MAY 2010

Paper I — ALGEBRA — I

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

PART A — (4 × 10 = 40 marks)

Answer any FOUR questions.

Each question carries 10 marks.

1. State and prove Lagrange's theorem.

2. State and prove Cauchy's theorem for Abelian groups.

3. Prove that the homomorphic image of a solvable group is solvable.

4. Define maximal ideal. Let R be a commutative ring with unity. Prove that an ideal M of R is maximal ideal of R if and only if R/M is a field.

5. Let $\alpha \in R$ be algebraic over F and $f(x)$ the minimum polynomial of α with deg $f(x) = n$. Then prove that $F(\alpha)$ has dimension n as a vector space over F.

6. Prove that two elements α and α' are conjugate over F if and only if they have the same minimum polynomial over F.

7. Let F be a field of characteristic O. Show that every algebraic extension of F is separable.

8. Let K/F be a Galois extension of degree n, $(n, \rho)=1$ such that $G=G(K/F)$ is a solvable group. Then there exists a radical extension L/F such that $K \subset L$.

PART B — $(3 \times 20 = 60$ marks)

Answer any THREE questions.

Each question carries 20 marks.

9. Define P-sylow subgroup of a group. If ρ is prime number and $\rho^{\alpha}/O(G)$ then prove that G has a subgroup of order p^{α}.

10. (a) Let R be an Euclidean ring. Suppose that for $a, b, c \in R$, a/bc but $(a, b)=1$ then prove that a/c.

(b) Prove that the field C of all complex numbers is algebraically closed.

11. (a) Prove that if R is a commutative ring with unit elements, so is $R[x]$. If R is an integral domain then prove that $R[x]$ is also an integral domain.

(b) If f is a field then prove that $f(x)$ is an Euclidean domain.

12. Let K be an extension field of F. Prove that the element $a \in K$ is algebraic over F if and only if $F(\alpha)$ is a finite extension of F.

13. State and prove Artin's theorem.

14. (a) Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by atmost n elements of G is a cyclic group.

(b) Prove that it is impossible by straight edge and compass alone to trisect 60°.

13. Show that a nonempty open set in the plane is connected $\Leftrightarrow$ any two of its points can be joined by a polygon which lies in the set.

14. State and prove the Cauchy theorem for rectangle.

4508/KA2

MAY 2010

Paper II — REAL AND COMPLEX ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

PART A — $(4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. (a) Define a continuous function on a metric space.

   (b) Show that f is a continuous function on a metric space $\Leftrightarrow$ Inverse image of an open set in y (under f) is open in X.

2. State and prove the Taylor's theorem for real functions on $[a,b]$.

3. (a) If f is continuous on $[a,b]$ show that $f \in \boldsymbol{R}(\alpha)$ on $[a,b]$.

   (b) If f is monotonic on $[a,b]$ and α is continuous on $[a,b]$ then show that $f \in \boldsymbol{R}(\alpha)$.

4. Show that countable Union of countable set is countable.

5. Show that the cross ratio (Z_1, Z_2, Z_3, Z_4) is real if and only if the four points lie on a circle or on a straight line.

6. State and prove the Local mapping theorem.

7. (a) Define Uniform Convergence.

(b) State and prove the Hurwitz's theorem.

8. Find the absolute values of

(a) $(-2i)(3-i)(2+4i)(1-i)$

(b) $\dfrac{(3-4i)(-1+2i)}{(-1-i)(3-i)}$.

PART B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. (a) State and prove the root test.

(b) State and prove the ratio test.

10. (a) Let E be a nonempty set in R. Then prove that

(i) There is continuous function on E which is not bounded

(ii) There is a continuous and bounded function on E which has no maximum

(iii) If E is bounded $\Rightarrow$ there is a continuous function on E, which is not uniformly continuous.

(b) Let f be a continuous real function on the internal $[a,b]$. If $f(a)<f(b)$ and C is a number satisfying $f(a)<C<f(b)$ show that $\exists$ a point $x \in (a,b)$ such that $f(x)=C$.

11. (a) Let f and g be two functions defined on $[a,b]$ and are differentiable at a point $x \in [a, b]$. Then show that $f+g, fg$, f/g are all differentiable at x.

(b) State and prove the Chain rule.

12. State and prove the Abel's theorem.

4509/KA3 **MAY 2010**

Paper III — TOPOLOGY AND MEASURE THEORY

(For those who joined in July after 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. Prove that the union of denumerable collection of denumerable sets is denumerable.

2. In a topological space X, let A be a subset of X, prove that (a) $\overline{A} = A \cup D(A)$ and (b) A is closed if and only if $A \equiv D(A)$.

3. Prove that any continuous image of a compact space is compact, Also prove that any closed sub space of a compact space is compact.

4. Prove that the spaces R^n and C^n are connected.

5. Let f be defined on I and assume that $\{f_n\}$ is a sequence F measurable functions on I such that $f_n(x) \to f(x)$ almost every where on I. Prove f is measurable on I.

6. Let ϕ and ψ be simple functions which vanish outside a set of finite measure, then prove that $\int(a\phi + b\phi) = a\int\phi + b\int\phi$ and if $\varphi \geq \psi \cdot$ a·e then $\int\phi \geq \int\psi$.

7. Show that there is a measurable set which is not a Borel set.

8. State and prove convergence theorem.

SECTION B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. (a) Prove that every separable metric space is second countable.

   (b) State and prove Lindelof's theorem.

10. Prove that every closed and bounded subspace of the real time is compact.

11. Prove that the product of any non-empty class of connected space is connected.

12. Let $\{A_n\}$ be a countable collection of sets of real numbers then prove that $m^*(UA_n) \leq \sum m^* A_n$.

13. State and prove Raden-Niceodyn theorem.

14. If $< f_n >$ is a sequence of non-negative functions then prove that $\int \underline{\lim} f_n \leq \underline{\lim} \int f_n$.

12. Evaluate $\int_0^1 \frac{1}{1+x}\,dx$, correct to three decimal places by

(a) Simpson's rule

(b) Romberg's rule.

13. (a) Solve the boundary value problem $y'' - 64y + 10 = 0$ with $y(0) = y(1) = 0$.

Compute the value of $y(0.5)$ and compare it with the true value.

(b) Solve the system of differential equations

$$\frac{dx}{dt} = y - t$$

$$\frac{dy}{dt} = x + t$$

When $x = 1$, $y = 1$ when $t = 0$, taking $\Delta t = h = 0.1$.

14. (a) Find the surface which is orthogonal to the one parameter system $z = cxy(x^2 + y^2)$ which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$.

(b) Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the parabola $x = 0$, $y^2 = z$.

Paper IV — NUMERICAL METHODS AND DIFFERENTIAL EQUATIONS

(For those who joined in July from the academic year 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. Explain Aitken Δ^2 method.

2. Use the Given's method find to eigen value of $\begin{pmatrix} 2 & -3 & 0 \\ -3 & 2 & -3 \\ 0 & -3 & 2 \end{pmatrix}$.

3. (a) Find the missing value of the following table:

x:	0	1	2	3	4
y:	1	2	4	–	16

Explain why $y(x = 3)$ is not $2^3 = 8$ in your answer.

(b) Find the polynomial of least degree passing through the points $(0,-1)$, $(1,1)$, $(2,1)$ and $(3,-2)$.

4. (a) Find the first and second derivative of the function tabulated below at $x = 3$

x :	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$:	–14	–10.032	–5.296	–0.256	6.672	14

(b) Find the first and second derivative of the function tabulated below at $x = 0.6$

x :	0.4	0.5	0.6	0.7	0.8
y :	1.5836	1.7974	2.0442	2.3275	2.6511

5. Solve $\frac{dy}{dx} = x + y$, given $y(1) = 0$ and get $y(1.1)$, $y(1.2)$ by Taylor series method. Compare your result with the explicit solution.

6. Show that e^{2x} and e^{3x} are linearly independent solutions of $y'' - 5y' + 6y = 0$. Find the solution $y(x)$ with the properly that $y(0) = 0$ and $y'(0) = 1$.

7. Explain Frobenius method.

8. Solve the Cauchy's problem for $zp + q = 1$, when the initial data curve is $x_0 = \mu$, $y_0 = \mu$, $z_0 = \mu/2$, $0 \le \mu \le 1$.

SECTION B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. (a) Find the root of the equation $y(x) = x^3 - 3x - 5 = 0$ which lies between 2 and 3, by Muller's method.

(b) Find the real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places using iteration method.

10. (a) Determine the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

(b) Find the inverse of $A = \begin{pmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{pmatrix}$.

11. (a) Find the cubic polynomial which takes the following values $y(0) = 1$, $y(1) = 0$, $y(2) = 1$ and $y(3) = 10$.

(b) Explain the Gram-Schmidt orthogonalization process.

11. (a) If $N(G)\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{\frac{-w^2}{2}}dw$, then prove that $N(-x)=1-N(x)$.

(b) Let X be $n(\mu,\sigma^2)$, so that $P_r(X<89)=0.90$ and $P_r(X<94)=0.95$ find μ and σ^2.

12. (a) Define F distribution.

(b) Derive F distribution.

13. Let Y_K be the K[th] order statistics of a random sample of size n from a distribution having probability density function $f(x)$. Then show that the joint density function of $Y_1,Y_2...Y_n$ is

$$g(y_1,y_2...y_n)=\begin{cases}\underline{|n}.f(y_1)...f(y_n); a<y_1,<y_2<...y_n b\\ 0 \text{ elsewhere}\end{cases}$$

Also find the probability density function of Y_K.

14. (a) State and prove the Rao - Creamer Inequality.

(b) Let $X_1,X_2...X_n$ demote a random sample from a Poisson distribution that has the mean $\theta>0$. Prove that $\bar{x}$ is an efficient estimator of θ.

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4511/KA5 **MAY 2010**

Paper V — MATHEMATICAL STATISTICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

Each question carries 10 marks.

1. (a) State and prove the Monotone Property.

(b) State and prove the Addition theorem of probability.

2. Let X, Y be two dimensional random variable and suppose that $E(X)=\mu_1$ and $E(Y)=\mu_2$ $Var(X)=\sigma^2$ and $Var(Y)=\sigma^2$. Let ρ be the correlation coefficient between X and Y. Prove that.

(a) If $E\left(\frac{y}{x}\right)$ is linear in x, then

$$E\left(\frac{y}{x}\right)=\mu_2+\rho\frac{\sigma_2}{\sigma_1}(x-\mu_1).$$

(b) If $E\left(\frac{X}{Y}\right)$ in linear in y then

$$E\left(\frac{X}{Y}\right)=\mu_1+\rho\frac{\sigma_1}{\sigma_2}(y-\mu_2).$$

3. Find the moment generating function of a bivariate normal distribution.

4. Let $X_1; X_2$ be a random sample from the normal distribution $n(0,1)$. Show that the marginal density function of $Y = \frac{X_1}{X_2}$ is the cauchy probability density function.

5. Let X_i denote a random variable with mean μ_i and variance σ_i^2, $i = 1,2,3...$ Let $X_1, X_2 ... X_n$ be mutually stochastically independent and let $K_1 ... K_n$ denote real constants. Find the mean and variance of $Y = K_1X_1 + K_2X_2 + ... + K_nX_n$.

6. Let Z_n be $\psi^2(n)$. Find the limiting distribution of the random variable $Y_n = \frac{Z_n - n}{\sqrt{2n}}$.

7. The length of life of bound X light bulbs is assumed to be $\psi(\mu_x, 784)$. The length of life of brand Y light bulbs is assumed to be $N(\mu_y, 627)$ and independent that of X. Of a random sample of 56 brand X light bulbs yielded a mean of 937.4 hours and a random sample of size 57 brand y light bulbs yielded a mean of 988.9 hours. Find a 90% confiderice internal of $\mu_x - \mu_y$.

8. Let $X_1, X_2 ... X_n$ denote a random sample from the distance that has probability density function

$$F(x;\theta) = \begin{cases} \theta^x (1-\theta)^{1-x}; x = 0,1; 0 < \theta < 1 \\ 0 \quad ; \text{elsewhere} \end{cases}$$

Prove that $Y_1 = Y_1 + X_2 + ... + X_n$ is sufficient statistic for θ.

SECTION B — (3 × 20 = 60 marks)

Answer any THREE questions.

Each question carries 20 marks.

9. For each of the following probability density function, compute $P_r(\mu - 2\sigma < X < \mu + 2\sigma)$

(a) $f(x) = \begin{cases} 6x(1-x); 0 < x < 1 \\ 0 \quad ; \text{elsewhere} \end{cases}$

(b) $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x ; x = 1,2,3... \\ 0 \ ; \text{elsewhere} \end{cases}$

10. Let $f(x,y) = \begin{cases} 2; 0 < x < y,\ 0 < y < 1 \\ 0; \text{elsewhere} \end{cases}$ be the joint probability density function of X and Y show that the conditional means are $\frac{1+x}{2}, 0 < x < 1$ and $\frac{y}{2}, 0 < y < 1$ respectively. Also show that the correlation coefficient X and Y is $\phi = \frac{1}{2}$.

(6 pages)

4512/KA6 **MAY 2010**

Paper VI — OPERATIONS RESEARCH

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. Find the optimum solution of LPP

$\text{Max } Z = 15X_1 + 45X_2$

subject to the condition

$X_1 + 16X_2 \le 240$

$5X_1 + 2X_2 \le 162$

$X_2 \le 50$

$X_1, X_2 \ge 0.$

If maximum $Z = \sum_j C_j X_i \quad j = 1, 2$ and C_2 in kept fixed at 45, determine how much can C_1 be changed without affecting the optimal solution.

2. The following table lists all the activities which constitute a small project. Also shows the necessary immediate predecessor for each activity.

Activity :	A	B	C	D	E	F	G
Immediate predecessor :	–	–	A	C	D	B, E	E, F
Activity duration (days) :	10	7	5	3	2	1	14

(a) Construct network

(b) Determine the optimum project length

(c) Find the critical path

(d) Total float for each activity.

3. Solve the LPP by dynamic programming

$\text{Min } Z = 8X_1 + 7X_2$

Subject to

$2X_1 + X_2 \le 8$

$5X_1 + 2X_2 \le 15$

and $X_1, X_2 \ge 0$.

4. State dominance property in Game theory and solve the following game by algebraic method

$$\begin{bmatrix} -1 & 2 & 1 \\ 1 & -2 & 2 \\ 3 & 4 & -3 \end{bmatrix}.$$

5. Derive Pollaczek-Khintchine formula.

6. State the necessary and sufficient condition for X_0 to be an extreme point of $f(X)$ and also examine the function $X_1 + 2X_3 + X_2X_3 - X_1^2 - X_2^2 - X_3^2$ for extreme points.

7. Define seperable convex programming and explain the method of solving the seperable convex programming.

8. Solve the following graphically

Max $Z = 3x_1 + 2X_2$

subject to

$2X_1 + 2X_2 \leq 9$

$3X_1 + 3X_2 \geq 18$

$X_1, X_2 \geq 0$

and integer n.

SECTION B — ($3 \times 20 = 60$ marks)

Answer any THREE questions.

9. Solve by using Big-M method :

Maximize $Z = 8X_2$

subject to

$X_1 - X_2 \geq 0$

$2X_1 + 3X_2 \leq -6$

X_1, X_2 are unrestricted.

10. Use dual simplex method to solve the following LPP

Min $Z = 10X_1 + 6X_2 + 2X_3$

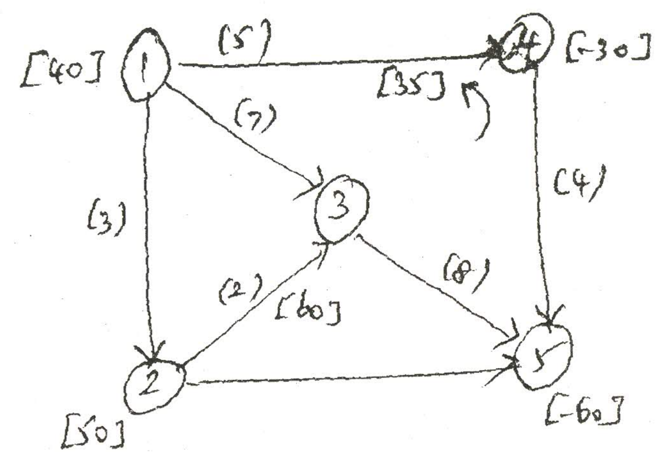
subject to

$-X_1 + X_2 + X_3 \geq 1$

$3X_1 + X_2 - X_3 \geq 2$

$X_1, X_2, X_3 \geq 0.$

11. Solve the network by simplex method



( ) unit cost

[ ] Arc capacity

12. Solve the 0–1 problem by the additive algorithm

Max $Z = 3y_1 + 2y_2 - 5y_3 - 2y_4 + 3y_5$

subject to

$$y_1 + y_2 + y_3 + 2y_4 + y_5 \le 4$$
$$7y_1 + 3y_3 - 4y_4 + 3y_5 \le 5$$
$$11y_1 - 6y_2 + 3y_4 - 3y_5 \ge 3$$

$Y_1, Y_2, Y_3, Y_4, Y_5 = (0, 1)$.

13. Explain the terms : traffic intensity, steady state, queue discipline a supermarket has two girls ringing up sales at the counters. If the service time for each customer in exponential with mean 4 minutes, and if people arrive in a poisson fashion at the rate of 10/hour.

(a) What is the probability of having to wait for the service?

(b) What is the expected percentage of idle time for each girl?

(c) Find the average queue length and average number of units in the system.

14. Define marker-process, Markov chain and various states of Markovchain. Derive Chapman-Kolomogorov equation. Find the mean recurrences time for each state of the Markov chain $\begin{bmatrix} 0.1 & 0.9 \\ 0.7 & 0.3 \end{bmatrix}$.

4513/KAA

MAY 2010

FUNCTIONAL ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. Define Banach space and give an example. Also give an example of a normal linear space but not Banach.

2. If N is named linear space and if x_0 is a non zero vector in N, Prove that there Exists a functional fo in N^* such that $f_0(x) = \|x_0\|$ and $\|f_o\| = 1$.

3. State and Prove Parallelogram Law.

4. Prove that the adjoint operator $T \to T^*$ on (BCH) has the following properties.

   (a) $\|T^*\| = \|T\|$.

   (b) $\|T^*T\| = \|T\|^2$.

5. Prove that the set of all eigen values of an operator T on a finite dimensional Hilbert space is

a non empty finite set and further show that the number of dements of this set does not exceed the dimension of the space.

6. If T is normal then prove that x is an eiqen vector of T with eigenvalue λ of and only of x in an egien vector of T^* with egien value $\overline{\lambda}$.

7. Let G be the set of all regular elements in a Banach algebra then prove that G is an open set.

8. If $1-x^r$ is regular then prove that $1-r_x$ is also regular.

SECTION B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. State and prove that Hahn Banach theorem.

10. State and prove Besse's inequality and prove that every non zreo Hilbert space contains a complete Orthonormal set.

11. State and prove the spector theorem for finite dimensional Hilbert space.

12. (a) Prove the resolvent equation and hence prove that $\sigma(x) \neq \phi$.

(b) Prove that $f(x) \lim_{n} \|x^n\|^{\frac{1}{n}}$.

13. Prove that the Gelfand mapping $x \rightarrow x^\wedge$ is a norm decreasing homomorphism of H into $C(m)$ and state the four properties that this mapping satisfy and prove the same.

14. (a) Prove that the following are equivalent.

(i) $\|x^2\| = \|x\|^2 \; \forall x.$

(ii) $r(x) = \|x\| \; \forall x.$

(iii) $\|x^\wedge\| = \|x\| \; \forall x.$

(b) If f_1 and f_2 are multiplicative functionals on A with the same nulls pace then prove that $f_1 = f_2$.

4514/KAB

MAY 2010

GRAPH THEORY AND DATA STRUCTURES

(For those who joined in July 2003 and after)

Time : Three hours

Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. (a) Show that in any graph G, $\sum_{i=1} \deg(v_i) = 2e$ and hence that the number of vertices of odd degree is always even.

   (b) Define Regular graph with an example.

2. (a) Prove that any connected graph G with n vertices and $n-1$ edges is a tree.

   (b) Define Binary Tree and State any two properties of a binary tree.

3. (a) Define a separable graph with an example.

   (b) Prove that a vertex v of a tree G is a cut vertex of G, if and only if $d(v) > 1$.

4. (a) Show that a graph G has a dual if and only if it is planar.

(b) Prove that $K_{3,3}$ cannot have a dual.

5. (a) In a graph G with $\delta > 0$, prove that $\alpha' + \beta' = \upsilon$.

(b) Prove that a set $S \subseteq \upsilon$ is an independent set of G if and only if $V - S$ is a covering of G.

6. (a) Show that the Chromatic Polynomial of a tree on n vertices is given by $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$.

(b) Write the Chromatic Polynomial of K_n.

7. State and prove Hall's Theorem.

8. Explain the greedy algorithm for the shortest path in a weighted graph.

SECTION B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. (a) Prove that a connected graph G is an Euler graph if and only it can be embedded into circuits.

(b) If a graph G has exactly two vertices of odd degree, then prove that there must be a putt joining their two vertices.

10. (a) State and prove the Necessary Condition for a simple graph G to be Hamiltonian.

(b) State and prove Dirac Theorem.

11. Prove that for any graph G, $K(G) \leq K'(G) \leq \delta(G)$.

12. Show that the graph G has a Perfect Matching if and if $0(G - S) \leq |S|$ for all $S \subseteq V$.

13. Explain the Sequential search in detail.

14. Explain the Depth-first and Breadth-first algorithms in detail.

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