right) &= -5
\end{aligned} \right\} \text{-----(6)}$$

must holds simultaneously for some 's' value.

If we choose  $s = -2(1 + 5\sqrt{2})$ , then both the conditions of (6) satisfied  
 $\therefore (5\sqrt{6}, -5, 0)$  lies on C'D'

$(5\sqrt{6}, -5, 0)$  is the point at intersection of A'B' and C'D'.

$(5\sqrt{6}, -5, 0)$  is the principal vanishing point w.r.t. the x-axis.

**Case 2:** Principal vanishing point w.r.t y-axis:-

From the 2<sup>nd</sup> Row of the matrix (Equation (3)), the principal vanishing point w.r.t y-axis will be:

$$\left(0, \frac{5}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \text{ in homogeneous system.}$$

The vanishing point in Cartesian system is:

$$\left(0, \frac{5/\sqrt{2}}{1/\sqrt{2}}, 0\right) = (0, 5, 0) \quad \text{-----(II)}$$

similar proof can be made to verify our claim:

**Case 3:** Principal vanishing point w.r.t z-axis:

From the 3<sup>rd</sup> row of matrix equation (3), we claim that the principal vanishing point

w.r.t z-axis will be:  $\left(-\frac{5}{2}, \frac{-5\sqrt{3}}{2\sqrt{2}}, 0, \frac{\sqrt{3}}{2\sqrt{2}}\right)$  in Homogeneous system.

In Cartesian system, the vanishing point is:

$$\left(\frac{(-5/2)}{\frac{\sqrt{3}}{2\sqrt{2}}}, \frac{\left(\frac{-5\sqrt{3}}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)}, 0\right) = \left(\frac{-5\sqrt{2}}{\sqrt{3}}, -5, 0\right) \quad \text{-----(III)}$$

A similar proof can be made to verify (III)

**Question 10:**

**(5 marks)**

Given  $p_0(1, 1)$ ;  $p_1(2, 3)$ ;  $p_2(4, 3)$ ;  $p_3(3, 1)$  as vertices of Bezier Curve. Determine 3 points on Bezier Curve.

**Answer:**

We know Cubic Bezier curve is

$$P(u) = \sum_{i=0}^3 p_i B_{3,i}(u)$$

$$\Rightarrow P(u) = p_0(1-u)^3 + 3p_1u(1-u)^2 + 3p_2u^2(1-u) + p_3u^3$$

$$P(u) = (1, 1)(1-u)^3 + 3(2, 3)u(1-u)^2 + 3(4, 3)u^2(1-u) + (3, 1)u^3$$

we choose different values of u from 0 to 1.

u = 0:  $P(0) = (1, 1)(1-0)^3 + 0 + 0 + 0 = (1, 1)$

u = 0.5:  $P(0.5) = (1, 1)(1-0.5)^3 + 3(2, 3)(0.5)(1-0.5)^2 + 3(4, 3)(0.5)^2(1-0.5) + (3, 1)(0.5)^3$

$$= (1, 1)(0.5)^3 + (2, 3)(0.375) + (0.375)(4, 3) + (3, 1)(0.125)$$

$$= (0.125, 0.125) + (0.75, 1.125) + (1.5, 1.125) + (1.125, 0.125)$$

$P(0.5) = (3.5, 2.5)$

u = 1:  $P(1) = 0 + 0 + 0 + (3, 1) \cdot 1^3 = (3, 1)$

Three points on Bezier curve are , P (0) = (1, 1); P (0.5) = (3.5, 2.5) and P (1) = (3, 1).

**Question 11:** (6 marks)

Prove the following:

- (a)  $\sum_{i=0}^n B_{n,i} = 1$       b)  $P(u=1) = p_n$       c)  $P(u=0) = P_0$

**Answer:**

(a) To Prove:  $\sum_{i=0}^n B_{n,i} = 1$

By Simple arithmetic we know,

$[(1-u) + u]^n = 1^n = 1$  .....(1)  
 expanding LHS of (1) binomially we find

$[(1-u) + u]^n = n_{C_0} (1-u)^n + n_{C_1} u(1-u)^{n-1} + n_{C_2} u^2(1-u)^{n-2} + \dots + n_{C_n} u^n$

$u^n$

$$= \sum_{i=0}^n n_{C_i} u^i (1-u)^{n-i}$$

$[(1-u) + u]^n = \sum_{i=0}^n B_{n,i}(u)$  .....(2)

by (1) & (2) we get

$$\sum_{i=0}^n B_{n,i}(u) = 1$$

(b) To Prove:  $P(u=1) = p_n$

if P (u) → Bezier curve of order n and Q (u) → Bezier curve of order m

$$B_{n,n}(u) = \frac{n!}{n!(n-n)!} u^n (1-u)^{n-n} = u^n$$

$P(u=1) = p_0 \cdot 0 + p_1 \cdot 0 + \dots + p_n \cdot 1^n = p_n$

(c) To Prove:  $P(u=0) = P_0$

$$\therefore P(u) = \sum_{i=0}^n p_i B_{n,i}(u) \\ = p_0 B_{n,0}(u) + p_1 B_{n,1}(u) + \dots + p_n B_{n,n}(u) \dots \dots \dots (1)$$

$$B_{n,i}(u) = nC_i u^i (1-u)^{n-i}$$

$$B_{n,0}(u) = nC_0 u^0 (1-u)^{n-0} = \frac{n!}{0!(n-0)!} \cdot 1 \cdot (1-u)^n = (1-u)^n$$

$$B_{n,1}(u) = nC_1 u^1 (1-u)^{n-1} = \frac{n!}{1!(n-1)!} \cdot u \cdot (1-u)^{n-1}$$

We observe that all terms except  $B_{n,0}(u)$  have multiple of  $u^i$  ( $i = 0$  to  $n$ ) using these terms with  $u = 0$  in (1) we get,

$$P(u=0) = p_0 (1-0)^n + p_1 \cdot 0 \cdot (1-0)^{n-1} + \dots + 0$$

$P(u=0) = p_0$ Proved
-----------------------

**Question 12:** (4 Marks)

Distinguish between Z-buffer method and scan-line method. What are the visibility test made in these methods?

**Answer:**

In z-buffer algorithm every pixel position on the projection plane is considered for determining the visibility of surfaces w. r. t. this pixel. On the other hand in scan-line method all surfaces intersected by a scan line are examined for visibility. The visibility test in z-buffer method involves the comparison of depths of surfaces w. r. t. a pixel on the projection plane. The surface closest to the pixel position is considered visible. The visibility test in scan-line method compares depth calculations for each overlapping surface to determine which surface is nearest to the view-plane so that it is declared as visible.

**Question 13:** (10 Marks)

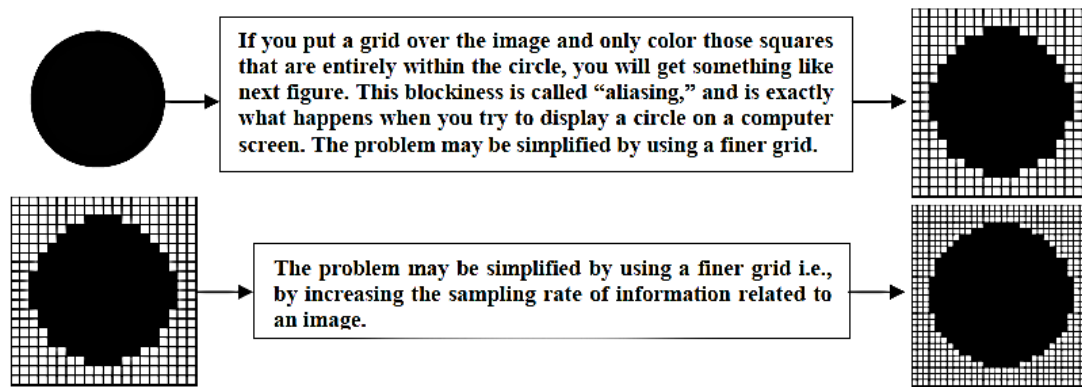
Explain the following:

- (i) Anti-aliasing (ii) Phong shading (iii) Specular reflection  
(iv) Ray tracking (v) Ray casting

**Answer:**

**(i) Anti-aliasing**

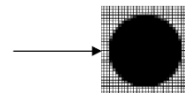
Anti-aliasing is a method for improving the realism of an image by removing the jagged edges from it. These jagged edges, or "jaggies", appear because a computer monitor has square pixels, and these square pixels are inadequate for displaying lines or curves that are not parallel to the pixels and other reason is low sampling rate of the image information, which in turn leads to these jaggies (quite similar to star casing discussed in previous blocks under DDA algorithm). For better understanding, take the following image of darkened circle:



It is not possible to completely eliminate aliasing because computers are digital (discrete) in nature. However, it is possible to minimize aliasing, the solutions used by ray tracers today involve treating each pixel as a finite square area (which, in fact, they are), rather than as a mere point on the screen. Instead the pixel should not be considered as a point or area but should be considered as a sample of image information (higher the sampling is lesser the aliasing is). Now let us discuss how appropriately the sampling can be done - Rays are fired into the scene through the centers of the pixels, and the intensities of adjacent rays are compared. If they differ by some pre-determined amount, more rays are fired into the surfaces of the pixels. The intensities of all the rays shot into a given pixel are then averaged to find a color that better fits what would be expected at that point.

**Note:** Do not treat a pixel as a square area, as this does not produce correct filtering behaviour, in fact a pixel is not a point, but it is a sample of information to be displayed.

**Anti-aliasing**, then, helps eliminate jagged edges and to make an image seem more realistic. Continuing the above example, the anti-aliased circle might, then, be represented.



### (ii) Phong shading

In Gouraud shading we were doing direct interpolation of intensities but a more accurate method for rendering a polygon surface is to interpolate normal vectors and then apply illumination model to each surface. This accurate method was given by Phong and it leads to Phong shading on Normal vector interpolation shading.

Calculations involved with Phong Shading:

- i) Determine average unit normal vector at each polygon vertex.
- ii) Linearly interpolate the vertex normals over the surface of polygon.
- iii) Apply illumination model along each scan line to calculate projected pixel intensities for surface points.

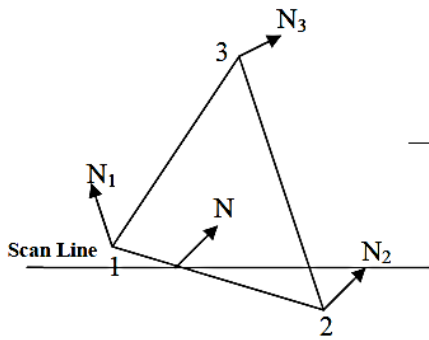


Figure A

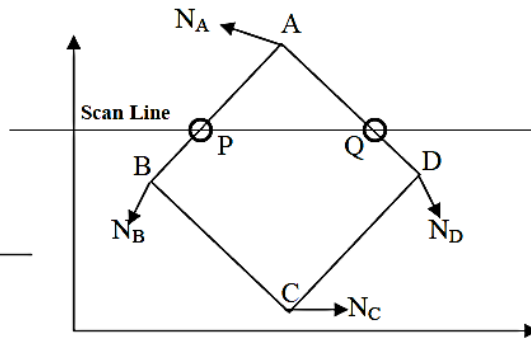
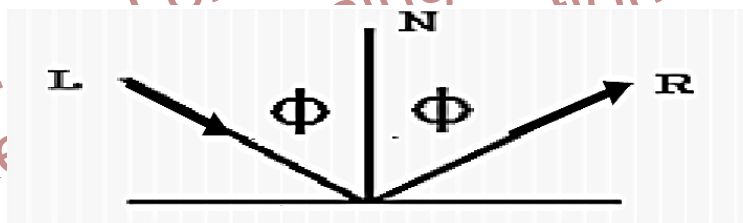


Figure B

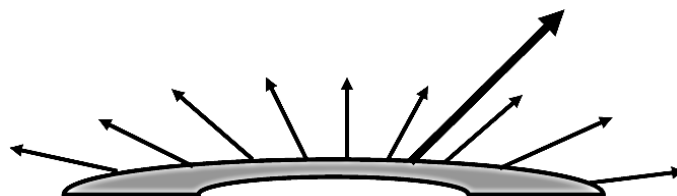
Interpolation of surface normals along the polygon edge between two vertices is shown above in Figure. The normal vector  $N$  for the scan line intersection point along the edge between vertices 1 and 2 can be obtained by vertically interpolating between edge end points normals. Then incremental methods are used to evaluate normal between scan lines and along each individual scan line. At each pixel position along a scan line, the illumination model is applied to determine the surface intensity at that point  $N = [((y-y_2) / (y_1-y_2)) N_1] + [((y_1-y) / (y_1-y_2)) N_2]$

### (iii) Specular reflection

Specular reflection is when the reflection is stronger in one viewing direction, i.e., there is a bright spot, called a specular highlight. This is readily apparent on shiny surfaces. For an ideal reflector, such as a mirror, the angle of incidence equals the angle of specular reflection, as shown below.



Light is reflected mainly in the direction of the reflected ray and is attenuated by an amount dependent upon the physical properties of the surface. *Since the light reflected from the surface is mainly in the direction of the reflected ray the position of the observer determines the perceived illumination of the surface.* Specular reflection models the light reflecting properties of shiny or mirror-like surfaces.



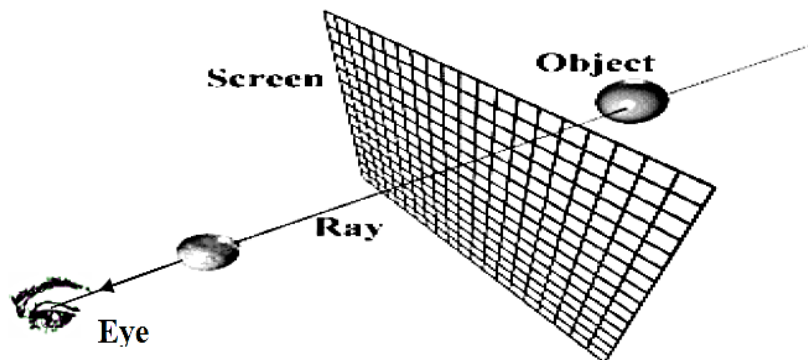
### (iv) Ray tracking

“Ray tracing” is a method of following the light from the eye to the light source. Whereas ray casting only concerns itself with finding the visible surfaces of objects, ray tracing takes that a few steps further and actually tries to determine what each visible surface looks like. Although it will cost your processor time spent in calculations you can understand the level of calculations involved in ray tracing by considering this example, Let’s say we are rendering (that is, ray

tracing) a scene at a resolution of 320 pixels wide by 240 pixels high, for a total of 76,800 pixels. Let it be of low complexity, with only 20 objects. That means, over the course of creating this picture, the ray tracer will have done 20 intersection tests for each of those 76,800 pixels, for a total of 1,536,000 intersection tests! In fact, most ray tracers spend most of their time calculating these intersections of rays with objects, anywhere from 75 to 95 % of a ray tracer's time is spent with such calculations. Apart from such hectic calculations, there is the good news that there are ways to decrease the number of intersection tests per ray, as well as increase the speed of each intersection test. In addition to this the bad news is that ray tracing complicates things much more than simply ray casting does. Ray tracing allows you to create several kinds of effects that are very difficult or even impossible to do with other methods. These effects include three items common to every ray tracer: reflection, transparency, and shadows. In the following paragraphs, we will discuss how these effects fit naturally into Ray tracing.

#### (v) Ray casting

Ray casting is a method in which the surfaces of objects visible to the camera are found by throwing (or casting) rays of light from the viewer into the scene. The idea behind ray casting is



to shoot rays from the eye, one per pixel, and find the closest object blocking the path of that ray – think of an image as a screen-door, with each square in the screen being a pixel. This is then the object the eye normally sees through that pixel. Using the material properties and the effect of the lights in the scene, this algorithm can determine the shading of this object. The simplifying assumption is made that if a surface faces a light, the light will reach that surface and not be blocked or in shadow. The shading of the surface is computed using traditional 3D computer graphics shading models. Ray casting is not a synonym for ray tracing, but can be thought of as an abridged, and significantly faster, version of the ray tracing algorithm. Both are image order algorithms used in computer graphics to render three dimensional scenes to two dimensional screens by following rays of light from the eye of the observer to a light source. Although ray tracing is similar to ray casting, it may be better thought of as an extension of ray casting we will discuss this in the next topic under this section.

#### Question 14:

(2 Marks)

How many key frames does a one minute animation film sequence with no duplications require, if there are five in between for each pair of key frames?

#### Answer:

One minute = 60 seconds

No. of frames required per second = 24

No. of in-between frames = 5

No. of frames required in entire film =  $(24 \times 60) / 5 = 288$

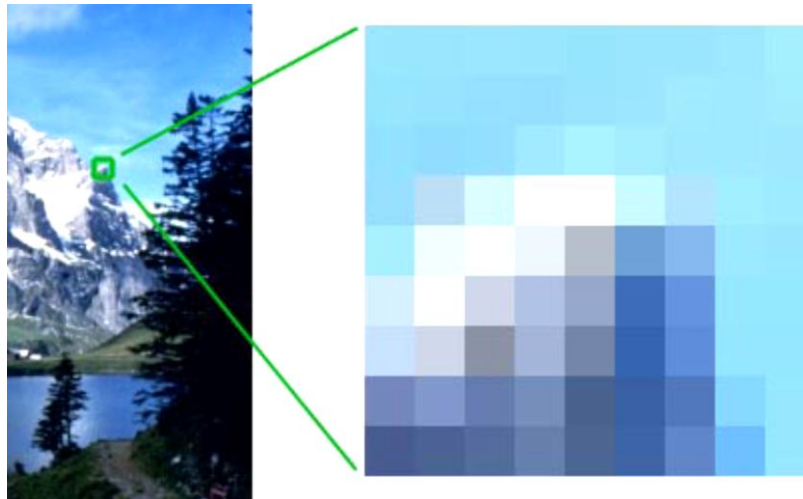
That is we would need 288 key frames for a one-minute animation film if the number of in-between frames is 5.

**Question 15:****(6 Marks)**

(i) Differentiate between Bitmap Graphics &amp; Vector Graphics

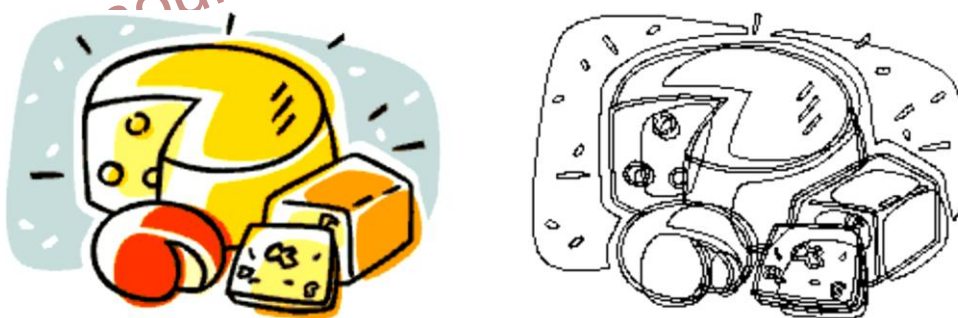
**Answer:**

**Bitmap graphics** are images which is the collection of bits that form an image. The image consists of a matrix of individual dots (or pixels) that have their own colour described using bits. Let's take a look at a typical bitmap image to demonstrate the principle:



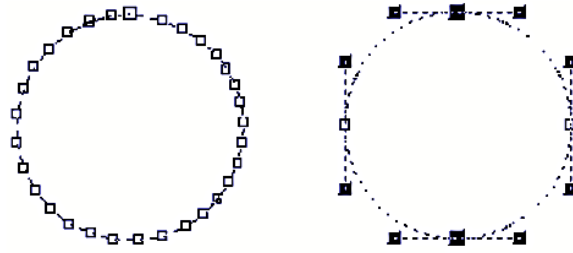
To the left you see an image and to the right a 250 percent enlargement of the top of one of the mountains. As you can see, the image consists of hundreds of rows and columns of small elements that all have their own colour. One such element is called a pixel. The human eye is not capable of seeing each individual pixel so we perceive a picture with smooth gradations.

**Vector graphics** are images that may be entirely described using mathematical definitions. The image below shows the principle. To the left you see the image itself and to the right you see the actual lines that make up the drawing.



Each individual line is made up a large number of small lines that interconnect a large number of or, just a few control points that are connected using Bezier curves. It is this latter method that generates the best results and that is used by most drawing programs.





This drawing demonstrates the two principles. To the left a circle is formed by connecting a number of points using straight lines. To the right, you see the same circle that is now drawn using 4 points (nodes) only.

(ii) Simulation of positive acceleration & Simulation of negative acceleration

**Answer:**

**Simulation of Positive Accelerations:** In order to incorporate increasing speed in an animation the time spacing between the frames should increase, so that greater change in the position occur, as the object moves faster. In general, the trigonometric function used to have increased interval size the function is  $(1 - \cos \Theta)$ ,  $0 < \Theta < \pi/2$ .

**Simulation of Negative Accelerations:** In order to incorporate decreasing speed in an animation the time spacing between the frames should decrease, so that there exist lesser change in the position as the object moves. In general, the trigonometric function used to have increased interval size the function is  $\sin \Theta$ ,  $0 < \Theta < \pi/2$ .

(iii) Analog Sound & Digital Sound

**Answer:**

**An analog recording** is one where the original sound signal is modulated onto another physical signal carried on some media or the groove of a gramophone disc or the magnetic field of a magnetic tape. A physical quantity in the medium (e.g., the intensity of the magnetic field) is directly related to the physical properties of the sound (e.g., the amplitude, phase and possibly direction of the sound wave.)

**A digital recording**, on the other hand is produced by first encoding the physical properties of the original sound as digital information which can then, be decoded for reproduction. While it is subject to noise and imperfections in capturing the original sound, as long as the individual bits can be recovered, the nature of the physical medium is of minimum consequence in the recovery of the encoded information

Don't Forget to Say Thanks!

E-mail: [lokeshchandrasingh@gmail.com](mailto:lokeshchandrasingh@gmail.com)

Web: [ignoumcaassignmentsolution.blogspot.in](http://ignoumcaassignmentsolution.blogspot.in)