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B.E./B.Tech. DEGREE EXAMINATIONS, NOV./DEC. 2011
Regulations 2008

Fourth Semester

Common to ECE & Bio Medical Engineering

MA 2261 Probability and Random Processes

Time: Three Hours Maximum: 100 marks

Answer ALL Questions

Part A - (10 x 2 = 20 marks)

1. The CDF of a continuous random variable is given by $F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-x/5}, & 0 \le x < \infty \end{cases}$. Find the PDF and mean of X.

2. If X is a normal random variable with mean zero and variance σ^2, find the PDF of $Y = e^X$.

3. If the joint pdf of (X, Y) is $f_{XY}(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$, check whether X and Y are independent.

4. The regression equations are $3x + 2y = 26$ and $6x + y = 31$. Find the correlation coefficient between X and Y.

5. When is a random process said to be mean ergodic?

6. If $\{X(t)\}$ is a normal process with $\mu(t) = 10$ and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$ find the variance of $X(10) - X(6)$.

7. The autocorrelation function of a stationary random process is $R_{XX}(\tau) = 25 + \dfrac{4}{1 + 6\tau^2}$. Find the mean and variance of the process.

8. Prove that for a WSS process $\{X(t)\}$, $R_{XX}(t, t+\tau)$ is an even function of τ.

9. State any two properties of a linear time-invariant system.

10. If $\{X(t)\}$ and $\{Y(t)\}$ in the system $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$ are WSS process, how are their auto correlation functions related.

Part B - (5 x 16 = 80 marks)

11. (a) (i) The probability function of an infinite discrete distribution is given by $P[X = j] = \frac{1}{2^j}$ $(j = 1, 2, 3, \cdots)$. Find
(1) Mean of X,
(2) $P[X$ is even$]$
(3) $P[X$ is divisible by 3$]$. (8)

(ii) A continuous R.V.X has the p.d.f. $f(x) = \begin{cases} \frac{k}{1+x^2}, & -\infty < x < \infty \\ 0, & \text{otherwise} \end{cases}$. Find
(1) the value of k,
(2) Distribution function of X,
(3) $P[X \geq 0]$. (8)

OR

11. (b) (i) Let X and Y be independent normal variates with mean 45 and 44 and standard deviation 2 and 1.5 respectively. What is the probability that randomly chosen values of X and Y differ by 1.5 or more? (8)

(ii) If X is a uniform random variable in the interval (-2, 2), find the probability density function $Y = |X|$ and $E[Y]$. (8)

12. (a) (i) The joint probability density function of random variable X and Y is given by $f(x, y) = \begin{cases} \frac{8xy}{9}, & 1 \leq x \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$. Find the conditional density functions of X and Y. (8)

(ii) The joint probability density function of the two dimensional random variable (X, Y) is $f(x, y) = \begin{cases} 2 - x - y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$. Find the correlation coefficient between X and Y. (8)

OR

12. (b) (i) If $X_1, X_2, X_3, ...X_n$ are uniform variates with mean = 2.5 and variance = 3/4, use CLT to estimate $P(108 \leq S_n \leq 12.6)$ where $S_n = X_1 + X_2 + X_3 + ... + X_n, n = 48$. (8)

(ii) X and Y are independent with a common PDF(exponential):

$$f(x) = \begin{cases} e^{-x} & , x \geq 0 \\ 0 & , x < 0 \end{cases} \quad \text{and} \quad f(y) = \begin{cases} e^{-y} & , y \geq 0 \\ 0 & , y < 0 \end{cases}.$$

Find the PDF for $X - Y$. (8)

13. (a) (i) Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide-sense stationary, if A and ω are constants and θ is a uniformly distributed in $(0, 2\pi)$ (8)

(ii) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2.. \\ \dfrac{at}{1+at}, & n = 0 \end{cases}.$$

Find the mean and variance of the process. Is the process first-order stationary? (8)

OR

13. (b) State the postulates of a Poisson process and derive the probability distribution. Also prove that the sum of two independent Poisson processes is a Poisson process. (16)

14. (a) (i) The autocorrelation function of a random process is given by

$$R(\tau) = \begin{cases} \lambda^2 & ; |\tau| > \varepsilon \\ \lambda^2 + \dfrac{\lambda}{\varepsilon}\left(1 - \dfrac{|\tau|}{\varepsilon}\right) & ; |\tau| \leq \varepsilon \end{cases}.$$

Find the power spectral density of the process. (8)

(ii) Given the power spectral density of a continuous process as

$$S_{xx}(\omega) = \frac{\omega^2 + 9}{\omega^4 + 5\omega^2 + 4}.$$

Find the mean square value of the process. (8)

OR

14. (b) (i) State and prove Weiner-Khintchine Theorem. (8)

(ii) The cross-power spectrum of real random processes $\{X(t)\}$ and $\{Y(t)\}$ is given by

$$S_{xy}(\omega) = \begin{cases} a + bj\omega, & \text{for } |\,\omega\,| < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Find the cross correlation function. (8)

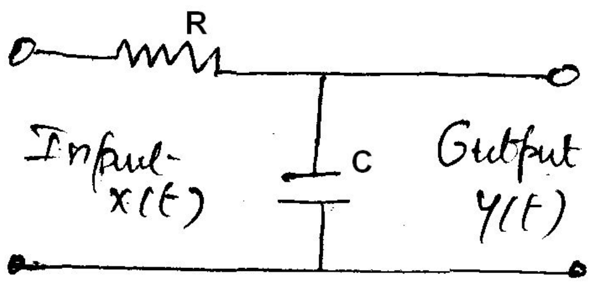
15. (a) (i) If the input to a time invariant, stable, linear system is a WSS process, prove that the output will also be a WSS process. (8)

(ii) Let $X(t)$ be a WSS process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$, then prove that $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$. (8)

OR

15. (b) (i) For a input - output linear system $(X(t), h(t), Y(t))$, derive the cross correlation function $R_{XY}(\tau)$ and the output autocorrelation function $R_{YY}(\tau)$. (8)

(ii) A white Gaussian noise $X(t)$ with zero mean and spectral density $\frac{N_0}{2}$ is applied to a low-pass RC filter shown in the figure.



Determine the autocorrelation of the output $Y(t)$. (8)