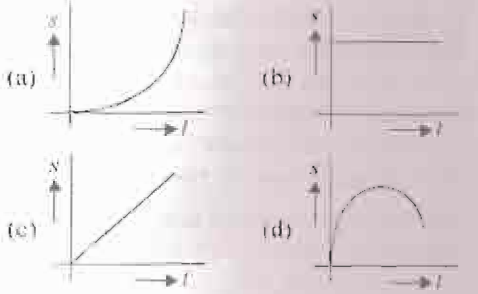
QUESTION PAPER

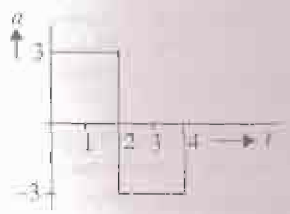
2009

PHYSICS

1. A body is travelling in a straight line with a uniformly increasing speed. Which one of the plot represents the changes in distance(s) travelled with time (t)?

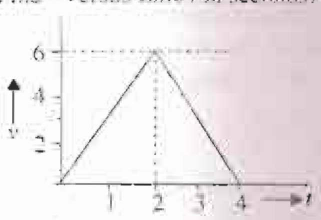


2. A particle starts from rest at $t = 0$ and undergoes an acceleration a in ms^{-2} with time t in seconds which is as shown.

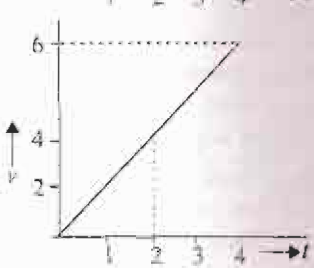


Which one of the following plot represents velocity v in ms^{-1} versus time t in seconds?

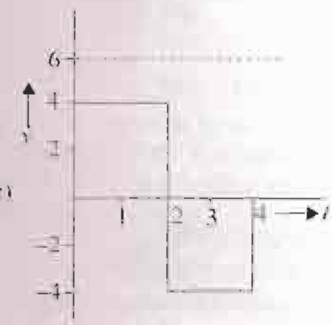
(a)



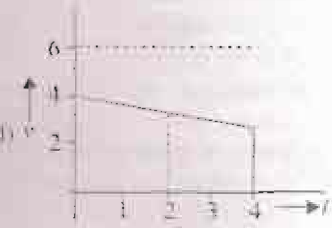
(b)



(c)



(d)



3. The acceleration a of a particle starting from rest varies with time according to relation $a = \alpha t + \beta$. The velocity of the particle after a time t will be

(a) $\frac{\alpha t^2}{2} + \beta$ (b) $\frac{\alpha t^2}{2} + \beta t$

(c) $\alpha t^2 + \frac{1}{2}\beta t$ (d) $\frac{(\alpha t^2 + \beta)}{2}$

4. A bullet hits and gets embedded in a solid block resting on a frictionless surface. In this process which one of the following is correct?
(a) Only momentum is conserved.
(b) Only kinetic energy is conserved.
(c) Neither momentum nor kinetic energy is conserved.
(d) Both momentum and kinetic energy are conserved.

5. Two masses of M and $4M$ are moving with equal kinetic energy. The ratio of their linear momenta is
(a) 1 : 8 (b) 1 : 4 (c) 1 : 2 (d) 4 : 1.

6. For an object thrown at 45° to horizontal, the

maximum height (H) and horizontal range (R) are related as

(a) $R = 16H$ (b) $R = 8H$
(c) $R = 4H$ (d) $R = 2H$.

7. If a simple pendulum of length L has maximum angular displacement α then the maximum kinetic energy of bob of mass M is

(a) $\frac{1}{2}\frac{ML}{g}$ (b) $\frac{Mg}{2L}$
(c) $MgL(1-\cos\alpha)$ (d) $\frac{MgL\sin\alpha}{2}$

8. The ratio of the vapour densities of two gases at a given temperature is 9 : 8. The ratio of the rms velocities of their molecules is

(a) $3 : 2\sqrt{2}$ (b) $2\sqrt{2} : 3$
(c) 9 : 8 (d) 8 : 9.

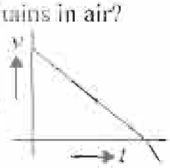
9. The excess pressure inside a spherical drop of radius r of a liquid of surface tension T is

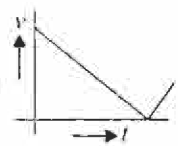
(a) directly proportional to r and inversely proportional to T
(b) directly proportional to T and inversely proportional to r
(c) directly proportional to the product of T and r
(d) inversely proportional to the product of T and r.

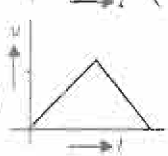
10. A black body at a high temperature T radiates energy at the rate of U (in W/m^2). When the temperature falls to half (*i.e.* $T/2$), the radiated energy (in W/m^2) will be

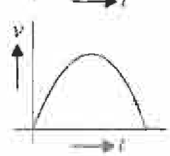
(a) $U/8$ (b) $U/16$ (c) $U/4$ (d) $U/2$.

11. A ball is thrown vertically upward. Ignoring the air resistance, which one of the following plot represent the velocity time plot for the period ball remains in air?

(a)


(b)


(c)


(d)


12. Moment of inertia of a ring of mass M and radius R about an axis passing through the centre and perpendicular to the plane is I. What is the moment of inertia about its diameter?

(a) I (b) $\frac{I}{2}$
(c) $\frac{I}{\sqrt{2}}$ (d) $I + MR^2$.

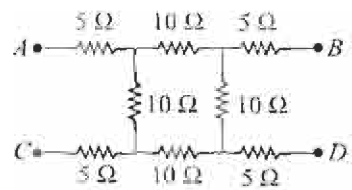
13. Two trains, each moving with a velocity of 30 ms^{-1}, cross each other. One of the trains gives a whistle whose frequency is 600 Hz. If the speed of sound is 330 ms^{-1}, the apparent frequency for passengers sitting in the other train before crossing would be

(a) 600 Hz (b) 630 Hz
(c) 920 Hz (d) 720 Hz.

14. A spherical shell of radius R has a charge $+q$ units. The electric field due to the shell at a point

(a) inside is zero and varies as r^{-1} outside it
(b) inside is constant and varies as r^{-2} outside it
(c) inside is zero and varies as r^{-2} outside it
(d) inside is constant and varies as r^{-1} outside it.

15. The equivalent resistance between the terminals A and D in the following circuit is



(a) 10 Ω (b) 20 Ω (c) 5 Ω (d) 30 Ω.

16. In a thermocouple, the neutral temperature is 270°C and the temperature of inversion is 525°C. The temperature of cold junction would be

(a) 30°C (b) 255°C (c) 15°C (d) 25°C.

17. R, L and C represent the physical quantities resistance, inductance and capacitance respectively. Which one of the following combination has dimensions of frequency?

(a) $\frac{1}{\sqrt{RC}}$ (b) $\frac{R}{L}$ (c) $\frac{1}{LC}$ (d) $\frac{C}{L}$

18. Which one of the following pair of quantities has same dimension?

(a) Force and work done
(b) Momentum and impulse
(c) Pressure and force
(d) Surface tension and stress.

19. The water flows from a tap of diameter 1.25 cm with a rate of 5×10^{-5} m^3 s^{-1}. The density and coefficient of viscosity of water are 10^3 kgm^{-3} and

10^{-3} Pas respectively. The flow of water is

(a) steady with Reynold's number 5100

(b) turbulent with Reynold's number 5100

(c) steady with Reynold's number 3900

(d) turbulent with Reynold's number 3900.

20. A circular disc rolls down an inclined plane. The ratio of rotational kinetic energy to total kinetic energy is

(a) 1/2 (b) 1/3 (c) 2/3 (d) 3/4.

21. If two parallel wires carry current in opposite directions

(a) the wires attract each other

(b) the wires repel each other

(c) the wires experience neither attraction nor repulsion

(d) the forces of attraction or repulsion do not depend on current direction.

22. If Young's double slit experiment of light, interference is performed in water, which one of the following is correct?

(a) Fringe width will decrease.

(b) Fringe width will increase.

(c) There will be no fringe.

(d) Fringe width will remain unchanged.

23. During the β^- decay

(a) an atomic electron is ejected

(b) an electron, already present within the nucleus, is ejected

(c) a proton in the nucleus decays emitting an electron

(d) a neutron in the nucleus decays emitting an electron

24. A ray of light strikes a material's slab at an angle of incidence 60°. If the reflected and refracted rays are perpendicular to each other, the refractive index of the material is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

25. A body is tied with a string and is given a circular motion with velocity v in radius r. The magnitude of the acceleration is

(a) v/r (b) v^2/r (c) v/r^2 (d) v^2/r^2.

26. The escape velocity of 10 g body from the earth is 11.2 kms^{-1}. Ignoring air resistance, the escape velocity of 10 kg of the iron ball from the earth will be

(a) 0.0112 kms^{-1} (b) 0.112 kms^{-1}

(c) 11.2 kms^{-1} (d) 0.56 kms^{-1}.

27. A particle is projected with certain velocity at two different angles of projections with respect to horizontal plane so as to have same range R on a horizontal plane. If t_1 and t_2 are the time taken for the two paths, then which one of the following relations is correct?

(a) $t_1t_2 = \frac{2R}{g}$ (b) $t_1t_2 = \frac{R}{g}$

(c) $t_1t_2 = \frac{R}{2g}$ (d) $t_1t_2 = \frac{4R}{g}$

28. A plane e.m. wave of frequency 30 MHz travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time $E = 6$ V/m along y-direction. Its magnetic field component B at this point would be

(a) 2×10^{-8} T along z-direction

(b) 6×10^{-8} T along x-direction

(c) 2×10^{-8} T along y-direction

(d) 6×10^{-8} T along z-direction.

29. A steady electric current is flowing through a cylindrical conductor.

(a) The magnetic field in the vicinity of the conductor is zero.

(b) The electric field in the vicinity of the conductor is non-zero.

(c) The magnetic field at the axis of the conductor is zero.

(d) The electric field at the axis of the conductor is zero.

30. A Carnot engine working between 450 K and 600 K has a work output of 300 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is

(a) 400 J (b) 800 J

(c) 1600 J (d) 3200 J

31. Two simple harmonic motions are represented by

$$y_1 = 5(\sin 2\pi t + \sqrt{3}\cos 2\pi t)$$

$$y_2 = 5\sin(2\pi t + \frac{\pi}{4})$$

The ratio of the amplitude of two S.H.M.'s is

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : $\sqrt{3}$

32. Soap bubbles can be formed floating in air by blowing soap solution in air, with the help of a glass tube, but not water bubbles. It is because

(a) the excess pressure inside water bubble being more due to large surface tension

(b) the excess pressure inside water bubble being less due to large surface tension

(c) the excess pressure inside water bubble being more due to large viscosity

(d) the excess pressure inside water bubble being less due to less viscosity.

33. A parallel plate capacitor is charged. If the plates are pulled apart

(a) the capacitance increases

(b) the potential difference increases

(c) the total charge increases

(d) the charge and potential difference remain the same.

34. A lead-acid battery of a car has an e.m.f. of 12 V. If the internal resistance of the battery is 0.5 ohm, the maximum current that can be drawn from the battery will be

(a) 30 A (b) 20 A (c) 6 A (d) 24 A.

35. A thin lens of glass ($\mu = 1.5$) of focal length -10 cm is immersed in water ($\mu = 1.33$). The new focal length is

(a) 20 cm (b) 40 cm (c) 48 cm (d) 12 cm.

36. For a transistor amplifier, the voltage gain

(a) is high at high and low frequencies and constant in the middle frequency range

(b) is low at high and low frequency and constant in the middle frequency range

(c) remains constant for all frequencies

(d) is high at high frequencies and low at low frequencies and constant in middle frequency range.

37. Which one of the following property of light does not support wave theory of light?

(a) Light obeys laws of reflection and refraction.

(b) Light waves get polarised.

(c) Light shows photoelectric effect.

(d) Light shows interference.

38. A block of mass M at the end of the string is whirled round a vertical circle of radius R. The critical speed of the block at the top of the swing is

(a) $(R/g)^{1/2}$ (b) g/R

(c) M/Rg (d) $(Rg)^{1/2}$

39. A metallic rod of length l and cross sectional area A is made of a material of Young's modulus Y. If the rod is elongated by an amount y, then the work done is proportional to

(a) y (b) $1/y$ (c) y^2 (d) $1/y^2$.

40. For vectors $\vec{A}$ and $\vec{B}$ making an angle θ which one of the following relations is correct?

(a) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ (b) $\vec{A} \times \vec{B} = AB \sin\theta$

(c) $\vec{A} \times \vec{B} = AB \cos\theta$ (d) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

41. A lift is moving upward with increasing speed with acceleration a. The apparent weight will be

(a) less than the actual weight

(b) more than the actual weight and have a fixed value

(c) more than the actual weight which increases as long as velocity increases

(d) zero.

42. According to Newton's law of cooling, the rate of cooling is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the temperature difference between the body and the surroundings and n is equal to

(a) three (b) two (c) one (d) four.

43. An open organ pipe of length l vibrates in its fundamental mode. The pressure variation is maximum

(a) at the two ends

(b) at the distance $l/2$ inside the ends

(c) at the distance $l/4$ inside the ends

(d) at the distance $l/8$ inside the ends.

44. A metallic wire of resistance 12 Ω is bent to form a square. The resistance between two diagonal points would be

(a) 12 Ω (b) 24 Ω (c) 6 Ω (d) 3 Ω.

45. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. The maximum torque experienced by the dipole is

(a) pE (b) p/E (c) E/p (d) $\vec{p} \cdot \vec{E}$

46. In a potentiometer arrangement, a cell of emf 1.5 V gives a balance point at 27 cm length of wire. If the cell is replaced by another cell and balance point shifts to 54 cm, the emf of the second cell is

(a) 3 V (b) 1.5 V (c) 0.75 V (d) 2.25 V.

47. In order to increase the angular magnification of a simple microscope, one should increase

(a) the object size

(b) the aperture of the lens

(c) the focal length of the lens

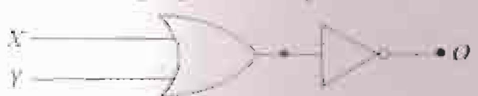
(d) the power of the lens.

48. A current carrying straight wire is kept along the axis of a circular loop carrying a current. The straight wire
(a) will exert an inward force on the circular loop
(b) will exert an outward force on the circular loop
(c) will exert a force on the circular loop parallel to itself
(d) will not exert any force on the circular loop.

49. Two lithium nuclei in a lithium vapour at room temperature do not combine to form a carbon nucleus because
(a) carbon nucleus is an unstable particle
(b) it is not energetically favourable
(c) nuclei do not come very close due to Coulombic repulsion
(d) lithium nucleus is more tightly bound than a carbon nucleus.

50. A typical optical fibre consists of a fine core of a material of refractive index μ_1 surrounded by a glass or plastic cladding with refractive index μ_2. Then
(a) μ_2 is slightly less than μ_1
(b) μ_2 is slightly higher than μ_1
(c) μ_2 should be equal to μ_1
(d) the difference $\mu_2 - \mu_1$ should be strictly equal to 1

51. The following logic circuit represents



(a) NAND gate with output $O = \overline{\overline{X}+\overline{Y}}$
(b) NOR gate with output $O = \overline{X+Y}$
(c) NAND gate with output $O = \overline{X \cdot Y}$
(d) NOR gate with output $O = \overline{X} \cdot \overline{Y}$

52. In an AC series circuit, the instantaneous current is maximum when the instantaneous voltage is maximum. The circuit element connected to the source will be
(a) pure inductor (b) pure capacitor
(c) pure resistor
(d) combination of a capacitor and an inductor.

53. A charged particle is moving along a magnetic field line. The magnetic force on the particle is
(a) along its velocity
(b) opposite to its velocity
(c) perpendicular to its velocity
(d) zero.

54. An air capacitor is charged with an amount of charge q and dipped into an oil tank. If the oil is pumped out, the electric field between the plates of capacitor will
(a) increase (b) decrease
(c) remain the same (d) become zero.

55. The mean kinetic energy of one mole of gas per degree of freedom (on the basis of kinetic theory of gases) is
(a) $\frac{1}{2}kT$ (b) $\frac{3}{2}kT$ (c) $\frac{3}{2}RT$ (d) $\frac{1}{2}RT$

56. The electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 20 V.m. The flux over a concentric sphere of radius 20 cm will be
(a) 20 V.m (b) 25 V.m
(c) 40 V.m (d) 200 V.m.

57. Light of wavelength λ falls on a metal having work function $\frac{hc}{\lambda_0}$. Photoelectric effect will take place only if
(a) $\lambda \geq \lambda_0$ (b) $\lambda \geq 2\lambda_0$
(c) $\lambda \leq \lambda_0$ (d) $\lambda = 4\lambda_0$

58. In the communication systems, AM is used for broadcasting because
(a) its use avoids receiver complexity
(b) it is more noise immune than other modulation systems
(c) it requires less transmitting power
(d) no other modulation system can give the necessary bandwidth for faithful transmission.

59. The rate of flow of water in a capillary tube of length l and radius r is V. The rate of flow in another capillary tube of length $2l$ and radius $2r$ for same pressure difference would be
(a) $16V$ (b) $9V$
(c) $8V$ (d) $2V$.

60. When a body is taken from poles to equator on the earth, its weight
(a) increases
(b) decreases
(c) remains the same
(d) increases at south pole and decreases at north pole.

61. If the various terms in the below given expressions have usual meanings, the van't Hoff factor (i) cannot be calculated by which of the following expression?
(a) $\pi V = \sqrt{i}\, nRT$
(b) $\Delta T_f = iK_f \cdot m$
(c) $\Delta T_b = iK_b \cdot m$
(d) $\dfrac{P^{\circ}_{solvent} - P_{solution}}{P^{\circ}_{solvent}} = i\left(\dfrac{n}{N+n}\right)$

62. A 5% solution of sugarcane (mol. wt. = 342) is isotonic with 1% solution of X under similar conditions. The mol. wt. of X is
(a) 136.2
(b) 68.4
(c) 34.2
(d) 171.2

63. For the reaction $A \to B$, $\Delta H = +24$ kJ/mole. For the reaction $B \to C$, $\Delta H = -18$ kJ/mole. The decreasing order of enthalpy of A, B, C follows the order
(a) A, B, C
(b) B, C, A
(c) C, B, A
(d) C, A, B

64. 3 moles of A and 4 moles of B are mixed together and allowed to come into equilibrium according to the following reaction:

$$A_{(g)} + 4B_{(g)} \rightleftharpoons 2C_{(g)} + 3D_{(g)}$$

When equilibrium is reached, there is 1 mole of C. The equilibrium extent of the reaction is
(a) 1/4
(b) 1/3
(c) 1/2
(d) 1.

65. In spectrochemical series chlorine is above water i.e., $Cl > H_2O$, this is due to
(a) good π-acceptor properties of Cl
(b) strong σ-donor and good π-acceptor properties of Cl
(c) good π-donor properties of Cl
(d) larger size of Cl than H_2O.

66. For square planar complex of platinum(II), $[Pt(NH_3)(Br)(Cl)py]^{+}$, how many isomeric forms are possible?
(a) Two
(b) Three
(c) Four
(d) Six.

67. If k_1 = rate constant at temperature T_1 and k_2 = rate constant at temperature T_2 for a first order reaction, then which of the following relations is correct? (E_a = activation energy)
(a) $\log \dfrac{k_1}{k_2} = \dfrac{2.303 E_a}{R}\left(\dfrac{T_2 - T_1}{T_1 T_2}\right)$
(b) $\log \dfrac{k_2}{k_1} = \dfrac{E_a}{2.303R}\left(\dfrac{T_2 - T_1}{T_1 T_2}\right)$
(c) $\log \dfrac{k_2}{k_1} = \dfrac{E_a}{2.303R}\left(\dfrac{T_1 T_2}{T_2 + T_1}\right)$
(d) $\log \dfrac{k_1}{k_2} = \dfrac{E_a}{2.303R}\left(\dfrac{T_1 T_2}{T_2 - T_1}\right)$

68. For a reaction between A and B, the initial rate of reaction is measured for various initial concentrations of A and B. The data provided are

	[A]	[B]	Initial reaction rate
(a)	0.20 M	0.30 M	5×10^{-5}
(b)	0.20 M	0.10 M	5×10^{-5}
(c)	0.40 M	0.05 M	7.5×10^{-5}

The overall order of the reaction is
(a) one (1)
(b) two (2)
(c) two and a half (2.5)
(d) three (3).

69. Two types of FXF angles are present in which of the following molecule (X = S, Xe, C)?
(a) SF_4
(b) XeF_4
(c) SF_6
(d) CF_4

70. Least active electrophile is
(a) $CH_3-\underset{\underset{O}{||}}{C}-OCH_3$
(b) $CH_3-\overset{\overset{O}{||}}{C}-Cl$
(c) $CH_3-\underset{\underset{O}{||}}{C}-NMe_2$
(d) $CH_3-\overset{\overset{O}{||}}{C}-SCH_3$

71. Given the hypothetical reaction mechanism,

$$A \xrightarrow{I} B \xrightarrow{II} C \xrightarrow{III} D \xrightarrow{IV} E$$

and the data as

Species formed	Rate of its formation
B	0.002 mole/h . per mole of A
C	0.030 mole/h . per mole of B
D	0.011 mole/h . per mole of C
E	0.420 mole/h . per mole of D

The rate determining step is
(a) step I
(b) step II
(c) step III
(d) step IV

72. For the given complex $[CoCl_2(en)(NH_3)_2]^+$, the number of geometrical isomers, the number of optical isomers and total number of isomers of all type possible respectively are

(a) 2, 2 and 4 (b) 2, 2 and 3

(c) 2, 0 and 2 (d) 0, 2, and 2

73. When EDTA solution is added to Mg^{2+} ion solution, then which of the following statements is not true?

(a) Four coordinate sites of Mg^{2+} are occupied by EDTA and remaining two sites are occupied by water molecules.

(b) All six coordinate sites of Mg^{2+} are occupied.

(c) pH of the solution is decreased.

(d) Colourless $[Mg\text{-}EDTA]^{2-}$ chelate is formed.

74. Ceric ammonium sulphate and potassium permanganate are used as oxidising agents in acidic medium for oxidation of ferrous ammonium sulphate to ferric sulphate. The ratio, of number of moles of ceric ammonium sulphate required per mole of ferrous ammonium sulphate to the number of moles of $KMnO_4$ required per mole of ferrous ammonium sulphate is

(a) 5.0 (b) 0.2

(c) 0.6 (d) 2.0

75. Strongest base is

(a)  (b) 

(c)  (d) 

76. In which of the following pairs, the constants/quantities are not mathematically related to each other?

(a) Gibb's free energy and standard cell potential.

(b) Equilibrium constant and standard cell potential.

(c) Rate constant and activation energy.

(d) Rate constant and standard cell potential.

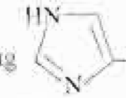
77. In which reaction, there will be increase in entropy?

(a) $Na_{(s)} + H_2O_{(l)} \longrightarrow NaOH_{(l)} + \frac{1}{2}H_{2(g)}$

(b) $Ag^+_{(aq)} + Cl^-_{(aq)} \longrightarrow AgCl_{(s)}$

(c) $H_{2(g)} + O_{2(g)} \longrightarrow H_2O_{(l)}$

(d) $Cu^{2+}_{(aq)} + 4NH_{3(g)} \longrightarrow [Cu(NH_3)_4]^{2+}_{(aq)}$

78. The drug  $-CH_2-CH_2-NH_2$ is used as

(a) antacid (b) analgesic

(c) antimicrobial (d) antiseptic

79. Which one of the following is not the correct reaction of aryl diazonium salts?

(a) $C_6H_5N_2^+Cl^- + Cu_2Cl_2 \longrightarrow C_6H_5Cl$

(b) $C_6H_5N_2^+Cl^- + HBF_4 \xrightarrow{Heat} C_6H_5F$

(c) $C_6H_5N_2^+Cl^- + H_3PO_2 \longrightarrow C_6H_5PO_4$

(d) $C_6H_5N_2^+Cl^- + SnCl_2/HCl \longrightarrow C_6H_5NHNH_2$

80. An aromatic compound X with molecular formula $C_9H_{10}O$ gives the following chemical tests.

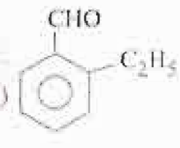
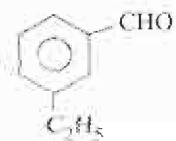
(i) Forms 2,4-DNP derivative

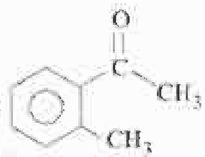
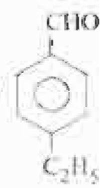
(ii) Reduces Tollen's reagent

(iii) Undergoes Cannizzaro reaction and

(iv) On vigorous oxidation 1,2-benzenedicarboxylic acid is obtained.

X is

(a)  (b) 

(c)  (d) 

81. Give the correct IUPAC name for

$$CH_3CH_2OCH(CH_3)CH_2CH_2CH_2Cl$$

(a) 2-ethoxy-5-chloropentane

(b) 1-chloro-4-ethoxy-4-methylbutane

(c) 1-chloro-4-ethoxypentane

(d) ethyl-1-chloropentylether.

82. Amino group is *ortho, para*-directing for aromatic electrophilic substitution. On nitration of aniline good amount of *m*-nitroaniline is obtained. This is

due to

(a) In nitration mixture, *ortho*-, *para*-activity of NH_2 group is completely lost.
(b) $-NH_2$ becomes $-NH_3^+$, which is *m*-directing
(c) $-NH_2$ becomes $-NH^+SO_4^-$, which is *m*-directing
(d) $-NH_2$ becomes $-NH^+NO_3^-$, which is *m*-directing.

83. The function of $ZnCl_2$ in Lucas test for alcohols is
(a) to act as an acid catalyst and react with HCl to form H_2ZnCl_4
(b) to act as a base catalyst and react with NaOH to form $Na_2Zn(OH)_4$
(c) to act as an amphoteric catalyst
(d) to act as a neutral catalyst.

84. Which sequence of reactions shows correct chemical relation between sodium and its compound?

(a) $Na + O_2 \longrightarrow Na_2O \xrightarrow{HCl(aq)} NaCl \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{\Delta} Na$

(b) $Na + O_2 \longrightarrow Na_2O \xrightarrow{H_2O} NaOH \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{\Delta} Na$

(c) $Na + H_2O \longrightarrow NaOH \xrightarrow{HCl} NaCl \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{\Delta} Na$

(d) $Na + H_2O \longrightarrow NaOH \xrightarrow{CO_2} Na_2CO_3 \xrightarrow{HCl} \underset{(molten)}{NaCl} \xrightarrow{electrolysis} Na$

85. Perxenate ion is
(a) XeO_6^{4-} (b) $HXeO_4^-$
(c) XeO_4^{2-} (d) XeO_6^{2-}

86. SF_2, SF_4 and SF_6 have the hybridisations at sulphur atom respectively as
(a) sp^2, sp^3, sp^3d^2 (b) sp^3, sp^3, sp^3d^2
(c) sp^3, sp^3d, sp^3d^2 (d) sp^3, spd^2, d^2sp^3

87. Given the limiting molar conductivity as

$\Lambda_m^\infty(HCl) = 425.9\ \Omega^{-1}\ cm^2\ mol^{-1}$

$\Lambda_m^\infty(NaCl) = 126.4\ \Omega^{-1}\ cm^2\ mol^{-1}$

$\Lambda_m^\infty(CH_3COONa) = 91\ \Omega^{-1}\ cm^2\ mol^{-1}$

The molar conductivity at infinite dilution of acetic acid (in $\Omega^{-1}\ cm^2\ mol^{-1}$) will be
(a) 481.5 (b) 390.5
(c) 299.5 (d) 516.9

88. For a chemical reaction, ΔG will always be negative if
(a) ΔH and $T\Delta S$ both are positive
(b) ΔH and $T\Delta S$ both are negative
(c) ΔH is negative and $T\Delta S$ is positive
(d) ΔH is positive and $T\Delta S$ is negative

89. In which pair of species, both species do have the similar geometry?
(a) CO_2, SO_2 (b) NH_3, BH_3
(c) CO_3^{2-}, SO_3^{2-} (d) SO_4^{2-}, ClO_4^-

90. If liquid is dispersed in solid medium, then this is called
(a) sol (b) emulsion
(c) liquid aerosol (d) gel.

91. Which is not correct order for the stated property?
(a) Ba > Sr > Mg : Atomic radius
(b) F > O > N : First ionisation energy
(c) Cl > F > I : Electron affinity
(d) O > Se > Te : Electronegativity

92. The solubility product of iron (III) hydroxide is 1.6×10^{-39}. If X is solubility of iron (III) hydroxide, then which one of the following expressions can be used to calculate X?
(a) $K_{sp} = X^4$ (b) $K_{sp} = 9X^3$
(c) $K_{sp} = 27X^3$ (d) $K_{sp} = 27X^4$

93. Given the following sequence of reaction.

$CH_3CH_2I \xrightarrow{NaCN} A \xrightarrow[\text{Partial hydrolysis}]{OH^-} B \xrightarrow{Br_2/NaOH} C$

The major product C is
(a) $CH_3CH_2NH_2$ (b) $CH_3CH_2-\underset{\underset{O}{\parallel}}{C}-NHBr$
(c) $CH_3CH_2-COONH_4$
(d) $CH_3CH_2-\underset{\underset{O}{\parallel}}{C}-NBr_2$

94. The number of unpaired electrons in gaseous species of Mn^{3+}, Cr^{3+} and V^{3+} respectively are .... and most stable species is ....
(a) 4, 3 and 2 and V^{3+}
(b) 3, 3 and 2 and Cr^{3+}
(c) 4, 3 and 2 and Cr^{3+}
(d) 3, 3 and 3 and Mn^{3+}

95. Freundlich equation for adsorption of gases (in amount of x g) on a solid (in amount of m g) at constant temperature can be expressed as

(a) $\log \frac{x}{m} = \log p + \frac{1}{n} \log k$

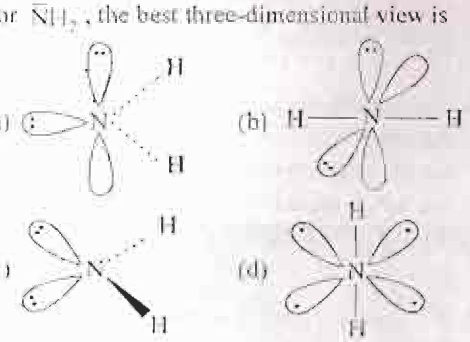
(b) $\log \frac{x}{m} = \log k + \frac{1}{n} \log p$

(c) $\frac{x}{m} = p^n$ (d) $\frac{x}{m} = \log p + \frac{1}{n} \log k$

96. In which case, the order of acidic strength is not correct?

(a) $HI > HBr > HCl$
(b) $HIO_4 > HBrO_4 > HClO_4$
(c) $HClO_4 > HClO_3 > HClO_2$
(d) $HF > H_2O > NH_3$.

97. For $\bar{N}H_2$, the best three-dimensional view is



98. The number of isomeric pentyl alcohols possible are

(a) two (b) four
(c) six (d) eight.

99. The following two reactions of HNO_3 with Zn are given as (equations are not balanced)

$Zn + \text{conc. } HNO_3 \rightarrow Zn(NO_3)_2 + X + H_2O$ - (A)
$Zn + \text{dil. } HNO_3 \rightarrow Zn(NO_3)_2 + Y + H_2O$ - (B)

In reactions A and B, the compounds X and Y respectively are

(a) NO_2 and NO (b) NO_2 and NO_2
(c) NO and NO_2 (d) NO_2 and NH_4NO_3.

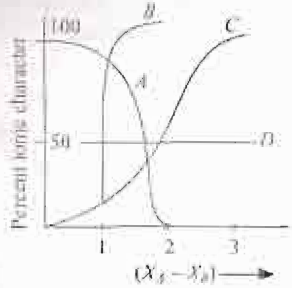
100. Which is not correct statement about the chemistry of $3d$ and $4f$ series elements?

(a) $3d$ elements show more oxidation states than $4f$-series elements.
(b) The energy difference between $3d$ and $4s$ orbitals is very little.
(c) Europium(II) is more stable than Ce(II).
(d) The paramagnetic character in $3d$-series elements increases from scandium to copper.

101. According to hard and soft acid base principle, a hard acid

(a) has low charge density
(b) shows preference for soft bases
(c) shows preference for donor atoms of low electronegativity
(d) is not polarizable.

102. For AB bond if percentage ionic character is plotted against electronegativity difference ($X_A - X_B$). The shape of the curve would look like as follows:



The correct curve is

(a) A (b) B
(c) C (d) D

103. Small quantities of solution of compounds TX, TY and TZ are put into separate test tubes containing X, Y and Z solution. TX does not react with any of these. TY reacts with both X and Z. TZ reacts with X. The decreasing order of ease of oxidation of the anions X^-, Y^- and Z^- is

(a) Y^-, Z^-, X^- (b) Z^-, X^-, Y^-
(c) Y^-, X^-, Z^- (d) X^-, Z^-, Y^-

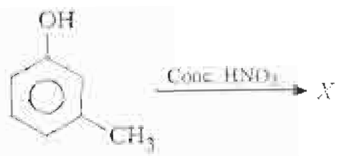
104. A mixture of salts ($Na_2SO_3 + K_2Cr_2O_7$) in a test tube is treated with dil. H_2SO_4 and resulting gas is passed through lime water. Which of the following observations is correct about this test?

(a) Solution in test tube becomes green and lime water turns milky.
(b) Solution in test tube is colourless and lime water turns milky.
(c) Solution in test tube becomes green and lime water remains clear.
(d) Solution in test tube remains clear and lime water also remains clear.

105. In a reaction $RCHO$ is reduced to RCH_3 using amalgamated zinc and concentrated HCl and warming the solution. The reaction is known as

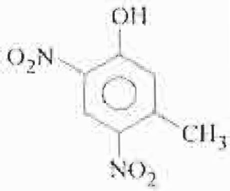
(a) Meerwein-Ponndorf reaction
(b) Clemmensen's reduction
(c) Wolff-Kishner reduction
(d) Schiff's reaction.

106. In the reaction for dinitration

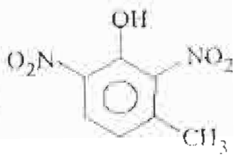


The major dinitrated product X is

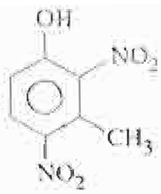
(a)



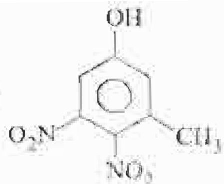
(b)



(c)



(d)



107. *Meso*-dibromobutane on debromination gives
   (a) *Trans*-2-butene (b) *Cis*-2-butene
   (c) 1-butene (d) 1-butyne

108. Which is correct statement about diborane structure?
   (a) All HBH bond angles are equal.
   (b) All H – B bond lengths are equal.
   (c) It has two three-centre-2-electron bonds.
   (d) All hydrogens and boron atoms are in one plane.

109. In pyrophosphoric acid, $H_4P_2O_7$, number of σ- and $d\pi$-$p\pi$ bonds respectively are
   (a) 8 and 2 (b) 6 and 2
   (c) 12 and zero (d) 12 and 2

110. Which is correct statement about σ- and π-molecular orbitals ? Statements are
   (i) π-bonding orbitals are ungerade
   (ii) π-antibonding orbitals are ungerade
   (iii) σ-antibonding orbitals are gerade
   (a) (i) only (b) (ii) and (iii) only
   (c) (iii) only (d) (ii) only

111. Match the chemicals in column I with their uses in column II

	Column I		Column II
A	Sodium perborate	I	Disinfectant
B	Chlorine	II	Antiseptic
C	Bithional	III	Milk bleaching agent
D	Potassium stearate	IV	Soap

   (a) A = I, B = II, C = III, D = IV
   (b) A = II, B = III, C = IV, D = I
   (c) A = III, B = I, C = II, D = IV
   (d) A = IV, B = I, C = II, D = III

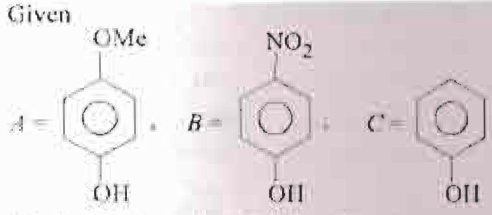
112. The stability of interhalogen compounds follows the order
   (a) $IF_3 > BrF_3 > ClF_3$
   (b) $BrF_3 > IF_3 > ClF_3$
   (c) $ClF_3 > BrF_3 > IF_3$
   (d) $ClF_3 > IF_3 > BrF_3$

113. When copper pyrites is roasted in excess of air, a mixture of CuO + FeO is formed. FeO is present as impurities. This can be removed as slag during reduction of CuO. The flux added to form slag is
   (a) SiO_2, which is an acid flux
   (b) Lime stone, which is a basic flux
   (c) SiO_2, which is a basic flux
   (d) CaO, which is a basic flux.

114. Which is not the correct statement for ionic solids in which positive and negative ions are held by strong electrostatic attractive forces ?
   (a) The radius ratio r^+/r^- increases as coordination number increases
   (b) As the difference in size of ions increases coordination number increases
   (c) When coordination number is eight, the r^+/r^- ratio lies between 0.225 to 0.414
   (d) In ionic solid of the type AX(ZnS, Wurtzite) the coordination number of Zn^{2+} and S^{2-} respectively are 4 and 4.

115. The correct order of acidic strength of carboxylic acids is
   (a) Formic acid < benzoic acid < acetic acid
   (b) Formic acid < acetic acid < benzoic acid
   (c) Acetic acid < formic acid < benzoic acid
   (d) Acetic acid < benzoic acid < formic acid

116. Given



The decreasing order of their acidic character is

(a) $A > B > C$ (b) $B > A > C$

(c) $B > C > A$ (d) $C > B > A$

117. Bauxite ore is made up of $Al_2O_3 + SiO_2 + TiO_2 + Fe_2O_3$. This ore is treated with conc. NaOH solution at 500 K and 35 bar pressure for few hours and filtered, the species present are

(a) $NaAl(OH)_4$ only (b) $Na_2Ti(OH)_6$ only

(c) $NaAl(OH)_4$ and Na_2SiO_3 both

(d) Na_2SiO_3 only

118. The relative penetrating power of α, β, γ and neutron (n) follows the order

(a) $\alpha > \beta > \gamma > n$ (b) $n > \gamma > \beta > \alpha$

(c) $\beta > \alpha > n > \gamma$ (d) none of these

119. In forming (i) $N_2 \rightarrow N_2^+$ and (ii) $O_2 \rightarrow O_2^+$; the electrons respectively are removed from

(a) (π^*2p_y or π^*2p_z) and (π^*2p_y or π^*2p_z)

(b) ($\pi 2p_y$ or $\pi 2p_z$) and ($\pi 2p_y$ or $\pi 2p_z$)

(c) ($\pi 2p_y$ or $\pi 2p_z$) and (π^*2p_y or π^*2p_z)

(d) (π^*2p_y or π^*2p_z) and ($\pi 2p_y$ or $\pi 2p_z$)

120. Given the polymers

A = Nylon 6,6 ; B = Buna-S ; C = Polythene

Arrange these in decreasing order of their intermolecular forces

(a) $A > B > C$ (b) $B > C > A$

(c) $B < C < A$ (d) $C < A < B$

MATHEMATICS

121. The sides BC, CA and AB of a triangle ABC are of lengths a, b and c respectively. If D is the mid-point of BC and AD is perpendicular to AC, then the value of $\cos A \cos C$ is

(a) $\frac{3(a^2-c^2)}{2ac}$ (b) $\frac{2(a^2-c^2)}{3bc}$

(c) $\frac{(a^2-c^2)}{3ac}$ (d) $\frac{2(c^2-a^2)}{3ac}$

122. The angle of elevation of the top of a hill from a point is α. After walking b meters towards the top up a slope inclined at an angle β to the horizontal, the angle of elevation of the top becomes γ. Then the height of the hill is

(a) $\frac{b\sin\alpha\sin(\gamma-\beta)}{\sin(\gamma-\alpha)}$ (b) $\frac{b\sin\alpha\sin(\gamma-\alpha)}{\sin(\gamma-\beta)}$

(c) $\frac{b\sin(\gamma-\beta)}{\sin(\gamma-\alpha)}$ (d) $\frac{b\sin(\gamma-\beta)}{\sin\alpha\sin(\gamma-\alpha)}$

123. The coefficient of the term independent of x in the expansion of $\left[\frac{(x+1)}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x-x^{1/2}}\right]^{10}$ is

(a) 210 (b) 105

(c) 70 (d) 112

124. $7^9 + 9^7$ is divisible by

(a) 128 (b) 24

(c) 64 (d) 72

125. The value of $C(47,4)+\sum_{r=1}^{5}C(52-r,3)$ is

(a) $C(52,4)$ (b) $C(51,4)$

(c) $C(52,3)$ (d) $C(51,3)$

126. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$ and $|\vec{c}-\vec{a}|=2\sqrt{2}$ and angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 30°, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ equals

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) $\frac{\sqrt{3}}{2}$

127. Let $\vec{a}=\hat{i}+\hat{j}-\hat{k}$; $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ be a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then $\vec{c}$ is

(a) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{6}}(2\hat{i}-\hat{j}+\hat{k})$

128. $ABCDEF$ is a regular hexagon with centre at the origin such that $\overline{AD}+\overline{EB}+\overline{FC}=\lambda\overline{ED}$. Then λ equals

(a) 2 (b) 4

(c) 6 (d) 3

129. A parallelopiped is formed by planes drawn through the points (2, 2, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelopiped is

(a) 7 (b) 9

(c) 11 (d) $\sqrt{155}$

130. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

131. If $P(x, y, z)$ is a point on the line segment joining $Q(2, 2, 4)$ and $R(3, 5, 6)$ such that the projections of OP on the axis are $\frac{13}{5}, \frac{19}{5}$ and $\frac{26}{5}$ respectively, then P divides QR in the ratio

(a) 1 : 2 (b) 3 : 2

(c) 2 : 3 (d) 1 : 3

132. The orthocentre of the triangle formed by the lines $xy = 0$ and $x + y = 1$ is

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$

(c) $\left(\frac{1}{4}, \frac{1}{4}\right)$ (d) $(0, 0)$

133. If the sum of the distances of a variable point from two perpendicular lines in a plane is 1, then its locus is

(a) a square (b) a circle

(c) a straight line (d) two intersecting lines

134. Point Q is symmetric to $P(4, -1)$ with respect to the bisector of the first quadrant. The length of PQ is

(a) $3\sqrt{2}$ (b) $5\sqrt{2}$

(c) $7\sqrt{2}$ (d) $9\sqrt{2}$

135. The radius of the circle, which is touched by the line $y = x$ and has its centre on the positive direction of x-axis and also cuts-off a chord of length 2 units along the line $\sqrt{3}y - x = 0$, is

(a) $\sqrt{5}$ (b) $\sqrt{3}$

(c) $\sqrt{2}$ (d) 1

136. Tangents drawn from the point (4, 3) to the circle $x^2 + y^2 - 2x - 4y = 0$ are inclined at an angle

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

137. If l denotes the semi-latus rectum of the parabola $y^2 = 4ax$ and SP and SQ denote the segments of any focal chord PQ, S being the focus, then SP, l and SQ are in the relation

(a) A.P. (b) G.P.

(c) H.P. (d) $l^2 = SP^2 + SQ^2$

138. The eccentricity of the ellipse $x^2 + 4y^2 + 8y - 2x + 1 = 0$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{5}}{2}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

139. The equation of the tangent parallel to $y = x$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

(a) $x - y + 1 = 0$ (b) $x - y + 2 = 0$

(c) $x - y + 3 = 0$ (d) $x - y - 2 = 0$

140. Which of the following statements is a tautology ?

(a) $(\sim q \wedge p) \wedge q$ (b) $(\sim q \wedge p) \wedge (p \wedge \sim p)$

(c) $(\sim q \wedge p) \vee (p \vee \sim p)$ (d) $(p \wedge q) \wedge (\sim (p \wedge q))$

141. If the variable takes the values 0, 1, 2, ..., n with frequencies proportional to the binomial coefficients $C(n, 0), C(n, 1), C(n, 2), \ldots, C(n, n)$ respectively, then the variance of the distribution is

(a) n (b) $\frac{\sqrt{n}}{2}$ (c) $\frac{n}{2}$ (d) $\frac{n}{4}$

142. Let A, B and C be three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then $P(B \cap C)$ satisfies

(a) $P(B \cap C) \leq 0.23$ (b) $P(B \cap C) \leq 0.48$

(c) $0.23 \leq P(B \cap C) \leq 0.48$

(d) $0.23 \leq P(B \cap C) \geq 0.48$

143. Out of $3n$ consecutive natural numbers, 3 natural numbers are chosen at random without replacement. The probability that the sum of the chosen numbers is divisible by 3 is

(a) $\frac{n(3n^2 - 3n + 2)}{2}$ (b) $\frac{(3n^2 - 3n + 2)}{2(3n-1)(3n-2)}$

(c) $\frac{(3n^2 - 3n + 2)}{(3n-1)(3n-2)}$ (d) $\frac{n(3n-1)(3n-2)}{3(n-1)}$

144. If X and Y are independent binomial variates $B\left(5, \frac{1}{2}\right)$ and $B\left(7, \frac{1}{2}\right)$, then $P(X + Y = 3)$ is

(a) $\frac{35}{47}$ (b) $\frac{55}{1024}$

(c) $\frac{220}{512}$ (d) $\frac{11}{204}$

145. $\int_0^{\pi/4} \log(1+\tan\theta)\,d\theta =$

(a) $\pi \log 2$
(b) $\frac{(\pi \log 2)}{2}$
(c) $\frac{(\pi \log 2)}{8}$
(d) $\log 2$

146. $\int_a^b \sqrt{(x-a)(b-x)}\,dx\,(b>a)$ is equal to

(a) $\frac{\pi(b-a)^2}{8}$
(b) $\frac{\pi(b+a)^2}{8}$
(c) $(b-a)^2$
(d) $(b-a)^2$

147. The area bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$ is

(a) $\left(\frac{5\pi}{4}-2\right)$ square units
(b) $\frac{(5\pi-2)}{4}$ square units
(c) $\frac{(5\pi-2)}{2}$ square units
(d) $\left(\frac{\pi}{2}-5\right)$ square units

148. $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\ldots+\frac{1}{3n}\right)=$

(a) log 2
(b) log 3
(c) log 5
(d) 0

149. The degree and order respectively of the differential equation of all the parabolas whose axis is x-axis, are

(a) 2, 1
(b) 1, 2
(c) 2, 2
(d) 1, 1

150. The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x+3y)$ with x-axis and which passes through (1, 2) is

(a) $6x+9y+2=26e^{3(x-1)}$
(b) $6x-9y+2=26e^{3(x-1)}$
(c) $6x-9y-2=26e^{3(x-1)}$
(d) $6x-9y-2=26e^{3(x-1)}$

151. If $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx = x[f(x)-g(x)]+c$, then

(a) $f(x)=\log(\log x)$; $g(x)=\frac{1}{\log x}$
(b) $f(x)=\log x$; $g(x)=\frac{1}{\log x}$
(c) $f(x)=\frac{1}{\log x}$; $g(x)=\log(\log x)$
(d) $f(x)=\frac{1}{x\log x}$; $g(x)=\frac{1}{\log x}$

152. If $f\left(\frac{3x-4}{3x+4}\right)=x+2$, then $\int f(x)\,dx$ is

(a) $e^{x+2}\log\left|\frac{3x-4}{3x+4}\right|+c$
(b) $-\frac{8}{3}\log|1-x|+\frac{2}{3}x+c$
(c) $\frac{8}{3}\log|1-x|+\frac{x}{3}+c$
(d) $e^{[(3x-4)/(3x+4)]}-\frac{x^2}{2}-2x+c$

153. The derivative of $f(\tan x)$ with respect to $g(\sec x)$ at $x=\frac{\pi}{4}$, where $f'(1)=2$ and $g'(\sqrt{2})=4$, is

(a) $\frac{1}{\sqrt{2}}$
(b) $\sqrt{2}$
(c) 1
(d) 0

154. If $y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)+\sin^{-1}\frac{2x}{1+x^2}$, then $\frac{dy}{dx}$ at $x=\frac{\pi}{2}$ is equal to

(a) $\frac{8}{(4+\pi^2)}$
(b) 0
(c) $-\frac{8}{(4+\pi^2)}$
(d) 1

155. If $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{1}{\sqrt{1-x^2}}$
(b) $\frac{1}{\sqrt{1-x^2}}$
(c) $\frac{3}{\sqrt{1-x^2}}$
(d) $\frac{x}{\sqrt{1-x^2}}$

156. The value of a in order that $f(x)=\sin x-\cos x-ax+b$ decreases for all real values of x is given by

(a) $a\geq\sqrt{2}$
(b) $a<\sqrt{2}$
(c) $a\geq 1$
(d) $a<1$

157. If $\log_{10}(x^3+y^3)-\log_{10}(x^2+y^2-xy)\leq 2$, then the maximum value of xy, for all $x\geq 0, y\geq 0$, is

(a) 2500
(b) 3000
(c) 1200
(d) 3500

158. Let ABC be an equilateral triangle formed by weightless inextensible strings, the side AB is horizontal. A and B are tied to fixed points D and E by equal weightless inextensible strings AD, BE; a weight of W gm is attached at C. The angle DAB,

ABE are each 150°. Then which of the following statements is true?

(a) The tensions in the strings AB, BC and CA are equal

(b) The tensions in the strings BE, AD are inversely proportional

(c) The tension in BE is less than the tension in AB

(d) The tension in AD is twice the tension in BC.

159. Two parabolic paths of angles α and β of projection aimed at a target on the horizontal plane through O, fall p units short and the other p units far from the target. If θ is the correct angle of projection so as to hit the target, then $\sin 2\theta$ is equal to

(a) $\frac{1}{2}(\tan\alpha+\tan\beta)$ (b) $\frac{1}{2}(\tan\alpha-\tan\beta)$

(c) $\frac{1}{2}(\sin 2\alpha+\sin 2\beta)$ (d) $\frac{1}{2}(\sin 2\alpha-\sin 2\beta)$

160. The linear programming problem :

Maximize $z = x_1 + x_2$

Subject to constraints

$$x_1 + 2x_2 \le 2000,\ x_1 + x_2 \le 1500,$$
$$x_2 \le 600,\ x_1 \ge 0 \text{ has}$$

(a) no feasible solution

(b) unique optimal solution

(c) a finite number of optimal solutions

(d) infinite number of optimal solutions

161. $A_1, A_2, \ldots, A_n$ are thirty sets, each with five elements and $B_1, B_2, \ldots, B_n$ are n sets, each with three elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$. If each element of S belongs to exactly ten of A_i's and exactly nine of the B_j's, then n is

(a) 45 (b) 35

(c) 40 (d) 30

162. Let R and S be two non-void relations on a set A. Which of the following statements is false?

(a) R and S are transitive implies $R\cap S$ is transitive

(b) R and S are transitive implies $R\cup S$ is transitive

(c) R and S are symmetric implies $R\cup S$ is symmetric

(d) R and S are reflexive implies $R\cap S$ is reflexive

163. Let $f : [4, \infty[\to [4, \infty[$ be defined by $f(x) = 5^{x(x-4)}$. Then $f^{-1}(x)$ is

(a) $2-\sqrt{4+\log_5 x}$

(b) $2+\sqrt{4+\log_5 x}$

(c) $\left(\frac{1}{5}\right)^{x(x-4)}$ (d) not defined

164. The number of solutions of the equation $z^2 + \bar{z} = 0$ is

(a) 2 (b) 4

(c) 6 (d) 8

165. If a, b, c are sides of a triangle, then $\frac{(a+b+c)^2}{(ab+bc+ca)}$ always belongs to

(a) [1, 2) (b) [2, 3]

(c) [3, 4) (d) [4, 5]

166. If α, β are the roots of the equation $\lambda(x^2 - x) + x + 5 = 0$ and if λ_1 and λ_2 are two values of λ obtained from $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{5}$, then $\frac{\lambda_1}{\lambda_2^2}+\frac{\lambda_2}{\lambda_1^2}$ equals

(a) 4192 (b) 4144

(c) 4096 (d) 4048

167. If $a, a_1, a_2, \ldots, a_{2n}, b$ are in arithmetic progression and $a, g_1, g_2, \ldots, g_{2n}, b$ are in geometric progression and h is the harmonic mean of a and b, then $\frac{a_1+a_{2n}}{g_1g_{2n}}+\frac{a_2+a_{2n-1}}{g_2g_{2n-1}}+\ldots+\frac{a_n+a_{n+1}}{g_ng_{n+1}}$ is equal to

(a) $2nh$ (b) n/h

(c) nh (d) $2n/h$

168. The sum of the products of the numbers $\pm1, \pm2, \ldots, \pm n$, taken two at a time is

(a) $\frac{-n(n+1)}{2}$

(b) $\frac{n(n+1)(2n+1)}{6}$

(c) $\frac{-n(n+1)(2n+1)}{6}$

(d) 0

169. $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$ is equal to

(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\left[\frac{n(n+1)}{2}\right]^2$

(c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n+1)(n+2)}{6}$

170. The sum of the series

$\log_4 2 - \log_8 2 + \log_{16} 2 - \log_{32} 2 + \ldots$ is

(a) e^2 (b) $\log_e 2 + 1$

(c) $\log_e 3 - 2$ (d) $1 - \log_e 2$

171. The sum of the series $\frac{1}{1.2}+\frac{1.3}{1.2.3.4}+\frac{1.3.5}{1.2.3.4.5.6}+\ldots$ is

(a) $e-1$ (b) $\sqrt{e}-1$

(c) $\sqrt{e}-2$ (d) $\sqrt{e}+e$

172. Range of the function $f(x)=\frac{x}{1+x^2}$ is

(a) $(-\infty, \infty)$ (b) $[-1, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-\sqrt{2}, \sqrt{2}]$

173. $\lim\limits_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{\pi}{2}$

174. The value of $f(0)$, so that the function

$$f(x)=\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$$ becomes

continuous for all x, is given by

(a) $a^{3/2}$ (b) $a^{1/2}$

(c) $-a^{1/2}$ (d) $-a^{3/2}$

175. The system of equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has no solution if

(a) $\lambda=3, \mu=10$ (b) $\lambda\neq 3, \mu=10$

(c) $\lambda\neq 3, \mu\neq 10$ (d) $\lambda=3, \mu\neq 10$

176. If $a_1, a_2, a_3, \ldots$ form a geometric progression and $a_i>0$ for all $i\geq 1$, then $\begin{vmatrix}\log a_m & \log a_{m+1} & \log a_{m+2}\\ \log a_{m+3} & \log a_{m+4} & \log a_{m+5}\\ \log a_{m+6} & \log a_{m+7} & \log a_{m+8}\end{vmatrix}$ is equal to

(a) $\log a_{m+8}-\log a_m$ (b) $\log a_m$

(c) $2\log a_{m+1}$ (d) 0

177. If x is a positive integer, then

$\begin{vmatrix}x! & (x+1)! & (x+2)!\\ (x+1)! & (x+2)! & (x+3)!\\ (x+2)! & (x+3)! & (x+4)!\end{vmatrix}$ is equal to

(a) $2x!(x+1)!$ (b) $2x!(x+1)!(x+2)!$

(c) $2x!(x+3)!$

(d) $2(x+1)!(x+2)!(x+3)!$

178. If $\tan\left(\frac{x}{2}\right)=\operatorname{cosec}x-\sin x$, then the value of $\tan^2\left(\frac{x}{2}\right)$ is

(a) $2-\sqrt{5}$ (b) $2+\sqrt{5}$

(c) $-2-\sqrt{5}$ (d) $-2+\sqrt{5}$

179. Consider the system of equations in x, y, z as $x\sin 3\theta-y+z=0$, $x\cos 2\theta+4y+3z=0$, $2x+7y+7z=0$. If this system has a non-trivial solution, then for integer n, values of θ are given by

(a) $\pi\left(n+\frac{(-1)^n}{3}\right)$ (b) $\pi\left(n+\frac{(-1)^n}{4}\right)$

(c) $\pi\left(n+\frac{(-1)^n}{6}\right)$ (d) $\frac{n\pi}{2}$

180. The value of

$\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{5\pi}{8}\right)+\cos^4\left(\frac{7\pi}{8}\right)$ is

(a) 0 (b) 1/2

(c) 3/2 (d) 1

SOLUTIONS

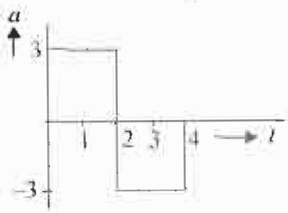
2009

1. **(a) :** $s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}at^2 \qquad (\because \; u = 0)$

It is an equation of parabola. Hence, graph (a) represents the correct option.

2. **(a) :**



Taking the motion from 0 to 2 s

$u = 0, a = 3 \text{ ms}^{-2}, t = 2 \text{ s}, v = ?$

$v = u + at = 0 + 3 \times 2 = 6 \text{ ms}^{-1}$

Taking the motion from 2 s to 4 s

$v = 6 + (-3)(2) = 0 \text{ ms}^{-1}$

Hence, graph (a) represents the correct option.

3. **(b) :** According to given relation acceleration $a = \alpha t + \beta$

As $a = \frac{dv}{dt} \Rightarrow \alpha t + \beta = \frac{dv}{dt}$

Since particle starts from rest, its initial velocity is zero *i.e.*, At time $t = 0$, velocity $= 0$.

$\Rightarrow \int_0^v dv = \int_0^t (\alpha t + \beta)dt \quad \text{or} \quad v = \frac{\alpha t^2}{2} + \beta t$

4. **(a) :** This is a case of a perfectly inelastic collision in which linear momentum is conserved but kinetic energy is not conserved.

5. **(c) :** Here, $m_1 = M, m_2 = 4M$

Kinetic energy of a particle of mass m having linear momentum p is given by $K = \frac{p^2}{2m}$

As K is same

$\therefore \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{M}{4M}} = \frac{1}{2}$

6. **(c) :** Here, angle of projection $\theta = 45°$

Maximum height $H = \frac{u^2 \sin^2\theta}{2g}$

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

For $\theta = 45°$

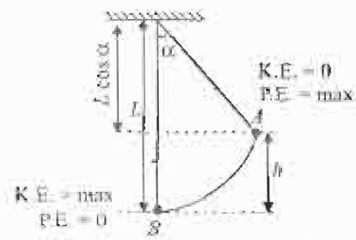
$H = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{4g} \qquad \left(\because \; \sin 45° = \frac{1}{\sqrt{2}}\right)$

$R = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \qquad (\because \; \sin 90° = 1)$

$\therefore \frac{R}{H} = \frac{u^2}{g} \times \frac{4g}{u^2} = 4 \quad \text{or} \quad R = 4H.$

7. **(c) :** Here, mass of the bob $= M$

Length of the pendulum $= L$



From figure,

$h = L - L\cos\alpha = L(1 - \cos\alpha)$

Potential energy at $A = Mgh = MgL(1 - \cos\alpha)$

Potential energy at $B = 0$

Kinetic energy at $A = \frac{1}{2}mv^2 = 0 \qquad (\because \; \text{As } v = 0)$

Mechanical energy at $A = 0 + MgL(1 - \cos\alpha)$

Let K_B is the kinetic energy at B

Mechanical energy at $B = K_B + 0$

According to conservation of mechanical energy at A and B, we get

$0 + MgL(1 - \cos\alpha) = K_B + 0$

or $K_B = MgL(1 - \cos\alpha)$

Kinetic energy is maximum at point B.

Therefore maximum kinetic energy of bob of mass M is $MgL(1 - \cos\alpha)$

8. **(b) :** Here, $\frac{\rho_1}{\rho_2} = \frac{9}{8}$

$v_{rms} = \sqrt{\frac{3RT}{M}}$

At a given temperature

$$\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

9. **(b) :** Excess pressure inside a spherical drop of radius r of a liquid of surface tension T is $P = \frac{2T}{r}$

10. **(b) :** According to Stefan's law, rate of energy radiated by a black body per unit area (in W/m²) at temperature T is given by

$U = \sigma T^4$ ....(i)

When the temperature falls to half, (*i.e.*, $T/2$) radiated energy (in W/m²) is U'

$\therefore \ U' = \sigma(T/2)^4$ ....(ii)

From (i) and (ii), we get

$$\frac{U'}{U} = \left(\frac{1}{2}\right)^4 = \frac{1}{16} \quad \text{or} \quad U' = \frac{U}{16}$$

11. **(a) :** During upward motion the velocity is decreasing and becomes zero at the highest point. While during downward motion the magnitude of velocity is increasing in downward direction. Hence, graph (a) represents the correct option.

12. **(b) :** Moment of inertia of a ring of mass M and radius R about an axis passing through the centre and perpendicular to the plane,

$I = MR^2$ ...(i)

Moment of inertia of a ring about its diameter

$$I_{diameter} = \frac{MR^2}{2}$$

$$I_{diameter} = \frac{1}{2}(MR^2) = \frac{I}{2} \quad \text{(Using (i))}$$

13. **(d) :** Here, velocity of source $v_s = 30$ ms^{-1}

velocity of observer $v_0 = 30$ ms^{-1}

velocity of sound $v = 330$ ms^{-1}

$\upsilon = 660$ Hz

As source and observer are moving towards each other with same speed, therefore apparent frequency is

$$\upsilon' = \upsilon\left[\frac{v+v_o}{v-v_s}\right] = 600\left[\frac{330+30}{330-30}\right]$$

$$= \frac{600 \times 360}{300} = 720 \text{ Hz}$$

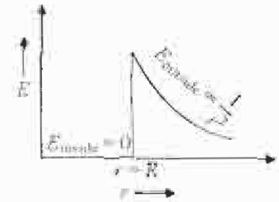
14. **(c) :** The electric field due to a spherical shell having a charge $+q$ units and radius R at a distance r from the centre is given by

(i) $E_{inside} = 0$ (For $r < R$)

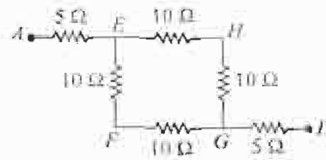
(ii) $E_{surface} = \frac{q}{4\pi\varepsilon_0 R^2}$ (For $r = R$)

(iii) $E_{outside} = \frac{q}{4\pi\varepsilon_0 r^2}$ (For $r > R$)

The variation of E with r for a spherical shell is shown in the figure.



15. **(b) :** The effective circuit is shown in the figure.



The resistance of arm EHG = 10 Ω + 10 Ω = 20 Ω, will be parallel to resistance of arm EFG = 10 Ω + 10 Ω = 20 Ω

$$\text{Their effective resistance} = \frac{20\,\Omega \times 20\,\Omega}{20\,\Omega + 20\,\Omega} = 10\,\Omega$$

Therefore, the equivalent resistance between A and D = 5 Ω + 10 Ω + 5 Ω = 20 Ω.

16. **(c) :** Here, neutral temperature $T_n = 270°C$

temperature of inversion $T_i = 525°C$

temperature of cold junction T_c = ?

As $T_n - T_c = T_i - T_n$

$\Rightarrow \ T_c = 2T_n - T_i = 2 \times 270°C - 525°C$

$T_c = 540°C - 525°C = 15°C$

17. **(b) :** Frequency $= \frac{1}{\text{second}} = \frac{1}{[T]}$

(a) $$\frac{1}{\sqrt{RC}} = \frac{1}{\sqrt{\text{ohm} \times \text{farad}}} = \frac{1}{\sqrt{\text{ohm} \times \frac{\text{coulomb}}{\text{volt}}}}$$

$$= \frac{1}{\sqrt{\frac{\text{volt}}{\text{ampere}} \times \frac{\text{coulomb}}{\text{volt}}}}$$

$$= \frac{1}{\sqrt{\frac{\text{volt}}{\text{ampere}} \times \frac{\text{ampere} \times \text{second}}{\text{volt}}}}$$

$$= \frac{1}{\sqrt{\text{second}}} = \frac{1}{[T]^{1/2}}$$

(b) $$\frac{R}{L} = \frac{\text{ohm}}{\text{henry}} = \frac{\text{ohm}}{\text{ohm} \times \text{second}} = \frac{1}{\text{second}} = \frac{1}{[T]}$$

(c) $\frac{1}{LC} = \frac{1}{\text{henry} \times \text{farad}} = \frac{1}{\text{ohm} \times \text{second} \times \frac{\text{coulomb}}{\text{volt}}}$

$= \frac{1}{\frac{\text{volt}}{\text{ampere}} \times \text{second} \times \frac{\text{ampere} \times \text{second}}{\text{volt}}}$

$= \frac{1}{(\text{second})^2} = \frac{1}{[T]^2}$

(d) $\frac{C}{L} = \frac{\text{farad}}{\text{henry}} = \frac{\text{coulomb/ volt}}{\text{ohm} \times \text{second}}$

$= \frac{\text{coulomb}}{\text{volt} \times \text{ohm} \times \text{second}}$

$= \frac{\text{ampere} \times \text{second}}{\text{volt} \times \frac{\text{volt}}{\text{ampere}} \times \text{second}}$

$= \frac{(\text{ampere})^2}{(\text{volt})^2} = \frac{[A]^2}{[ML^2T^{-3}A^{-1}]^2}$

$= [M^{-2}L^{-4}T^6A^4]$

18. **(b) :** Impulse = Change in momentum
Hence, impulse and momentum both have same dimensions *i.e.*, $[MLT^{-1}]$.

19. **(b) :** Here, diameter $D = 1.25$ cm $= 1.25 \times 10^{-2}$ m
density of water $\rho = 10^3$ kgm^{-3}
coefficient of viscosity $\eta = 10^{-3}$ Pas
rate of flow of water $Q = 5 \times 10^{-5}$ m^{3}s^{-1}

Reynold's number $N_R = \frac{v\rho D}{\eta}$ ...(i)

where v is the speed of flow.
Rate of flow of water Q = Area of cross section × speed of flow

$Q = \frac{\pi D^2}{4} \times v$ or $v = \frac{4Q}{\pi D^2}$

Substituting the value of v in eqn (i), we get

$N_R = \frac{4Q\rho D}{\pi D^2 \eta} = \frac{4Q\rho}{\pi D \eta}$

Substituting the values, we get

$N_R = \frac{4 \times 5 \times 10^{-5} \times 10^3}{\left(\frac{22}{7}\right) \times 1.25 \times 10^{-2} \times 10^{-3}} = 5100$

For $N_R > 3000$, the flow is turbulent.
Hence, the flow of water is turbulent with Reynold's number 5100.

20. **(b) :** Rotational kinetic energy $K_R = \frac{1}{2} I\omega^2$

$K_R = \frac{1}{2} \times \frac{MR^2}{2} \times \omega^2$

$K_R = \frac{1}{4} Mv^2$ $(\because\ v = R\omega)$

Translational kinetic energy $K_T = \frac{1}{2} Mv^2$
Total kinetic energy $= K_T + K_R$

$= \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2$

$\frac{\text{Rotational kinetic energy}}{\text{Total kinetic energy}} = \frac{\frac{1}{4}Mv^2}{\frac{3}{4}Mv^2} = \frac{1}{3}$

21. **(b) :** If two parallel wires carry current in opposite directions, they repel each other whereas if two parallel wires carry current in same direction, they attract each other.

22. **(a) :** If Young's double slit experiment is performed in water of refractive index μ instead of air, the wavelength of light will change from λ to (λ/μ)

$\therefore\ \beta' = \frac{D}{d}\left(\frac{\lambda}{\mu}\right) = \frac{\beta}{\mu}$

Fringe width reduces and becomes $(1/\mu)$ times of its value in air.

23. **(d) :** For beta minus (β^-) decay, a neutron transforms into proton within the nucleus according to

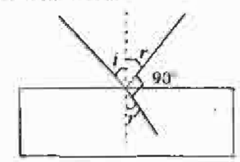
n	$\rightarrow$	p	+	e^-	+	$\bar{\upsilon}$
neutron		proton		electron		antineutrino

For beta plus (β^+) decay, a proton transforms into neutron within the nucleus according to

p	$\rightarrow$	n	+	e^+	+	υ
proton		neutron		positron		neutrino

24. **(d) :** Here, angle of incidence $i = 60°$

From figure,



$r + 90° + r' = 180$

or $r' = 90° - r$

$r' = 90° - i$ (As $\angle i = \angle r$) ...(i)

According to Snell's law
$1 \sin i = \mu \sin r'$

$\Rightarrow\ \mu = \frac{\sin i}{\sin r'} = \frac{\sin i}{\sin(90° - i)}$ (Using (i))

$\mu = \frac{\sin i}{\cos i} = \tan i = \tan 60° = \sqrt{3}$.

25. **(b) :** Centripetal acceleration $a_c = v^2/r$
It acts along the radius and directed towards the centre of the circular path.

26. **(c) :** Escape velocity is independent of the mass of

the projected body provided the air resistance is neglected. Hence, the escape velocity of 10 kg of the iron ball projected from the earth is same that of escape velocity of 10 g body projected from the earth *i.e.*, 11.2 kms^{-1}.

27. (a) : Horizontal range is same for angle of projection θ and $90^\circ - \theta$.

$\therefore$ Horizongal range $R = \frac{u^2 \sin 2\theta}{g}$ ....(i)

$$\therefore \quad t_1 = \frac{2u\sin\theta}{g}, \; t_2 = \frac{2u\sin(90^\circ - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore \quad t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left(\frac{2u^2\sin\theta\cos\theta}{g}\right)$$

$$= \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) \quad (\because \; \sin 2\theta = 2\sin\theta\cos\theta)$$

$$= \frac{2R}{g} \quad \text{(Using (i))}$$

28. (a) : Here, $E = 6$ V/m, $c = 3 \times 10^8$ ms^{-1}

$$B = \frac{E}{c} = \frac{6 \text{ V/m}}{3\times10^8 \text{ ms}^{-1}} = 2\times10^{-8} \text{ T}$$

E is along the y-direction and the plane e.m.wave propagate along x-direction. Therefore, B should be in a direction perpendicular to both x and y-axes. Using vector algebra $\vec{E}\times\vec{B}$ should be along x-direction. Since $(+\hat{j})\times(+\hat{k}) = \hat{i}$, B is along the z-direction.

Thus, magnetic field component B would be 2×10^{-8} T along z-direction.

29. (c) : As $B_{inside} = \frac{\mu_0 2Ir}{4\pi R^2}$

At the axis of the conductor $r = 0$

$\therefore \; B_{inside} = 0$

30. (*) : Here, source temperature $T_1 = 600$ K

sink temperature $T_2 = 450$ K

work done per cycle $W = 300$ J

Efficiency of a Carnot engine $\eta = 1 - \frac{T_2}{T_1}$

$$\eta = 1 - \frac{450}{600} = \frac{150}{600} = \frac{1}{4} \quad \text{....(i)}$$

Also,

$$\eta = \frac{\text{Work done per cycle } (W)}{\text{Amount of heat absorbed from the source per cycle } (Q_1)}$$

$$\text{or} \quad Q_1 = \frac{W}{\eta} = \frac{W}{\left(\frac{1}{4}\right)} \quad \text{(Using (i))}$$

$Q_1 = 4W = 4 \times 300$ J $= 1200$ J

* None of the given option is correct.

31. (c) : Here, $y_1 = 5(\sin 2\pi t + \sqrt{3}\cos 2\pi t)$

$y_2 = 5\sin(2\pi t + \frac{\pi}{4})$

$y_1 = 5\sin 2\pi t + 5\sqrt{3}\cos 2\pi t$

As of the form of $y_1 = \alpha \sin 2\pi t + \beta \cos 2\pi t$

Let $\alpha = r\cos\theta = 5$, $\beta = r\sin\theta = 5\sqrt{3}$

$\therefore \; y_1 = r\cos\theta \sin 2\pi t + r\sin\theta\cos 2\pi t$

$= r\sin(2\pi t + \theta)$

Also, $\alpha^2 + \beta^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2$

$$\text{or} \quad r = \sqrt{\alpha^2+\beta^2} = \sqrt{(5)^2 + (5\sqrt{3})^2}$$

$$= 5\sqrt{1^2 + (\sqrt{3})^2} = 10$$

$$\therefore \; y_1 = 10\sin(2\pi t + \theta) \quad \therefore \; \frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$$

32. (a)

33. (b) : Capacitance of a parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

A parallel plate capacitor is charged (battery is disconnected) then the plates are pulled apart, the capacitance decreases while the charge remains the same.

$\because$ Potential difference $= \frac{\text{Charge}}{\text{Capacitance}}$

$\therefore$ Potential difference increases.

34. (d) : Here, e.m.f. $\varepsilon = 12$ V

internal resistance $r = 0.5\ \Omega$.

$$I_{max} = \frac{\varepsilon}{r} = \frac{12 \text{ V}}{0.5\ \Omega} = 24 \text{ A}.$$

35. (b) : Here, $f_a = +10$ cm

$\mu_g = 1.5 = 3/2$, $\mu_w = 1.33 = 4/3$

According to lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ with } \mu = \frac{\mu_L}{\mu_M}$$

$$\therefore \; \frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{(3/2)}{1} - 1\right)C \text{ where } C = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{or} \quad \frac{1}{f_a} = \frac{C}{2} \quad \text{....(i)}$$

$$\text{Also, } \frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{(3/2)}{(4/3)} - 1\right)C$$

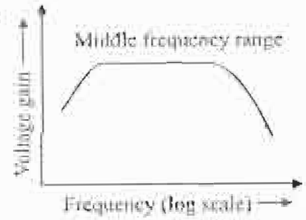
$$\frac{1}{f_e} = \frac{C}{8} \qquad ...(ii)$$

From (i) and (ii), we get

$$\frac{f_e}{f_o} = \frac{C}{2} \times \frac{8}{C} = 4$$

or $f_e = 4f_o = 4 \times 10$ cm $= 40$ cm.

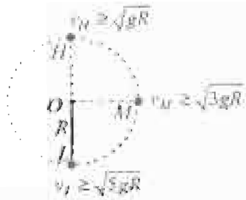
36. **(b) :**



From the graph, we can conclude that voltage gain decreases both at low frequencies as well as at high frequencies. At the middle frequencies, voltage gain remains the constant.

37. **(c) :** Laws of reflection and refraction, polarisation and interference are the phenomena which support the wave theory of light whereas photoelectric effect support the particle nature of light.

38. (d) :



The critical speed of the block at the top is $\sqrt{Rg}$.

39. **(c) :** Volume V = cross sectional A × length l

or $V = Al$

$$\text{strain} = \frac{\text{Elongation}}{\text{Original length}} = \frac{y}{l}$$

$$\text{Young's modulus } Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Work done, } W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$W = \frac{1}{2} \times Y \times (\text{strain})^2 \times Al$$

$$= \frac{1}{2} \times Y \times \left(\frac{y}{l}\right)^2 \times Al = \frac{1}{2}\left(\frac{YA}{l}\right)y^2 \text{ or } W \propto y^2$$

40. **(d):** $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

$\hat{n}$ is a unit vector indicating the direction of $\vec{A} \times \vec{B}$

Vector product or cross product is anticommutative

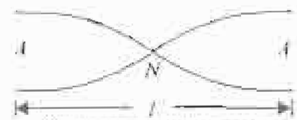
i.e., $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

41. **(b) :** Apparent weight $W' = m(g + a)$.

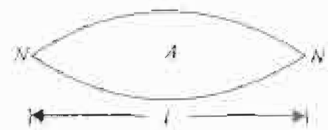
42. **(c) :** According to Newton's law of cooling, as the rate of cooling is directly proportional to temperature difference between the body and the surroundings

i.e., $\frac{-d\theta}{dt} \propto \Delta\theta \Rightarrow n = 1$

43. **(b) :**



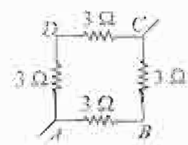
Displacement wave pattern



Pressure wave pattern

The pressure variation is maximum at a distance $l/2$ because the displacement node is pressure antinode.

44. **(d) :** When a metallic wire of resistance 12 Ω is bent to form a square, the equivalent circuit diagram is as shown in the figure.



Resistance of arm ABC = 3 Ω + 3 Ω = 6 Ω will be parallel to resistance of arm ADC = 3 Ω + 3 Ω = 6 Ω.

Their effective resistance is

$$= \frac{6\ \Omega \times 6\ \Omega}{6\ \Omega + 6\ \Omega} = 3\ \Omega$$

Hence, the resistance between diagonal points *i.e.*, between A and C is 3 Ω.

45. **(a) :** The torque on a electric dipole is given by

$\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = pE \sin\theta$

The torque will be maximum, when $\theta = 90°$

$\tau_{max} = pE$

46. **(a) :** $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$

where $\varepsilon_1, \varepsilon_2$ = e.m.f's of two cells.

l_1, l_2 = lengths of the potentiometer wire on which e.m.f ε_1 and ε_2 are balanced

Here,

$\varepsilon_1 = 1.5$ V, $l_1 = 27$ cm

$\varepsilon_2 = ?, l_2 = 54$ cm

$\therefore \quad \varepsilon_2 = \varepsilon_1 \frac{l_2}{l_1} = 1.5 \text{ V} \times \frac{54 \text{ cm}}{27 \text{ cm}} = 3 \text{ V}.$

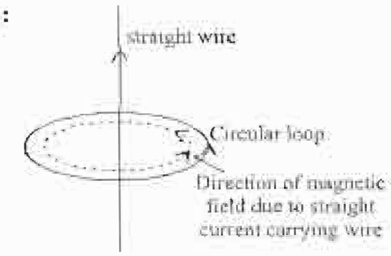
47. **(d) :** When the image is formed at infinity

$$m = \frac{D}{f} = DP$$

When the image is formed at the near point

$$m = \left(1 + \frac{D}{f}\right) = 1 + DP$$

48. **(d) :**



The figure shows that, magnetic field due to straight wire is either parallel or antiparallel to the current flow in loop depending on direction of current in wire. Thus force F exerted by this magnetic field B is,

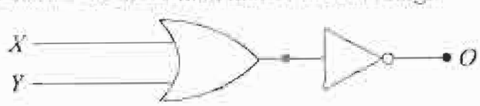
$\vec{F} = Id\vec{l} \times \vec{B}$

$= IdlB \sin\theta = 0 \qquad [\because \theta = 0 \text{ or } 180°]$

49. **(c)**

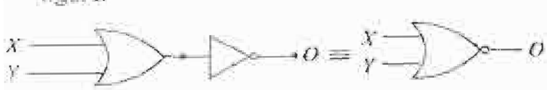
50. **(a) :** An optical fibre consists of a fine core of a material of refractive index μ_1 surrounded by a glass or a plastic cladding with refractive index μ_2 such that $\mu_2 < \mu_1$. The difference $\Delta\mu = \mu_1 - \mu_2$ is typically very small of the order of 10^{-3}. An optical fibre is based on the principle of total internal reflection. For total internal reflection to take place refractive index of core should be more than that of the cladding.

51. **(b) :**



$O = \overline{X + Y}$

The logic symbol of NOR gate as shown in the figure.



Hence, the given logic circuit represents NOR gate with output $O = \overline{X + Y}$

$\because \quad \overline{X + Y} = \overline{X} \cdot \overline{Y}$, therefore (d) is also correct.

52. **(c) :** The circuit element connected to the AC source will be pure resistor. In pure resistive AC circuit, voltage and current are in the same phase.

53. **(d) :** Force on a charged particle in the magnetic field is

$\vec{F} = q(\vec{v} \times \vec{B})$ or $F = qvB\sin\theta$

Here, $\theta = 0°$ *i.e.*, $\vec{v}$ is parallel to $\vec{B}$

$\therefore \quad F =$ zero

54. **(a)**

55. **(d) :** On the basis of kinetic theory of gases, the mean kinetic energy of one mole of gas per degree of freedom is $\frac{1}{2}RT$

where R is the universal gas constant.

On the basis of kinetic theory of gases, the mean kinetic energy of a molecule of gas per degree of freedom is $\frac{1}{2}kT$

where k is the Boltzmann constant.

56. **(a) :** According to Gauss's law total flux coming out of a closed surface enclosing charge q is given by

$$\phi = \oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

From this expression, it is clear the total flux linked with a closed surface only depends on the enclosed charge and independent of the shape and size of the surface.

$$\phi = \oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0} = 20 \text{ Vm given}$$

This $\frac{q}{\varepsilon_0}$ is constant as long as the enclosed charge is constant

$\Rightarrow$ The flux over a concentric sphere of radius 20 cm = 20 Vm.

57. **(c) :** For photoelectric effect to take place

$$\frac{hc}{\lambda} \geq \frac{hc}{\lambda_0} \Rightarrow \lambda \leq \lambda_0$$

58. **(a)**

59. **(c) :** Rate of flow of water through a capillary tube is

$$V = \frac{\pi P r^4}{8\eta l}$$

where

P = pressure difference at the two ends of tube

l = length of tube, r = radius of tube

η = coefficient of viscosity of water

As P, η remain the same

$\therefore \quad \frac{V'}{V} = \frac{(2r)^4}{(r)^4} \times \frac{(l)}{(2l)} = \frac{16}{2} = 8$

or $V' = 8V$

60. **(b)** : $g' = g - R\omega^2 \cos^2\lambda$

where ω is the angular velocity of rotation of earth about its polar axis, R is the radius of the earth and λ is the latitude of a place.

At poles, $\lambda = 90°$

$\therefore \quad g_{pole} = g - R\omega^2 \cos^2 90° = g$

At equator $\lambda = 0°$

$g_{equator} = g - R\omega^2 \cos^2 0° = g - R\omega^2 < g$

Thus, the acceleration due to gravity decreases from poles to equator

Hence, when a body is taken from poles to equator on the earth, its weight decreases.

61. **(a)** : van't Hoff factor is simply a ratio. It is a ratio of abnormal colligative property to normal colligative property.

$i = \frac{\text{Abnormal colligative property}}{\text{Normal colligative property}}$

It can be calculated by the given formula

$i = \frac{\pi V}{nRT}$; $i = \frac{\Delta T_f}{K_f\, m}$; $i = \frac{\Delta T_b}{K_b m}$ and

$i = \left(\frac{P° - P_s}{P°}\right)\left(\frac{n+N}{n}\right)$

62 **(b)** : $\pi_{sugarcane} = \pi_X$

or $C_{sugarcane} = C_X$

$\left(\frac{w_B \times 1000}{m_B \times V}\right)_{sugarcane} = \left(\frac{w_B \times 1000}{m_B \times V}\right)_X$

$\left(\frac{5 \times 1000}{342 \times 100}\right)_{sugarcane} = \left(\frac{1 \times 1000}{m_B \times 100}\right)_X$

m_B of $X = \frac{1000 \times 100 \times 342}{5 \times 1000 \times 100} = 68.4$

63. **(b)** : $A \rightarrow B$, $\Delta H = 24$ kJ/mole

i.e., $H_B - H_A = 24$ kJ/mole ... (i)

$B \rightarrow C$, $\Delta H = -18$ kJ/mole

i.e., $H_C - H_B = -18$ kJ/mole ... (ii)

Adding eqn. (i) and (ii),

$H_C - H_A = 6$ kJ/mole ... (iii)

From eqns. (i), (ii) and (iii),

$H_B > H_C > H_A$.

64. **(c)** :

	A	$+\ 4B$	$\rightleftharpoons 2C$	$+\ 3D$
Initial	3	4	0	0
At equilibrium	$3 - x$	$4 - 4x$	$2x$	$3x$

It is given that $2x = 1$ or $x = 1/2$

Equilibrium extent

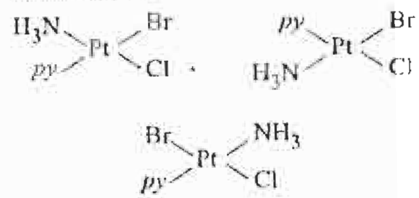
$= \frac{\text{Amount of product formed}}{\text{Stoichiometric coefficient of product}}$

$= \frac{\left(2 \times \frac{1}{2}\right)}{2} = \frac{1}{2}$.

65. None of the given options is correct.

H_2O is a stronger ligand than Cl and the order of the ligands in spectrochemical series is determined experimentally.

66. **(b)** : Square planar complexes of the type [$MABXY$] occur in three isomeric forms.



67. **(b)** : Arrhenius gave a mathematical expression to deduce the relationship between rate constant and temperature.

$$k = Ae^{-E_a/RT}$$

where A is frequency factor and it is a constant.

E_a is activation energy

R is gas constant, T is temperature

On taking log on both sides,

$\ln k = \ln A - \frac{E_a}{RT}$

If k_1 and k_2 are rate constants at temperatures T_1 and T_2 respectively, then

$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{T_2 - T_1}{T_2 T_1}\right]$

68. Suppose order with respect to A is x and with respect to B is y.

From (a), $[0.20]^x\,[0.30]^y = 5 \times 10^{-5}$ ... (i)

From (b), $[0.20]^x\,[0.10]^y = 5 \times 10^{-5}$ ... (ii)

Dividing (i) by (ii), $1 = [3]^y$ or $y = 0$.

From (c), $7.5 \times 10^{-5} = [0.40]^x\,[0.05]^y$ ... (iii)

As $y = 0$

So, $[0.40]^x = 7.5 \times 10^{-5}$

Taking log on both sides,

$x \log(0.4) = \log(7.5 \times 10^{-5})$ or $x = 10.56$.

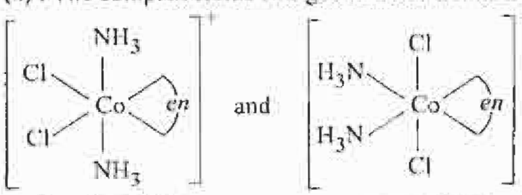
None of the options is correct.

69. **(a) :** $SF_4 \rightarrow$ Trigonal bipyramidal structure
(Two types of angle)
$XeF_4 \rightarrow$ Square planar structure
(One type of angle)
$SF_6 \rightarrow$ Octahedral structure
(One type of angle)
$CF_4 \rightarrow$ Tetrahedral structure
(One type of angle)

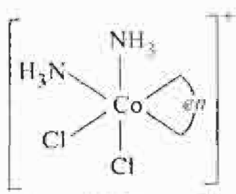
70. **(c) :** $-NMe_2$ (secondary amine) is strongest electron donating group among the given options and it increases electron density on carbonyl carbon thereby dispersing the positive charge on it.

71. **(a) :** As the slowest step is rate determining step and from the given data formation of B is the slowest step, so, step I is the rate determining step.

72. **(a) :** The complex forms two geometrical isomers.



trans (w.r.t. NH_3) *cis* (w.r.t. NH_3)



This exists in *d*- and *l*-forms. Thus there are 2 optically active isomer. Hence a total of 4 isomers are possible for the complex.

73. **(a) :** EDTA (ethylenediaminetetraacetic acid) has four donor oxygen atoms and two donor nitrogen atoms in each molecule. As H^+ is released in this process. So, pH will decrease.

$$Mg^{2+} + [H_2EDTA]^{2-} \longrightarrow [Mg(EDTA)]^{2-} + 2H^+$$

74. **(a) :**

$$2KMnO_4 + 8H_2SO_4 + 10FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O \longrightarrow K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 10(NH_4)_2SO_4 + 68H_2O$$

Consumption of $KMnO_4$ per mole of $FeSO_4 \cdot (NH_4)_2SO_4$ is 1/5 mole.

$$Fe^{2+} + Ce^{4+} \longrightarrow Fe^{3+} + Ce^{3+}$$

So, ratio $= \frac{1}{1/5} = 5.0$

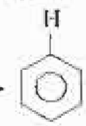
75. **(a) :** Amongst the given options N in option (b) is sp^2 hybridised. So, it is relatively more electronegative than N in options (a) and (c). In option (c) lone pair of N is involved in aromaticity. So, it is also not available for protonation. Out of options (a) and (d) oxygen is more electronegative than N. So, (a) is most basic.

76. **(d) :** $\Delta G = -nFE°$

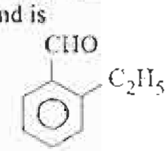
$$E°_{cell} = -\frac{RT}{nF}\ln K \qquad k = Ae^{-E_a/RT}.$$

77. **(a) :** Entropy order is as follows:
gas > liquid > solid
In option (a), a solid and liquid reactant give a liquid and gaseous product. So entropy is increasing from reactant to product side.

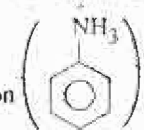
78. It is a derivative of histidine. It is not used as a drug. So, none of the given options is correct.

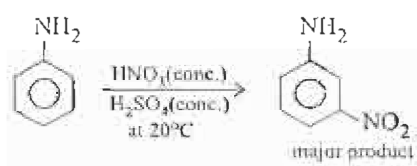
79. **(c) :** $C_6H_5N_2^+Cl^- + H_3PO_2 \xrightarrow{H_2O}$ 

80. **(a) :** From the formula it is clear that it is an unsaturated compound. As on vigorous oxidation it gives 1,2-benzenedicarboxylic acid, so, it is an aromatic compound having substituents at *o*-position.
It undergoes Cannizzaro's reaction and Tollen's reactions, so, it is an aromatic aldehyde.
From the above observations, it can be concluded that the compound is



81. **(a) :** Alkoxy group will get preference over halogen.

82. **(b) :** Nitration involves acidic medium, so protonation of aniline takes place forming anilinium ion  which being *m*-directing forms mainly *m*-nitroaniline.



83. **(a)**

84. **(d) :** As $NaCl + CO_2 \not\longrightarrow Na_2CO_3$

and $Na_2CO_3 \xrightarrow{\Delta}\!\!\!\!\!\not\quad Na$

So, reactions (a), (b) and (c) are not correct.

$$2Na + 2H_2O \longrightarrow 2NaOH + H_2$$
$$2NaOH + CO_2 \longrightarrow Na_2CO_3 + H_2O$$
$$Na_2CO_3 + HCl \longrightarrow NaHCO_3 + NaCl$$
$$NaHCO_3 + HCl \longrightarrow NaCl + H_2O + CO_2$$
$$NaCl \xrightarrow{\text{electrolysis}} Na$$

85. **(a) :** XeO_3 reacts with aqueous alkali to form the hydrogen xenate ion $(HXeO_4^-)$ which slowly disproportionates to give xenon and perxenate ion in which Xe is present in +8 oxidation state.

$$XeO_3 + OH^- \longrightarrow HXeO_4^-$$
$$2HXeO_4^- + 2OH^- \longrightarrow XeO_6^{4-} + Xe + 2H_2O + O_2$$

86. **(c) :** No. of electron pair = No. of atoms bonded to it + 1/2(Group number of central atom – valency of the central atom)

$$SF_2 = 2 + \frac{1}{2}(6-2) = 4$$

Hybridization = sp^3

$$SF_4 = 4 + \frac{1}{2}(6-4) = 5$$

Hybridization = sp^3d

$$SF_6 = 6 + \frac{1}{2}(6-6) = 6$$

Hybridization = sp^3d^2.

87. **(b) :** According to Kohlrausch's law: Each ion makes a definite contribution to the total molar conductivity of an electrolyte irrespective of the nature of the other ion.

e.g., for A_xB_y

$$\Lambda_m^\circ = x\lambda^\circ_{(A^{y+})} + y\lambda^\circ_{(B^{x-})}$$

Similarly

$$\Lambda^\infty_{m(CH_3COOH)} = \Lambda^\infty_{m(CH_3COONa)} + \Lambda^\infty_{m(HCl)} - \Lambda^\infty_{m(NaCl)}$$
$$= 91 + 425.9 - 126.4$$
$$= 390.5\ \Omega^{-1}\ cm^2\ mol^{-1}.$$

88. **(c)**

89. **(d) :** $CO_2 \rightarrow$ linear structure

$SO_2 \rightarrow$ angular structure

$NH_3 \rightarrow$ tetrahedral structure

$BH_3 \rightarrow$ does not exist

$CO_3^{2-} \rightarrow$ triangular planar structure

$SO_3^{2-} \rightarrow$ pyramidal structure

$SO_4^{2-} \rightarrow$ tetrahedral structure

$ClO_4^- \rightarrow$ tetrahedral structure

90. **(d) :** Gels are the colloidal system in which liquid is dispersed phase and dispersion medium is solid.

91. **(b) :** Electronic configuration of O is $1s^2 2s^2 2p^4$.
Electronic configuration of N is $1s^2 2s^2 2p^3$.
As electron is to be removed from half filled shell $(2p^3)$ in case of N. So, first ionization energy of N is more than that of oxygen.

92. **(d) :** $Fe(OH)_3 \rightleftharpoons Fe^{3+} + 3OH^-$

$K_{sp} = (X)(3X)^3 = 27X^4$

93. **(a) :** $CH_3CH_2I \xrightarrow{NaCN} H_3CCH_2-CN$

$\downarrow$ Partial hydrolysis, OH^-

$H_3CCH_2CONH_2$

$\downarrow$ Br_2, KOH | Hofmann bromamide reaction

$H_3CCH_2NH_2$

94. **(c) :** $Mn^{3+} \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^4$

$Cr^{3+} \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^3$

$V^{3+} \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^2$

No. of unpaired electron = $Mn^{3+} \rightarrow 4$,
$Cr^{3+} \rightarrow 3$ and $V^{3+} \rightarrow 2$.

95. **(b)**

96. **(b) :** As the electronegativity of central atom increases acidic strength of oxo-acids increases. So, correct order is

$HClO_4 > HBrO_4 > HIO_4$

97. **(c) :** Hybridization of NH_2^- = No. of atoms bonded to it + 1/2(Group number of central atom – valency of the central atom ± no. of electrons)

$2 + 1/2(5 - 2 + 1) = 4 = sp^3$

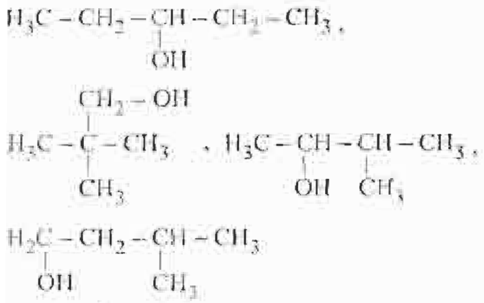
So it is having a tetrahedral structure.

98. (d) : $CH_3CH_2CH_2CH_2CH_2-OH$,

$CH_3-CH_2-CH(CH_3)-CH_2-OH$,

$H_3C-CH_2-C(CH_3)_2-OH$,

$H_3C-CH_2-CH_2-CH(OH)-CH_3$.



Total number = 8

99. **(d) :** Zn + conc. HNO_3 :

$Zn + 4HNO_3 \longrightarrow Zn(NO_3)_2 + 2NO_2 + 2H_2O$

Zn + dil. HNO_3 :

$4Zn + 10HNO_3 \longrightarrow 4Zn(NO_3)_2 + NH_4NO_3 + 3H_2O$

100. **(d) :** Paramagnetic character in $3d$ series elements increases from d^1 to d^5 configuration and then decreases as the number of electron decreases.

101. **(d) :** Hard acids are not very polarizable. They show preference for hard bases or donor atoms of high electronegativity.

102. **(c) :** Percentage of ionic character

$= 16(X_A - X_B) + 3.5(X_A - X_B)^2$.

50% ionic character occurs when the electro-negativity difference between the atoms is about 1.7. This is evident from curve C and hence is the most appropriate.

103. **(a) :** According to the question, TX does not react with any of the solutions, thus its ease of oxidation is minimum.

TY reacts with both X and Z.

i.e., $Y^- + X \longrightarrow X^- + Y$

$Y^- + Z \longrightarrow Z^- + Y$

TZ reacts only with X.

i.e., $Z^- + X \longrightarrow X^- + Z$

$\therefore$ Out of Y^- and Z^-, ease of oxidation of Y^- is more than that of Z^-, hence overall order is

$Y^- > Z^- > X^-$.

104. **(a) :** Reactions taking place inside the test tube

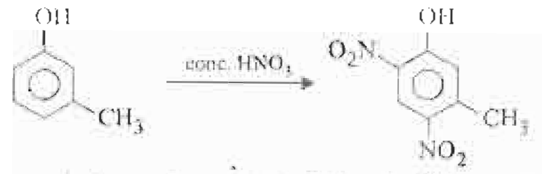
(i) $Na_2SO_3 + H_2SO_4 \longrightarrow Na_2SO_4 + H_2O + SO_2$

(ii) $K_2Cr_2O_7 + 3Na_2SO_3 + 4H_2SO_4 \longrightarrow 3Na_2SO_4 + K_2SO_4 + Cr_2(SO_4)_3 + 4H_2O$ (green)

(iii) $Ca(OH)_2 + SO_2 \longrightarrow CaSO_3\downarrow + H_2O$ (lime water turns milky)

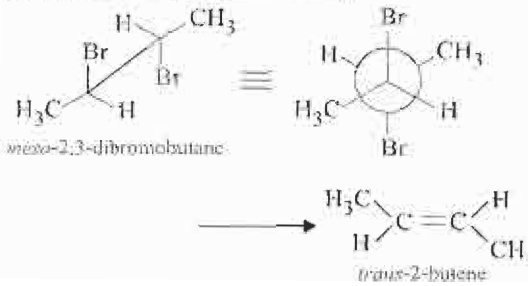
105. **(b)**

106. **(a) :**

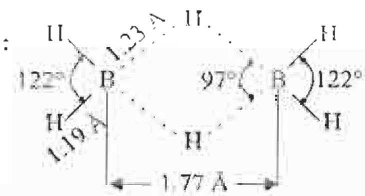


In m-hydroxytoluene, both –OH and $-CH_3$ groups are o-, p-directing, but since –OH is a more powerful group than $-CH_3$, the incoming $-NO_2$ attacks o-, p-position with respect to the –OH group.

107. **(a) :** Elimination can occur *via* a conformation of the starting compound which places the two bromine atoms in an anti-periplanar arrangement.

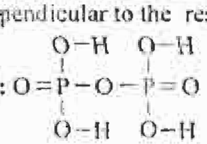


108. **(c) :**



Four H atoms; two on the left and two on the right, known as terminal hydrogens and two boron atoms lie in the same plane. Two hydrogen atoms forming bridges one above and other below lie in a plane perpendicular to the rest of molecule.

109. **(d) :**



110. **(a) :** The symmetry of molecular orbital is determined by rotating the orbitals about a line perpendicular to it. If the sign of the lobes remains the same, the orbital is gerade and if the sign changes, the orbital is ungerade. π-bonding orbitals are ungerade whereas π-antibonding orbitals are gerade. Also σ-antibonding orbitals are ungerade.

111. **(c)**

112. **(a) :** Stability of interhalogen compounds increases

with increse in electronegativity difference between halogens. So, the order is $IF_3 > BrF_3 > ClF_3$

113.(a) : Ferrous oxide combines with silica and forms ferrous silicate. By this reaction most of the iron is removed as slag

$FeO + SiO_2 \longrightarrow \underset{\text{Ferrous silicate (slag)}}{FeSiO_3}$

114.(c) : When radius ratio lies between 0.225 to 0.414 then co-ordination number is 4.

115.(d) : $-CH_3$ group shows $+I$ effect and increases electron density on $\overset{\overset{O}{||}}{C}-$ and so, decreases acidic strength.

Ph group shows $+R$ and $-I$ effect. So, it also increases electron density but less than $-R$ group. So, the order is

$CH_3-COOH < Ph-COOH < HCOOH$

116.(c) : $B > C > A$ as $-OCH_3$ exerts $+I$ effect and NO_2 exerts $-I$ effect.

117.(c) : TiO_2 and Fe_2O_3 are basic oxides. So they will not react with NaOH. Al_2O_3 is amphoteric oxide and SiO_2 is acidic oxide. So they will react with NaOH.

$Al_2O_3 + NaOH \longrightarrow NaAl(OH)_4$

$SiO_2 + NaOH \longrightarrow Na_2SiO_3$.

118.(d) : γ has maximum penetrating power.

119. $O_2 \rightarrow KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2$
$\pi(2p_y)^2\ \pi^*(2p_x)^1\ \pi^*(2p_y)^1$

$O_2^- \rightarrow KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2$
$\pi(2p_y)^2\ \pi^*(2p_z)^1$

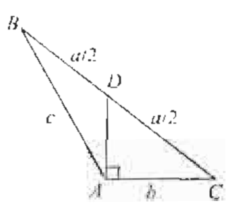
$N_2 \rightarrow KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \sigma(2p_z)^2$

$N_2^- \rightarrow KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \sigma(2p_z)^1$

So, none of the given options is correct.

120.(a) : Polyamides are having polar bond so, they are strongest. Between polyethylene and Buna-S, Buna-S has larger surface area, so, it will be more stable. So order is $C < B < A$.

121.(d) :



Given D is mid-point of BC and $AD \perp AC$

From right angle ΔDAC

$$\cos C = \frac{b}{a/2} = \frac{2b}{a} \quad \text{....(i)}$$

also from cosine formulae

$$\cos C = \frac{a^2+b^2-c^2}{2ab} \quad \text{....(ii)}$$

From (i) and (ii) we get $\dfrac{2b}{a} = \dfrac{a^2+b^2-c^2}{2ab}$

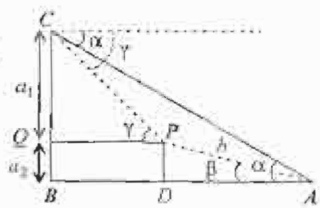
$$\Rightarrow 4b^2 = a^2+b^2-c^2 \Rightarrow b^2 = \frac{a^2-c^2}{3}$$

$$\therefore \cos A \cos C = \frac{b^2+c^2-a^2}{2bc}\cdot\frac{2b}{a}$$

$$= \frac{\left[\left(\frac{a^2-c^2}{3}\right)+c^2-a^2\right]}{ca}$$

$$= \frac{a^2-c^2+3c^2-3a^2}{3ca} = \frac{2c^2-2a^2}{3ca} = \frac{2}{3}\left(\frac{c^2-a^2}{ca}\right)$$

122.(a) :



Let BC be the hill and C be the top of hill.

$\angle CAB = \alpha$

$PA = b$

From right angle ΔPAD

$$\sin\beta = \frac{a_2}{b} \Rightarrow a_2 = b\sin\beta \quad \text{...(1)}$$

also from ΔCQP, $a_1 = PC\sin\gamma$ ...(2)

$\angle PCA = \gamma - \alpha$

$$\therefore \frac{b}{\sin(\gamma-\alpha)} = \frac{PC}{\sin(\alpha-\beta)} \Rightarrow PC = \frac{b\sin(\alpha-\beta)}{\sin(\gamma-\alpha)}$$

$$\text{from (2) } a_1 = \frac{b\sin(\alpha-\beta)\sin\gamma}{\sin(\gamma-\alpha)} \quad \text{...(3)}$$

from (1) and (3)

$$\text{height of hill} = a_2 + a_1 = b\sin\beta + \frac{b\sin(\alpha-\beta)\sin\gamma}{\sin(\gamma-\alpha)}$$

$$= \frac{b}{\sin(\gamma-\alpha)}[\sin\beta\sin(\gamma-\alpha) + \sin(\alpha-\beta)\sin\gamma]$$

$$= \frac{b}{\sin(\gamma-\alpha)}[\sin\beta\sin\gamma\cos\alpha - \sin\beta\cos\gamma\sin\alpha + \sin\gamma\sin\alpha\cos\beta - \sin\gamma\cos\alpha\sin\beta]$$

$$= \frac{b\sin\alpha}{\sin(\gamma-\alpha)}[\sin\gamma\cos\beta - \sin\beta\cos\gamma]$$

$= \dfrac{h\sin\alpha\sin(\gamma-\beta)}{\sin(\gamma-\alpha)}$

$\therefore$ Required height $= \dfrac{h\sin\alpha\sin(\gamma-\beta)}{\sin(\gamma-\alpha)}$

123.(a) : Given expression

$$\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x-x^{1/2}}\right]^{10} = \left[\frac{(x^{1/3})^3+1}{x^{2/3}-x^{1/3}+1}-\frac{[(x^{1/2})^2-1]}{x-x^{1/2}}\right]^{10}$$

$$=\left[\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1}-\frac{(x^{1/2}-1)(x^{1/2}+1)}{x^{1/2}(x^{1/2}-1)}\right]^{10}$$

$= [x^{1/3}+1-1-x^{-1/2}]^{10} = [x^{1/3}-x^{-1/2}]^{10}$

General term in the given expansion is

$T_{r+1} = {}^nC_r(x)^r(y)^{n-r}$

$= {}^{10}C_r(x)^{1/3(10-r)}(-x)^{-(1/2)r}$

$= (-1)^{-r/2}\,{}^{10}C_r(x)^{\frac{10-r}{3}+\left(\frac{-1}{2}r\right)}$

$= (-1)^{-r/2}\,{}^{10}C_r(x)^{\frac{20-2r-3r}{6}}$

Since required term is independent of x

$\therefore\ \dfrac{20-2r-3r}{6} = 0 \Rightarrow -5r = -20 \Rightarrow r = 4$

$\therefore$ coefficient of term independent of x is

$T_{4+1} = T_5 = {}^{10}C_4(-1)^{-4/2}$

$= {}^{10}C_4(-1)^{-2} = \dfrac{10!}{6!\times 4!} = \dfrac{10\times 9\times 8\times 7}{4\times 3\times 2} = 210.$

124. (c) : $7^9 + 9^7 = (8-1)^9 + (8+1)^7$

$= [{}^9C_0 8^9 - {}^9C_1 8^8 + {}^9C_2 8^7 - \dots + {}^9C_8 8^1 - {}^9C_9(1)]$

$\quad + [{}^7C_0 8^7 + {}^7C_1 8^6 + {}^7C_2 8^5 + \dots + {}^7C_6 8^1 + {}^7C_7(1)]$

$= 8^2[{}^9C_0 8^7 - {}^9C_1 8^6 + ({}^9C_2 + {}^7C_0)8^5 - ({}^9C_3 - {}^7C_1)8^4$

$\quad + ({}^9C_4 + {}^7C_2)8^3 - ({}^9C_5 - {}^7C_3)8^2 + {}^9C_6 8 - {}^9C_7 + {}^7C_5]$

$\quad + {}^9C_8 8 - {}^9C_9 + {}^7C_6 8 + {}^7C_7$

$= 8^2k + 72 - 1 + 56 + 1 = 8^2k + 128 = 8^2(k+2)$

which is divisible by 64

125.(a) : ${}^{47}C_4 + \sum\limits_{r=1}^{5} {}^{52-r}C_3 = {}^{47}C_4 + \sum\limits_{r=1}^{5} {}^{52-r}C_3$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{47}C_4 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= ({}^{47}C_4 + {}^{47}C_3) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= ({}^{48}C_4 + {}^{48}C_3) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r]$

$= ({}^{49}C_4 + {}^{49}C_3) + {}^{50}C_3 + {}^{51}C_3$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 = {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$

126.(a) : we have $\vec a\cdot\vec c = |\vec c|$ and $|\vec c - \vec a| = 2\sqrt 2$

$\Rightarrow \vec a\cdot\vec c = |\vec c|$ and $|\vec c|^2 + |\vec a|^2 - 2\vec a\cdot\vec c = 8$

$\Rightarrow |\vec c|^2 + 9 - 2|\vec c| = 8$

$\Rightarrow (|\vec c|-1)^2 = 0 \Rightarrow |\vec c| = 1$

Now $|(\vec a\times\vec b)\times\vec c| = |\vec a\times\vec b|\cdot|\vec c|\sin 30^\circ$

$= \dfrac12|\vec a\times\vec b| = \dfrac12\times 3 = \dfrac32$

$$\left[\because\ \vec a\times\vec b = \begin{vmatrix}\hat i & \hat j & \hat k\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix} = 2\hat i - 2\hat j + \hat k\right]$$

$\therefore\ |\vec a\times\vec b| = 3$

127.(d) : Required unit vector $= \pm\dfrac{\vec a\times(\vec a\times\vec b)}{|\vec a\times(\vec a\times\vec b)|}$

Now $\vec a\times(\vec a\times\vec b) = (\vec a\cdot\vec b)\vec a - (\vec a\cdot\vec a)\vec b$

$$\therefore\ \vec a\times\vec b = \begin{vmatrix}\hat i & \hat j & \hat k\\ 1 & 1 & -1\\ 1 & -1 & 1\end{vmatrix} = \hat i(1-1) - \hat j(1+1) + \hat k(-1-1)$$

$= -2\hat j - 2\hat k$

$$\therefore\ \vec a\times(\vec a\times\vec b) = \begin{vmatrix}\hat i & \hat j & \hat k\\ 1 & 1 & -1\\ 0 & -2 & -2\end{vmatrix}$$

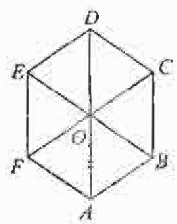
$= \hat i(-2-2) - \hat j(-2) + \hat k(-2) = -4\hat i + 2\hat j - 2\hat k$

$|\vec a\times(\vec a\times\vec b)| = \sqrt{24} = 2\sqrt 6$

$\therefore$ Unit vector $= \pm\dfrac{\vec a\times(\vec a\times\vec b)}{|\vec a\times(\vec a\times\vec b)|} = \pm\dfrac{(2\hat i - \hat j + \hat k)}{\sqrt 6}$

(taking positive for option)

128.(b) :



$\overline{AD} = 2\overline{OD},\ \overline{EB} = 2\overline{EO}$

Since $\overline{FC}\parallel\overline{ED}\ \ \therefore\ \overline{FC} = 2\overline{ED}$

$\therefore\ \overline{AD} + \overline{EB} + \overline{FC}$

$= 2\overline{OD} + 2\overline{EO} + 2\overline{ED}$

$= 2(\overline{OD} + \overline{EO}) + 2\overline{ED} = 2\overline{ED} + 2\overline{ED} = 4\overline{ED}$

Give $\overline{AD} + \overline{EB} + \overline{FC} = \lambda\overline{ED} \Rightarrow \lambda = 4$

129.(a) : The lengths of the edges are given by

$a = 5-2 = 3,\ b = 9-3 = 6,\ c = 7-5 = 2$

$\therefore$ length of the diagonal $= \sqrt{3^2+6^2+2^2}$

$= \sqrt{9+36+4} = \sqrt{49} = 7$

130. (d) : Clearly, $\vec a$ is perpendicular to the normals to the two planes determined by the given pairs of vectors.

$\vec{n}_1$ = Normal vector to the plane determined by $\hat{i}$ and $\hat{i}+\hat{j} = \hat{i}\times(\hat{i}+\hat{j}) = \hat{k}$

$\vec{n}_2$ = Normal vector to the plane determined by $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k} = (\hat{i}-\hat{j})\times(\hat{i}+\hat{k}) = -\hat{i}-\hat{j}+\hat{k}$

Since $\vec{a}$ is $\perp$ to $\vec{n}_1$ and $\vec{n}_2$

$\therefore\ \vec{a} = \lambda(\vec{n}_1\times\vec{n}_2) = \lambda(\hat{k}\times(-\hat{i}-\hat{j}+\hat{k})) = \lambda(-\hat{j}+\hat{i})$

let θ be the angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$

Then $\cos\theta = \dfrac{\lambda(1+2+0)}{\lambda\sqrt{2}\cdot\sqrt{1+4+4}} = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4}$

131. (b) : Since $\overline{OP}$ has projections $\frac{13}{5}, \frac{19}{5}$ and $\frac{26}{5}$ on the coordinate axes,

$\therefore\ \overline{OP} = \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k}$

Suppose P divides the line segment joining of $Q(2, 2, 4)$ and $R(3, 5, 6)$ in the ratio $\lambda : 1$.

Then the position vector of P is

$\left(\dfrac{3\lambda+2}{\lambda+1}\right)\hat{i} + \left(\dfrac{5\lambda+2}{\lambda+1}\right)\hat{j} + \left(\dfrac{6\lambda+4}{\lambda+1}\right)\hat{k}$

$\therefore\ \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k}$

$= \left(\dfrac{3\lambda+2}{\lambda+1}\right)\hat{i} + \left(\dfrac{5\lambda+2}{\lambda+1}\right)\hat{j} + \left(\dfrac{6\lambda+4}{\lambda+1}\right)\hat{k}$

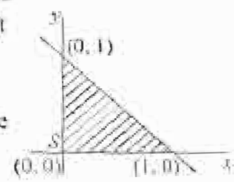
$\Rightarrow 2\lambda = 3 \Rightarrow \lambda = 3/2$

$\therefore$ Required ratio in which P divides QR is $3 : 2$.

132. (d) : Δ is formed by the lines $xy = 0$ and $x + y = 1$ is right angle at origin.

$\therefore$ orthocentre is $(0, 0)$.

($\because$ orthocentre of right angle Δ is the vertex at which angle S is $90°$)



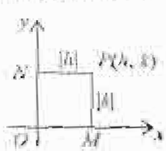
133. (a) : Let P is a variable point whose coordinates are (h, k) and x-axis and y-axis are two perpendicular lines.

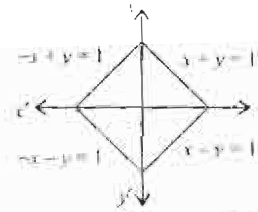
So $PM + PN = 1$

$\Rightarrow |k| + |h| = 1$

$\Rightarrow |x| + |y| = 1$

which is required locus and is square





134. (b) : Equation of the bisector of the first quadrant is $y = x$.

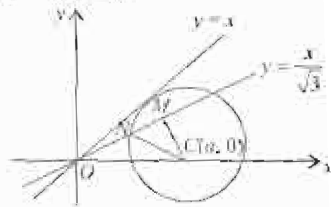
$\therefore$ The coordinates of point Q which is symmetric to $P(4, -1)$ is $(-1, 4)$.

$\therefore\ PQ = \sqrt{(4+1)^2 + (-1-4)^2} = \sqrt{25+25} = 5\sqrt{2}$

135. (c) : Since the required circle has its centre on x-axis. So, let its coordinates be $(a, 0)$. Also circle touches $y = x$.

$\therefore$ Radius = length of $\perp$ from $(a, 0)$ to $y = x$ is $a/\sqrt{2}$

Also the circle cuts off a chord of length 2 units along $x - \sqrt{3}y = 0$



$\therefore\ CN^2 = MN^2 + CM^2$

or $\left(\dfrac{a}{\sqrt{2}}\right)^2 = 1^2 + \left[\dfrac{a - \sqrt{3}\times 0}{\sqrt{1^2 + (\sqrt{3})^2}}\right]^2$

$(\because CN^2 = CM^2 + MN^2)$

$\therefore$ radius $= \dfrac{a}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

136. (d) : $(4, 3)$ lies outside the circle, so combined equation of the tangents drawn from $(4, 3)$ to the circle $x^2 + y^2 - 2x - 4y = 0$ is $SS_1 = T^2$

or $(x^2 + y^2 - 2x - 4y)(16 + 9 - 8 - 12)$

$= [4x + 3y - (x + 4) - 2(y + 3)]^2$

$\Rightarrow 4x^2 - 4y^2 - 50x + 6xy + 100 = 0$

Since coefficient of x^2 + coefficient of $y^2 = 4 + (-4) = 0$

$\therefore$ Pair of tangents is perpendicular.

137. (c) : Let PQ be a focal chord of the parabola $y^2 = 4ax$ with focus $S(a, 0)$

$P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ are such that SP and SQ are segments of focal chord.

$\Rightarrow t_1t_2 = -1$

Now $SP = a + at_1^2 = a(1+t_1^2)$

$SQ = a(t_2^2+1)$

$$= a\left(\frac{1}{t_1^2}+1\right) = a\left(\frac{1+t_1^2}{t_1^2}\right) \quad [\because t_1t_2 = -1]$$

$$\therefore \left(\frac{1}{SP}+\frac{1}{SQ}\right) = \frac{1}{a(t_1^2+1)} + \frac{t_1^2}{a(t_1^2+1)}$$

$$= \frac{1}{a}\left[\frac{t_1^2+1}{t_1^2+1}\right] = \frac{1}{a}$$

$$\Rightarrow \frac{SQ+SP}{SP\cdot SQ} = \frac{1}{a}$$

$$\Rightarrow \frac{2SP\cdot SQ}{SQ+SP} = 2a = l \text{ (semi-latus rectum} = 2a)$$

$\therefore$ Hence SP, l, SQ are in H.P.

138. (a) : Given ellipse $x^2 + 4y^2 + 8y - 2x + 1 = 0$

$\Rightarrow (x^2 - 2x + 1) + (4y^2 + 2\cdot 2y\cdot 2 + 4) - 4 = 0$

$\Rightarrow (x-1)^2 + 4(y+1)^2 = 4$

$\Rightarrow (x-1)^2 + 4(y+1)^2 = 4$

$\Rightarrow \frac{(x-1)^2}{4} + \frac{(y+1)^2}{1} = 1$ is the ellipse.

$\therefore a^2 = 4, b^2 = 1$

also $b^2 = a^2(1-e^2) \Rightarrow 1 = 4(1-e^2)$

$$\Rightarrow \frac{1}{4} = 1 - e^2 \Rightarrow e^2 = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow e = \frac{\sqrt{3}}{2}$$

139. (a) : Let $y = x + k$ be a tangent to $\frac{x^2}{3} - \frac{y^2}{2} = 1$

($\because$ tangent is parallel to $y = x$)

$\therefore k^2 = 3 - 2 = 1, a^2 = 3, b^2 = 2$ ($\because k^2 = a^2m^2 - b^2$)

$\Rightarrow k = \pm 1$

$\therefore y = x \pm 1$ is the required tangent.

140. (c) : (a) $(\sim q \wedge p) \wedge q$

q	$\sim q$	p	$(\sim q \wedge p)$	$(\sim q \wedge p) \wedge q$
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F

(b) $(\sim q \wedge p) \wedge (\sim p \wedge \sim p)$

q	$\sim q$	p	$\sim p$	$\sim q \wedge p$	$p\wedge \sim p$	$(\sim q\wedge p)\wedge(p\wedge \sim p)$
T	F	T	F	F	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	T	F	T	F	F	F

(c) $(\sim q \wedge p) \vee (p \vee \sim p)$

q	$\sim q$	p	$\sim p$	$\sim q \wedge p$	$p\vee \sim p$	$(\sim q\wedge p)\vee(p\vee \sim p)$	
T	F	T	F	F	T	T	Hence a tautology
T	F	F	T	F	T	T	
F	T	T	F	T	T	T	
F	T	F	T	F	T	T	

(d) $(p \wedge q) \wedge (\sim(p \wedge q))$

q	p	$p \wedge q$	$\sim(p \wedge q)$	$(p\wedge q)\wedge(\sim(p\wedge q))$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

141. (d) : $\bar{X} = \frac{0\cdot{}^nC_0 + 1\cdot{}^nC_1 + 2\cdot{}^nC_2 + \dots + n\cdot{}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n}$

$$= \frac{1}{2^n}\sum_{r=1}^{n} r\cdot\frac{n}{r}\,{}^{n-1}C_{r-1}$$

$$\left[\because \sum_{r=0}^{n} {}^nC_r = 2^n, {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}\right]$$

$$= \frac{1}{2^n} n\cdot 2^{n-1} = \frac{n}{2}$$

also $\frac{1}{N}\sum f_i x_i^2 = \frac{1}{2^n}\sum_{r=0}^{n} r^2\,{}^nC_r$

$$= \frac{1}{2^n}\sum_{r=0}^{n} r(r-1)\,{}^nC_r + \frac{1}{2^n}\sum_{r=0}^{n} r\,{}^nC_r$$

$$= \frac{1}{2^n}\left\{\sum_{r=0}^{n}\frac{r(r-1)n(n-1)}{r(r-1)}\,{}^{n-2}C_{r-2} + n\,2^{n-1}\right\}$$

$$= \frac{1}{2^n}[n(n-1)2^{n-2} + n\cdot 2^{n-1}]$$

$$= \frac{n(n-1)}{4} + \frac{n}{2} = \frac{n}{2}\left[\frac{n-1}{2}+1\right] = \frac{n}{2}\left(\frac{n+1}{2}\right)$$

$$\text{var}(x) = \frac{1}{N}\sum f_i x_i^2 - \bar{X}^2$$

$$= \frac{n}{2}\left(\frac{n+1}{2}\right) - \frac{n^2}{4} = \frac{n}{4}(n+1-n) = \frac{n}{4}$$

142. (c) : Since $P(A \cup B \cup C) \ge 0.75$

$\therefore 0.75 \le P(A \cup B \cup C) \le 1$

$\Rightarrow 0.75 \le P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \le 1$

$\Rightarrow 0.75 \le 0.3 + 0.4 + 0.8 - 0.08 - P(B \cap C) - 0.28 + 0.09 \le 1$

$\Rightarrow 0.75 \le 1.23 - P(B \cap C) \le 1$

$\Rightarrow -0.48 \le -P(B \cap C) \le -0.23$

$\Rightarrow 0.23 \le P(B \cap C) \le 0.48$

143. (c) : Let the sequence of $3n$ consecutive integers

begins with m. Then $3n$ consecutive integers are
$m, m+1, m+2, \ldots\ldots, m+(3n-1)$

3 integers from $3n$ can be selected in ${}^{3n}C_3$ ways

$\therefore$ Total no. of outcomes $= {}^{3n}C_3$ ....(i)

Now $3n$ integers can be divided into 3 groups.

G_1 : n numbers of form $3p$

G_2 : n numbers of form $3p+1$

G_3 : n numbers of form $3p+2$

The sum of 3 integers chosen from $3n$ integers will be divisible by 3 if either all the three are from same group or one integer from each group. The no. of ways that the three integers are from same group is ${}^nC_3 + {}^nC_3 + {}^nC_3$ and no. of ways that the integers are from different group is ${}^nC_1 \times {}^nC_1 \times {}^nC_1$

$\therefore$ favourable cases $= ({}^nC_3 + {}^nC_3 + {}^nC_3) + ({}^nC_1 \times {}^nC_1 \times {}^nC_1)$

$\therefore$ Required probability $= \dfrac{3\cdot {}^nC_3 + ({}^nC_1)^3}{{}^{3n}C_3}$

$= \dfrac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$

144.(b) : $B\left(5, \frac{1}{2}\right) \Rightarrow n = 5,\ p = \frac{1}{2},\ q = \frac{1}{2}$

$B\left(7, \frac{1}{2}\right) \Rightarrow n = 7,\ p = \frac{1}{2},\ q = \frac{1}{2}$

Since x and y are independent events

$\Rightarrow x + y = 3$ means

$x = 0, y = 3;\ x = 1, y = 2;\ x = 2, y = 1;\ x = 0, y = 3$

$\therefore\ P(x+y=3) = {}^5C_0\left(\frac{1}{2}\right)^5 \cdot {}^7C_3\left(\frac{1}{2}\right)^7 + {}^5C_1\left(\frac{1}{2}\right)^5$
${}^7C_2\left(\frac{1}{2}\right)^7 + {}^5C_2\left(\frac{1}{2}\right)^5 {}^7C_1\left(\frac{1}{2}\right)^7 + {}^5C_3\left(\frac{1}{2}\right)^5 {}^7C_0\left(\frac{1}{2}\right)^7$

$= \dfrac{55}{1024}$ (on simplification)

145. (c) : $I = \displaystyle\int_0^{\pi/4} \log(1+\tan\theta)\,d\theta$ ....(1)

$I = \displaystyle\int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - \theta\right)\right) d\theta$

$\left(\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right)$

$= \displaystyle\int_0^{\pi/4} \log\left(1 + \frac{\tan(\pi/4) - \tan\theta}{1 + \tan(\pi/4)\tan\theta}\right) d\theta$

$= \displaystyle\int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$

$= \displaystyle\int_0^{\pi/4} [\log 2 - \log(1+\tan\theta)]\,d\theta$ ....(2)

Adding (1) and (2) we get

$2I = \displaystyle\int_0^{\pi/4} \log(1+\tan\theta)\,d\theta + \int_0^{\pi/4} \log 2\,d\theta - \int_0^{\pi/4} \log(1+\tan\theta)\,d\theta$

$\Rightarrow\ 2I = \left(\frac{\pi}{4} - 0\right)\log 2 \Rightarrow I = \frac{\pi}{8}\log 2$

146.(a) : $I = \displaystyle\int_a^b \sqrt{(x-a)(b-x)}\,dx\ \ (b > a)$

let $x = a\cos^2\theta + b\sin^2\theta$

$\therefore\ dx = (-2a\cos\theta\sin\theta + 2b\sin\theta\cos\theta)\,d\theta$

$= 2(b-a)\cos\theta\sin\theta\,d\theta$

when $x = a$

$a = a\cos^2\theta + b\sin^2\theta$

$a(1-\cos^2\theta) = b\sin^2\theta \Rightarrow a\sin^2\theta = b\sin^2\theta$

$(a-b)\sin^2\theta = 0 \Rightarrow \theta = 0$

when $x = b$, $\theta = \pi/2$

$\therefore\ I = \displaystyle\int_0^{\pi/2} \sqrt{(b\sin^2\theta - a\sin^2\theta)(b\cos^2\theta - a\cos^2\theta)} \times 2\cos\theta\sin\theta(b-a)\,d\theta$

$= \displaystyle\int_0^{\pi/2} \sqrt{(b-a)^2\sin^2\theta\cos^2\theta}\ 2\sin\theta\cos\theta(b-a)\,d\theta$

$= \displaystyle\int_0^{\pi/2} 2\cdot(b-a)\sin\theta\cos\theta\,(b-a)\sin\theta\cos\theta\,d\theta$

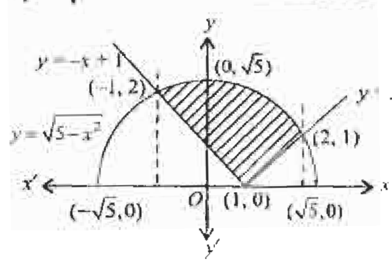
$= \displaystyle\int_0^{\pi/2} \frac{(b-a)^2}{2}\sin^2 2\theta\,d\theta = \frac{(b-a)^2}{2}\int_0^{\pi/2} \sin^2 2\theta\,d\theta$

$\Rightarrow\ I = \dfrac{(b-a)^2}{2}\displaystyle\int_0^{\pi/2}\left[\frac{1-\cos 4\theta}{2}\right]d\theta$

$\Rightarrow\ I = \dfrac{(b-a)^2}{2} \times 2 \times \dfrac{1}{2} \times \dfrac{\pi}{2} = \dfrac{\pi}{8}(b-a)^2$

147.(b) : $y = |x-1| \Rightarrow y = x - 1$ when $x > 1$ and
$y = -(x-1)$ when $x < 1$

and $y = \sqrt{5 - x^2}$ is the semicircle above x-axis



Finding intersection points of $y=\sqrt{5-x^2}$ and $y=x-1$ which gives (2, 1) and $y=\sqrt{5-x^2}$ with $y=-x+1$ which gives (−1, 2).

∴ Required bounded area

$$=\int_{-1}^{2}\sqrt{5-x^2}\,dx-\int_{-1}^{2}|(x-1)|\,dx$$

$$=\left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt5}\right]_{-1}^{2}-\left[\int_{-1}^{1}(-x+1)dx+\int_{1}^{2}(x-1)dx\right]$$

$$=\frac{5\pi}{4}-\frac{1}{2}\text{ sq. units}$$

148.(b) : $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\ldots+\frac{1}{3+n}\right)=\lim_{n\to\infty}\sum_{r=0}^{2n}\frac{1}{n+r}$

$$=\lim_{n\to\infty}\sum_{r=0}^{2n}\left(\frac{1}{1+\left(\frac{r}{n}\right)}\right)\frac{1}{n}$$

put $\frac{r}{n}=x\Rightarrow\frac{1}{n}=dx$

$$=\int_0^2\frac{1}{1+x}dx=[\log(1+x)]_0^2=\log3-\log1=\log3$$

149. (b) : Equation of all parabolas whose axis is x-axis are $y^2=4a(x+c)$.

Differentiating w.r.t. x we get $2yy'=4a$

Again differentiating we get $2(yy''+y'y')=0$

$\Rightarrow 2yy''+2(y')^2=0$

∴ Degree is the natural power on highest order differential coefficient = 1

Order is the highest order differential coefficient = 2

150.(a) : Given $\frac{dy}{dx}=(2x+3y)$

let $2x+3y=z$

$\Rightarrow 2+3\frac{dy}{dx}=\frac{dz}{dx}\Rightarrow\frac{1}{3}\left(\frac{dz}{dx}-2\right)=\frac{dy}{dx}$

$\Rightarrow\frac{1}{3}\left(\frac{dz}{dx}-2\right)=z\ \Rightarrow\ \frac{dz}{dx}=3z+2$

$\Rightarrow\int\frac{dz}{3z+2}=\int dx\quad\Rightarrow\quad\log(3z+2)=3x+c$

$\Rightarrow 3z+2=e^{3x+c}$

$\Rightarrow 6x+9y+2=e^{3x+c}$ ...(1)

$6+18+2=e^{3+c}$ (∵ (1, 2) lies on it)

$26=e^{3+c}\quad\Rightarrow\quad e^c=26\cdot e^{-3}$

Then equation is $6x+9y+2=26e^{3(x-1)}$

151.(a) : Let $I=\int\left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$

Put $\log x=t$ or $x=e^t\quad\Rightarrow\quad dx=e^t\,dt$

$$\therefore\ I=\int\left(\log t+\frac{1}{t^2}\right)e^t dt=\int\left(\log t+\frac{1}{t}-\frac{1}{t}+\frac{1}{t^2}\right)e^t dt$$

$$=\int\left(\log t+\frac{1}{t}\right)e^t dt+\int\left(-\frac{1}{t}+\frac{1}{t^2}\right)e^t dt$$

$$=\int\underset{\text{I}}{\log t}\cdot\underset{\text{II}}{e^t}\,dt+\int\frac{e^t}{t}dt+\int\underset{\text{I}}{\left(-\frac{1}{t}\right)}\underset{\text{II}}{e^t}\,dt+\int e^t\frac{1}{t^2}dt$$

Integrating by parts, we get

$$=(\log t)e^t-\int\frac{1}{t}e^t dt+\int e^t\cdot\frac{1}{t}dt+\left(\frac{-1}{t}\right)e^t-\int\frac{1}{t^2}e^t dt+\int e^t\frac{1}{t^2}dt+c$$

$$=e^t\log t-\frac{1}{t}e^t+c=x\log(\log x)-\frac{x}{\log x}+c$$

$$=x\left(\log(\log x)-\frac{1}{\log x}\right)+c=x(f(x)-g(x))+c$$

∴ $f(x)=\log(\log x)$ and $g(x)=\frac{1}{\log x}$

152.(b) : Let $z=\frac{3x-4}{3x+4}$

$$\therefore\ \frac{z+1}{z-1}=\frac{\frac{3x-4}{3x+4}+1}{\frac{3x-4}{3x+4}-1}=\frac{3x-4+3x+4}{3x-4-3x-4}$$

$$=\frac{6x}{-8}=\frac{-6x}{8}=\frac{-3}{4}x$$

$$\therefore\ f(z)=\frac{4}{3}\left(\frac{z+1}{1-z}\right)+2=\frac{4z+4+6-6z}{3(1-z)}=\frac{10-2z}{3(1-z)}$$

$$=\frac{10+2(1-z)-2}{3(1-z)}=\frac{8}{3(1-z)}+\frac{2}{3}$$

$$\therefore\ f(x)=\frac{8}{3(1-x)}+\frac{2}{3}$$

$$\therefore\ \int f(x)dx=\int\frac{8}{3(1-x)}dx+\int\frac{2}{3}dx$$

$$=\frac{-8}{3}\log|1-x|+\frac{2}{3}x+c$$

153.(a) : $\frac{d\,f(\tan x)}{d\,g(\sec x)}=\frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x\tan x}$

$$=\frac{f'(\tan x)\sec x}{g'(\sec x)\tan x}$$

$$=\frac{f'(\tan x)}{g'(\sec x)}\frac{1}{\cos x}\times\frac{\cos x}{\sin x}=\frac{f'(\tan x)}{g'(\sec x)}\frac{1}{\sin x}$$

$$\therefore\ \left.\frac{d\,f(\tan x)}{d\,g(\sec x)}\right|_{x=\pi/4}=\frac{f'(1)}{g'(\sqrt2)}\cdot\sqrt2=\frac{2}{4}\times\sqrt2=\frac{1}{\sqrt2}$$

154.(a) : $y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)+\sin^{-1}\frac{2x}{1+x^2}$

$$y=\frac{\log\sin x}{\log\cos x}\cdot\frac{\log\cos x}{\log nx}+\sin^{-1}\frac{2x}{1+x^2}$$

$$y=\frac{\log\sin x}{\log nx}+\sin^{-1}\frac{2x}{1+x^2}$$

$$\left(\text{Let } u=\frac{\log\sin x}{\log nx},\ v=\frac{2x}{1+x^2}\right)$$

$\Rightarrow y=u+v$

$$\left[\frac{dy}{dx}\right]_{\pi/2}=\left.\frac{du}{dx}\right]_{\pi/2}+\left.\frac{dv}{dx}\right]_{\pi/2}\qquad\text{....(1)}$$

$$u=\frac{\log\sin x}{\log nx}\Rightarrow\frac{du}{dx}=\frac{(\log nx)\cot x-\dfrac{\log\sin x}{x}}{(\log nx)^2}$$

$$\therefore\ \left.\frac{du}{dx}\right]_{x=\pi/2}=0$$

$$v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)\quad \text{let } x=\tan\theta$$

$\therefore\ v=\sin^{-1}(\sin 2\theta)=2\theta=2\tan^{-1}x$

$$\therefore\ \frac{dv}{dx}=\frac{2}{1+x^2}\quad\therefore\ \left.\frac{dv}{dx}\right]_{x=(\pi/2)}=\frac{2}{1+(\pi^2/4)}=\frac{8}{\pi^2+4}$$

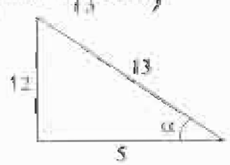
$$\therefore\ \frac{dy}{dx}\text{ at }\left(x=\frac{\pi}{2}\right)=\frac{8}{\pi^2+4}$$

155.(b) : $y=\sin^{-1}\left(\dfrac{5x+12\sqrt{1-x^2}}{13}\right)$

let $x=\sin\theta$

$$\therefore\ y=\sin^{-1}\left(\frac{5\sin\theta+12\cos\theta}{13}\right)$$

$$=\sin^{-1}\left(\frac{5}{13}\sin\theta+\frac{12}{13}\cos\theta\right)$$



$\therefore\ y=\sin^{-1}(\sin\theta\cos\alpha+\sin\alpha\cos\theta)$

where $\alpha=\tan^{-1}\left(\dfrac{12}{5}\right)$

$=\sin^{-1}\sin(\theta+\alpha)=\theta+\alpha$

$$\therefore\ y=\sin^{-1}x+\alpha\quad\therefore\ \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

156. (a) : Any function $f(x)$ is decreasing if $f'(x)\le 0$

Given $f(x)=\sin x-\cos x-ax+b$

Differentiating w.r.t. x we get

$f'(x)=\cos x+\sin x-a$

Since $f'(x)\le 0\ \Rightarrow\ \cos x+\sin x-a\le 0$

$\Rightarrow a\ge(\sin x+\cos x)$

$\Rightarrow\ a\ge\sqrt{2}\quad(\because \max(\sin x+\cos x)=\sqrt{2}\ \text{at } x=\frac{\pi}{4})$

157. (a) : Given $\log_{10}(x^3+y^3)-\log_{10}(x^2+y^2-xy)\le 2$

$$\Rightarrow\ \log_{10}\left(\frac{x^3+y^3}{x^2+y^2-xy}\right)\le 2$$

$$\Rightarrow\ \log_{10}\left[\frac{(x+y)(x^2+y^2)-xy}{x^2+y^2-xy}\right]\le 2$$

$\Rightarrow\log_{10}(x+y)\le 2\quad\Rightarrow\quad(x+y)\le 10^2=100$

Since A.M $\ge$ G.M

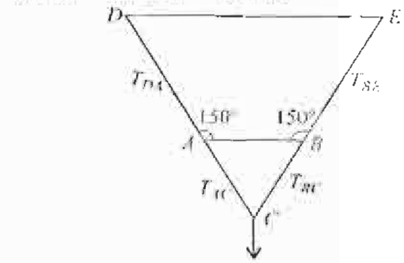
$$\Rightarrow\ \frac{x+y}{2}\ge\sqrt{xy}\quad\Rightarrow\sqrt{xy}\le\frac{x+y}{2}\le 50$$

$\Rightarrow xy\le 2500\Rightarrow$ Max. $(xy)=2500$

158.(a) : $T_{AC}\sin 30^\circ=T_{BC}\sin 30^\circ$

$T_{AC}=T_{BC}\quad\therefore\ 2T_{AC}\cos 30^\circ=W$

$$\frac{T_{DA}}{\sin 60^\circ}=\frac{T_{AB}}{\sin 150^\circ}=\frac{T_{AC}}{\sin 150^\circ}\quad\therefore\quad T_{AB}=T_{AC}=\frac{W}{\sqrt{3}}$$



159. (c) : From the figure,

$$OA=R-p=\frac{u^2\sin 2\alpha}{g}\qquad\text{...(i)}$$

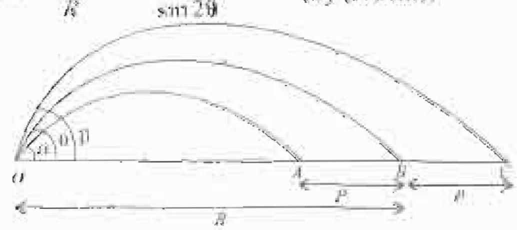
$$OC=R+p=\frac{u^2\sin 2\beta}{g}\qquad\text{...(ii)}$$

$$OB=R=\frac{u^2\sin 2\theta}{g}\qquad\text{...(iii)}$$

Adding (i) and (ii)

$$\Rightarrow\ 2R=\frac{u^2}{g}(\sin 2\alpha+\sin 2\beta)\qquad\text{...(iv)}$$

$$\therefore\ \frac{2R}{R}=\frac{\sin 2\alpha+\sin 2\beta}{\sin 2\theta}\qquad\text{(by (iv)/(iii))}$$



$$\Rightarrow\ \sin 2\theta=\frac{1}{2}(\sin 2\alpha+\sin 2\beta)$$

160.(d) : Max $z = x_1 + x_2$

such that $x_1 + 2x_2 \le 2000$

$x_1 + x_2 \le 15000$

$x_1 \ge 0, x_2 \le 600$

$OABCD$ is the feasible region.

$O(0, 0)$; $A(0, 600)$; $B(800, 600)$;

$C(1000, 500)$; $D(1500, 0)$

So maximum value will exist for all points on CD

$\therefore$ Infinite optimal solutions will exist.

161.(a) : Since each A_i has 5 elements, we have

$$\sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150 \quad \text{....(1)}$$

Suppose S has m distinct elements. Since each element of S belongs to exactly 12 of A_i's, we also have

$$\sum_{i=1}^{n} n(A_i) = 10\, m \quad \text{...(2)}$$

From (1) and (2) $10\,m = 150 \Rightarrow m = 15$

Since 3 elements each of B_j has and each element of S belongs to exactly 9 of the B_j's we have

$$\sum_{j=1}^{n} n(B_j) = 3n \text{ and } \sum_{j=1}^{n} n(B_j) = 9m$$

$\Rightarrow 3n = 9m \Rightarrow n = 3m \Rightarrow n = 3 \times 15 = 45.$

162. (b) : Let $A = \{1, 2, 3\}$

$R = \{(1, 1), (1, 2)\}$

$S = \{(2, 2), (2, 3)\}$ be two transitive relations on A.

Thus $R \cup S = \{(1, 1)(1, 2)(2, 2)(2, 3)\}$

then $(1, 2) \in R \cup S$ and $(2, 3) \in R \cup S$

but $(1, 3) \notin R \cup S$

$\therefore R \cup S$ is not transitive.

163. (d) : A function $f(x)$ has inverse function $f^{-1}(x)$ when $f(x)$ is bijective (one-one and onto) function

$f(x) = 5^{x(x-4)}$

$f'(x) = (2x - 4)5^{x(x-4)} \log 5 > 0$ in $[4, \infty)$

$\therefore f(x)$ is one-one and also strictly increasing in $[4, \infty)$

When $x = 4 \Rightarrow f(4) = 5^0 = 1$

when $x = \infty \Rightarrow f(\infty) = 5^{\infty} = \infty$

$\therefore$ Range $= [1, \infty[$

$\therefore$ Range $\ne$ codomain

i.e. $[1, \infty[\ne [4, \infty[$

$\therefore f(x)$ is not onto

$f^{-1}(x)$ is not defined. Therefore $f^{-1}(x)$ does not exist

164. Let $z = x + iy$ then $z^2 + z = 0$

$\Rightarrow x^2 - y^2 + 2ixy + x - iy = 0$

$\Rightarrow x^2 - y^2 + x + 2ixy - iy = 0$

$\Rightarrow x^2 - y^2 + x + i(2xy - y) = 0$

$\Rightarrow x^2 - y^2 + x = 0$ and $y(2x - 1) = 0$

$\Rightarrow y = 0$ or $x = 1/2$

if $y = 0$

$x^2 + x = 0 \Rightarrow x(x + 1) = 0 \Rightarrow x = 0, x = -1$

if $x = 1/2$

$$\frac{1}{4} + \frac{1}{2} - y^2 = 0 \Rightarrow \frac{3}{4} = y^2 \Rightarrow y = \pm \frac{\sqrt{3}}{2}$$

$\therefore 0 + 0i, -1 + 0i, \frac{1}{2} + \frac{\sqrt{3}}{2} i$ and $\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are four solutions.

165. (c) : Since $a, b, c > 0$ as they are sides of a Δ

We have

$$a^2 + b^2 + c^2 - ab - bc - ac = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] \ge 0$$

$\therefore a^2 + b^2 + c^2 \ge ab + bc + ca$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} \ge 1 \quad \text{...(1)}$$

Also $b^2 + c^2 - a^2 = 2bc \cos A < 2bc$

Similarly $c^2 + a^2 - b^2 < 2ac$

$a^2 + b^2 - c^2 < 2ab$

Adding these three inequations, we get

$a^2 + b^2 + c^2 < 2(bc + ac + ab)$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{bc + ac + ab} < 2 \quad \text{...(2)}$$

from (1) and (2), we have

$$1 \le \frac{a^2 + b^2 + c^2}{ab + bc + ac} < 2$$

$$\Rightarrow 1 \le \frac{(a+b+c)^2}{ab + bc + ca} - 2 < 2$$

$$\Rightarrow 3 \le \frac{(a+b+c)^2}{ab + bc + ca} < 4$$

166. (d) : Given α and β are the roots of

$\lambda(x^2 - x) + x + 5 = 0$

$\Rightarrow \lambda x^2 - \lambda x + x + 5 = 0$

$\lambda x^2 + (1 - \lambda)x + 5 = 0$

$$\therefore \quad \alpha + \beta = \frac{-(1-\lambda)}{\lambda} = \frac{\lambda - 1}{\lambda}$$

$\alpha\beta = 5/\lambda$

$$\text{Now } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5} \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{4}{5}$$

$$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5} \Rightarrow \frac{\left(\frac{\lambda-1}{\lambda}\right)^2 - \frac{2 \times 5}{\lambda}}{5/\lambda} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{\lambda^2 - 2\lambda + 1 - 10\lambda}{5\lambda} = \frac{4}{5}$$

$\Rightarrow \lambda^2 - 16\lambda + 1 = 0$

If λ_1 and λ_2 are the roots then

$\lambda_1 + \lambda_2 = 16, \lambda_1\lambda_2 = 1$

$$\therefore \quad \frac{\lambda_1}{\lambda_2^2} + \frac{\lambda_2}{\lambda_1^2} = \frac{\lambda_1^3 + \lambda_2^3}{(\lambda_1\lambda_2)^2}$$

$$= \frac{(\lambda_1 + \lambda_2)^3 - 3\lambda_1\lambda_2(\lambda_1 + \lambda_2)}{(\lambda_1\lambda_2)^2} = \frac{(16)^3 - 3(16)}{(1)^2}$$

$= 16(256 - 3) = 16 \times 253 = 4048$

167. (d) : Since $a, a_1, a_2, a_3, ..., a_{2n}, b$ are in arithmetic progression.

$\therefore a_1, a_2, ..., a_{2n}$ are A.M.'s between a and b

$\Rightarrow a_1 + a_{2n} = a_2 + a_{2n-1} = ... = a + b$

Similarly $a, g_1, g_2, ..., g_{2n}, b$ are in G.P.

$\therefore g_1g_{2n} = g_2g_{2n-1} = ... = g_ng_{n+1} = ab$

$$\therefore \quad \frac{a_1 + a_{2n}}{g_1g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2g_{2n-1}} + ... + \frac{a_n + a_{n+1}}{g_ng_{n+1}}$$

$$= \frac{a+b}{ab} + \frac{a+b}{ab} + ... + \frac{a+b}{ab}$$

$$= \frac{n(a+b)}{ab} = \frac{2n}{h}$$

where h is H.M. between a and b

$$\Rightarrow \quad h = \frac{2ab}{a+b}$$

168.(c) : $[\pm 1, \pm 2, \pm 3, ..., \pm n]^2$ is equal to

$[(1 + 2 + ... + n) + (-1) + (-2) + ... + (-n)^2]$

$= 2[1^2 + 2^2 + ... + n^2] + 2$[Product taken two at a time]

[Apply $(a + b + c + d + ...)^2 = (a^2 + b^2 + c^2 + d^2 + ...)$ $+ 2(ab + bc + ac + ad + ...)$]

$\Rightarrow 0 = 2[1^2 + 2^2 + ... + n^2] +$ 2[Product taken two at a time]

$\Rightarrow$ 2[Product taken two at a time] $= -2[1^2 + 2^2 + ... + n^2]$

$$= -2\left[\frac{n(n+1)(2n+1)}{6}\right] = -\left(\frac{n(n+1)(2n+1)}{6}\right)$$

169.(d) : $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n}\sum_{j=1}^{i} j$

$$= \sum_{i=1}^{n}\left(\frac{i(i+1)}{2}\right) = \frac{1}{2}\left[\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{4}n(n+1)\left[\frac{2n+1}{3} + 1\right] = \frac{n(n+1)}{4}\frac{[2n+1+3]}{3}$$

$$= \frac{n(n+1)(n+2)}{6}$$

170. (d) : We have,

$\log_4 2 - \log_8 2 + \log_{16} 2 - \log_{32} 2 + ...$

$$= \frac{1}{\log_2 4} - \frac{1}{\log_2 8} + \frac{1}{\log_2 16} - \frac{1}{\log_2 32} + ...$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + ...$$

$$= 1 - \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots\right) = 1 - \log_e 2$$

171. (b) : The n^{th} term of the given series is

$$T_n = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdots (2n)}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdots (2n-2)(2n-1)(2n)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdots (2n-1)2n} \times \frac{1}{2 \cdot 4 \cdot 6 \cdots (2n-2)(2n)}$$

$$= \frac{1}{2^n n!}$$

$$\therefore \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \frac{(1/2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(1/2)^n}{n!} - 1 = e^{1/2} - 1$$

172. (c) : Let $f(x) = y$

$$\Rightarrow \quad \frac{x}{1+x^2} = y \Rightarrow x^2y - x + y = 0$$

$$x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

Now $\frac{1 \pm \sqrt{1-4y^2}}{2y}$ is a real number iff

$1 - 4y^2 \geq 0$ and $y \neq 0 \Rightarrow 4y^2 - 1 \leq 0$ and $y \neq 0$

$$\Rightarrow \quad \left(y^2 - \frac{1}{4}\right) \leq 0 \text{ and } y \neq 0$$

$$\Rightarrow \quad \left(y - \frac{1}{2}\right)\left(y + \frac{1}{2}\right) \leq 0 \text{ and } y \neq 0$$

$$\Rightarrow \quad -\frac{1}{2} \leq y \leq \frac{1}{2} \text{ and } y \neq 0$$

$$\Rightarrow \quad y \in \left[-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$$

For $x = 0$, $y = 0$ $\therefore$ range is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

173.(a) : $\lim_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}=\lim_{x\to\pi/2}\frac{\cot x(1-\sin x)}{(\pi-2x)^3}$

$=\lim_{x\to\pi/2}\frac{\tan((\pi/2)-x)[1-\cos((\pi/2)-x)]}{(\pi-2x)^3}$

$=\lim_{x\to\pi/2}\frac{\tan((\pi/2)-x)\cdot 2\sin^2((\pi/4)-(x/2))}{2((\pi/2)-x)\cdot 16((\pi/4)-(x/2))^2}$

$=\frac{2}{32}\lim_{x\to\pi/2}\frac{\tan((\pi/2)-x)}{\pi/2-x}\times\lim_{x\to\pi/2}\left[\frac{\sin((\pi/4)-(x/2))}{(\pi/4)-(x/2)}\right]^2$

$=\frac{2}{32}=\frac{1}{16}$

174.(c) : $f(x)=\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$

$f(x)$ is continuous for all x so it is continuous at $x=0$

$\therefore\ f(0)=\lim_{x\to 0}f(x)$

$=\lim_{x\to 0}\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$

$=\lim_{x\to 0}\frac{(\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2})}{(\sqrt{a+x}-\sqrt{a-x})}$

$\times\frac{(\sqrt{a+x}+\sqrt{a-x})}{(\sqrt{a+x}+\sqrt{a-x})}\times\frac{(\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2})}{(\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2})}$

$=\lim_{x\to 0}\frac{-2ax}{2x}\lim_{x\to 0}\frac{(\sqrt{a+x}+\sqrt{a-x})}{(\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2})}$

$=\frac{-a\times 2\sqrt{a}}{2a}=-\sqrt{a}$

175. (d) : $x+y+z=6,\ x+2y+3z=10$

$x+2y+\lambda z=\mu$ has no solution

$$\left[\begin{array}{ccc:c}1&1&1&6\\1&2&3&10\\1&2&\lambda&\mu\end{array}\right]$$

$$\left[\begin{array}{ccc:c}1&1&1&6\\1&2&3&10\\0&0&3-\lambda&10-\mu\end{array}\right]$$

Now $|A|=0$ if $\lambda=3$

for $\lambda=3$ either infinite solutions exist or no solution exist for no solution $\mu\neq 10$ if $\mu=10$ infinite solutions exist.

176. (d) : Since $a_1, a_2, a_3, \ldots$ are in G.P.

$\therefore\ \log a_1, \log a_2, \log a_3, \ldots$ are in G.P.

$\Rightarrow \log a_m+\log a_{m+2}=2\log a_{m+1}$

$\log a_{m+3}+\log a_{m+5}=2\log a_{m+4}$

and $\log a_{m+6}+\log a_{m+8}=2\log a_{m+7}$

Applying $C_2\to C_2-(1/2)(C_1+C_3)$, we get

$$\Delta=\begin{vmatrix}\log a_m&0&\log a_{m+2}\\\log a_{m+3}&0&\log a_{m+5}\\\log a_{m+6}&0&\log a_{m+8}\end{vmatrix}=0$$

177.(b) :

$$\begin{vmatrix}x!&(x+1)!&(x+2)!\\(x+1)!&(x+2)!&(x+3)!\\(x+2)!&(x+3)!&(x+4)!\end{vmatrix}$$

$=x!(x+1)!(x+2)!$

$$\begin{vmatrix}1&1&1\\x+1&x+2&x+3\\(x+2)(x+1)&(x+2)(x+3)&(x+3)(x+4)\end{vmatrix}$$

$$=x!(x+1)!(x+2)!\begin{vmatrix}1&0&0\\x+1&1&1\\(x+2)(x+1)&(x+2)\times 2&(x+3)\times 2\end{vmatrix}$$

$(C_3\to C_3-C_2, C_2\to C_2-C_1)$

$=x!(x+1)!(x+2)!\,[1(2(x+3)-2(x+2))]$

$=2(x!)(x+1)!(x+2)!$

178. (d) : $\sin x=\frac{2\tan x/2}{1+\tan^2 x/2}$

Let $\tan\left(\frac{x}{2}\right)=y$

$\therefore\quad \tan\frac{x}{2}=\operatorname{cosec}x-\sin x=\frac{1}{\sin x}-\sin x$

$\Rightarrow\ y=\frac{1+y^2}{2y}-\frac{2y}{1+y^2}$

$\Rightarrow 2y^2(1+y^2)=1+y^4+2y^2-4y^2$

$\Rightarrow\ 2y^2+2y^4=1+y^4-2y^2$

$\Rightarrow 1+y^4-2y^2-2y^4-2y^2=0$

$\Rightarrow 1-y^4-4y^2=0\ \Rightarrow\ y^4+4y^2-1=0$

$\Rightarrow\ y^2=\frac{-4\pm\sqrt{16+4}}{2}=\frac{-4\pm\sqrt{20}}{2}=-2\pm\sqrt{5}$

$y^2=-2+\sqrt{5}$ and $-2-\sqrt{5}$

but $y^2\neq -2-\sqrt{5}$ $\quad(\because y^2>0)$

$\Rightarrow\ y^2=\tan^2(x/2)=-2+\sqrt{5}$

179. (c) : Given system of equations

$x\sin 3\theta-y+z=0$

$x\cos 2\theta+4y+3z=0$

$2x+7y+7z=0$

are homogeneous system of linear equations

Since system has non trivial solution

$$\therefore \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

$\Rightarrow \sin 3\theta[28 - 21] + 1[7\cos 2\theta - 6] + [7\cos 2\theta - 8] = 0$

$\Rightarrow 7\sin 3\theta + 7\cos 2\theta + 7\cos 2\theta - 14 = 0$

$\Rightarrow 3\sin\theta - 4\sin^3\theta + 2(1 - 2\sin^2\theta) - 2 = 0$

$\Rightarrow \sin\theta(4\sin^2 + 4\sin\theta - 3) = 0$

either $\sin\theta = 0$ or $4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\therefore \sin\theta = \frac{1}{2}, \sin\theta \neq -\frac{3}{2} \quad (\because \sin\theta > -1)$

$\therefore \theta = n\pi$ or $\theta = n\pi + (-1)^n \frac{\pi}{6}$

$\Rightarrow \quad \theta = \pi\left[n + \frac{(-1)^n}{6}\right]$

180. (c) : Since $\frac{1+\cos 2\theta}{2} = \cos^2\theta$

$\Rightarrow \cos^4\frac{\pi}{8} = \left(\cos^2\frac{\pi}{8}\right)^2$

$= \left[\frac{1+\cos(2\pi/8)}{2}\right]^2 = \left[\frac{1+\cos(\pi/4)}{2}\right]^2$

Similarly $\cos^4\frac{3\pi}{8} = \left[\frac{1+\cos(3\pi/4)}{2}\right]^2$

$\cos^4\frac{5\pi}{8} = \left[\frac{1+\cos(5\pi/4)}{2}\right]^2$

$\cos^4\frac{7\pi}{8} = \left[\frac{1+\cos(7\pi/4)}{2}\right]^2$

$\therefore$ Given expression will become

$$\frac{1}{4}\left[2\left(1+\frac{1}{\sqrt{2}}\right)^2 + 2\left(1-\frac{1}{\sqrt{2}}\right)^2\right]$$

$$= \frac{1}{4}\times 2[2+1] = \frac{1}{4}\times 2\times 3 = \frac{3}{2}$$

$$\left(\because \cos\frac{7\pi}{4} = -\cos\frac{3\pi}{4},\quad \cos\frac{5\pi}{4} = -\cos\frac{\pi}{4}\right)$$

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