MATHEMATICAL SCIENCES

This Test Booklet will contain 120 (20 Part `A'+40 Part `B+60 Part 'C') Multiple Choice Questions (MCQs) Both in Hindi and English. Candidates are required to answer 15 in part 'A', 25 in Part 'B' and 20 questions in Part 'C' respectively (No. of questions to attempt may vary from exam to exam). In case any candidate answers more than 15, 25 and 20 questions in Part A, B and C respectively only first 15, 25 and 20 questions in Parts A, B and C respectively will be evaluated. Each questions in Parts `A' carries two marks, Part 'B' three marks and Part 'C' 4.75 marks respectively. There will be negative marking @0.5 marks in Part 'A' and 0.75 in part 'B' for each wrong answers. Below each question in Part 'A' and Part 'B', four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question. Part 'C' shall have one or more correct options. Credit in a question shall be given only on identification of ALL the correct options in Part 'C'. No credit shall be allowed in a question if any incorrect option is marked as correct answer. No partial credit is allowed.

MODEL QUESTION PAPER

PART A

May be viewed under heading "General Science"

PART B

21. The sequence $a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \ldots + \frac{1}{(2n)^2}$

1 converges to 0

2 converges to 1/2

3 converges to 1/4

4 does not converge.

22. Let $x_n = n^{1/n}$ and $y_n = (n!)^{1/n}$, $n \geq 1$ be two sequences of real numbers. Then

1 (x_n) converges, but (y_n) does not converge

2 (y_n) converges, but (x_n) does not converge

3 both (x_n) and (y_n) converge

4 Neither (x_n) nor (y_n) converges

23. The set $\{ x \in \mathbb{R} : x \sin x \leq 1 , x \cos x \leq 1 \} \subset \mathbb{R}$ is

1 a bounded closed set

2 a bounded open set

3 an unbounded closed set.

4 an unbounded open set.

24. Let f:[0,1] → $\mathbb{R}$ be continuous such that $f(t) \geq 0$ for all t in [0, 1]. Define

$g(x) = \int_0^x f(t)dt$ then

1 g is monotone and bounded

2 g is monotone, but not bounded

3 g is bounded, but not monotone

4 g is neither monotone nor bounded

25. Let f be a continuous function on [0, 1] with f(0) =1. Let G(a) = $\frac{1}{a}\int_0^a f(x)dx$

1 $\lim_{a\to 0} G(a) = \frac{1}{2}$

2 $\lim_{a\to 0} G(a) = 1$

3 $\lim_{a\to 0} G(a) = 0$

4 The limit $\lim_{a\to 0} G(a)$ dose not exist

26. Let $\alpha_n = \sin(\frac{1}{n^2})$, n = 1,2, …. Then

1 $\sum_{n=1}^{\infty} \alpha_n$ converges

2 $\limsup_{n\to\infty} \alpha_n \neq \liminf_{n\to\infty} \alpha_n$

3 $\lim_{n\to\infty} \alpha_n = 1$

4 $\sum_{n=1}^{\infty} \alpha_n$ diverges

27. If, for $x \in \mathbb{R}$, $\varphi(x)$ denotes the integer closest to x (if there are two such integers take the larger one), then $\int_{10}^{12} \varphi(x)dx$ equals

1 22

2 11

3 20

4 12

28. Let P be a polynomial of degree k > 0 with a non-zero constant term. Let $f_n(x) = P(\frac{x}{n})$ $\forall$ $x \in (0,\infty)$

1 $\lim\limits_{n\to\infty} f_n(x) = \infty$ $\forall$ $x \in (0, \infty)$

2 $\exists$ $x \in (0, \infty)$ such that $\lim\limits_{n\to\infty} f_n(x) > P(0)$

3 $\lim\limits_{n\to\infty} f_n(x) = 0$ $\forall$ $x \in (0, \infty)$

4 $\lim\limits_{n\to\infty} f_n(x) = P(0)$ $\forall$ $x \in (0, \infty)$

29. Let C [0, 1] denote the space of all continuous functions with supremum norm.

Then, $K = \left\{ f \in [0,1] : \lim\limits_{n\to\infty} f\left(\frac{1}{n}\right) = 0 \right\}$ is a

1. vector space but not closed in C[0,1].
2. closed but does not form a vector space.
3. a closed vector space but not an algebra.
4. a closed algebra.

30. Let u, v, w be three points in $\mathbb{R}^3$ not lying in any plane containing the origin.

Then

1 $\alpha_1 u + \alpha_2 v + \alpha_3 w = 0 \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$

2 u, v, w are mutually orthogonal

3 one of u, v, w has to be zero

4 u, v , w cannot be pairwise orthogonal

31. Let x, y be linearly independent vectors in $\mathbb{R}^2$ suppose T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation such that Ty = α x and Tx =0 Then with respect to some basis in $\mathbb{R}^2$, T is of the form

1 $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$, a > 0

2 $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, a , b > 0; a $\neq$ b

3 $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

4 $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

32. Suppose A is an n x n real symmetric matrix with eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$ then

1 $\prod_{i=1}^{n} \lambda_i < \det(A)$

2 $\prod_{i=1}^{n} \lambda_i > \det(A)$

3 $\prod_{i=1}^{n} \lambda_i = \det(A)$

4 $if\ \det(A) = 1$ then $\lambda_j = 1$ for j =1, … n.

33. Let f be analytic on D = { z$\in \mathbb{C}$: |z | < 1} and f(0) =0 .

Define

$$g(z) = \begin{cases} \dfrac{f(z)}{z};\ z \neq 0 \\ f'(0);\ z = 0 \end{cases}$$

Then

1 g is discontinuous at z = 0 for all f

2 g is continuous, but not analytic at z = 0 for all f

3 g is analytic at z = 0 for all f

4 g is analytic at z = 0 only if f ' (0) = 0

34. Let $\Omega \subseteqq \mathbb{C}$ be a domain and let f(z) be an analytic function on Ω such that

$|f(z)| = |\sin z|$ for all $z \in \Omega$ then

1 f(z) = $\overline{\sin z}$ for all $z \in \Omega$

2 f(z) = $\sin\left(\overline{z}\right)$ for all $z \in \Omega$.

3 there is a constant $c \in \mathbb{C}$ with $|c| = 1$ such that f(z) = c sin z for all $z \in \Omega$

4 such a function f(z) does not exist

35. The radius of convergence of the power series

$$\sum_{n=0}^{\infty}(4n^4 - n^3 + 3)\ z^n \text{ is}$$

1 0

2 1

3 5

4 ∞

36. Let $\mathbb{F}$ be a finite field such that for every $a \in \mathbb{F}$ the equation $x^2 = a$ has a solution in $\mathbb{F}$. Then

1 the characteristic of $\mathbb{F}$ must be 2

2 $\mathbb{F}$ must have a square number of elements

3 the order of $\mathbb{F}$ is a power of 3

4 $\mathbb{F}$ must be a field with prime number of elements

37. Let $\mathbb{F}$ be a field with 5^{12} elements. What is the total number of proper subfields of $\mathbb{F}$?

1 3

2 6

3 8

4 5

38. Let K be an extension of the field Q of rational numbers

1 If K is a finite extension then it is an algebraic extension

2 If K is an algebraic extension then it must be a finite extension

3 If K is an algebraic extension then it must be an infinite extension

4 If K is a finite extension then it need not be an algebraic extension

39. Consider the group S_9 of all the permutations on a set with 9 elements. What is the largest order of a permutation in S_9 ?

1 21

2 20

3 30

4 14

40. Suppose V is a real vector space of dimension 3. Then the number of pairs of linearly independent vectors in V is

1 one

2 infinity

3 e^3

4 3

41. Consider the differential equation

$$\frac{dy}{dx} = y^2, (x, y) \in \mathbb{R} \times \mathbb{R}.$$

Then,

1. all solutions of the differential equation are defined on $(-\infty,\infty)$.
2. no solution of the differential equation is defined on $(-\infty,\infty)$.
3. the solution of the differential equation satisfying the initial condition $y(x_0) = y_0$, $y_0 > 0$, is defined on $\left(-\infty, x_0 + \frac{1}{y_0}\right)$.
4. the solution of the differential equation satisfying the initial condition $y(x_0)=y_0$, $y_0>0$, is defined on $\left(x_0 - \frac{1}{y_0}, \infty\right)$.

42. The second order partial differential equation

$$\left(1-\sqrt{xy}\right)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \left(1+\sqrt{xy}\right)\frac{\partial^2 u}{\partial y^2} = 0 \text{ is}$$

1. hyperbolic in the second and the fourth quadrants
2. elliptic in the first and the third quadrants

3. hyperbolic in the second and elliptic in the fourth quadrant
4. hyperbolic in the first and the third quadrants

43. A general solution of the equation $\frac{\partial u(x,y)}{\partial x} + u(x,y) = e^{-x}$ is

1. $u(x,y) = e^{-x} f(y)$
2. $u(x,y) = e^{-x} f(y) + xe^{x}$
3. $u(x,y) = e^{+x} f(y) + xe^{+x}$
4. $u(x,y) = e^{-x} f(y) + xe^{-x}$

44. Consider the application of Trapezoidal and Simpson's rules to the following integral

$$\int_0^4 (2x^3 - 3x^2 + 5x + 1)dx$$

1. Both Trapezoidal and Simpson's rules will give results with same accuracy.
2. The Simpson's rule will give more accuracy than the Trapezoidal rule but less accurate than the exact result.
3. The Simpson's rule will give the exact result.
4. Both Trapezoidal rule and Simpson's rule will give the exact results.

45. The integral equation

$$g(x)y(x)=f(x)+\lambda\int_{\alpha}^{\beta} k(x,t)y(t)dt$$

with f(x), g(x) and k(x,t) as known functions, α and β as known constants, and λ as a known parameter, is a

1. linear integral equation of Volterra type
2. linear integral equation of Fredholm type
3. nonlinear integral equation of Volterra type
4. nonlinear integral equation of Fredholm type

46. Let $y(x) = f(x)+\lambda\int_a^b k(x,t)y(t)dt$, where f(x) and k(x,t) are known functions, a and b are known constants and λ is a known parameter. If λ_i be the eigenvalues of the corresponding homogeneous equation, then the above integral equation has in general,

1. many solutions for $\lambda \neq \lambda_i$
2. no solution for $\lambda \neq \lambda_i$
3. a unique solution for $\lambda = \lambda_i$
4. either many solutions or no solution at all for $\lambda = \lambda_i$, depending on the form of f(x)

47. The equation of motion of a particle in the x-z plane is given by

$$\frac{d\vec{v}}{dt} = -\vec{v} - \hat{k}$$

with $\vec{v} = \alpha\hat{k}$, where $\alpha = \alpha(t)$ and $\hat{k}$ is the unit vector along the z-direction. If initially (i.e., t = 0) $\alpha = 1$, then the magnitude of velocity at t = 1 is

1. 2/e
2. (2+e)/3
3. (e–2)/e
4. 1

48. Consider the functional

$$F(u,v) = \int_0^{\pi/2} \left[\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2 + 2u(x)v(x) \right] dx$$

with

$u(0) = 1, v(0) = -1$ and

$u\left(\frac{\pi}{2}\right) = 0, v\left(\frac{\pi}{2}\right) = 0$.

Then, the extremals satisfy

1. $u(\pi) = 1, v(\pi) = -1$
2. $u(\pi) + v(\pi) = 0, u(\pi) - v(\pi) = 2$
3. $u(p) = -1, v(p) = 1$
4. $u(\pi) + v(\pi) = -2, u(\pi) - v(\pi) = 0$

49. The pairs of observations on two random variables X and Y are

$X:$	2	5	7	11	13	19
$Y:$	0	15	25	45	55	85

Then the correlation coefficient between X and Y is

1 0

2 1/5

3 1/2

4 1

50. Let X_1, X_2, X_3 be independent random variables with $P(X_i = +1) = P(X_i = -1) = 1/2$. Let $Y_1 = X_2X_3$, $Y_2 = X_1X_3$ and $Y_3 = X_1X_2$.

Then which of the following is NOT true?

1. Y_i and X_i have same distribution for i = 1, 2, 3
2. (Y_1, Y_2, Y_3) are mutually independent
3. X_1 and (Y_2, Y_3) are independent
4. (X_1, X_2) and (Y_1, Y_2) have the same distribution

51. Let X be an exponential random variable with parameter λ. Let Y = [X] where [x] denotes the largest integer smaller than x. Then

1. Y has a Geometric distribution with parameter λ.
2. Y has a Geometric distribution with parameter 1- e^{-1}.
3. Y has a Poisson distribution with parameter λ
4. Y has mean [1/λ]

52. Consider a finite state space Markov chain with transition probability matrix P=((p_{ij})). Suppose p_{ii} =0 for all states i. Then the Markov chain is

1. always irreducible with period 1.
2. may be reducible and may have period > 1.
3. may be reducible but period is always 1.
4. always irreducible but may have period > 1.

53. Let X_1, X_2, …. X_n be i.i.d. Normal random variables with mean 1 and variance 1. and let $Z_n = (X_1^2 + X_2 + \ldots + X_n)/n$ Then

1. Z_n converges in probability to 1
2. Z_n converges in probability to 2
3. Z_n converges in distribution to standard normal distribution
4. Z_n converges in probability to Chi-square distribution.

54. Let X_1, X_2, …. X_n be a random sample of size n ($\geq$4) from uniform (0,θ) distribution. Which of the following is NOT an ancillary statistic?

1. $\dfrac{X_{(n)}}{X_{(1)}}$
2. $\dfrac{X_n}{X_1}$
3. $\dfrac{X_4 - X_1}{X_3 - X_2}$
4. $X_{(n)} - X_{(1)}$

55. Suppose $X_1, X_2, \ldots X_n$ are i.i.d, Uniform $(0,q)$, $\theta \in \{1, 2....\}$.

Then the MLE of θ is

1. $X_{(n)}$
2. $\bar{X}$
3. $[X_{(n)}]$ where [a] is the integer part of a.
4. $[X_{(n)}+1]$ where [a] is the integer part of a.

56. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with common continuous distribution function F(x). Let R_i = Rank(X_i), i= 1, 2, …, n. Then P (| $R_n - R_1$ | ³ n-1) is

1. 0
2. $\frac{1}{n(n-1)}$
3. $\frac{2}{n(n-1)}$
4. $\frac{1}{n}$

57. A simple random sample of size n is drawn without replacement from a population of size N (> n). If π_i (i=1,2,…N) and p_{ij} (i$\neq$j. i, j =1, 2, … N) denote respectively, the first and second order inclusion probabilities, then which of the following statements is NOT true?

1 $\text{å}_{i=1}^{N} p_i = n$

2 $\text{å}_{j ^1 i}^{N} p_{ij} = (n - 1)p_i$

3 $p_i p_j$ £ p_{ij} for each pair (i, j)

4 $p_{ij} < p_i$ for each pair (i, j) .

58. Consider a balanced incomplete block design with usual parameters v, b, r, k (≥ 2), λ . Let t_i be the effect of the i[th] treatment (i = 1, 2,…,v) and σ^2 denote the variance of an observation. Then the variance of the best linear unbiased estimator of $\sum_{i=1}^{v} p_i t_i$ where $\sum_{i=1}^{v} p_i = 0$ and $\sum_{i=1}^{v} p_i^2 = 1$, under the intra-block model, is

1 $\left(\lambda v / k\right)\sigma^2$

2 $2\sigma^2 / r$

3 $\left(k / \lambda v\right)\sigma^2$

4 $\left(2k / \lambda v\right)\sigma^2$

59. An aircraft has four engines – two on the left side and two on the right side. The aircraft functions only if at least one engine on each side functions.
If the failures of engines are independent, and the probability of any engine failing in equal to p, then the reliability of the aircraft is equal to

1. $p^2(1-p^2)$
2. ${}^4C_2 p^2(1-p)^2$
3. $(1-p^2)^2$
4. $1-(1-p^2)^2$

60. A company maintains EOQ model for one of its critical components. The setup cost is k , unit production cost is c , demand is a units per unit time, and h is the cost of holding one unit per unit time. In view of the criticality of the component the company maintains a safety stock of s units at all times. The economic order quantity for this problem is given by.

1. $\sqrt{\frac{2ak}{h}+s}$
2. $s+\sqrt{\frac{2ak}{h}}$
3. $\sqrt{\frac{2ak}{h}}$
4. $\sqrt{\frac{2ak+s}{h}}$

PART C

61. Suppose $\{a_n\}$, $\{b_n\}$ are convergent sequences of real numbers such that $a_n > 0$ and $b_n > 0$ for all n.

Suppose $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$. Let $c_n = a_n/b_n$. Then

1. $\{c_n\}$ converges if $b > 0$
2. $\{c_n\}$ converges only if $a = 0$
3. $\{c_n\}$ converges only if $b > 0$
4. $\limsup_{n\to\infty} c_n = \infty$ if $b = 0$.

62. Consider the power series $\sum_{n=0}^{\infty} a_n x^n$

where $a_0 = 0$ and $a_n = \sin(n!)/n!$ for $n \geq 1$. Let R be the radius of convergence of the power series. Then

1. $R \geq 1$
2. $R \geq 2\pi$
3. $R \leq 4\pi$
4. $R \geq \pi$.

63. Suppose f is an increasing real-valued function on $[0, \infty)$ with $f(x) > 0$ $\forall x$ and let

$$g(x) = \frac{1}{x}\int_0^x f(u)\,du; \quad 0 < x < \infty.$$

Then which of the following are true:

1. $g(x) \leq f(x)$ for all $x \in (0,\infty)$
2. $xg(x) \leq f(x)$ for all $x \in (0,\infty)$
3. $xg(x) \geq f(0)$ for all $x \in (0,\infty)$
4. $yg(y) - xg(x) \leq (y-x)f(y)$ for all $x < y$.

64. Let $f: [0, 1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x\cos(\pi/(2x)) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Then

1. f is continuous on [0, 1]
2. f is of bounded variation on [0, 1]
3. f is differentiable on the open interval (0, 1) and its derivative f' is bounded on (0,1)
4. f is Riemann integrable on [0, 1].

65. For any positive integer n, let $f_n : [0, 1] \to \mathbb{R}$ be defined by

$f_n(x) = \frac{x}{nx+1}$ *for* $x \in [0,1]$.

Then

1. the sequence $\{f_n\}$ converges uniformly on [0, 1]
2. the sequence $\{ f_n' \}$ of derivatives of $\{f_n\}$ converges uniformly on [0, 1]
3. the sequence $\left\{ \int_0^1 f_n(x) dx \right\}$ is convergent
4. the sequence $\left\{ \int_0^1 f_n'(x) dx \right\}$ is convergent.

66. Let f: $[0, \infty) \to \mathbb{R}$ and g : $[0, \infty) \to \mathbb{R}$ be continuous functions satisfying

$\int_0^{f(x)} t^2 dt = x^3(1+x)^2$ *and* $\int_0^{x^2(1+x)} g(t) dt = x$ for all $x \in [0, \infty)$.

Then f(2) + g(2) is equal to

1. 0
2. 5
3. 6
4. 11.

67. Consider f: $\mathbb{R}^2 \to \mathbb{R}$ defined by f(0, 0) = 0 and

$f(x, y) = \frac{x^2 y}{x^4 + y^2}$ for $(x, y) \neq (0, 0)$.

Then which of the following statements is correct?

1. Both the partial derivatives of f at (0, 0) exist
2. The directional derivative $D_{\underline{u}} f(0, 0)$ of f at (0, 0) exists for every unit vector $\underline{u}$
3. f is continuous at (0, 0)
4. f is differentiable at (0, 0).

68. Let f: $\mathbb{R}^2 \to \mathbb{R}$ and g: $\mathbb{R}^2 \to \mathbb{R}$ be defined by

$f(x, y) = |x| + |y|$ and $g(x, y) = |xy|$.

Then

1. f is differentiable at (0, 0), but g is not differentiable at (0, 0)
2. g is differentiable at (0, 0), but f is not differentiable at (0,0)
3. Both f and g are differentiable at (0, 0)
4. Both f and g are continuous at (0, 0).

69. Decide for which of the functions $F : \mathbb{R}^3 \to \mathbb{R}^3$ given below, there exists a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $(\nabla f)(x) = F(x)$.

1. $(4xyz - z^2 - 3y^2, 2x^2z - 6xy + 1, 2x^2y - 2xz - 2)$
2. (x, xy, xyz)
3. $(1,1,1)$
4. (xyz, yz, z).

70. Let $f: \mathbb{R}^n \to \mathbb{R}$ be the function defined by the rule $f(x) = x.b$, where $b \in \mathbb{R}^n$ and $x.b$ denotes the usual inner product. Then

1. $[f'(x)](b) = b \cdot b$
2. $[f'(x)](x) = \frac{x.x}{2}$, $x \in \mathbb{R}^n$
3. $[f'(0)](e_1) = b.e_1$, where $e_1 = (1, 0, \cdots, 0) \in \mathbb{R}^n$.
4. $[f'(e_1)](e_j) = 0$, $j \neq 1$, where $e_j = (0, \cdots, 1, \cdots 0)$ with 1 in the j^{th} slot.

71. Consider the subsets A and B of $\mathbb{R}^2$ defined by

$$A = \left\{ \left(x, x \sin \frac{1}{x} \right) : x \in (0,1] \right\} \ and \ B = A \cup \{(0,0)\}.$$

Then
1. A is compact
2. A is connected
3. B is compact
4. B is connected.

72. Lef $f = \mathbb{R} \to \mathbb{R}$ be a continuous function.Which of the following is always true?

1. $f^{-1}(U)$ is open for all open sets $U \subseteq \mathbb{R}$
2. $f^{-1}(C)$ is closed for all closed sets $C \subseteq \mathbb{R}$
3. $f^{-1}(K)$ is compact for all compact sets $K \subseteq \mathbb{R}$
4. $f^{-1}(G)$ is connected for all connected sets $G \subseteq \mathbb{R}$.

73. Let A be an $n \times n$ matrix, $n \geq 2$, with characteristic polynomial $x^{n-2}(x^2 - 1)$. Then
1. $A^n = A^{n-2}$
2. Rank of A is 2
3. Rank of A is at least 2
4. There exist nonzero vectors x and y such that $A(x + y) = x - y$.

74. Let A, B and C be real $n \times n$ matrices such that $AB + B^2 = C$. Suppose C is nonsingular. Which of the following is always true?

1. A is nonsingular
2. B is nonsingular
3. A and B are both nonsingular
4. A + B is nonsingular.

75. Let V be a real vector space and let $\{x_1, x_2, x_3\}$ be a basis for V. Then

1. $\{x_1 + x_2, x_2, x_3\}$ is a basis for V
2. The dimension of V is 3
3. x_1, x_2, x_3 are pairwise orthogonal
4. $\{x_1 - x_2, x_2 - x_3, x_1 - x_3\}$ is a basis for V.

76. Consider the system of m linear equations in n unknowns given by Ax = b, where A= (a_{ij}) is a real $m \times n$ matrix, x and b are $n \times 1$ column vectors. Then

1. There is at least one solution
2. There is at least one solution if b is the zero vector
3. If m = n and if the rank of A is n, then there is a unique solution
4. If m < n and if the rank of the augmented matrix [A: b] equals the rank of A, then there are infinitely many solutions.

77. Let V be the set of all real $n \times n$ matrices $A = (a_{ij})$ with the property that $a_{ij} = -a_{ji}$ for all i, j = 1, 2,⋯,n. Then

1. V is a vector space of dimension $n^2 - n$
2. For every A in V, $a_{ii} = 0$ for all i = 1,2, ⋯, n
3. V consists of only diagonal matrices
4. V is a vector space of dimension $\frac{n^2 - n}{2}$.

78. Let W be the set of all 3×3 real matrices $A = (a_{ij})$ with the property that $a_{ij} = 0$ if i > j and $a_{ii} = 1$ for all i. Let $B = (b_{ij})$ be a 3×3 real matrix that satisfies AB = BA for all A in W. Then

1. Every A in W has an inverse which is in W.
2. $b_{12} = 0$
3. $b_{13} = 0$
4. $b_{23} = 0$.

79. Let f(z) be an entire function with $\mathrm{Re}(f(z)) \geq 0$ for all $z \in \mathbb{C}$. Then

1. $\mathrm{Im}(f(z)) \geq 0$ for all $z \in \mathbb{C}$
2. Im (f(z)) = a constant
3. f is a constant function
4. $\mathrm{Re}(f(z)) = |z|$ for all $z \in \mathbb{C}$.

80. Let f be an analytic function defined on $D = \{z \mid |z| < 1\}$ such that $|f(z)| \leq 1$ for all $z \in D$. Then

1. there exists $z_0 \in D$ such that $f(z_0) = 1$
2. the image of f is an open set
3. f(0) = 0

4. f is necessarily a constant function.

81. Let $f(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z}$. Then

1. f has a pole of order 2 at z = 0
2. f has a simple pole at z = 0
3. $\oint_{|z|=1} f(z)dz = 0$, where the integral is taken anti-clockwise
4. the residue of f at z = 0 is -2πi.

82. Let f be an analytic function defined on D = { $z \in \mathbb{C}$: $|z| < 1$}.Then $g : D \to \mathbb{C}$ is analytic if

1. g(z) = $f(\bar{z})$ for all $z \in D$
2. g(z) = $\overline{f(z)}$ for all $z \in D$
3. g(z) = $\overline{f(\bar{z})}$ for all $z \in D$
4. g(z) = $\bar{i} f(z)$ for all $z \in D$.

83. Which of the following statements involving Euler's function ϕ is/are true?

1. ϕ(n) is even as many times as it is odd
2. ϕ(n) is odd for only two values of n
3. ϕ(n) is even when n > 2
4. ϕ(n) is odd when n = 2 or n is odd.

84. Let p be a prime number and $d \mid (p-1)$. Then which of the following statements about the congruence $x^d \equiv 1$ (mod p) is/are true?

1. It does not have any solution
2. It has at most d incongruent solutions
3. It has exactly d incongruent solutions
4. It has at least d incongruent solutions.

85. Let K be a field, L a finite extension of K and M a finite extension of L. Then

1. [M:K] = [M:L] + [L:K]
2. [M:K] = [M:L] [L:K]
3. [M:L] divides [M:K]
4. [L:K] divides [M:K] .

86. Let R be a commutative ring and R[x] be the polynomial ring in one variable over R.

1. If R is a U.F.D., then R [x] is a U.F.D.
2. If R is a P.I.D., then R [x] is a P.I.D.
3. If R is an Euclidean domain, then R[x] is an Euclidean domain
4. If R is a field, the R[x] is an Euclidean domain.

87. Let G be a group of order 56. Then

1. All 7-sylow subgroups of G are normal
2. All 2-Sylow Subgroups of G are normal
3. Either a 7-Sylow subgroup or a 2-Sylow subgroup of G is normal
4. There is a proper normal subgroup of G.

88. Which of the following statements is/are true?

1. 50! ends with an even number of zeros
2. 50! ends with a prime number of zeros
3. 50! ends with10 zeros
4. 50! ends with 12 zeros.

89. Let $X = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$

$Y = \{(x, y) \in \mathbb{R}^2 \mid |x| + |y| = 1\}$, and

$Z = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 = 1\}$.

Then

1. X is not homeomorphic to Y
2. Y is not homeomorphic to Z
3. X is not homeomorphic to Z
4. No two of X, Y or Z are homeomorphic.

90. Let τ_1, τ_2 and τ_3 be topologies on a set X such that $\tau_1 \subset \tau_2 \subset \tau_3$ and (X, τ_2) is a compact Hausdorff space. Then

1. $\tau_1 = \tau_2$ if (X, τ_1) is a Hausdorff space
2. $\tau_1 = \tau_2$ if (X, τ_1) is a compact space
3. $\tau_2 = \tau_3$ if (X, τ_3) is a Hausdorff space
4. $\tau_2 = \tau_3$ if (X,τ_3) is a compact space.

91. The initial value problem $\dot{x}(t) = 3x^{2/3}, \ x(0) = 0$;

in an interval around t = 0, has

1. no solution
2. a unique solution
3. finitely many linearly independent solutions
4. infinitely many linearly independent solutions.

92. For the system of ordinary differential equations:

$$\frac{d}{dt}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

1. every solution is bounded
2. every solution is periodic
3. there exists a bounded solution
4. there exists a non periodic solution.

93. The kernel $p(x, y) = \frac{y}{y^2+x^2}$ is a solution of

1. the heat equation
2. the wave equation
3. the Laplace equation
4. the Lagrange equation.

94. The solution of the Laplace equation on the upper half plane, which takes the value $\varphi(x) = e^x$ on the real line is

1. the real part of an analytic function
2. the imaginary part of an analytic function
3. the absolute value of an analytic function
4. an infinitely differentiable function.

95. Which of the following polynomials interpolate the data

x	1	1/2	3
y	3	-10	2

1. $3 + 26\,(x-1) - \frac{53}{5}\,(x-)\,(x-)$
2. $3\,(x-1)\,(x-\frac{1}{2})\;\; 1-10(x-\frac{1}{2})\,(x-3) + 10\,(x-3)\,(x-1)$
3. $3(x-\frac{1}{2})\,(x-3) - 8\,(x-1)\,(x-3) + \frac{2}{5}(x-1)(x-\frac{1}{2})$
4. $(x-3)\,(x+10) + \frac{1}{2}(x+10)\,(x-2) + 3\,(x-2)\,(x-3).$

96. The evaluation of the quantity $\sqrt{x+1}$ – 1 near x = 0 is achieved with minimum loss of significant digits if we use the expression

1. $\sqrt{x+1}\ -1$

2. $\dfrac{x}{\sqrt{x+1}+1}$

3. $\left(1-\dfrac{1}{\sqrt{x+1}}\right)\sqrt{x+1}$

4. $\dfrac{x+2\sqrt{x+1}}{\sqrt{x+1}-1}$.

97. If x(t) is an extremal of the functional $\int_a^b \left(\frac{1}{2}m(\dot{x})^2 - cx^2\right)dt,$ where a, b,c are arbitrary constants and $\dot{x}$ = dx/dt, then the function x(t) satisfies

1. $m\ddot{x}+2cx=0$

2. $m\ddot{x}-2cx=0$

3. $m(\dot{x})^2+2cx^2=k_1$ with k_1 as an arbitrary constant

4. $x(t)=k_1\sin(\sqrt{\frac{2c}{m}}\,t+k_2)$ with k_1 and k_2 as arbitrary constants.

98. If u(x) and v(x) satisfying u(0) = 1, v(0) = –1, u (π/2) = 0 and v((π/2)=0 are the extremals of the functional $\int_0^{\frac{\pi}{2}} \left\{\left(\frac{du}{dx}\right)^2+\left(\frac{dv}{dx}\right)^2+2uv\right\}dx,$ then

1. $u(\frac{\pi}{4})+v(\frac{\pi}{4})=0$

2. $u(\frac{\pi}{3})-v(\frac{\pi}{3})=0$

3. $u(\frac{\pi}{4})-v(\frac{\pi}{4})=1$

4. $u(\frac{\pi}{3})+v(\frac{\pi}{3})=0.$

99. Consider the integral equation $y(x)=x^2+\lambda\int_o^1 xty(t)dt,$

where λ is a real parameter. Then the Neumann series for the integral equation converges for all values of λ

1. except for λ=3
2. lying in the interval -3 < λ < 0
3. lying in the interval -3 < λ < 3
4. lying in the interval 0 < λ < 3.

100. The solution of the integral equation $\phi(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\phi(t)dt$ satisfies

1. $\phi(0) + \phi(1) = 1$
2. $\phi\left(\frac{1}{2}\right) + \phi\left(\frac{1}{3}\right) = 1$
3. $\phi\left(\frac{1}{4}\right) + \phi\left(\frac{1}{2}\right) = 1$
4. $\phi\left(\frac{3}{4}\right) + \phi\left(\frac{1}{4}\right) = 1$.

101. A particle of unit mass is constrained to move on the plane curve xy=1 under gravity g. Then

1. the kinetic energy of the system is $\frac{1}{2}(\dot{x}^2 + \dot{y}^2)$
2. the potential energy of the system is $\frac{g}{x}$
3. the Lagrangian of the particle is $\frac{1}{2}\dot{x}^2\left(1 + x^{-4}\right) - \left(\frac{g}{x}\right)$
4. the Lagrangian of the particle is $\frac{1}{2}\dot{x}^2\left(1 + x^{-4}\right) + \left(\frac{g}{x}\right)$.

102. Suppose a mechanical system has the single coordinate q and Lagrangian $L = \frac{1}{4}\dot{q}^2 - \frac{q^2}{9}$. Then

1. the Hamiltonian is $p^2 + (\frac{q^2}{9})$

2. Hamilton's equations are $\dot{q} = 2p,\ \dot{p} = -(2/9)q$

3. q satisfies $\ddot{q} + (4/9)q = 0$

4. the path in the Hamiltonian phase-space, i.e. $q - p$ plane is an ellipse.

103. Let $X_1, \ldots\ldots, X_n$ be i.i.d. observations from a distribution with variance $\sigma^2 (< \infty)$. Which of the following is/are unbiased estimator(s) of σ^2?

1. $\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$

2. $\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$

3. $\binom{n}{2}^{-1}\frac{1}{2}\sum_{i\le 1<j\le n}\left(X_i - X_j\right)^2$

4. $\frac{1}{n}\sum_{i=1}^{n}X_i^2 - n\overline{X}^2$.

104. Let $X_1, X_2, \ldots\ldots\ldots$ be i.i.d. N(0,1) and let $S_n = \sum_{i=1}^{n} X_i$ be the partial sums.

Which of the following is/are true?

1. $\frac{S_n}{n} \to 0$ almost surely

2. $E\left(\frac{S_n}{n}\right) \to 0$

3. $Var\left(\frac{S_n}{n}\right) \to 0$

4. $Var\left(\frac{S_n^2}{n^2}\right) \to 0$

105. Let (X, Y) be a pair of independent random variables with X having exponential distribution with mean 1 and Y having uniform distribution on $\{1,2,\ldots,m\}$. Define Z=X+Y. Then

1. E(Z|X) = X + $\frac{m+1}{2}$

2. $E(Z|Y) = 1+\frac{m+1}{2}$

3. $Var(Z|X) = \frac{m^2-1}{2}$

4. $Var(Z|Y) = 2$.

106. A simple symmetric random walk on the integer line is a Markov chain which is

1. recurrent
2. null recurrent
3. irreducible
4. positive recurrent.

107. Suppose X and Y are random variables with E(X) = E(Y) =0, V(X)=V(Y)=1 and Cov (X,Y) = 0.25. Then which of the following is/are always true?

1. $P\{|X+2Y| \geq 4\} \leq \frac{4}{16}$
2. $P\{|X+2Y| \geq 4\} \leq \frac{5}{16}$
3. $P\{|X+2Y| \geq 4\} \leq \frac{6}{16}$
4. $P\{|X+2Y| \geq 4\} \leq \frac{7}{16}$.

108. Let $X_1,\ldots\ldots, X_n$ be a random sample from uniform $(\theta, \theta+1)$ distribution. Which of the following is/are maximum likelihood estimator(s) of θ?

1. $X_{(1)}$
2. $X_{(n)}$
3. $X_{(n)}-1$
4. $\frac{X_{(n)}+X_{(1)}}{2}-0.5$.

109. Let $\underline{X} = (X_1,\ldots\ldots,X_n)$ be a random sample from uniform

$(0, \theta)$. Which of the following is/are uniformly most powerful size $\alpha\left(0<\alpha<\frac{1}{2}\right)$ test(s) for testing H_0: $\theta=\theta_o$ against H_1: $\theta>\theta_o$?

1. $\phi_1(\underline{X})=1$, if $X_{(n)}>\theta_0$ or $X_{(n)}<\theta_o\,\alpha^{1/n}$

   $= 0$, otherwise

2. $\phi_2(\underline{X})=1$, if $X_{(n)}>\theta_0$

   $= \alpha$, if $X_{(n)}\le\theta_0$

3. $\phi_3(\underline{X})=1$, if $X_{(n)}>\theta_0\alpha^{1/n}$

   $= 0$, if $X_{(n)}\le\theta_0\alpha^{1/n}$

4. $\phi_4(\underline{X})=1$, if $X_{(n)}<\theta_0(\alpha/2)^{1/n}$ or $X_{(n)}>\theta_0(1-\alpha/2)^{1/n}$

   $= 0$, otherwise

110. Suppose $\underline{X}_{p\times1}$ has a $N_p(\underset{\sim}{O}, I_p)$ distribution. The distribution of $\underline{X}^T A\underline{X}$ is chi-square with r degrees of freedom only if

1. A is idempotent with rank r
2. Trace (A) = Rank (A) = r
3. A is positive definite
4. A is non-negative definite with rank r.

111. Let $X_1, X_2, \ldots\ldots, X_m$ be iid random variables with common continuous cdf F(x). Also let $Y_1, Y_2, \ldots\ldots, Y_n$ be iid random variables with common continuous cdf G(x) and X's & Y's are independently distributed. For testing H_o: F(x) = G(x) for all x against H_1: F(x) $\neq$ G(x) for at least one x, which of the following test is/are used?

1. Wilcoxon signed rank test
2. Kolmogorov-Smirnov test
3. Wald-Wolfowitz run test
4. Sign test.

112. Random variables X and Y are such that E(X)=E(Y) = 0, V(X)=V(Y)=1, correlation (X,Y) = 0.5. Then the

1. conditional distribution Y given X = x is normal with mean 0.5x and variance 0.75

2. least-squares linear regression of Y on X is $y=0.5x$ and of X on Y is $x=2y$

3. least-squares linear regression of X on Y is $x = 0.5y$ and of Y on X is $y = 2x$.

4. least-squares linear regression of Y on X is $y=0.5x$ and of X on Y is $x = 0.5y$.

113. X has a binomial (5,p) distribution on which an observation x=4 has been made. In a Bayesian approach to the estimation of p, a beta (2,3) prior distribution (with density proportional to $p(1-p)^2$) has been formulated. Then the posterior

1. distribution of p is uniform on (0.1)

2. mean of p is $\frac{6}{10}$

3. distribution of p is beta (6,4)

4. distribution of p is binomial (10,0.5).

114. In a study of voter preferences in an election, the following data were obtained

Gender	Party voting for C	Party voting for B	Total
Male	250	250	500
Female	250	250	500
Total	500	500	1000

Then the

1. chi-square statistic for testing no association between party and gender is 0.

2. expected frequency under the hypothesis of no association is 250 in each cell.

3. log-linear model for cell frequency m_{ij}, $\log(m_{ij})$ = constant, i,j=1,2, fits perfectly to the data.

4. chi-square test of no gender-party association with 1 degree of freedom has a p-value of 1.

115. Let X,Y and N be independent random variables with P(X=0) = ½=1-P(X=1) and Y following Poisson with parameter $\lambda > 0$ and N following normal with mean 0 and variance 1. Define

$$Z = \begin{cases} Y & if \quad X = 0 \\ N & if \quad X = 1 \end{cases}$$

Then, the characteristic function of Z is given by

1. $\left(\frac{1}{2}+\frac{1}{2}e^{it}\right)e^{-\lambda\left(1-e^{it}\right)}e^{-t^2/2}$
2. $e^{-\lambda\left(1-e^{it}\right)}e^{-t^2/2}$
3. $\dfrac{e^{-\lambda\left(1-e^{it}\right)}+e^{-t^2/2}}{2}$
4. $\left(\frac{1}{2}+\frac{1}{2}e^{it}\right)\left(\dfrac{e^{-\lambda\left(1-e^{it}\right)}+e^{-t^2/2}}{2}\right).$

116. A simple random sample of size n is drawn from a finite population of N units, *with* replacement. The probability that the i^{th} ($1 \le i \le N$) unit is included in the sample is

1. n/N
2. $1-\left(1-\frac{1}{N}\right)^n$
3. $\left(\frac{N-1}{N}\right)^n$
4. $\dfrac{n(n-1)}{N(N-1)}.$

117. Under a balanced incomplete block design with usual parameters v, b, r, k, λ, which of the following is/are true?

1. All treatment contrasts are estimable if k ≥ 2
2. The variance of the best linear unbiased estimator of any normalized treatment contrast is a constant depending only on the design parameters and the per observation variance
3. The covariance between the best linear unbiased estimators of two mutually orthogonal treatment contrasts is strictly positive
4. The variance of the best linear unbiased estimator of an elementary treatment contrast is strictly smaller than that under a randomized block design with replication r.

118. Consider a randomized (complete) block design with v (>2) treatments and r (≥2) replicates. Which of the following statements is/are true?

1. The design is connected
2. The variance of the best linear unbiased estimator (BLUE) of every normalized treatment contrast is the same
3. The BLUE of any treatment contrast is uncorrelated with the BLUE of any contrast among replicate effects
4. The variance of the BLUE of any elementary treatment contrast is $2\sigma^2/r$, where σ^2 is the variance of an observation.

119. The starting and optimal tableaus of a minimization problem are given below. The variables are x_1, x_2 and x_3. The slack variables are S_1 and S_2.

<u>Starting Tableau</u>

	Z	x_1	x_2	x_3	S_1	S_2	RHS
Z	1	a	1	-3	0	0	0
S_1	0	b	2	2	1	0	6
S_2	0	-1	2	-1	0	1	1

<u>Optimal Tableau</u>

	Z	x_1	x_2	x_3	S_1	S_2	RHS
Z	1	0	-1/3	-11/3	-2/3	0	-4
x_1	0	c	2/3	2/3	1/3	0	e
S_2	0	d	8/3	-1/3	1/3	1	3

Which of the following are the correct values of the unknowns a, b, c, d and e

1. a = 2, b = 3, c = 1, d = 0, e = 2
2. a = 2, b = -3, c = 1, d = 0, e = -2
3. a = -2, b = 3, c = 1, d = 0, e = 2
4. a = -2, b = 3, c = -1, d = 0, e = 2

120. Consider the following linear programming problem.

Minimize $Z = x_1 + x_2$
subject to $sx_1 + tx_2 \geq 1$
$x_1 \geq 0$
x_2 unrestricted.

The necessary and sufficient condition to make the LP

1. feasible is $s \leq 0$ and $t = 0$
2. unbounded is $s > t$ or $t < 0$
3. have a unique solution is $s = t$ and $t > 0$
4. have a finite optimal solution is $x_2 \geq 0$.